TECHNISCHE UNIVERSITÄT WIEN

Dissertation

Stability behaviour of electromagnetic track brakes: self-excitation mechanisms and control

carried out for the purpose of obtaining the degree of Doctor technicae (Dr. techn.), submitted at TU Wien, Faculty of Mechanical and Industrial Engineering

Bernhard EBNER

Mat. Nr.: 01327254

under the supervision of

Univ.Prof. Dipl.-Ing. Dr.techn. Johannes Edelmann
Institute of Mechanics and Mechatronics, TU Wien
Vienna, Austria

Vienna, October 2025

reviewed by

Prof. Maksym Spiryagin PhD, MSc, BSc
Centre for Railway Engineering, CQUniversity
Queensland, Australia

and

Ao. Univ.Prof. Dipl.-Ing. Dr.techn. Manfred Plöchl
Institute of Mechanics and Mechatronics, TU Wien
Vienna, Austria

I confirm, that going to press of this thesis needs the confirmation of the examination committee.

Affidavit

I declare in lieu of oath, that I wrote this thesis and performed the associated research myself, using only literature cited in this volume. I confirm that this work is original and has not been submitted elsewhere for any examination, nor is it currently under consideration for a thesis elsewhere. I acknowledge that the submitted work will be checked using suitable and state-of-the-art means (plagiarism detection software).

City, date	Signature

This thesis received financial support from the Austrian Federal Ministry for Labour and Economy, the National Foundation for Research, Technology and Development, and the Christian Doppler Research Association. This support was provided within the framework of the *CD Laboratory for Enhanced Braking Behaviour of Railway Vehicles*, which is gratefully acknowledged.

Danksagung

Der Abschluss dieser Dissertation markiert das Ende eines arbeitsintensiven und prägenden Kapitels in meinem Leben. Dieses war nicht nur vom Erwerb fachlicher Kompetenzen bestimmt, sondern in hohem Maße auch von persönlicher Entwicklung: durch erprobte Beharrlichkeit und Durchhaltevermögen, den Umgang mit neuen und komplexen Themengebieten sowie durch das ständige Hinterfragen sowohl von Ergebnissen als auch von mir selbst.

"Wissenschaft beginnt im Staunen, aber sie lebt von der Kritik."

Für mich drückt dieses Zitat von Karl Popper über wissenschaftliche Disziplinen hinaus eine Haltung aus, die sowohl zur Reflexion des Gehörten als auch zur Selbstreflexion auffordert.

Das Ende dieses Kapitels und damit meines Universitätsstudiums wäre ohne die Unterstützung vieler wichtiger Menschen in meinem Leben nicht möglich gewesen. Ihnen möchte ich im Folgenden meinen aufrichtigen Dank aussprechen.

Ein ganz besonderer Dank gilt meinen beiden Doktorvätern, Professor Dr. Johannes Edelmann und Professor Dr. Manfred Plöchl. Sie haben durch ihre eigene Begeisterung meine Leidenschaft für die technischen Dynamik geweckt und mich durch zahlreiche fruchtbare Fachdiskussionen, herausfordernden Fragestellungen und stets konstruktiver Kritik fortwährend unterstützt und gefordert. Ihr Vertrauen, das freundschafliche und von Humor geprägte Arbeitsklima und die Förderung meiner selbstständigen Arbeitsweise haben ein für mich freies und produktives Arbeitsumfeld geschaffen. Auch in schwierigen und sehr intensiven Phasen war ich dadurch stets motiviert, mein Bestes zu geben. Durch ihre umfassende Expertise konnte die vorliegende Arbeit in vielerlei Hinsicht an Qualität gewinnen. Mein Dank gilt außerdem Professor Dr. Alois Steindl für seine Unterstützung und seine wertvollen Beiträge zu Homotopiemethoden und Bifurkationen.

Ein großes Danke gilt auch meinen KollegInnen bei Knorr-Bremse GmbH. Ohne sie wäre diese Arbeit in dieser Form nicht möglich gewesen. Besonders danke ich Dr. Daniel Tippelt für seine Begeisterung an komplexen Systemen und der lehrreichen Zusammenarbeit im Zuge von Fahrversuchen, sowie Dr. Michael Jirout für seine stete Unterstützung und für sein Vertrauen und Förderung meiner Kompetenzen. Beide haben mir mit ihrer Expertise und jahrelanger Erfahrungen zur Magnetschienenbremse in vielen Gesprächen wertvolle Einblicke auch aus Unternehmensperspektive eröffnet.

Ein wesentlicher Teil des positiven Arbeitsumfelds entstand durch meine KollegInnen am Institut für Mechanik und Mechatronik, welche is als engagierte Experten in unterschiedlichsten Themenbereichen kennenlernen durfte. Dank gilt meinen ersten Bürokollegen, Dr. Florian Zehetbauer und Dr. Andreas Fichtinger, für die herzliche Aufnahme in die Gruppe, Philipp Mandl mit dem ich über die gesamte Zeit der Dissertation viele schöne Momente erleben durfte, und meinen jüngeren Kollegen für das großartige Miteinander auch in der gewachsenen Gruppe. Ich bedanke mich außerdem bei meinen aktuellen

Zimmer- und CD-Labor Kollegen: Lorenz Klimon, mit dem ich meine Kaffeeliebhaberei teilen darf und an dem ich seine hilfsbereite und umsichtige Art schätze, und Dr. Emin Koçbay, dessen Humor mich täglich zum Lachen bringt. Gemeinsam mit allen anderen Institutsangehörigen bleiben mir die zahlreichen fachlichen, gesellschaftlichen und politischen Diskussionen, die gemeinsame Zeit in den Mittagspausen oder bei Ausflügen und Feierlichkeiten sowie die stete Hilfsbereitschaft in bester Erinnerung.

Darüber hinaus gilt mein Dank für viele schöne Erinnerungen meinen Freunden außerhalb der TU-Wien: meinen ehemaligen Studienkollegen, meinen Freunden aus der alten Heimat in Salzburg sowie den neu gewonnenen Freunden in Wien. Die gemeinsamen Feiern, Treffen am Grillplatz, Spiele- und Kochabende haben mir stets wohltuende Abwechslung vom oftmals fordernden Alltag geschenkt.

Spezieller Dank gilt meiner gesamten Familie, die mir stets das Gefühl von Zusammenhalt und Sicherheit vermittelt und so entscheidend zum Gelingen meines Studiums beigetragen hat. Besonders dankbar bin ich meiner Verlobten, Corinna Paulus, für ihren beständigen Rückhalt, ihrem Glauben an mich und für die vielen gemeinsame Erinnerungen. Abschließend gilt mein Dank und Anerkennung meinen Eltern, Eva und Anton Ebner, ohne deren Unterstützung ich diesen Weg niemals beschreitet und erfolgreich absgeschlossen hätte.

Contents

Abstract

Magnetic track brakes (*MTB*s) are additional braking systems for railway vehicles, primarily employed under low-adhesion conditions and in emergencies. While their effectiveness in such situations was shown frequently, experiments have revealed that *MTB*s are prone to self-excited vibrations, particularly at low velocities. These vibrations compromise the braking performance, increase structural loads, and pose potential safety risks. Although the phenomenon has been observed in experiments and was addressed in recent studies, the underlying excitation mechanisms and their dependence on the frictional contact and electromagnetic interactions remain insufficiently understood. This lack of understanding hinders the ability to interpret existing measurement data and limits the implementation of effective control strategies, both passive and active.

This cumulative thesis aims to enhance the understanding of the excitations involved. It examines the onset of self-excited vibrations, the dynamic behaviour of the coupled electromagnetic system following a loss of stability, and offers recommendations to improve the robustness of the system through a combination of experimental investigations and theoretical analysis. Experimental studies provide essential insights into the velocity dependence of the friction coefficient and electromagnetic attraction forces, both of which are shown to critically influence stability. Building on these findings, electromagnetic-mechanical models are developed and utilised to analyse the vibrational behaviour of *MTB*s. A minimal model utilised to study fundamental excitation mechanisms, an *MTB* brake frame model applied to investigate the interactions through the mechanical structure and the shared power supply, and a corner model to examine influences of distributed contact points and velocity-dependent attraction forces. Using energy considerations and nonlinear stability theory, the thesis demonstrates how frictional properties, electromagnetic coupling effects, and motional eddy currents interact with structural damping to govern the emergence and growth of self-excited vibrations. Applied bifurcation analysis and continuation methods reveal the transitions from stable operation to periodic and quasi-periodic vibrations, including the coexistence of multiple attractors and the onset of stickslip motions. Based on the revealed mechanisms, the influence of changing friction conditions, design parameters, and properties of the electromagnetic system on the stability behaviour is examined. Beyond investigations on the fundamental dynamics, control strategies to actively mitigate self-excited vibrations are introduced. Two state-feedback approaches are proposed that directly exploit the available actuator by modulating the electric voltage, which either suppresses an identified excitation mechanism or actively dissipates energy. Nonlinear stability analysis of the closed-loop system reveals that both strategies significantly increase stable regions. However, when friction levels are high and the friction characteristics exhibit large negative gradients, self-excited vibrations can only be avoided through active dissipation.

The findings provide theoretical insights into the nonlinear stability behaviour and the influence of system parameters on vibrational behaviour, which is fundamental for avoiding self-excited vibrations in future lightweight designs. The results advance the fundamental knowledge at *MTB*s and deliver insights into coupled electromagnetic-mechanical systems.

Kurzfassung

Magnetschienenbremsen (MG-Bremsen) sind zusätzliche Bremssysteme für Schienenfahrzeuge, die bei geringem Kraftschluss und Notbremsungen wirksam eingesetzt werden. Versuche bei Anwendung bis zum Stillstand haben gezeigt, dass MG-Bremsen zu selbsterregten Schwingungen neigen. Diese Schwingungen können die Bremswirkung verringern und die Beanspruchung der Struktur erhöhen und stellen somit ein Sicherheitsrisiko dar. Dieses Phänomen wurde durch Messfahrten und ersten Studien beleuchtet, jedoch sind die zugrunde liegenden Erregungsmechanismen und deren Wechselwirkungen durch den Reibkontakt und dem gekoppelten elektromagnetischen System nicht vollständig geklärt. Dieses fehlende Verständnis verhindert vorhandene Messdaten vollständig zu interpretieren und limitiert die Entwicklung effektiver Gegenmaßnahmen. Diese kumulative Dissertation zielt darauf ab die zugrundeliegenden Mechanismen besser zu verstehen und daraus Lösungsvorschläge für robuste MG-Bremsen abzuleiten. Dabei wird der Stabilitätsverlust und das dynamische Verhalten nach einsetzen selbsterregter Schwingungen, durch eine Kombination aus experimentellen Untersuchungen und theoretischen Analysen untersucht. Experimente liefern wesentliche Erkenntnisse über die Geschwindigkeitsabhängigkeit des Reibungskoeffizienten sowie der elektromagnetischen Anziehungskräfte, die beide einen entscheidenden Einfluss auf das Stabilitätsverhalten haben. Darauf aufbauend wird ein Minimalmodell, zur Untersuchung fundamentaler Erregungsmechanismen, ein Rahmenmodell, zur Analyse der mmechanischen und elektrischen Kopplung, sowie ein Viertelmodell, zur Betrachtung von verteilten Kontaktpunkten und geschwindigkeitsabhängigen Anzugskräften, abgeleitet. Durch Energiebilanzen am schwingenden System und mithilfe nichtlinearer Stabilitätstheorie wird gezeigt, wie Reibungseigenschaften, elektromagnetische Kopplungseffekte und bewegungsinduzierte Wirbelströme mit der strukturellen Dämpfung zusammenwirken und das Entstehen sowie das Anwachsen selbsterregter Schwingungen bestimmen. Mithilfe von Bifurkationsanalysen und Fortsetzungsmethoden werden Übergänge vom stabilen Betrieb zu periodischen und quasi-periodischen Schwingungen, die Koexistenz mehrerer Attraktoren und das Einsetzen von Haft-Gleit-Bewegungen aufgezeigt. Aufbauend auf dem analysierten dynamischen Verhalten erfolgt eine systematische Untersuchung des Einflusses veränderlicher Reibungsbedingungen, Konstruktionsparameter und elektromagnetischer Eigenschaften auf das Stabilitätsverhalten. Darüber hinaus werden aktive Regelungsstrategien zur Minderung selbsterregter Schwingungen eingeführt. Zwei Ansätze mit Zustandsrückführung werden vorgestellt, die den vorhandenen Aktor über eine Spannungsmodulation nutzen. Nichtlineare Stabilitätsanalysen des geschlossenen Regelkreises zeigen, dass beide Strategien den stabilen Bereich deutlich erweitern, wobei nur der Ansatz, der aktiv Energie dissipiert, Schwingungen über den gesamten Geschwindigkeitsbereich verhindern kann. Die Ergebnisse liefern theoretische Einsichten in das nichtlineare Stabilitätsverhalten und zeigen den Einfluss von Systemparametern auf das Schwingungsverhalten und bilden damit die Grundlage zur Vermeidung selbsterregter Schwingungen in zukünftigen Leichtbaukonstruktionen. Damit wird nicht nur das Grundlagenverständnis zu MG-Bremsen erweitert, sondern neue Erkenntnisse für gekoppelte elektromagnetisch-mechanische Systeme vorgestellt.

1. Motivation and scientific context

Rail transport is widely regarded as one of the most sustainable modes of mobility. It is known for its high energy efficiency and low greenhouse gas emissions. Accordingly, it is commonly recognised as one of the most environmentally friendly high-volume transport systems, capable of safely, quickly, and efficiently moving passengers and freight. These advantages are enabled by the small rolling resistance of the wheel–rail contact. A drawback of this steel-steel contact with comparable small adhesion is the limitation in acceleration values. To keep efficiency high and ensure a safe operation, unfavourable operational situations such as platform overruns and passing signals at danger must be avoided. Preventing such circumstances is especially challenging when low and extremely low adhesion conditions emerge, for example, from morning dew or fallen leaves in autumn. In these situations, brake systems utilising the wheel-rail contact may not be sufficient to meet operators' requirements. Braking systems independent from the wheel–rail contact, such as the magnetic track brake (*MTB*), are then deployed to develop additional braking forces.

Increasing travelling velocities and loaded mass of modern railway vehicles at unchanged infrastructure put high demands on the overall braking system, especially at low adhesion. As one consequence, operators often use *MTB*s until full-stop nowadays, increasing the currently restricted velocity range of the brake system, bounded below 25 km/h due to high deceleration peaks. However, it was shown by experiments in the field in [1] that the electromagnetic-mechanical coupled system is prone to self-excited vibrations when used at such small vehicle velocities. Moreover, striving for lightweight design is noticeable at bogies as it directly reduces the energy consumption, the costs during operation and manufacturing, and the wear on infrastructure and vehicles. A direct consequence is the requirement for lightweight design on brake systems, which are mounted either directly on the wheelset, which adds unsprung mass, or in the bogie between primary and secondary suspension. Demands increase especially for additional brake systems as they are only applied occasionally. Developing future technical solutions for the simultaneous requirements of extended operational velocity range and reduced weight while ensuring high safety standards requires fundamental knowledge of the *MTB*'s system dynamics and the influence of design parameters and environmental conditions. This cumulative thesis seeks to explain fundamental influences on the dynamics and the stability behaviour by providing necessary insights into the electromagnetic-mechanical coupling, which is not yet fully understood. Finally, this ends with profound suggestions on mitigation strategies, both passive and active.

The *MTB* of a mainline vehicle, considered in this study, is illustrated in Fig. 1. Fig. 1a shows the track brake suspended in the bogie, depicted in its ready position, awaiting an actuation. During the activation process, the rectangular-shaped brake frame, illustrated with main components in Fig. 1b, is lowered by four pneumatic cylinders onto the rail. Simultaneously, electric voltage is applied to the coils of each corner electromagnet, illustrated in Fig. 2, which creates a magnetic field and is guided by the horseshoe-shaped brake elements to the rail head. At the interface between brake element and rail, magnetic

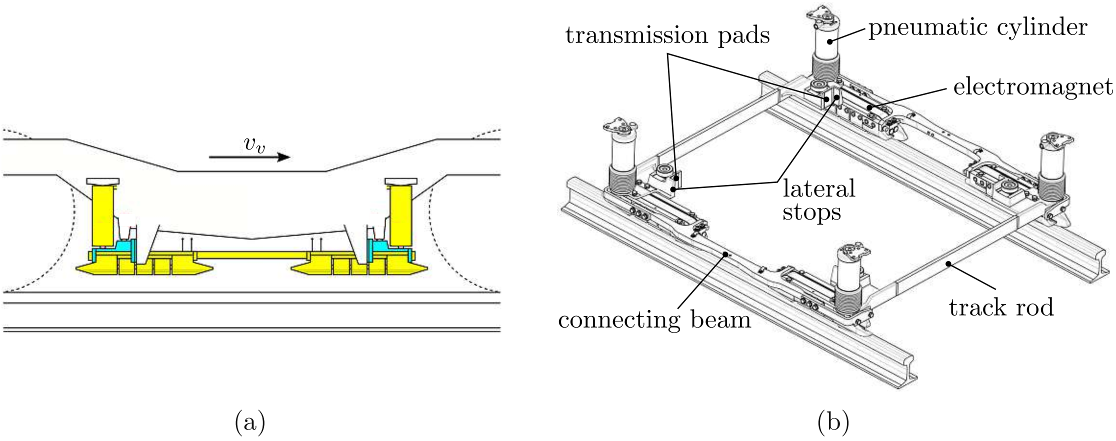


Figure 1: (a) *MTB* in ready position mounted in the bogie, adapted from [1], (b) mechanical structure of the *MTB* without bogie on the rail.

attraction forces, striving to minimise the air gap, arise and build the normal force of the *MTB*–rail contact. Based on the frictional contact between the two contact partners - individual brake element (endpiece, respectively intermediate element, see Fig. 2) and the rail head - sliding friction forces are created which are transmitted through the structure and finally by the transmission pads into the bogie as an added up brake force. The magnets stay centred on the rail during typical operation of the *MTB*. They are guided by the vehicle only in the longitudinal direction, with the lateral stops at the transmission pads staying untouched.

Such track brakes equipped with electromagnets have been used since the early nineteenth century, with the first patent published in 1905, [2], while they were first restricted to the usage in trams. However, since the 30-40s in the last century *MTB*s are also applied in mainline vehicles with velocities greater than 140 km/h, [3]. Since then, several scientific papers have been published addressing the braking behaviour of *MTB*s. A first detailed investigation on the static and dynamic behaviour of *MTB*s was published in 1988 in [4–6]. The influence of different parameters, such as the electric current, air gap between brake element and rail, wear, and rail profiles, on the magnetic attraction force at standstill was analysed experimentally. The dynamic behaviour of the coupled

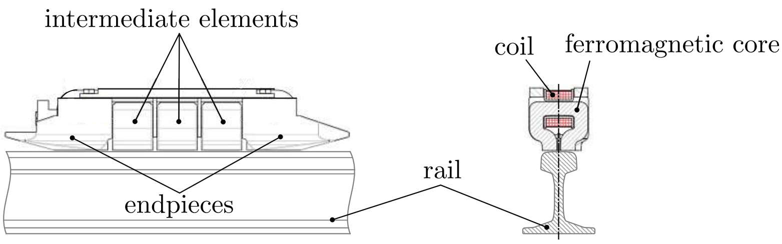


Figure 2: Corner electromagnet with individual elements, adapted from [1].

electromagnetic system during the vertical attraction process was discussed by a simple electric circuit with the back-coupling from the magnetic field considered by additional damping, accompanied by experiments. Further, measurements of the brake force and the sliding coefficient of friction over the vehicle velocity were presented, showing a significant decrease with increasing velocity and a pronounced variance. The authors of [7] and [8] showed from field tests that using an *MTB* positively affects reduced wheel-rail adhesion from different types of contaminations. Further, they pointed out the influences of slippery tracks on the braking forces of *MTB*s and the whole braking performance of the used multiple-unit train. Calculations in [9] on the necessity of brakes that work independently from the wheel-rail contact (in particular *MTB*s), were based on the assumptions of steady-state magnetic forces and a given velocity-dependent friction coefficient. In [10], a basic rigid multibody simulation model of an *MTB* for light rail vehicles (LRV) was developed and validated. The movement of an *MTB* in its ready position was investigated in [11] with a rigid multibody dynamics model. In [12], two simulation models were developed, representing the electric and mechanical behaviour of the *MTB*. The mechanical (contact) interaction between brake shoe and rail was modelled with constant stiffness and damping coefficients in the vertical direction, neglecting lateral forces. The friction force was derived with a Coulomb friction model based on a velocity-dependent friction coefficient. The reluctance of the magnetic circuit was a function of vertical distance and was calculated with steady-state FEM analysis, resulting in an analytical look-up table. The simulation model was validated with measurements for motions in the vertical direction. In [13] and [14], a simulation model for an *MTB* utilised in LRV was developed, to map the first brake impact during activation. For that purpose, the magnetic attraction and reluctance were described as a function of the current and the vertical position derived by experiments, neglecting eddy currents and magnetic leakage. The coefficient of friction was a function of the longitudinal velocity and of time to account for the transient buildup of frictional forces. In [15], the buildup of the braking forces of an *MTB*, including the first brake impact transferred to the bogie, and the vertical jerk after a power shutdown, was analysed, utilising a multibody dynamics model with rigid magnets and locked lateral displacement. The transmission link and bogie frame were modelled as spring-damper elements with parameters from quasi-static finite element analysis or experimental data. As shown from the previous *MTB* specific literature research, the published work either focused on measurements and discussed the basic braking behaviour, or on modelling the vertical motion during activation, respectively shutdown and the beginning of the braking process.

First investigations on the vibrational and stability behaviour of an *MTB* were published in [1, 16, 17]. It was shown by measurements in the field that *MTB*s are prone to self-excited vibrations at low velocities, which can cause structural problems and even fatigue damage. The oscillatory instability was investigated at a linearised electromagnetic-mechanical minimal model of the *MTB*. Possible self-excitation mechanisms due to the negative gradient of the velocity-dependent friction characteristic and the electromagnetic coupling were revealed, based on the Hurwitz criterion, [18], applied to the linearised system. It was shown that if the gradient of the velocity-dependent friction coefficient exhibits a sufficiently large negative value or system parameters of the electromagnetic coupling are tuned accordingly, the system becomes oscillatory unstable. The authors further validated their findings of the linearised minimal model by a full multibody system model, and presented a few passive remedies and their positive effects to mitigate vibrations.

Fundamental insights into mitigating friction-induced self-excitations are presented in [19–22]. While passive mitigation techniques are advantageous as they don't need additional sensors, controllers, etc., they may not always be feasible due to possible constraints in the mechanical design or possible drawbacks regarding weight. In such cases, active vibration control can be beneficial, especially if vibrations can be mitigated or even suppressed with available actuators, i.e. the electromagnet, which generates the normal force of the frictional contact at *MTB*s. While active vibration control is not investigated in this application, it is addressed in several publications for different systems. Optimal approaches concerning the energy fed into the system were proposed, e.g. with a state feedback control at a basic friction oscillator by a fluctuating normal force in [21, 23]. In this case, the energy input into the oscillating system is minimised if the normal force exhibits a phase shift of 180° to the mechanical displacement. In [24], a state feedback control was implemented for a magnetic levitation (MAGLEV) vehicle, utilising a voltage modulation to emulate the interaction of a dynamic vibration absorber with the track by the available electromagnetic actuator. Similarly, a virtual electromagnetic energy harvester was implemented in [25], and a virtual sky-hooked damper was realised in [26], aiming for a reduced energy input of the oscillating system. To propose an effective strategy to actively mitigate vibrations at *MTB*s with the already available coupled electromagnetic system is a challenging task, as it was proposed in the literature as a possible excitation mechanism in the open-loop case itself. However, a detailed explanation of the root cause of the onset of vibrations of the coupled electromagnetic-mechanical system is not yet resolved. Hence, a thorough understanding of the fundamental excitation mechanisms is indispensable for utilising a meaningful control strategy and must be obtained first.

Different mechanisms and applications of self-excited vibrations beyond the *MTB* were introduced, e.g. in [27]. A State-of-the-Art paper, giving an overview and revealing all kinds of phenomena related to self-oscillation, was published in [28]. Friction-induced self-excitations from sliding contacts, particularly relevant here, were covered broadly in [29]. In particular, for basic understanding, modelling, and experimental validation of different mechanisms resulting in friction induced vibrations, brake squeal was frequently addressed, e.g. [30], and has been studied in detail, e.g. [31] on spragging, [32] on stick-slip, [33] on Hopf bifurcation due to negative damping, [34] on instability with mode-coupling in friction induced vibrations. The negative gradient of the velocity-dependent friction characteristic, also called the falling regime in the friction characteristic, is often present as a well-known and well-studied reason for self-excited vibrations, [19], because of a positive energy flow into the oscillating system, [35]. Investigations in [21] showed that a positive energy flow into the oscillating system is also possible without a falling regime in the friction characteristic if the normal force fluctuates, which peaks at a diminished phase shift between normal force and oscillating motion. In [36], it was pointed out that friction-induced vibrations are intrinsically sensitive to parameter variations and proposed a modelling approach to consider the effects of sensitivity and uncertainty on predictions. The basic mechanism of inertial self-excitation, which may result from the coupled differential equations of the electromagnetic system in *MTB*s, was addressed in [37], including an electric-mechanical vibrator example, with more details in [38]. At MAGLEV vehicles with electromagnetic suspension (EMS), similar to the *MTB* concerning the electromagnetic actuator interacting with the track, self-excited vibrations were also observed at low velocities and even at a standstill, [39, 40]. Investigations on the coupled system in [24] pointed out that the fundamental frequency of the track is a key factor in the occurrence of self-excited vibrations. Investigations on EMS-MAGLEV vehicles in [41] found additionally that the relative motion of the electromagnet with respect to the electric conductive rail results in a significant distortion and weakening of the magnetic field, i.e. the attraction force shows a significant velocity dependency due to motion-induced eddy currents. Investigations in [42] revealed that this velocity dependency significantly influences the vertical dynamics. The mentioned investigations from the literature on basic phenomena causing self-excited vibrations focus either on the electromagnetic-mechanical coupling or a velocity-dependent sliding friction coefficient. However, the interaction of both causes as emerging at *MTB*s has not yet attracted much research.

Published research regarding the vibrational behaviour of the *MTB* was studied within the simplification of a linearised system. Hence, no statements on the nonlinear stability behaviour and the vibrational behaviour after the loss of stability could be drawn, leading to a lack of interpretation of existing measurement data. The nonlinear stability behaviour of systems incorporating frictional contacts is addressed in various scientific publications. In [43], the stick-slip vibration of a self-excited smooth and discontinuous friction oscillator of a moving belt type is investigated. Different belt velocities and normal force excitations revealed distinguished shapes of oscillations, and a path to chaos from a period-doubling bifurcation was shown. The influence of an alternating normal force was studied in [44] on the vibrational characteristic of a 1 degree-of-freedom (DOF) friction oscillator with changing dynamic and static friction coefficients. It was found that the amplitude and frequency of the excitation can change the oscillatory motion from single-periodic, multi-periodic, to chaotic stick-slip motions. In [45], the vibrational behaviour of a friction oscillator on a moving belt is examined under various damping values, which determines the classification of oscillator motions. Investigations in [46] highlight stick-slip motions at a linear moving cart driven by a continuous rotating direct current motor via scotch yoke by studying a basic electromagnetic-mechanical coupled model, and an analytical approximation for the stick duration was proposed. In [47], it is found that the stick-slip pattern of a coupled friction oscillator depends on the velocity of the energy source, the normal contact load, and the eigenfrequencies of the involved modes. Authors of [48] studied the stability behaviour of a pad-disk brake system, considering changing friction characteristics, and ascertained that based on the angular velocity, the vibrational behaviour can change from stable periodic orbits emerging after a Hopf bifurcation to the occurrence of period-doubling bifurcation and chaotic dynamics. In [49], the dynamic behaviour of a two-layered brake pad system is examined, where a significant influence of the brake pad mass and connection stiffness on the stability behaviour of the system has been identified. Different types of systems and structures related to stick-slip motions and their nonlinear dynamics are introduced in reviews of [50, 51]. Following the literature review on the nonlinear dynamics of systems incorporating dry friction, it becomes evident that the friction characteristic (static and kinetic), the normal force, geometric dimensions regarding the contact point, involved stiffness and damping, and the dynamics of coupled systems are mainly responsible for structural changes of the related stability behaviour. The presented insights are primarily gained from minimal models, which are indispensable for the basic understanding of dynamical phenomena.

Concluding from the literature overview, most scientific research addressed the braking and actuation behaviour of *MTB*s in modelling and experiments. However, the combination of *MTB* and friction enhancers, such as sand, and the following change in the braking behaviour is not yet published. As the utilisation of *MTB*s at lowest velocities is a comparatively new requirement, the related problem of occurring self-excited vibrations at these velocities is only addressed in very few publications. Possible excitation mechanisms are published beyond the application of *MTB*s. Still, the interaction of the electromagnetic coupling and the frictional contact on the onset of vibrations is not yet fully understood. Based on the missing understanding of the mechanisms involved at the onset of self-excited vibrations, strategies to actively mitigate self-excited vibrations at *MTB*s are still pending. The influence of motion-induced eddy currents on the attraction force and stability behaviour is experimentally and theoretically unexplored, although they show significant influences in adjacent applications. The nonlinear stability behaviour of the coupled system, and how variations in system and environmental parameters influence it, has yet to be addressed. Gaining insight into this nonlinear stability behaviour is crucial for evaluating whether self-excited vibrations will occur due to unavoidable perturbations and determining whether the structure can endure the resulting amplitudes when vibrations become inevitable.

2. Research Objectives and Scope

This thesis aims to provide insights into the stability behaviour of *MTB*s, suggest improvements for the design, and active control strategies to avoid severe self-excited vibrations. Hence, a thorough understanding of the underlying mechanisms involved at the onset of vibrations and the nonlinear stability behaviour of the coupled system must be resolved. Fundamental dependencies on the loss of stability and the behaviour afterwards shall be examined with the known issues of *MTB*s and revealed dependencies of adjacent applications in mind.

Specifically, the objectives of this cumulative thesis, based on the identified research gap above, are:

1. **Experimental foundation:** Establish an empirical basis for understanding the velocity dependence of frictional and electromagnetic forces in *MTB*s, and for characterising the qualitative behaviour of occurring self-excited vibrations.

2. **Excitation mechanisms in coupled systems:** Provide simplified yet representative dynamic models of the coupled electromagnetic mechanical system with dry friction, clarify the fundamental mechanisms responsible for the onset of self-excited vibrations, and determine their sensitivity to operating conditions.

3. **Mitigation and active vibration control:** Contribute strategies for suppressing self-excited vibrations, including active concepts that exploit the electromagnetic system itself, with the goal of extending the stability range and ensuring safe and robust braking performance.

4. **Nonlinear stability and post-critical dynamics:** Characterise the nonlinear stability behaviour of *MTB*s by analysing transitions from stable to unstable operation, the emergence of limit cycles, and the sensitivity of vibrational behaviour to design parameters and environmental conditions.

By addressing these objectives, this cumulative thesis, comprising five peer-reviewed scientific contributions, *Paper A*–*Paper E*, aims to contribute to gaining fundamental insights into the stability behaviour of *MTB*s and consequently provide information on possible practical improvements to design a robust and vibration-free braking system.

The experimental basis for the investigations are provided by a measurement campaign presented in *Paper A* with corresponding results on the velocity dependencies of the frictional contact, *Paper A*, and of the magnetic attraction forces, *Paper E*, and by the discussed qualitative behaviour of measured vibrations in *Paper B* and *Paper E*. Fundamental excitation and dissipation mechanisms are studied by a minimal model in *Paper C* and are extended for distributed contact points and velocity-dependent attraction forces in *Paper E*. The feasibility of an active vibration control is shown in *Paper B* with a more detailed discussion on control strategies and their effectiveness in *Paper C*. Dependencies of occurring vibrations by changing operating conditions and design parameters are presented in *Paper E*. The vibrational behaviour after a loss of stability is presented for a minimal model of the *MTB* in *Paper C*, for a full *MTB*-brake frame model in *Paper D*, and utilising an augmented corner model in *Paper E*.

In the following Sections, the underlying methods to resolve open research questions are described briefly. Section 3.1 presents the basic approach and fundamental findings from experiments. In Section 3.2, the electromagnetic-mechanical system models are introduced. A description of the applied energy consideration in the context of self-excited vibrations is given in Section 3.3. With the identified excitation mechanisms, the idea of active vibration control with the available electromagnetic actuator is presented in Section 3.4. Methods used to investigate the dynamic behaviour after the loss of stability are addressed in Section 3.5. Section 4 briefly describes the scientific contributions published within this thesis. Finally, the scientific impact of the investigations is summarised in Section 5

3. Methods

3.1. Experimental investigations on the velocity dependencies of the *MTB*–rail contact

Experiments with the real application and measurements of meaningful states, inputs and outputs of the system during operation are indispensable for the basic understanding of the coupled dynamic behaviour. The obtained insights are necessary to support reasonable modelling and restrict parameters within bounds of practical relevance. In Fig. 3a, measurements of the *MTB*, depicted in Fig. 1b, are shown for braking manoeuvres until very low velocities. Fig. 3a depicts the vehicle velocity v_v in black, the braking forces measured in the left and right transmission pads $F_{Bx,j}$ in dark and light green and the electric current in the coils i_{el} in red for a braking behaviour from approx. 100 km/h until standstill in the upper diagram. In the lower diagram, the corresponding amplitudes of the braking forces $\hat{F}_{Bx,j} = F_{Bx,j} - \bar{F}_{Bx,j}$ with respect to the mean average $\bar{F}_{Bx,j}$ are plotted with the envelopes in dashed lines. The vertical red lines indicate the

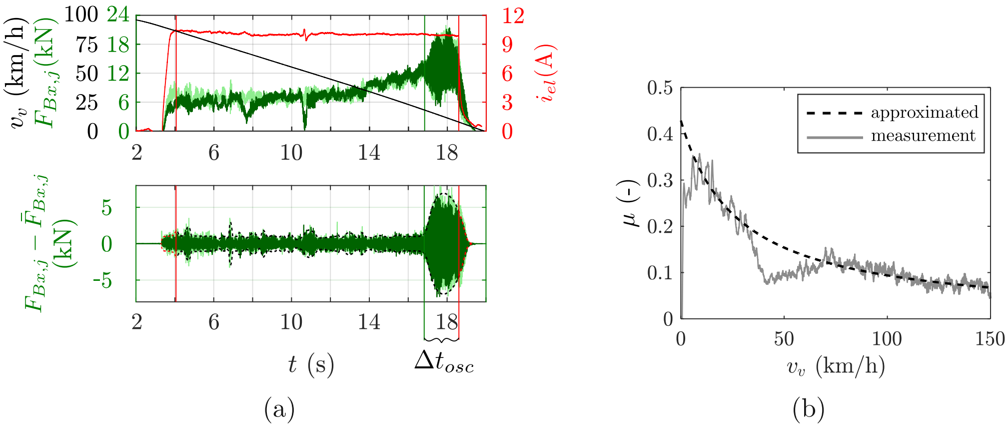
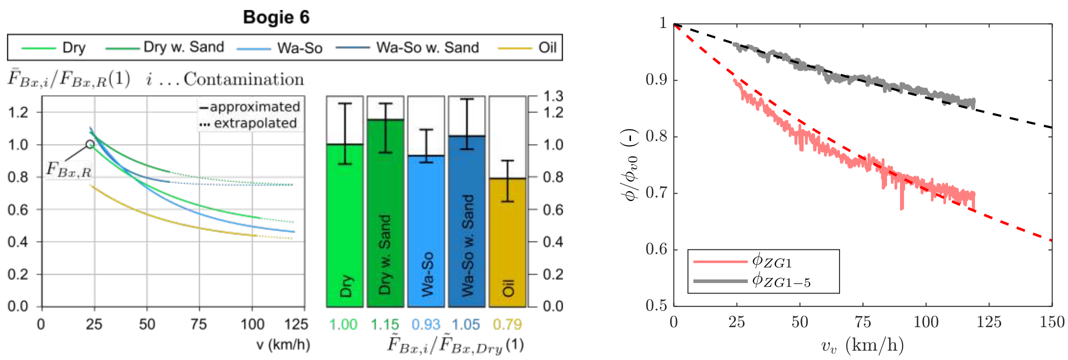


Figure 3: (a) Measurements of the braking forces $F_{Bx,j}$ in the left and right transmission links, electric current i_{el} and velocity v_v, during a braking manoeuvre from $100 - 0$ km/h, *Paper E* (b) Measurements from [17] and approximated sliding friction coefficients, *Paper C*.

start and the end of the considered quasi-static braking manoeuvre, as the electric current reaches its steady state value and the vehicle velocity reduces with a constant and small acceleration of -1.5m/s^2. During the quasi-static braking manoeuvre, braking forces increase as vehicle velocity decreases. This behaviour can be partially explained by the velocity-dependent sliding coefficient of friction μ, which is illustrated in Fig. 3b. The shown measurement of the coefficient of friction was obtained from experiments conducted without the electromagnet being active, since the magnetic attraction forces present at the *MTB*–rail contact are internal forces and cannot be measured directly. As a result, the sliding coefficient of friction is calculated using the directly measured braking force and the normal force, which is derived from the measured air pressures in the pneumatic cylinders. If a certain velocity is reached in Fig. 3a, vibrations start to grow from available disturbances present during the whole velocity range. Focusing on the onset of increasing amplitudes, e.g. depicted in *Paper B*, a small time range of rapidly growing vibration amplitudes may be noticed with a distinct slower change in amplitudes afterwards. In Fig. 3a, a distinct maximum of the measured amplitudes is visible where vibrations decay until the electric current is shut down, where amplitudes start to diminish. For a detailed understanding of this behaviour, the postcritical behaviour of the coupled system is investigated, while basic excitation mechanisms must be understood first.

To address the dynamic and stability behaviour of the *MTB* by simulation, reasonable changes of the environmental conditions, especially the *MTB*–rail contact conditions, must be considered. As summarised in the literature review, the velocity dependency on different contaminations has already been published; however, the influence of sand on the qualitative behaviour has not been resolved. Hence, a measurement campaign was performed with two magnetic track brakes mounted on a diesel multiple unit train for repeated braking manoeuvres on a closed track contaminated with different friction modifiers. The results for the *MTB* in the sixth, out of eight, bogies are depicted in Fig. 4a. In the left diagram, the averaged longitudinal braking force $\bar{F}_{Bx,i}$ is plotted over the vehicle velocity, relative to the braking force at 25 km/h for a dry rail $F_{Bx,R}$, for each contamination i. The right bar plot shows the mean value of the equivalent brake force $\tilde{F}_{Bx,i}$, as a constant force providing an equivalent brake energy for an assumed quasi-static braking manoeuvre as the integrated velocity-dependent behaviour over the brake distance. The values are presented relative to those on a dry track $\tilde{F}_{Bx,Dry}$, with whiskers indicating the maximum and minimum measured values. The sliding



(a) (b)

Figure 4: (a) Qualitative velocity dependency and relative equivalent values of the *MTB*'s brake force for different contaminations, *Paper A* (b) Measured and approximated magnetic flux characteristic in solid respectively dashed lines for the leading endpiece ϕ_{ZG1} and the leading five pole shoes ϕ_{ZG1-5}, *Paper E*.

coefficient of friction, the only systematic difference in the presented data, does not differ significantly between dry and low-adhesion conditions (Wa-So refers to water with soap). When sand is added, minor decreases in the brake forces with increasing velocity may be noticed, implying smaller gradients in the friction characteristics. This pattern also occurs in extremely low-adhesion conditions (oil), where the friction level is reduced additionally.

Further influences on the velocity dependency of the brake force can appear due to a velocity dependency of the normal force, i.e. the magnetic attraction force, as it appears at EMS-MAGLEV vehicles. In this application, the forward velocity significantly affects the suspension's normal force due to motion-induced eddy currents, [41]. Hence, field tests were executed as part of this thesis to determine the effect of motion-induced eddy currents on the attraction force at *MTB*s, which is unexplored experimentally and theoretically. As mentioned earlier, the magnetic forces acting in the *MTB*–rail contact are inner forces and, therefore, not measurable during regular operation. Hence, an indirect method to measure this influence is necessary, which is done by measuring the magnetic flux ϕ_p through the pole, which is proportional to the attraction force $F_{EM,p} \propto \phi_p^2$. Therefore, a secondary coil of N_{sec} windings is applied around each brake element of the electromagnet (endpieces and intermediate elements), and the magnetic flux ϕ_p through this closed loop is derived through the correlation with the measured induced voltage in the winding:

$$U_{ind} = -N_{sec}\frac{\mathrm{d}\phi_p}{\mathrm{d}t}. \tag{1}$$

The results are depicted in Fig. 4b for the leading brake element ϕ_{ZG1} and as an average over the first five elements ϕ_{ZG1-5}. Hence, it is obtained that the magnetic flux is significantly reduced for increasing vehicle velocities v_v, which means an even pronounced velocity dependency for the attraction force. Additionally, it is found that the reduction of the attraction force, based on motion-induced eddy currents, is dependent on the relative position of the considered brake element in the whole magnet, with an increased reduction at the leading element compared to the rear elements.

3.2. Modelling the electromagnetic-mechanical coupled system

To investigate the dynamic behaviour of mechatronic systems and to study the influence of main parameters on the stability behaviour, lumped parameter models are often utilised, [52]. In the case of coupled systems, as emerging at *MTB*s with the electric power supply of the electromagnet, the magnetic field generated by the electromagnet, and the motion of the mechanical structure, models for each subsystem are necessary. Further, a defined relation between lumped elements of one system, which are dependent on state variables of the other system, is required, e.g. the air gap between the *MTB* and the rail in the magnetic system, is dependent on the motion of the mechanical system. Such relations can be obtained by semi-analytical methods, look-up tables, or pre-defined dependencies, [53]. As different open questions arise from the literature review, models of varying model accuracy are developed and utilised at various stages of investigation.

Minimal Model

First theoretical investigations, *Paper B* and *Paper C*, address the self-excitation mechanism based on the electromagnetic-mechanical coupling, the underlying phenomena causing a positive energy flow into the oscillating system, and an active vibration control utilising this coupling. As the impact of the electromagnetic system on the loss of stability, combined with frictional contact, is not yet understood, a minimal model is aimed at the first stage.

Spectrum analysis of measured bending moments on the *MTB* structure from 132 braking manoeuvres, covering a velocity range of 150-8 km/h, in [1], identifies a single excited frequency of approx. 28 Hz of the brake-frame structure. This frequency corresponds together with the measured time signals, presented in [1], to the second asymmetric in-plane structural mode of the *MTB*-frame from a conducted eigenmode analysis in [17]. Each corner electromagnet exhibits an almost rigid body rotation at this mode, with the track rod and the connecting beams deforming in between. Hence, a similar model as applied in [17] for a linear stability analysis is assessed here, mapping only the excited eigenmode of the *MTB* with one degree of freedom. The minimal model describing a quarter of the full *MTB* contains the electric-, magnetic-, and mechanical subsystems with the couplings marked in grey, is depicted in Fig. 5. The nonlinear autonomous differential equations of second order, as the mathematical description of the minimal model, are given with parameters in *Paper C*.

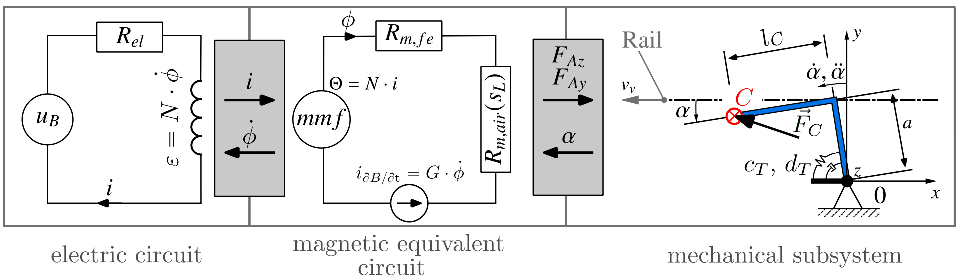


Figure 5: Minimal model of the electromagnetic-mechanical coupled system with individual electric-, magnetic, and mechanical subsystems, *Paper C*.

The **electric circuit** describes the electric power supply of the electromagnet with the battery voltage u_B, the ohmic resistance of the wiring R_{el}, and the electromotive force of the coil $\varepsilon = N \cdot \dot{\phi}$, with N the number of windings.

The **magnetic equivalent circuit (MEC)** is a lumped parameter approximation of the 2D magnetic field through the electromagnet's cross-section, with the approximation of an a priori known magnetic flux path. This description of the magnetic field by a magnetic equivalent circuit allows the reduction of the initially complicated calculation of a generally inhomogeneous spatial magnetic field to a simpler set of equations, describing a network of concentrated elements (magnetic resistances and magnetomotive force), [53]. The MEC of the minimal model, shown in Fig. 5, is excited by the magnetomotive force (MMF) of the coil $\Theta = N \cdot i$ with i the electric current of the electric circuit. It further contains an eddy current loss element $i_{\partial B/\partial \mathrm{t}} = G \cdot \dot{\phi}$, describing the weakening of the magnetic field by eddy currents induced from the time-varying magnetic field, with the electric conductivity of the eddy currents G. The magnetic resistance from the involved iron of the pole shoe and the rail is considered with $R_{m,fe}$, and the magnetic resistance $R_{m,air}$ from an effective air gap s_L. This air gap is a hyperbolic function of the lateral displacement and a linear function of the vertical lift-off with respect to a perfect alignment over the non-flat rail head, parametrised by the results of a magnetostatic finite element analysis in [16].

The **mechanical subsystem** describes the rotation, with states $\alpha, \dot{\alpha}$ of the rigid corner electromagnet with inertia $I_{Mg,0}$, over the rail which is in translational motion v_v, and is attached to the reference point 0 by a spring-damper element approximating the stiffness and damping of the structure. Electromagnetic forces, generated in the *MTB*–rail interface, act in vertical F_{EM_z} and lateral F_{EM_y} direction in the fictitious contact point C. The friction force F_R occurring at this sliding contact is derived within the approximation of Coulomb friction, with a velocity-dependent sliding coefficient of friction. The additional damping due to a lateral friction force component is neglected in this model, which is utilised for the first investigations.

MTB brake frame model

To investigate the stability behaviour of the full *MTB* brake frame in *Paper D*, considering the mechanical coupling of the individual corner electromagnets through the elastic frame structure and the electrical coupling through a shared electric wiring of the coils from the individual corner electromagnets, the *MTB* brake frame model is introduced. The individual subsystems are depicted in Fig. 6. The electric circuit now incorporates four inductances, one for each corner electromagnet, with two electric branches which couple the magnets on the same rail in series with the ones on the opposite rail in parallel. The electric currents i_{14} and i_{23} of each branch excite the corresponding magnetic circuit, generating the magnetic fluxes ϕ_1, ϕ_4, respectively ϕ_2, ϕ_3. The time-varying fluxes $\dot{\phi}_j$ of each MEC j interact through the shared electric circuit. The mechanical subsystem extends the rigid corner system from the minimal model to a full *MTB*. However, the stiffness of the beams in between, which exhibit a significantly lower stiffness than the corner magnets, is now considered by influence coefficients, which enables the possibility to regard the interaction of a force or moment acting on the corner i with the corner j. Based on the approximation of linear system behaviour of the mechanical structure, i.e. linear elastic material, the principle of superposition holds, and the influence coefficient δ_{ij} is defined as the displacement in the i-direction due to a moment M_j in the j-direction, as described in [54]. With Euler's second law (balance of angular momentum) applied at each corner j to derive the moments M_j, the equation of motion is derived for the mechanical structure with the relation of the generalised coordinates β_i and the moment M_j

$$\beta_i = \sum_{j=1}^{4} \delta_{ij} \; M_j; \quad i = [1, 4]. \tag{2}$$

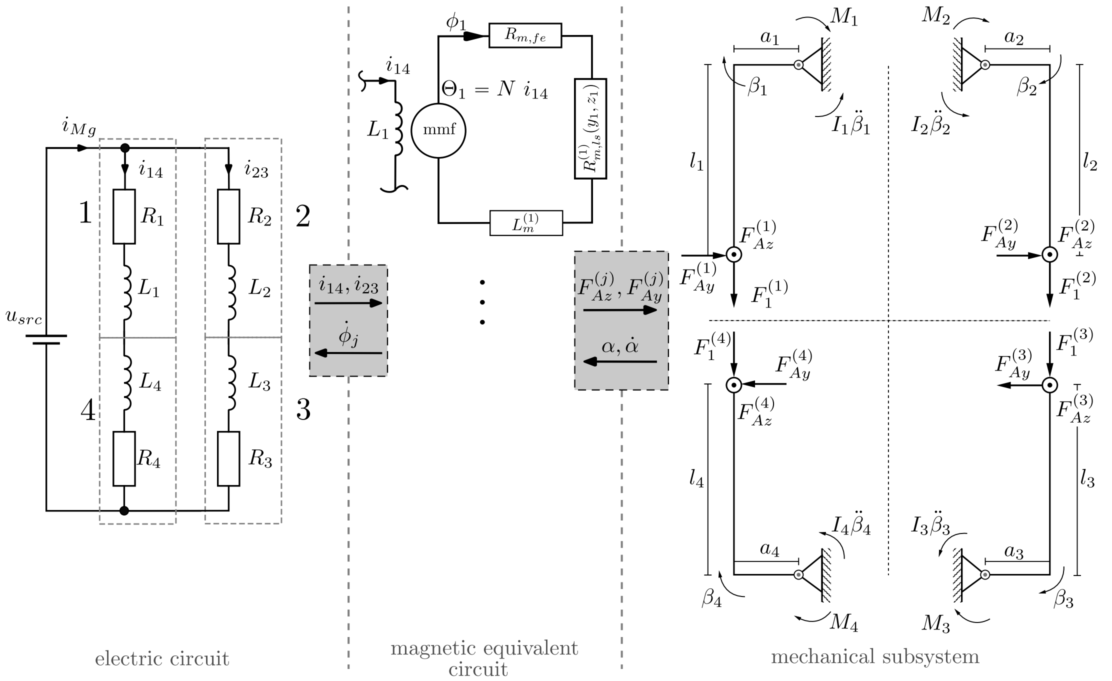


Figure 6: Simplified mechanical model of the four-corner electromagnets, a) Electric circuit of the magnetic track brake; (b) Magnetic subsystem of the (1) corner electromagnet; (c) Lateral and vertical displacements at one corner electromagnet over the rail.

Where the influence coefficients are obtained by introducing a unity moment M_j to only one corner j and computing the resulting rotation at the other corners β_i, which in this case is equal to the influence coefficient δ_{ij}. The computation of the response by applied moments was calculated with a finite element model of the *MTB* brake frame, which was also used during the corresponding eigenmode analysis of the structure, necessary for the parametrisation of the lumped mechanical system, described in *Paper D*.

Corner *MTB* model

In both models described above, each electromagnet is considered as one rigid body with one fictitious contact point. The model assumptions in this case do not consider different brake elements, typical for mainline *MTB*s, depicted in Fig. 2, and velocity-dependent attraction forces. Hence, a third model is introduced to investigate the influence of changing contact and friction conditions at segmented brake magnets with distributed contact points and velocity-dependent attraction forces. This model is further utilised to investigate the influence of the contact conditions and main system parameters on the post-critical behaviour, i.e. the emerging limit cycles, which necessitates considering the damping effect due to friction force components lateral to the vehicle velocity. For these reasons, the corner *MTB* model with its subsystems is introduced, depicted in Fig. 7.

The nonlinear stability behaviour is analysed, and dependencies of contact conditions and system parameters on the vibrational behaviour are discussed in *Paper E*.

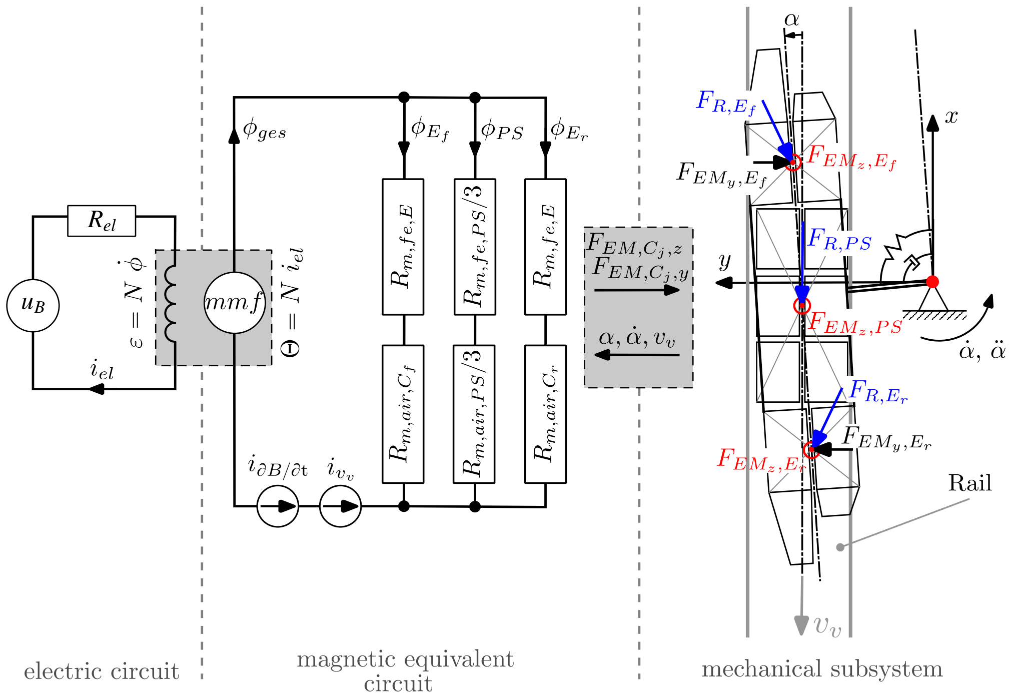


Figure 7: Electric circuit, magnetic equivalent circuit of the system model, and mechanical subsystem of the corner electromagnet.

The shown electric subsystem is similar to the minimal model, as only a quarter of the *MTB* is considered here. The MEC contains three magnetic branches with related magnetic fluxes ϕ_{PS}, as the sum through all intermediate elements, and ϕ_{E_f} and ϕ_{E_r} through the front and rear endpiece. The depicted branch of the intermediate brake elements in Fig. 7 is already a condensed branch from the three parallel circuits, assuming perfectly aligned intermediate elements on the rail head, due to the lateral and vertical clearance to the magnets' body. Each branch consists of a magnetic resistance of the effective air gap s_{L,C_j} between the individual brake segment and the rail head at the related contact point C_j, and the magnetic resistance of the involved iron $R_{m,fe,PS}$ for the intermediate elements and $R_{m,fe,E}$ for the endpieces. Considered eddy currents are induced 1) by the time derivative of the magnetic field $\partial \boldsymbol{B}/\partial t$, similar to the minimal model, and 2) by the velocity term $(\boldsymbol{v} \times \boldsymbol{B})$, describing the electromotive force of the translational moving conductor (Rail) through a magnetic field. The modelling of motion-induced eddy currents in lumped parameter models of translational moving conductors and quasi-static magnetic fields is not addressed for the case of *MTB*s in the literature. However, for rotating, [55], and linear, [56], eddy current brakes, and induction machines [57], models describing the induction of eddy currents and the weakening of the main magnetic field are presented by an analytical expressions of an MMF in the

MEC or by a coupled magnetic and electrical network with interacting electro- and magnetomotive forces. To keep a computationally efficient minimal model and due to the lack of a more detailed knowledge on how eddy currents are induced and how they are affecting the magnetic field, a MMF i_{v_v} as an analytical function, to match the behaviour from experiments in Fig. 4b, is introduced in the MEC. This model considers the phenomenological influence of the involved dynamics of the coupled electromagnetic system and the velocity-dependent normal force, and the respective influences on the stability behaviour are examined. However, the magnetic field's spatial distortion and the interaction of the induced eddy currents in the rail are not considered here. Hence, a linear superposition of both influences is assumed, neglecting the interaction in more detail, and both MMFs $i_{\partial B/\partial t}$ and i_{v_v} are considered effectively with the total magnetic flux. The mechanical subsystem describes a quarter of the *MTB*, similar to the minimal model. However, the endpieces and the intermediate elements have individual contact points here, $C_j = \{C_{Ef}, C_{Er}, C_{PS}\}$, with acting electromagnetic forces $F_{EM_z,C_j}, F_{EM_y,C_j}$ and the resulting friction force F_{R,C_j} in the $x-y$ plane. Considering individual contact points enables investigations into the influence of changing contact conditions at the individual brake element–rail contact. The friction force is no longer constrained in the longitudinal direction, and the components are derived by assuming Coulomb friction with a velocity-dependent sliding coefficient of friction. Further, a smooth transition between static and kinetic friction is introduced by a continuously differentiable approximation of the signum function, as artificial smoothing, [58]. To avoid singularities when the relative velocity of the *MTB* and the rail becomes zero, during possible stick-slip transitions, a case differentiation is introduced. The detailed description with underlying approximations and the nonlinear system equations are given in *Paper E*.

3.3. Energy considerations and self-excitation mechanisms

Self-excited vibrations are oscillations without external excitation, while the alternating force that sustains the motion is controlled by the motion itself. If the energy input from an available energy source is greater than the dissipated energy during one oscillation period, amplitudes will rise, indicating that the current state is unstable, [35]. This method to investigate the system's stability behaviour by an energy balance through one oscillating period is addressed to investigate fundamental excitation mechanisms and for mitigation strategies, both actively and passively, in e.g. [19, 21, 23]. The negative gradient of the velocity-dependent friction characteristic, as a well-known cause for self-excited vibrations, can be studied by a basic friction oscillator on a moving band with constant velocity v_0 and constant normal force. In this case, the friction force F_R increases with increasing oscillator motion $\dot{x}$, because of a reduced relative velocity $v_{rel} = v_0\dot{x}$ at the contact point. When integrating the friction power on the oscillator over one period, a positive input of mechanical energy results during each period T,

$$\Delta E_{in} = \int_0^T F_R \cdot \dot{x} \; \mathrm{d}t > 0. \tag{3}$$

Hence, amplitudes will increase until a stable limit cycle is reached, [23]. A positive energy flow into the oscillating system is also possible for a constant sliding friction coefficient if the normal force fluctuates, which peaks if the normal force and the oscillating motion are in phase, [21].

For the minimal model of the *MTB*, depicted in Fig. 5, considered during the basic investigations on excitation mechanisms for the onset of self-excited vibrations, the mechanical power reads,

$$P_P = \boldsymbol{F}_C\ \boldsymbol{v}_C - d_T \cdot \dot{\alpha}^2 = F_{R,x}\ v_{C,x} + F_{A,z}\ v_{C,z} + F_{A,y}\ v_{C,y} - d_T \cdot \dot{\alpha}^2, \qquad (4)$$

with $F_{A,z}$ the vertical magnetic attraction force, $F_{A,y}$ the lateral magnetic self-centring force, $F_{R,x}$ the longitudinal friction force, and $\boldsymbol{v}_C$ the velocity of the contact point C. For the directions x, y, z please refer to the coordinate system in the mechanical subsystem of Fig. 5. For the linearised system with respect to the quasi-steady state (α_0, ϕ_0, $\dot{\alpha}_0 = \ddot{\alpha}_0 = \dot{\phi}_0 = 0$) at $v_v = v_s$, the behaviour of the mechanical oscillation $\dot{\alpha}$, the magnetic flux ϕ, and the effective energy input $\Delta E_P = \int_0^T P_p\ dt$ obtained by a numerical integration of Eq. 4 is shown in Fig. 8. The diagrams indicate that the magnetic flux ϕ

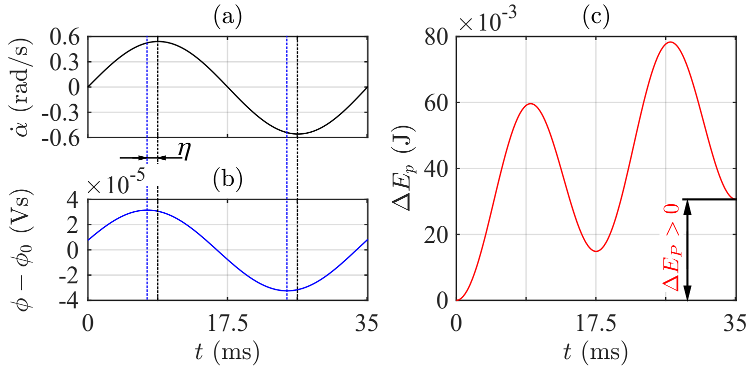


Figure 8: (a) angular velocity $\dot{\alpha}$, (b) magnetic flux ϕ, and (c) effective energy input ΔE_P (c) during one oscillation period T at an unstable equilibrium state, *Paper B*.

lags by η in phase relative to the mechanical motion $\dot{\alpha}$, and a positive effective energy can be observed over one oscillation period. Hence, the quasi-steady state in Fig. 8 is unstable, i.e. amplitudes will grow over time.

For an assumed harmonic behaviour, reasonably near the quasi-steady state, the mechanical power of Eq. (4) can be integrated analytically over one oscillating period, and the effective energy input is obtained

$$\Delta E_P = T\ \frac{(\hat{\alpha}\ \omega)^2}{2}\ \left[\left(K_{EM,1} - K_{EM,2}\right)\ \frac{1}{\omega}\ \frac{\hat{\phi}}{\hat{\alpha}}\ \cos(\eta) - K_{d_T} - K_{\partial\mu/\partial v}\right], \qquad (5)$$

with coefficients $K_{EM,1}, K_{EM,2}$, K_{d_T}, and $K_{\partial\mu/\partial v}$ depending on the quasi-steady-state and system parameters, provided in *Paper C*. If this effective energy is positive, amplitudes will increase; otherwise, they will diminish. Hence, each term with a positive sign identifies a possible self-excitation mechanism, and each term with a negative sign indicates a damping mechanism. A detailed discussion on deriving the analytical equation, a description of the individual coefficients, and restrictions of the phase shift and amplitude ratio for the considered application is provided in *Paper C*.

The first term in Eq. (5) describes the effective energy from the electromagnetic coupling. It shows dependencies of the phase shift η, the amplitude ratio of the magnetic flux and the mechanical displacement $\hat{\phi}/\hat{\alpha}$, the inverse of the eigenfrequency $\omega = 2\pi/T$ and the two coefficients $K_{EM,1}, K_{EM,2}$. This term implies that without dynamics of the electromagnetic system and therefore being ϕ in phase with α ($\eta = \pi/2$), the first term yields zero, and the effective energy would not be affected through the electromagnetic terms $K_{EM,1}$ and $K_{EM,2}$. The same holds if the magnetic flux were constant $\hat{\phi} = 0$ during the mechanical oscillations, representing a constant normal force. In the general open-loop case, the amplitude ratio fulfils $\hat{\phi}/\hat{\alpha} > 0$ and the phase shift $0 < \eta < \pi/2$. Hence, η is identified as a key parameter of stability for electromagnetic-mechanical coupled systems and the first coefficient with $K_{EM,1}$ in Eq. (5) is identified as a source of self-excited vibrations and the second coefficient $K_{EM,2}$ as a dissipation mechanism. The energy input described by $K_{EM,1}$ includes the partial derivative of the vertical attraction force by the magnetic flux, and the magnitude of the friction coefficient at the considered quasi-steady-state, i.e. it describes the contribution from the current friction coefficient and the oscillating normal force. The energy dissipation described by $K_{EM,2}$ includes the partial derivative of the lateral magnetic force with respect to the magnetic flux, i.e. the energy dissipation from the self-centring force F_{EM_y}. For realistic parameter ranges and friction characteristics, the energy input exceeds the energy dissipation ($K_{EM,1} > K_{EM,2}$). The resulting positive energy input into the mechanical system peaks if the magnetic flux is in phase with the mechanical oscillations ($\eta = 0$), is amplified by a high friction level and is dampened by the self-centring force F_{Ay}. The term including K_{d_T} in Eq. (5) describes the energy dissipated by damping of the structure. The term with $K_{\mathrm{d}\mu/\mathrm{d}v}$ includes the normal force at the quasi-steady state $F_{EM_z|0}$, and the gradient of the friction characteristic k_μ. If the gradient is positive, energy is dissipated; if a falling regime at the operating point is present, energy is fed into the system, indicating the well-known excitation mechanism at friction oscillators, [35].

To examine whether the equilibrium states, i.e. the quasi-steady states of the linearised system, are asymptotically unstable, the term in brackets of the found analytical Eq. (5) can be evaluated, for changing velocities, friction characteristics, etc. The contributions of the individual excitation and damping mechanisms are evaluated for varying vehicle velocities and different friction characteristics in *Paper C*. Further, the analytical equation provides direct information on how the energy input can be reduced to mitigate or even diminish emerging amplitudes, which can be passively realised by, e.g. changes in the mechanical and electrical design. In addition, the effectiveness of the individual mitigation strategy can be evaluated with Eq. (5).

3.4. Active vibration control

Active vibration control at pure friction oscillators was frequently addressed in the literature, e.g. by a state feedback control at a basic friction oscillator with a fluctuating normal force in [19, 23] to minimise the energy input.

Analysis of the effective energy input for the *MTB* in the previous Section 3.3 suggests a minimisation of the factor $\left[\hat{\phi}/\hat{\alpha} \cdot \cos(\eta)\right]$ in Eq. (5), associated with the excitation mechanism from the electromagnetic coupling, to mitigate oscillations actively. Although the amplitude ratio $\hat{\phi}/\hat{\alpha}$ and the phase shift η between the magnetic flux and the mechanical motion are both reasonable as possible control variables, there are restrictions on the demanded amplitude ratio. For $\eta > \pi/2$, the term $\cos(\eta)$ gets negative, corresponding to active damping. Hence, an optimal control strategy minimising the effective energy input due to a forced phase shift of $\eta = \pi$ leads to the quickest and most efficient mitigation of occurring amplitudes. This approach is similar to investigations on pure friction oscillators, where the phase shift between a prescribed alternating normal force and the oscillating motion is used to reduce the occurring amplitudes. However, in the considered application of an *MTB*, the open-loop phase shift is in a range where a positive energy flow is generated into the system.

Consequently, a state feedback is proposed which enforces a specific phase shift η_d and utilises the amplitude ratio $k = \hat{\phi}/\hat{\alpha}$ as a free controller gain. The small active modulation Δu_B of the electric voltage, with respect to the steady-state value u_{B0}

$$u_B = u_{B0} + \Delta u_B \tag{6}$$

is considered to be the control input, and the control law reads

$$\Delta u_B = -\boldsymbol{K} \cdot \Delta \boldsymbol{x} \tag{7}$$

with small deviations $\Delta \boldsymbol{x} = [\Delta\alpha, \Delta\dot{\alpha}, \Delta\phi]^T$ from the quasi-steady state indicated with index $|0$. When a harmonic oscillation with an identified eigenfrequency ω, valid at the onset of vibrations, is considered, the control gain $\boldsymbol{K}$ can be derived for a demanded phase shift η_d and amplitude ratio $k = \hat{\phi}/\hat{\alpha}$ from the linearised electromagnetic system. Then, the gain $\boldsymbol{K}$ results for $\eta_d = \pi$ in

$$\boldsymbol{K}_{\eta_d=\pi} = \left[-\frac{1}{C_u}\ k\ \omega - \frac{(\partial C_\phi/\partial\alpha)_{|0}}{C_u},\ k\ \frac{1}{\omega}\ \frac{(\partial C_\phi/\partial\phi)_{|0}}{C_u},\ 0 \right]^T \tag{8}$$

with C_u and C_ϕ as coefficients of Δu respectively $\Delta\phi$ of the linearised electromagnetic system.

From the derived feedback gain, it becomes evident that only the mechanical states are necessary for the developed state feedback. The second term in the first element remains if the amplitude ratio $k = \hat{\phi}/\hat{\alpha}$ is set to zero, which indicates a constant magnetic flux with $\hat{\phi} = 0$ but $\hat{\alpha} \neq 0$ during oscillations. In this case, the term corresponding to the effective energy input from the electromagnetic coupling is eliminated, and only the feedback of the mechanical displacement α is sufficient. However, the self-excitation mechanism based on the negative gradient of the friction characteristics remains unchanged in this case. Energy is only actively dissipated with the suggested state feedback if a demanded amplitude ratio $k > 0$ is implemented, see *Paper C* for more details. The feasibility of the active vibration control, with the implemented state feedback, resulting in a negative energy input $\Delta E_P < 0$ over each oscillation period for the same quasi-steady state as depicted for the open-loop case in Fig. 8, is shown in Fig. 9.

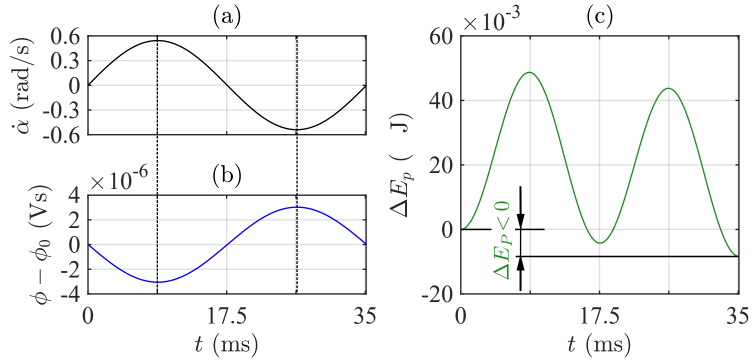


Figure 9: (a) angular velocity $\dot{\alpha}$, (b) magnetic flux ϕ, and (c) effective energy input ΔE_P (c) during one oscillation period T with implemented state feedback, *Paper B*.

3.5. Nonlinear stability and post-critical behaviour

Published research regarding the vibrational behaviour of the *MTB* and the previous energy consideration examines the stability behaviour within the simplification of a linearised system. Consequently, no conclusions can be made regarding the nonlinear stability behaviour and the vibrational characteristics following a loss of stability. This lack of understanding hinders the ability to interpret existing measurement data, e.g. the shown behaviour in Fig. 3a. Further, it can impede making meaningful statements regarding safe operations when vibrations cannot be avoided. Hence, different methods are applied to investigate the nonlinear stability behaviour of the suggested system models in Section 3.2.

3.5.1. Bifurcation analysis and numerical continuation

The appearance of a topologically nonequivalent phase portrait, i.e. a structural change in the system behaviour, under a variation of parameters, e.g. a stable focus point (equilibrium) becomes an unstable focus, is called a bifurcation, [59, 60]. In the present cumulative thesis, occurring bifurcations are analysed to investigate structural changes of the system behaviour of autonomous ordinary differential equations, describing the proposed system models, under parameter variation to examine the stability behaviour.

The parametrised autonomous ODEs representing the system model read

$$\dot{\boldsymbol{x}}(t) = F(\boldsymbol{x},\ \boldsymbol{\lambda}), \tag{9}$$

with the state vector $\boldsymbol{x} \in \mathbb{R}^j$, the nonlinear function $F(\boldsymbol{x},\ \boldsymbol{\lambda}) \in \mathbb{R}^j$ describing the coupled system equations, with j the number of states depending on the considered system model, and the vector of parameters $\boldsymbol{\lambda} \in \mathbb{R}^k$.

Depending on the structural change, different types of bifurcations are distinguished. For example, a Fold bifurcation of stationary solutions (equilibria) occurs if a real eigenvalue of the Jacobian matrix of stationary solutions crosses the imaginary axis, and a Hopf bifurcation arises if a conjugate pair of complex eigenvalues crosses it. The Fold bifurcation typically leads to a change in the stability behaviour of the system under parameter perturbations, whether the Hopf bifurcation leads to the emergence of another type of solutions, namely periodic orbits or limit cycles if they are isolated (generically the case), [61]. For the Hopf bifurcation, we further distinguish between a supercritical or subcritical bifurcation. The supercritical Hopf bifurcation is called soft loss of stability or soft generation of limit cycles, as it describes the smooth change of a stable stationary branch into a stable periodic branch. In the subcritical case, a hard generation of limit cycles or a hard loss of stability is obtained, as an unstable periodic branch follows from the stationary branch, which turns back and gains stability. When the varied parameter is increased beyond the bifurcation value, the amplitude suddenly undergoes a jump and large periodic orbits occur, [58]. Besides the bifurcations of stationary solutions, a structural change can obviously occur for solutions of Eq. 9 with $\dot{\boldsymbol{x}} \neq 0$, during parameter variations. Common bifurcations of periodic orbits are, e.g. the Fold bifurcations of cycles, if a stable and unstable limit cycle collide and disappear at this bifurcation, [59], which can also be interpreted as a loss of stability of the periodic orbit when considering the branch of periodic orbits, [61]. The transition from stable to unstable periodic orbit signifies the presence of a second Floquet multiplier $\zeta_1 = 1$. Floquet multipliers are eigenvalues of the monodromy matrix $\boldsymbol{M}(T_0)$, derived by the fundamental matrix solution $\boldsymbol{M}(t)$ of Eq. 9 for constant parameters $\boldsymbol{\lambda}$, which is described by,

$$\dot{\boldsymbol{M}} = \boldsymbol{A}(t)\boldsymbol{M}, \tag{10}$$

with $\boldsymbol{M}(0) = \boldsymbol{I}$ the identity matrix as initial condition. There $\boldsymbol{A}(t) = \partial F/\partial \boldsymbol{x}(\boldsymbol{x}^0(t))$, describing the linear system governing the evolution of perturbations $\boldsymbol{z}(t) = \boldsymbol{x}(t) - \boldsymbol{x}^0(t)$ near the cycle L_0 of the nonlinear system $F(\boldsymbol{x}, \boldsymbol{\lambda})$ at a parameter set $\boldsymbol{\lambda}_0$ with $\boldsymbol{x}^0(t)$ denote the corresponding periodic solution $x^0(t + T_0) = x^0(t)$,

$$\dot{\boldsymbol{z}} = \boldsymbol{A}(t)\boldsymbol{z}. \tag{11}$$

Any solution $\boldsymbol{z}(t)$ to the Eq. 11 satisfies

$$\boldsymbol{z}(T_0) = \boldsymbol{M}(T_0)\boldsymbol{z}(0). \tag{12}$$

Naturally the stability of the cycle depends on the properties of the variational equation of the cycle Eq. 11, hence the eigenvalues of the monodromy matrix $\boldsymbol{M}(T_0)$, denoted $1, \zeta_1, \zeta_2, \ldots, \zeta_{j-1}$, are applied to evaluate the stability of cycles, [59].

A further bifurcation of periodic orbits is a Neimark-Sacker bifurcation occurring if a conjugate pair of complex Floquet multipliers exists and lies on the unit cycle, which gives birth to quasi-periodic orbits, i.e. the phase trajectory corresponds to a torus, [59].

To analyse bifurcations of a system described by Eq. 9, diagrams depicting a scalar measure $[\boldsymbol{x}]$ of the vector $\boldsymbol{x}$ as solution of Eq. 9 versus a varied parameter $\lambda_i \in \boldsymbol{\lambda}$ can be studied, which are called bifurcation diagrams, [60]. The continua of such solutions are called branches, and can be obtained by applying continuation methods to calculate the solution $\boldsymbol{x}(\lambda)$ starting at a solution $[\boldsymbol{x}](\lambda_i)$, which is obtained, e.g. by time integration at a fixed set of parameters until a stationary or a periodic solution of Eq. 9 is reached. Then, a parameter is freed and the branch is calculated based on numerical path continuation, typically utilising a predictor-corrector scheme. To investigate the nonlinear stability behaviour of the *MTB*, bifurcation diagrams are studied within *Paper C*, and *Paper E*, utilising the continuation toolbox MatCont, [62]. There, a tangent predictor step $\boldsymbol{X}_{n,0} = \boldsymbol{x}_n + \kappa \boldsymbol{v}_n$ with the first prediction $\boldsymbol{X}_{n,0}$ at $\boldsymbol{x}_n$ with the stepsize κ and the normalised tangent vector $\boldsymbol{v}_n$ is followed by a correction using a Newton-like iteration with the Moore-Penrose condition[1], is applied. A detailed description of the underlying equations to derive the solutions $\boldsymbol{x}_n$ depends on the studied solutions, i.e. equilibrium or periodic orbits, and is given in *Paper E*.

With the method described, bifurcation diagrams are observed for the minimal model and depicted in Fig. 10. In Subfigure (a), the equilibrium states of α, corresponding to a

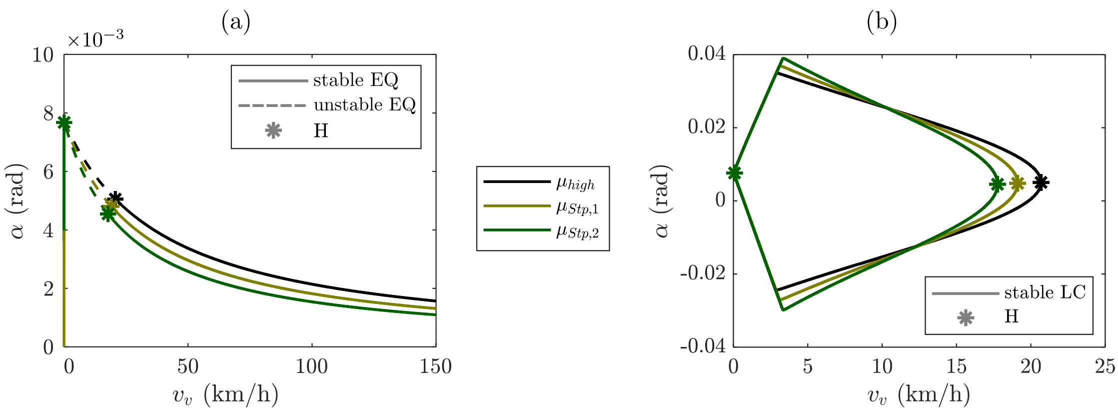


Figure 10: (a) Stable and unstable equilibrium states, (b) maxima and minima values during emerging periodic orbits for three different friction characteristics μ_{high}, $\mu_{Stp,1}$, $\mu_{Stp,2}$, *Paper C*.

quasi-steady braking manoeuvre from 150 km/h until standstill, are depicted for three different friction characteristics, which exhibit the same static friction coefficient and increasing negative gradients from μ_{high} to $\mu_{Stp,2}$, $\mathrm{d}\mu_{high}/\mathrm{d}v_v > \mathrm{d}\mu_{Stp,1}/\mathrm{d}v_v > \mathrm{d}\mu_{Stp,2}/\mathrm{d}v_v$.

[1]Modified pseudo-arc-length condition, aiming in the solution nearest to the predicted point $\boldsymbol{X}_{n,0}$, [59].

Two supercritical Hopf bifurcations, indicated with H, are revealed, corresponding to a soft loss of stability and unstable equilibrium states in between. When compared to the energy considerations, the Hopf-bifurcations coincide with the points where the energy input from the identified excitation mechanism exceeds the involved damping mechanism, corresponding to a loss of asymptotic stability afterwards, validating the energy considerations in Section 3.3. In Fig. 10(b), the solution branch of occurring limit cycles after the Hopf bifurcations is depicted with the mechanical displacement's enveloping max and min values. In contrast to pure friction-induced self-excited vibrations, where the unstable region is increased for steeper sliding friction gradients, the considered electromagnetic-mechanical coupled system described by the minimal model in Fig. 5 shows the opposite behaviour. The Hopf bifurcations are shifted to lower velocities for friction characteristics exhibiting an increasing negative gradient and decreasing magnitudes. The behaviour of the stable limit cycles in Fig. 10(b) is not qualitatively affected by the different friction characteristics. Still, the growth rate of emerging amplitudes increases with steeper curves of the friction coefficient, leading to the largest amplitudes for $\mu_{Stp,2}$.

3.5.2. Poincare sections: Interaction of the whole *MTB*-frame

The dynamic behaviour of occurring vibrations, i.e. identifying occurring limit cycles and related stability analysis, can be investigated by appropriate cross-sections Σ in the state space, so-called Poincaré sections. This cross-section Σ is a smooth hypersurface with dimension $j-1$, one less than the dimension of the state space $\boldsymbol{x} \in \mathbb{R}^j$, and is defined for the *MTB*-brake frame model, in *Paper D*:

$$\Sigma = \{\boldsymbol{x} = (\beta_1, \ldots, \beta_4, \dot{\beta}_1, \ldots, \dot{\beta}_4, \phi_1, \ldots, \phi_4)^T \in \mathbb{R}^{12} \, | \beta_3 = 0; \dot{\beta}_3 > 0\}. \tag{13}$$

When a periodic orbit L_0 is considered, it starts at the point $\boldsymbol{x}_0$ on Σ and returns to Σ at the same point $\boldsymbol{x}_0(t) = \boldsymbol{x}_0(t + T_0)$, hence periodic motions corresponds to fixed points at Poincaré Sections. Moreover, nearby orbits starting at a point $\boldsymbol{x} \in \Sigma$ close to $\boldsymbol{x}_0$, will return at some point $\tilde{\boldsymbol{x}} \in \Sigma$ near $\boldsymbol{x}_0$ and intersect Σ transversally. Thus, the Poincaré map is constructed

$$\boldsymbol{x} \to \tilde{\boldsymbol{x}} = P(\boldsymbol{x}). \tag{14}$$

Further, the stability of the cycle L_0 is equivalent to the stability of the fixed point of the Poincaré map, [59]. Hence, the cycle is stable if all eigenvalues of the Jacobian of the Poincaré map are located inside the unit circle.

The intersections with the defined Poincaré Section Σ detected during vibrations of the considered system can be applied to identify the type of steady-state motions, [60]. While the periodic orbit corresponds to a fixed point, a quasi-periodic motion corresponds to an invariant closed curve as depicted for the coupled full *MTB*-brake frame model, Fig. 6, at a vehicle velocity of $v_v = 2$ m/s for the generalised coordinate β_1 in Fig. 11. When the vehicle velocity increases, the Poincaré section shrinks to a few discrete points, describing periodic vibrations, and indicates a Neimark-Sacker bifurcation.

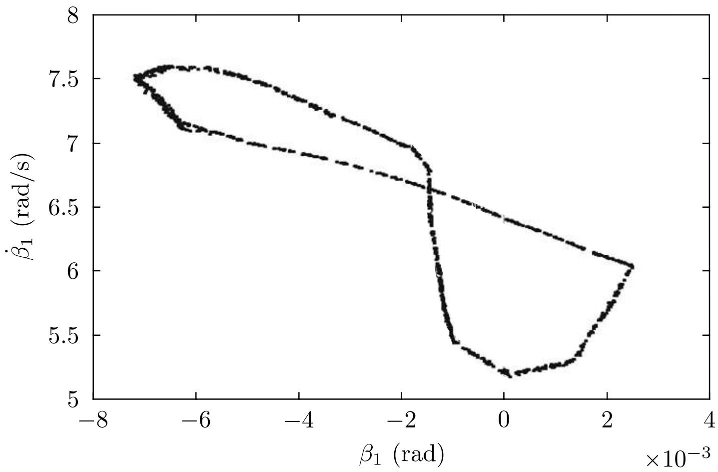


Figure 11: Poincaré section of quasi-periodic oscillations at a vehicle velocity of $v_v = 2$ m/s in $(\beta_1, \dot{\beta}_1)$, *Paper D*.

3.5.3. Continuation of bifurcations in two parameters

The above methods observe and investigate emerging bifurcations and full solution branches for varying system parameters. However, it is of further interest how the revealed bifurcations, obtained by numerical continuation of equilibria and periodic orbits, change when parameters characterising the frictional contact, motion-induced eddy currents or the mechanical design are varied. For this reason, continuation of bifurcation in two parameters is applied, which in the case of the Hopf-bifurcation results in the stability border of equilibria and in the case of the Fold bifurcation of limit cycles in the loss of stability of the related periodic solutions.

For these investigations, the corner *MTB* model is utilised to consider the contact conditions of a mainline magnetic track brake with detached contact points of several brake elements and the weakening of the magnetic field due to motion-induced eddy currents. Further, the component of the friction force orthogonal to the forward motion is considered, adding an additional damping mechanism, and the lateral and vertical clearance to the structure is regarded by decoupled intermediate elements.

Fig. 12 depicts the solution space of the post-critical behaviour with freed vehicle velocity v_v, the velocity-dependent friction parameter δ_μ, and the period T at the periodic solutions. This plot characterises the post-critical behaviour for different shapes of the friction characteristic, while the static friction coefficient remains constant. Subfigure (a) presents a 3D plot, (b) the related front, and (c) the top view of several bifurcation diagrams with the maximum mechanical displacement $\max(\alpha)$ as the scalar measure for varied velocities v_v at selected and constant values of δ_μ, highlighting the continuation of occurring cycles in grey. The red line represents the continuation of Hopf bifurcations, $\mathrm{H}(\delta_\mu, v_v)$, encasing parameter regions $\boldsymbol{\lambda}$ associated with unstable equilibrium states. This branch reveals a minimal necessary velocity-dependent friction parameter δ_{μ,H^*} for Hopf-bifurcations to be present, and for values $\delta_\mu > \delta_{\mu *}$, periodic orbits follow from the shown Hopf curve depicted in red. When δ_μ is increased further, occurring amplitudes

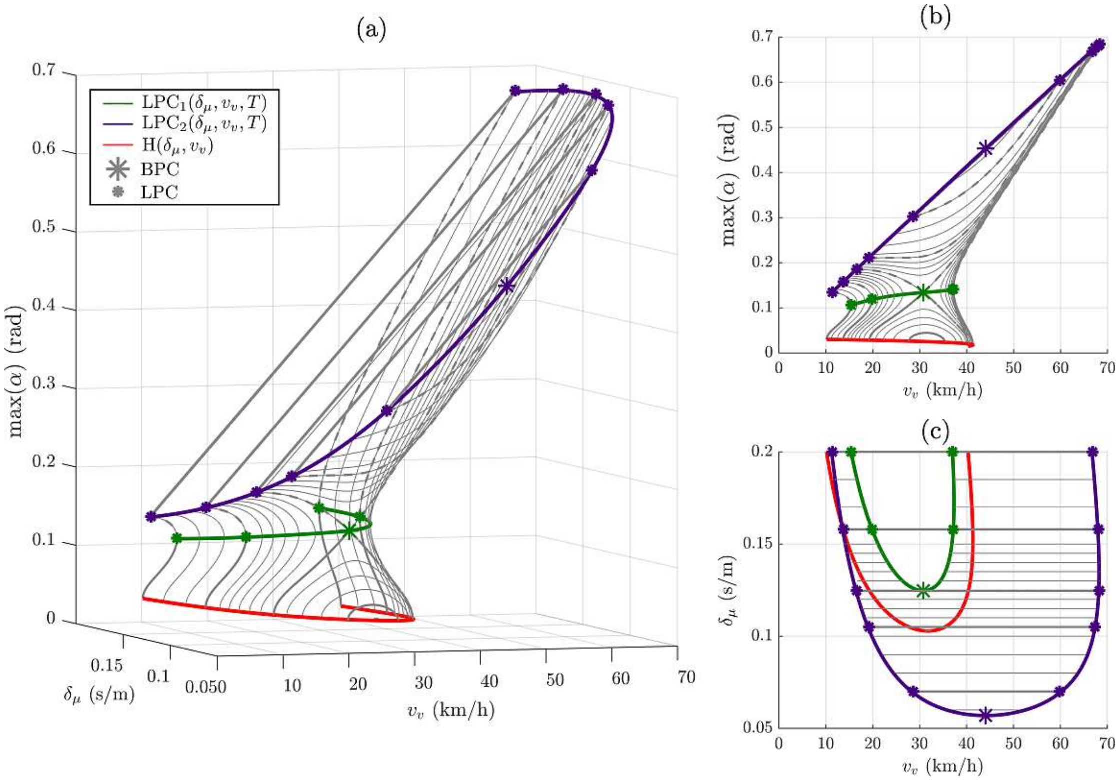


Figure 12: Full solution space of occurring periodic orbits by variation of δ_μ and v_v, including Hopf and both LPC branches, *Paper E*.

rise, and at a specific value δ_{μ,BPC_1}, a transcritical bifurcation of limit cycles, indicating a branch point (BPC) is reached. This value describe a seperation of a no-jump situation for $\delta_\mu < \delta_{\mu,BPC_1}$ from a two-jump situation for $\delta_\mu > \delta_{\mu,BPC_1}$, when considering the bifurcation diagrams and is called a gap center in a two-parameter setting, [58]. Hence, unstable periodic orbits are created at the related continuation of Fold bifurcations of limit cycles, LPC$_1(\delta_\mu, v_vT)$, depicted in green. When the unstable limit cycles are followed, proceeding from LPC$_1(\delta_\mu, v_vT)$, further Fold bifurcations are revealed, indicating an additional branch of stable limit cycles. When those Fold bifurcations are continued, a curve LPC$_2(\delta_\mu, v_v, T)$, shown in purple, which delineates a region of stable periodic solutions which exhibit stick-slip motions due to large amplitudes, corresponding to the second branch of stable periodic solutions, is found. The related branch point of this two-parameter continuation, with δ_{μ,BPC_2}, indicates an isola formation, with stable limit cycles coexisting with stable equilibriums for $\delta_{\mu,H^*} > \delta_\mu > \delta_{\mu,BPC_2}$, with unstable limit cycles describing the region of attraction. For values $\delta_{\mu,BPC_1} > \delta_\mu > \delta_{\mu,H^*}$ stable limit cycles coexist, and at the branch point with δ_{μ,BPC_1} stable and unstable limit cycles attach and annihilate each other for $\delta_\mu > \delta_{\mu,BPC_1}$, and the described two-jump situation occur. With this method of studying two-parameter continuation, different parameters in the system can be examined by considering the post-critical behaviour, which is described in detail in *Paper E*.

4. Summary of the scientific papers

Paper A

Investigations of Degraded Adhesion Conditions and Interrelated Methods for Improving Braking Performance using the Advanced TrainLab (aTL)

Marcus Fischer, Thomas Rasel, Bernhard Ebner, Felix Kröger, Sebastian Heinz

Proceedings of EuroBrake2023
https://doi.org/10.46720/EB2023-BSY-020

In this first publication, the influence of contaminants and friction enhancers on the *MTB*–rail sliding contact is investigated experimentally. Multiple series of brake tests with different initial velocities, up to 120 km/h, were carried out on different conditions: dry conditions, without detected spin-downs, low adhesion conditions, with a water/soap solution on the rail, and extremely low adhesion conditions, created by applying paper tape combined with pure water or plant-based oil to the rail head. The evaluated quasi-static braking manoeuvre showed that the sliding coefficient of friction in the *MTB*–rail contact does not vary significantly between dry and low adhesion conditions, both qualitatively and quantitatively. When sand is applied to the wheel–rail contact of the wheelset in front of the considered *MTB*, a significant increase in the averaged brake force is detected, and the velocity dependency is reduced. With oil applied to the rail, the qualitative behaviour of the coefficient of friction stays unaffected; however, the quantitative values are decreased significantly in this case. Things change with applied paper tape, as the measured brake force decreases with decreasing velocities in the first stage, reaching a minimum, and is increased afterwards. This behaviour is explained by gathering paper tape by the *MTB* first, and at a specific velocity, the velocity-dependent friction coefficient becomes dominant, leading to an increase in the brake force afterwards, which is not observed in experiments with real leaves on the rail. With these results, it is reasonable to consider a monotonous decay of the sliding friction coefficient with increasing velocity during the dynamic and stability behaviour of the *MTB*, *Paper B*-*Paper E*; however, the influence of changing gradients and the level of the friction characteristic should be examined which is changed significantly during the operation of an *MTB*.

Bernhard Ebner is responsible for: planning and implementing the measurement and testing concept of the magnetic track brake, monitoring *MTB* during test runs, evaluation and plausibility check of measurement data regarding *MTB*, and writing, reviewing & editing the article.

Paper B

Active mitigation of self-excited vibrations of a magnetic track brake

Bernhard Ebner, Daniel Tippelt, Johannes Edelmann, Manfred Plöchl — Journal of Physics: Conference Series
https://doi.org/10.1088/1742-6596/2647/15/152007

The behaviour of self-excited vibrations occurring in *MTB*s during braking manoeuvres at low velocities by depicting measurements and analysing the feasibility of an active vibration control is addressed in this publication. Measurements present the excited eigenmode and eigenfrequency of the vibrations with the loss of stability at a certain velocity, where amplitudes rise rapidly and reach a respective limit cycle. Utilising a linearised minimal *MTB*-model at a constant vehicle velocity, the effective energy input in the mechanical subsystem by the electromagnetic coupling and the frictional contact is analysed numerically. A positive energy input at the considered operation point and a phase shift between the magnetic flux and mechanical oscillations are identified. While previous work has focused mainly on passive countermeasures, this study introduces an active vibration control strategy based on a modulation of the electric input voltage of the *MTB*. A state feedback control is utilised to force the identified phase shift to a specific value, ensuring a negative effective energy over an oscillation period at the linearised *MTB* model. The developed control is applied to the nonlinear quarter *MTB* model, starting at a respective limit cycle of the considered operation point with constant vehicle velocity. Occurring amplitudes are effectively reduced, and self-excited vibrations were suppressed completely for a defined friction characteristic, illustrating that active vibration control with the available electromagnetic actuator is feasible.

Bernhard Ebner is responsible for: conceptualisation, methodology, derivation of the model equations, visualisation and presentation of the experimental and theoretical results, numerical calculations, and writing - the original draft.

Paper C

System analysis and active vibration control of a simplified electromagnetic track brake model

Bernhard Ebner, Johannes Edelmann, Manfred Plöchl — Journal of Sound and Vibration
https://doi.org/10.1016/j.jsv.2025.119307

In this third paper, a more detailed investigation of the energy input leading to the onset of self-excited vibrations at *MTB*s, the behaviour after the loss of stability, and active vibration control strategies are presented. The minimal model from *Paper B* is utilised in parametrised form to analyse the system behaviour with changing vehicle velocities and friction conditions, based on *Paper A*, extending the previous investigations focusing on a chosen operating point. An analytical expression for the effective energy input is derived in this paper, based on the linearised system with respect to the quasi-steady states, and the assumption of a harmonic behaviour, reasonable near the onset of vibrations. With this expression, two main self-excitation mechanisms are identified: (1) the well-known falling friction mechanism, based on the negative gradient of the velocity-dependent friction coefficient. (2) the electromagnetic coupling mechanism, which leads to a phase-shifted alternating normal force with a positive energy input into the oscillating system, even for a constant sliding coefficient of friction. With the analytical expression of the energy input, mitigation strategies can be found, and two active control strategies based on a modulation of the electric voltage are proposed. First, a state feedback control to keep a constant normal force, which suppresses the second self-excitation mechanism and requires only the mechanical displacement for the feedback loop. Second, similar to the approach in *Paper B*, a control forcing the magnetic flux to be in antiphase with the mechanical oscillation, which then actively dissipates energy and corresponds to an optimal control strategy with respect to the energy input. Nonlinear stability analysis of the closed-loop system revealed that only the second control strategy is able to prevent a loss of stability over the whole velocity region when the friction characteristic becomes very steep. The minimal necessary controller gain to keep stable operation over the whole velocity range is derived through the numerical continuation of occurring Hopf bifurcations of the closed-loop system. The post-critical behaviour of the electromagnetic-mechanical coupled system is analysed by a continuation of periodic orbits emerging at the Hopf bifurcation and shows a significant reduction of the braking performance of the *MTB* if vibrations can not be avoided. Additionally, it is shown that harmonic mechanical oscillations change to stick-slip vibrations at very low velocities, where the relative velocity at the contact point becomes zero during the stick phases.

Bernhard Ebner is responsible for: conceptualisation, methodology, derivation of the model equations, formal analysis, numerical calculations, investigation, interpretation and visualisation of the results, and writing - the original draft.

Paper D

Self-sustained oscillations of a magnetic track brake frame

Konstantin Avramov, Bernhard Ebner, Johannes Edelmann, Yuri V. Mikhlin, Borys Uspensky

Nonlinear Dynamics
https://doi.org/10.1007/s11071-024-10643-6

The dynamic behaviour of the full *MTB* during self-excited vibrations at low velocities is investigated in this fourth publication. For this purpose, a novel nonlinear mathematical model of the *MTB* is presented, integrating both the mechanical and the electromagnetic interaction of the individual corner electromagnets into an in-plane approximation of the full *MTB*-frame with four mechanical and four electromagnetic degrees of freedom. The mechanical subsystem accounts for the interaction through the flexible brake frame along with dry friction between *MTB* and the rail. The structure is modelled by four rigid corner electromagnets, each having one fictitious contact point, which are coupled based on influence coefficients determined through finite element analysis. The electromagnetic subsystem is modelled by a lumped-parameter approach that captures the oscillatory magnetic fluxes induced by the mechanical displacements and the interaction through the shared electric circuit of the electric power supply. The coupled *MTB*-brake frame model is utilised for numerical analysis of the equilibrium states, periodic oscillations, and bifurcations for a range of vehicle velocities and several initial conditions in the case of oscillatory solutions. For the undamped case without a component of friction lateral to the direction of motion, unstable equilibrium states are present throughout the whole velocity range. Two distinct types of stable self-excited vibrations with different oscillation frequencies and shapes emerge: one with large amplitudes performing an asymmetrical motion of the brake frame and one with small amplitudes with a symmetrical motion. Stickslip motions arise from dry friction, especially at low velocities, where sections of constant generalised velocities are observed. At very low velocities, a NeimarkSacker bifurcation occurs, leading to quasiperiodic vibrations underneath. As velocity increases, multiharmonic oscillations transition into simpler single-harmonic vibrations. The study highlights the complex interaction between dry friction and electromagnetic coupling in the case of a full *MTB* with interactions especially important at the lowest velocities.

Bernhard Ebner is responsible for: derivation of the electromagnetic model equations, providing the finite element model, discussion on numerical results, visualisation and presentation of the system model, and writing parts of the original draft, review & and editing.

Paper E

Stability behaviour of a basic magnetic track brake model: Influences of system parameters and motion-induced eddy currents

Bernhard Ebner, Manfred Plöchl, Johannes Edelmann — Nonlinear Dynamics
https://doi.org/10.1007/s11071-025-11574-6

In this final publication, building upon the prior publications *Paper A*-*Paper E*, a nonlinear electromagneticmechanical model of a quarter *MTB* is utilised to investigate the onset and evolution of self-excited vibrations, to explain still open questions from experiments and to suggest improvements. The model incorporates experimentally observed velocity dependencies of the friction coefficient from *Paper A*, the newly found velocity dependency of the attraction force, which is presented in this paper, and considers segmented brake elements with individual frictional contacts. Although it was presented in *Paper D*, that the interaction through the structure can lead to quasi-periodic oscillation at the lowest velocities, basic mechanisms, including damping, are studied on a quarter model here, enabling a clear explanation of coherences and practical improvements. The method of energy considerations from *Paper B* and *Paper C* are further utilised to explain the dynamic behaviour. The velocity-dependent attraction forces arise from motion-induced eddy currents and are incorporated phenomenologically by an analytical approximation of measurements. By utilising bifurcation analysis and numerical continuation of limit cycles and bifurcations, the study explores the nonlinear stability behaviour and occurring oscillation patterns of the system across varying system parameters and environmental conditions. It is obtained that specific friction levels and negative gradients of the friction and attraction force characteristics are necessary to reach unstable equilibria, explaining that severe vibrations were only measurable in 1 out of 10 experiments. A second branch of stable limit cycles is revealed and emerges as an isola formation coexisting with stable equilibrium states. With critical parameters being increased, unstable equilibria are reached, emerging between two super-critical Hopf bifurcations, which are connected by stable limit cycles with a distinct maximum of vibration amplitudes. This behaviour also matches well with observations from experiments, where amplitudes of measured braking forces increase in the first stage of severe vibrations, while they are decreasing after a specific velocity. When, for example, the friction characteristic is steepened further, stable limit cycles emerging at the Hopf bifurcations attach with unstable limit cycles of the isola and annihilate each other, forming a transcritical bifurcation. When exceeding this branch point, two-jump situations from stable limit cycles with small amplitudes to limit cycles with large amplitudes and stick-slip motions and back are the consequence. Hence, involved damping mechanisms shall be employed to prevent unstable equilibria, and a velocity region with a two-jump situation must be avoided, which is suggested by changing design parameters, which can reduce excitation mechanisms or increase damping when appropriately tuned. Further, it is proposed that avoiding weld formations at the endpieces reduces the risk of oscillations as well.

Bernhard Ebner is responsible for: conceptualisation, methodology, planning and execution of experiments regarding motion-induced eddy currents, derivation of the model equations, numerical analysis, interpretation and visualisation of experimental and numerical results, and writing - the original draft.

5. Scientific impact

This cumulative thesis enhances the understanding of self-excited vibrations in magnetic track brakes (*MTB*s) and provides effective strategies for their mitigation. The investigations demonstrate that the *MTB* is prone to self-excited vibrations at low velocities with rapidly growing amplitudes, which subsequently decay after reaching a global maximum. These behaviours are explained by distinct excitation mechanisms interacting with dissipation mechanisms that had not previously been classified in this context.

Three fundamental excitation mechanisms are identified. The first is the well-known falling-friction effect, analogous to classical friction oscillators. The second is an electromagnetic coupling mechanism, arising from the dynamics of the electromagnetic system, leading to a positive energy input even for constant friction coefficients, due to a varying and phase-shifted normal force. The third is a motional eddy current mechanism, where velocity-dependent normal forces act analogously to the friction mechanism, but originate from the back-coupling of motion-induced eddy currents in the rail. Their interaction with unavoidable damping, such as structural damping, self-centring electromagnetic forces, and lateral friction, explains both the onset and the amplitude evolution of self-excited vibrations at *MTB*s. The analysis demonstrates that the friction characteristic, through its gradient and magnitude, determines which mechanism dominates, offering an explanation for the pronounced sensitivity of stability to environmental and operational conditions.

The nonlinear dynamics of the *MTB* are further characterised by a range of bifurcation phenomena, including Hopf, Fold, and NeimarkSacker bifurcations, as well as the coexistence of periodic attractors and stickslip oscillations. These findings clarify how stability is lost and regained, and how multiple vibration regimes may exist for the same operating conditions. The significance of segmented brake elements and distributed *MTB*–rail contact points on the stability behaviour is highlighted, revealing both their potential to reduce and their risk to promote large-amplitude oscillations and degraded braking forces. The dual impact of increased structural loads and compromised braking performance underscores the safety relevance of the identified mechanisms.

Beyond explaining the vibrational behaviour and providing a correlation between design parameters and occurring vibrations, the thesis introduces novel active control strategies that exploit the electromagnet itself as an actuator. By modulating the electrical input, the self-excitation mechanism based on the electromagnetic coupling can be suppressed or inverted, enabling active energy dissipation. These simple, yet effective strategies represent a step beyond passive countermeasures and demonstrate the feasibility of embedding active vibration control directly into the brake design.

References

[1] D. Tippelt. Self-excited vibrations of magnetic track brakes: Modelling, analysis and mitigation. PhD thesis. TU Wien, 2022. DOI: 10.34726/hss.2022.33382.

[2] AT11554B. Patent of: The Westinghouse brake company limited. Elektromagnetische Bremse für Eisenbahnfahrzeuge mit mehreren, über den Fahrschienen angeordneten Elektromagneten. https://worldwide.espacenet.com/publicationDetails/biblio?locale=de_EP&CC=AT&NR=11554B. 1903.

[3] G. Gfatter, S. Haas and G. Vohla. Schienenbremsen Track Brakes. Knorr-Bremse GmbH. Vol. 1st. Mödling: Knorr-Bremse GmbH, 2004.

[4] W. Hendrichs. Das statische, dynamische und thermische Verhalten von Magnetschienenbremsen, Teil 1. *Elektrische Bahnen* 86.Heft 7 (1988), 224–228.

[5] W. Hendrichs. Das statische, dynamische und thermische Verhalten von Magnetschienenbremsen, Teil 2. *Elektrische Bahnen* 86.Heft 10 (1988), 324–330.

[6] W. Hendrichs. Das statische, dynamische und thermische Verhalten von Magnetschienenbremsen, Teil 3. *Elektrische Bahnen* 86.Heft 11 (1988), 357–360.

[7] O. Arias-Cuevas and Z. Li. Field investigations into the performance of magnetic track brakes of an electrical multiple unit against slippery tracks. Part 1: Adhesion improvement. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 225.6 (2011), 613–636. DOI: 10.1177/0954409711401515.

[8] O. Arias-Cuevas and Z. Li. Field investigations into the performance of magnetic track brakes of an electrical multiple unit against slippery tracks. Part 2: Braking force and side effects. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 226.1 (2012), 72–94. DOI: 10.1177/0954409711408375.

[9] C. Cruceanu and C. Crciun. Necessity and conditionality regarding the electromagnetic track brake-parametric study. *MATEC Web of Conferences*. Vol. 178. EDP Sciences. 2018, 06020. DOI: 10.1051/matecconf/201817806020.

[10] R. Rathammer. Mehrkörpersimulation einer Magnetschienenbremse in Tiefaufhängung. MA thesis. TU Wien, 2000. DOI: 20.500.12708/182348.

[11] D. Tippelt. Dynamische Simulation einer Magnetschienenbremse in Hochlage. MA thesis. TU Wien, 2011. DOI: 20.500.12708/14801.

[12] E. Galardi et al. Development of efficient models of Magnetic Braking Systems of railway vehicles. *International Journal of Rail Transportation* 3.2 (2015), 97–118. DOI: 10.1080/23248378.2015.1015219.

[13] M. Jirout, P. Lugner and W. Mack. Dynamic simulation of the contact forces in the transmission link of a magnetic track-brake in low suspension. *10th Mini conference on vehicle system dynamics, identification and anomalies*. Vehicle system dynamics, identification and anomalies (VSDIA). 2008, 199–206. DOI: 20.500.12708/65813.

[14] M. Jirout, W. Mack and P. Lugner. Non-smooth dynamics of a magnetic track brake. *Regular and Chaotic Dynamics* 14.6 (2009), 673–681. DOI: 10.1134/S1560354709060057.

[15] M. Jirout. Mechanische Auslegung von Magnetschienenbremsen. PhD thesis. Technische Universität Wien, 2006. DOI: 20.500.12708/185498.

[16] D. Tippelt et al. Analysis of self-excited vibrations of an electromagnetic track brake. *The IAVSD International Symposium on Dynamics of Vehicles on Roads and Tracks*. Springer. 2019, 442–451. DOI: 10.1007/978-3-030-38077-9_52.

[17] D. Tippelt et al. Modelling, analysis and mitigation of self-excited vibrations of a magnetic track brake. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 236.6 (2021), 684–694. DOI: 10.1177/0954409721103.

[18] P. C. Müller and W. Schiehlen. Linear vibrations. A theoretical treatment of multi-degree-of-freedom vibrating systems. Vol. 7. Springer Science & Business Media, 2012, 327. DOI: 10.1007/978-94-009-5047-4.

[19] K. Popp. Modelling and control of friction-induced vibrations. *Mathematical and Computer Modelling of Dynamical Systems* 11.3 (2005), 345–369. DOI: 10.1080/13873950500076131.

[20] J. J. Thomsen. Using fast vibrations to quench friction-induced oscillations. *Journal of sound and vibration* 228.5 (1999), 1079–1102. DOI: 10.1006/jsvi.1999.2460.

[21] M. Kröger, M. Neubauer and K. Popp. Experimental investigation on the avoidance of self-excited vibrations. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 366.1866 (2008), 785–810. DOI: 10.1098/rsta.2007.2127.

[22] G. Spelsberg-Korspeter. Robust structural design against self-excited vibrations. Springer, 2013. DOI: 10.1007/978-3-642-36552-2.

[23] K. Popp and M. Rudolph. Vibration control to avoid stick-slip motion. *Journal of Vibration and Control* 10.11 (2004), 1585–1600. DOI: 10.1177/1077546304042.

[24] D. Zhou, C. Hansen and J. Li. Suppression of maglev vehicle–girder self-excited vibration using a virtual tuned mass damper. *Journal of sound and Vibration* 330.5 (2011), 883–901. DOI: 10.1016/j.jsv.2010.09.018.

[25] J. Li et al. The active control of maglev stationary self-excited vibration with a virtual energy harvester. *IEEE Transactions on Industrial Electronics* 62.5 (2014), 2942–2951. DOI: 10.1109/TIE.2014.2364788.

[26] J.-h. Li et al. Maglev self-excited vibration suppression with a virtual sky-hooked damper. *Journal of Central South University* 23.6 (2016), 1363–1371. DOI: 10.1007/s11771-016-3188-8.

[27] W. Ding. Self-excited vibration. Theory, Paradigms, and Research Methods. Springer Berlin, Heidelberg, 2009. DOI: 10.1007/978-3-540-69741-1.

[28] A. Jenkins. Self-oscillation. *Physics Reports* 525.2 (2013), 167–222. DOI: 10.1016/j.physrep.2012.10.007.

[29] G. S. Chen. Handbook of friction-vibration interactions. Elsevier, 2014.

[30] U. von Wagner, D. Hochlenert and P. Hagedorn. Minimal models for disk brake squeal. *Journal of Sound and Vibration* 302.3 (2007), 527–539. DOI: https://doi.org/10.1016/j.jsv.2006.11.023.

[31] J. Kang and C. M. Krousgrill. The onset of friction-induced vibration and spragging. *Journal of Sound and Vibration* 329.17 (2010), 3537–3549. DOI: 10.1016/j.jsv.2010.03.002.

[32] N. Hoffmann. Transient growth and stick-slip in sliding friction. *Transaction of ASME* 76 (2006), 642–647. DOI: 10.1115/1.2165233.

[33] H. Hetzler. On the effect of nonsmooth Coulomb friction on Hopf bifurcations in a 1-DoF oscillator with self-excitation due to negative damping. *Nonlinear Dynamics* 69.1 (2012), 601–614. DOI: 10.1007/s11071-011-0290-1.

[34] N. Hoffmann et al. A minimal model for studying properties of the mode-coupling type instability in friction induced oscillations. *Mechanics research communications* 29.4 (2002), 197–205. DOI: 10.1016/S0093-6413(02)00254-9.

[35] J. P. Den Hartog. Mechanical vibrations. Vol. 4. Dover Publications, New York, 1985.

[36] T. Butlin and J. Woodhouse. Friction-induced vibration: Quantifying sensitivity and uncertainty. *Journal of Sound and Vibration* 329.5 (2010), 509–526. DOI: 10.1016/j.jsv.2009.09.026.

[37] V. Babitsky and P. Landa. Auto-oscillation Systems with Inertial Self-excitation. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik* 64.8 (1984), 329–339. DOI: 10.1002/zamm.19840640803.

[38] P. S. Landa. Nonlinear oscillations and waves in dynamical systems. Vol. 360. Springer Science & Business Media, 1996. DOI: 10.1007/978-94-015-8763-1.

[39] K. Magnus, K. Popp and W. Sextro. Schwingungen. Eine Einführung in physikalische Grundlagen und die theoretische Behandlung von Schwingungsproblemen. Vieweg+Teubner Verlag Wiesbaden, 2013. DOI: 10.1007/978-3-8351-9227-0.

[40] C. Hansen et al. Active Control of Noise and Vibration Second Edition Volume 2. CRC press, 2013. DOI: 10.1201/b15923.

[41] G. Liu et al. Multiphysics analysis of a hybrid suspension system for middle-low-speed maglev trains. *The European Physical Journal Applied Physics,* 90.1 (2020), 8. DOI: 10.1051/epjap/2020200015.

[42] B. Buth and B. Lu. Dynamic analysis of vehicle-guideway interaction in a maglev cargo transportation system. *ASME International Mechanical Engineering Congress and Exposition*. Vol. 11. American Society of Mechanical Engineers. 2012, 1–8. DOI: 10.1115/IMECE2012-85552.

[43] Z. Li, Q. Cao and Z. Nie. Stick-slip vibrations of a self-excited SD oscillator with Coulomb friction. *Nonlinear Dynamics* 102.3 (2020), 1419–1435. DOI: 10.1007/s11071-020-06009-3.

[44] Y. Zhu et al. The effect of dynamic normal force on the stick–slip vibration characteristics. *Nonlinear Dynamics* 110.1 (2022), 69–93. DOI: 10.1007/s11071-022-07614-0.

[45] H.-I. Won and J. Chung. Stick–slip vibration of an oscillator with damping. *Nonlinear Dynamics* 86.1 (2016), 257–267. DOI: 10.1007/s11071-016-2887-x.

[46] R. Lima and R. Sampaio. Stick–slip oscillations in a multiphysics system. *Nonlinear Dynamics* 100.3 (2020), 2215–2224. DOI: 10.1007/s11071-020-05677-5.

[47] J. Kang, C. M. Krousgrill and F. Sadeghi. Oscillation pattern of stick–slip vibrations. *International Journal of Non-Linear Mechanics* 44.7 (2009), 820–828. DOI: 10.1016/j.ijnonlinmec.2009.05.002.

[48] D. Wei et al. Properties of stability, bifurcation, and chaos of the tangential motion disk brake. *Journal of Sound and Vibration* 375 (2016), 353–365. DOI: 10.1016/j.jsv.2016.04.022.

[49] D. Wei et al. Analysis of the stick-slip vibration of a new brake pad with double-layer structure in automobile brake system. *Mechanical Systems and Signal Processing* 118 (2019), 305–316. DOI: 10.1016/j.ymssp.2018.08.055.

[50] R. A. Ibrahim. Friction-induced vibration, chatter, squeal, and chaospart II: dynamics and modeling. *Appl. Mech. Rev.* (1994). DOI: 10.1115/1.3111080.

[51] K. Popp. Non-smooth mechanical systemsan overview. *Forschung im Ingenieurwesen* 64.9 (1998), 223–229. DOI: 10.1007/PL00010860.

[52] R. Darula and S. Sorokin. Simplifications in modelling of dynamical response of coupled electro-mechanical system. *Journal of Sound and Vibration* 385 (2016), 402–414. DOI: 10.1016/j.jsv.2016.08.036.

[53] E. Kallenbach et al. Elektromagnete. Vol. 5. Springer, 2018, 438. DOI: 10.1007/978-3-658-14788-4.

[54] J. H. Argyris, S. Kelsey et al. Energy theorems and structural analysis. Vol. 60. Springer, 1960, 81. DOI: 10.1007/978-1-4899-5850-1.

[55] J. Wang and J. Zhu. A Simple Method for Performance Prediction of Permanent Magnet Eddy Current Couplings Using a New Magnetic Equivalent Circuit Model. *IEEE Transactions on Industrial Electronics* 65.3 (2018), 2487–2495. DOI: 10.1109/TIE.2017.2739704.

[56] H. Gholizad, B. Funieru and A. Binder. Direct Modeling of Motional Eddy Currents in Highly Saturated Solid Conductors by the Magnetic Equivalent Circuit Method. *IEEE Transactions on Magnetics* 45.3 (2009), 1016–1019. DOI: 10.1109/TMAG.2009.2012546.

[57] J. Perho. Reluctance network for analysing induction machines. PhD thesis. Helsinki University of Technology, 2002.

[58] R. Seydel. Practical bifurcation and stability analysis. Vol. 5. Springer Science & Business Media, 2009. DOI: 10.1007/978-1-4419-1740-9.

[59] Y. A. Kuznetsov. Elements of applied bifurcation theory. Vol. 4. Springer, 1998, 703. DOI: 10.1007/978-3-031-22007-4.

[60] J. Guckenheimer and P. Holmes. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Vol. 42. Springer Science & Business Media, 2013. DOI: 10.1007/978-1-4612-1140-2.

[61] W. Govaerts. Numerical bifurcation analysis for ODEs. *Journal of computational and applied mathematics* 125.1-2 (2000), 57–68. DOI: 10.1016/S0377-0427(00)00458-1.

[62] W. Govaerts et al. MATCONT: Continuation toolbox for ODEs in Matlab. Utrecht University, 2019.

Paper A

Investigations of Degraded Adhesion Conditions and Interrelated Methods for Improving Braking Performance using the Advanced TrainLab (aTL)

Marcus Fischer, Thomas Rasel, Bernhard Ebner, Felix Kröger, Sebastian Heinz
EuroBrake 2023, Conference Proceedings, Art.Nr. BSY-020
https://doi.org/10.46720/EB2023-BSY-020

Keywords: degraded adhesion, vehicle test runs, wheel slide protection, adhesion management, magnetic track brake

Abstract: The article describes the results of on-train tests performed by Knorr-Bremse and DB Systemtechnik within the Shift2Rail PIVOT2 technology initiative under the HORIZON 2020 European Research Framework Programme. The tests aimed first, to carry out an in-depth investigation of degraded wheel/rail adhesion conditions, and second, to validate the effectiveness of technical solutions to optimally master such conditions. The solutions are especially important for later use with ATO (Automatic Train Operation) at Grade of Automation (GoA) level 3 and 4, meaning driverless/ unattended train operation, to avoid restrictions that would otherwise reduce rail system performance. Performed in spring 2022 aboard the DB advanced TrainLab (aTL), the tests concentrated on two solutions: WheelGrip adapt (a WSP function), and a train-wide Adhesion Management (ADM) function using sanding systems. The tests demonstrated the reliability of the adaptive response of the new WSP towards different adhesion conditions maintaining the same braking performance in regular low adhesion conditions (UIC) and improved braking performance in extremely low adhesion conditions. The advanced train-wide ADM was able to achieve even shorter stopping distances. With the support of the Vienna University of Technology (TU Wien), the study also investigated the behavior of magnetic track brakes (MTBs) and their conditioning effects, concluding that MTBs not only provide additional braking force, but also improve the performance of other brakes acting on downstream wheelsets. However, the effect differs depending on the third-body layer, achieving better results on paper-based versus oil-based contamination. The collected data will be used by Knorr-Bremse to design further solutions, and by DB Systemtechnik to produce a mathematical model to subsequently configure a WSP test rig for performing enhanced certification tests.

References of Paper A

[1] RAIB et al. Autumn adhesion investigation Part 3: Review of adhesion-related incidents Autumn 2005. *Report 25 (part3)* (2007).

[2] P. Hubbard, G. Amarantidis and C. Ward. Leaves on the line: low adhesion detection in railways. *IFAC-PapersOnLine* 49.21 (2016), 467–472.

[3] K. Büker, M. Fischer and J. Gräber. Verbesserung von Kapazität und Betriebsqualität durch reduzierte Streuung der Bremswege. *ETR* Ausgabe 11 (2022).

[4] O. Arias-Cuevas and Z. Li. Field investigations into the performance of magnetic track brakes of an electrical multiple unit against slippery tracks. Part 1: Adhesion improvement. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 225.6 (2011), 613–636. DOI: 10.1177/0954409711401515.

[5] O. Arias-Cuevas and Z. Li. Field investigations into the performance of magnetic track brakes of an electrical multiple unit against slippery tracks. Part 2: Braking force and side effects. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 226.1 (2012), 72–94. DOI: 10.1177/0954409711408375.

[6] M. Fischer et al. Mehr Mobilität auf der Schiene: Erhöhung der Transportkapazität Durch Optimierung des Kraftschlusses. 2020.

Paper B

Active mitigation of self-excited vibrations of a magnetic track brake

Bernhard Ebner, Daniel Tippelt, Johannes Edelmann, Manfred Plöchl
Journal of Physics: Conference Series, Volume 2647, Art.Nr. 152007
https://doi.org/10.1088/1742-6596/2647/15/152007

Keywords: vehicle dynamics, self-excited vibrations, active vibration control, electromagnetic-mechanical coupling, friction-induced vibrations, electromagnetic track brake

Abstract: Magnetic track brakes (mtbs) are additional braking systems used in railway vehicles at emergency situations and low adhesion conditions. Operation at low velocities can cause harmful self-excited vibrations, which must be avoided at all circumstances. Few passive countermeasures are already published, but active vibration control of an mtb lacks in literature so far. In this paper an active vibration control to diminish self-excited vibrations, based on reducing the energy, drawn by the oscillating system, is studied. Considering a minimal model of the mtb, the energy input depends on the electromagnetic-mechanical coupling and the friction force in the mtbrail contact. The obtained equation of this energy reveal a dependency of the phase shift, between magnetic flux and the oscillatory mechanical motion. A control law for the input voltage is obtained to reach a specific phase shift reducing the energy input and oscillating amplitudes.

References of Paper B

[1] International Union of Railways. UIC-Kodex 546, Vorschriften für den Bau der verschiedenen Bremsteile - Hochleistungsbremsen für Personenzüge. *UIC-Kodex 546* (2014), 8.

[2] G. Gfatter, S. Haas and G. Vohla. Schienenbremsen Track Brakes. Knorr-Bremse GmbH. Vol. 1st. Mödling: Knorr-Bremse GmbH, 2004.

[3] O. Arias-Cuevas and Z. Li. Field investigations into the performance of magnetic track brakes of an electrical multiple unit against slippery tracks. Part 2: Braking force and side effects. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 226.1 (2012), 72–94. DOI: 10.1177/0954409711408375.

[4] D. Tippelt et al. Analysis of self-excited vibrations of an electromagnetic track brake. *The IAVSD International Symposium on Dynamics of Vehicles on Roads and Tracks*. Springer. 2019, 442–451. DOI: 10.1007/978-3-030-38077-9_52.

[5] D. Tippelt et al. Modelling, analysis and mitigation of self-excited vibrations of a magnetic track brake. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 236.6 (2021), 684–694. DOI: 10.1177/0954409721103.

[6] K. Popp and M. Rudolph. Vibration control to avoid stick-slip motion. *Journal of Vibration and Control* 10.11 (2004), 1585–1600. DOI: 10.1177/1077546304042.

[7] J. P. Den Hartog. Mechanical vibrations. Vol. 4. Dover Publications, New York, 1985.

Paper C

System analysis and active vibration control of a simplified electromagnetic track brake model

Bernhard Ebner, Johannes Edelmann, Manfred Plöchl
Journal of Sound and Vibration, Volume 618, 119307
https://doi.org/10.1016/j.jsv.2025.119307

Keywords: Self-excited vibrations, Active vibration control, Electromagneticmechanical coupling, Friction induced vibrations, Bifurcation analysis, Electromagnetic track brake

Abstract: Magnetic track brakes are prone to self-excited vibrations at low velocities, which can harm the mechanical structure and need to be mitigated. To avoid severe vibrations, active control strategies utilising the available electromagnetic system are investigated in this study. As the electromagneticmechanical coupling is a cause for self-excited vibrations itself, energy-based analyses are conducted on a simplified model to examine the underlying excitation mechanisms first. They are then assessed to identify effective countermeasures. An analytical equation for the effective energy input into the oscillating system is obtained, highlighting that the negative gradient of the sliding friction coefficient and the involved dynamics of the coupled electromagnetic system are reasons for the loss of stability. It is shown that the dominating phenomenon can change with the vehicle velocity and that each cause can initiate self-excited vibrations without the other being present. Conducted bifurcation analyses confirm analytical findings and reveal a significant reduction in the magnetic attraction force by increasing oscillating amplitudes, leading to decreased braking performance during oscillations. Building on the acquired findings, an active vibration control strategy is presented to force the phase shift between the electromagnetic and mechanical states to a particular value, minimising the effective energy input. Results from the nonlinear stability analysis of the closed-loop system show that the implemented state feedback can suppress the excitation mechanism associated with the electromagnetic coupling, further actively dissipate energy, and prevent vibrations over the whole velocity range.

References of Paper C

[1] M. Jirout, P. Lugner and W. Mack. Dynamic simulation of the contact forces in the transmission link of a magnetic track-brake in low suspension. *10th Mini conference on vehicle system dynamics, identification and anomalies*. Vehicle system dynamics, identification and anomalies (VSDIA). 2008, 199–206. DOI: 20.500.12708/65813.

[2] M. Fischer et al. Investigations of Degraded Adhesion Conditions and Interrelated Methods for Improving Braking Performance using the Advanced TrainLab (aTL). *EuroBrake 2023*. FISITA. 2023.

[3] DIN, EN and 16207:2014+a1:2019. Bahnanwendungen Bremse funktion und leistungsfähigkeit von magnetschienenbremssystemen für schienenfahrzeuge (2014).

[4] D. Tippelt. Self-excited vibrations of magnetic track brakes: Modelling, analysis and mitigation. PhD thesis. TU Wien, 2022. DOI: 10.34726/hss.2022.33382.

[5] D. Tippelt et al. Modelling, analysis and mitigation of self-excited vibrations of a magnetic track brake. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 236.6 (2021), 684–694. DOI: 10.1177/0954409721103.

[6] J. P. Den Hartog. Mechanical vibrations. Vol. 4. Dover Publications, New York, 1985.

[7] M. Oestreich, N. Hinrichs and K. Popp. Bifurcation and stability analysis for a non-smooth friction oscillator. *Archive of Applied Mechanics* 66.5 (1996), 301–314. DOI: 10.1007/BF00795247.

[8] K. Avramov et al. Self-sustained oscillations of a magnetic track brake frame. *Nonlinear Dynamics* 113.10 (2025), 11121–11142. DOI: 10.1007/s11071-024-10643-6.

[9] M. Kröger, M. Neubauer and K. Popp. Experimental investigation on the avoidance of self-excited vibrations. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 366.1866 (2008), 785–810. DOI: 10.1098/rsta.2007.2127.

[10] K. Popp and M. Rudolph. Vibration control to avoid stick-slip motion. *Journal of Vibration and Control* 10.11 (2004), 1585–1600. DOI: 10.1177/1077546304042.

[11] K. Popp. Modelling and control of friction-induced vibrations. *Mathematical and Computer Modelling of Dynamical Systems* 11.3 (2005), 345–369. DOI: 10.1080/13873950500076131.

[12] K. Magnus, K. Popp and W. Sextro. Schwingungen. Eine Einführung in physikalische Grundlagen und die theoretische Behandlung von Schwingungsproblemen. Vieweg+Teubner Verlag Wiesbaden, 2013. DOI: 10.1007/978-3-8351-9227-0.

[13] C. Hansen et al. Active Control of Noise and Vibration Second Edition Volume 2. CRC press, 2013. DOI: 10.1201/b15923.

[14] D. Zhou, C. Hansen and J. Li. Suppression of maglev vehicle–girder self-excited vibration using a virtual tuned mass damper. *Journal of sound and Vibration* 330.5 (2011), 883–901. DOI: 10.1016/j.jsv.2010.09.018.

[15] J. Li et al. The active control of maglev stationary self-excited vibration with a virtual energy harvester. *IEEE Transactions on Industrial Electronics* 62.5 (2015), 2942–2951. DOI: 10.1109/TIE.2014.2364788.

[16] J.-h. Li et al. Maglev self-excited vibration suppression with a virtual sky-hooked damper. *Journal of Central South University* 23.6 (2016), 1363–1371. DOI: 10.1007/s11771-016-3188-8.

[17] B. Ebner et al. Active mitigation of self-excited vibrations of a magnetic track brake. *Journal of Physics: Conference Series*. Vol. 2647. 15. IOP Publishing. 2024, 152007. DOI: 10.1088/1742-6596/2647/15/152007.

[18] T. Butlin and J. Woodhouse. Friction-induced vibration: Quantifying sensitivity and uncertainty. *Journal of Sound and Vibration* 329.5 (2010), 509–526. DOI: 10.1016/j.jsv.2009.09.026.

[19] R. Darula and S. Sorokin. Simplifications in modelling of dynamical response of coupled electro-mechanical system. *Journal of Sound and Vibration* 385 (2016), 402–414. DOI: 10.1016/j.jsv.2016.08.036.

[20] X. Li et al. Angular-based modeling of unbalanced magnetic pull for analyzing the dynamical behavior of a 3-phase induction motor. *Journal of Sound and Vibration* 494 (2021), 22. DOI: 10.1016/j.jsv.2020.115884.

[21] E. Kallenbach et al. Elektromagnete. Vol. 5. Springer, 2018, 438. DOI: 10.1007/978-3-658-14788-4.

[22] Y. A. Kuznetsov. Elements of applied bifurcation theory. Vol. 4. Springer, 1998, 703. DOI: 10.1007/978-3-031-22007-4.

[23] H. Troger and A. Steindl. Nonlinear stability and bifurcation theory: an introduction for engineers and applied scientists. Springer Science & Business Media, 2012. DOI: 10.1007/978-3-7091-9168-2.

Paper D

Self-sustained oscillations of a magnetic track brake frame

Konstantin Avramov, Bernhard Ebner, Johannes Edelmann, Yuri V. Mikhlin, Borys Uspensky
Nonlinear Dynamics, Volume 113, pp. 1112111142
https://doi.org/10.1007/s11071-024-10643-6

Keywords: Electromagnetic track brake, Self-excited vibration, Poincaré sections, Nonlinear normal mode, Bifurcation, Electromagneticmechanical coupling

Abstract: Magnetic track brakes (MTBs) are additional brake systems for railway vehicles used in low adhesion and emergency conditions. In particular, the frame of this brake may exhibit self-sustained vibrations. To study the underlying mechanisms of these oscillations, a nonlinear mathematical model of the MTB is derived that consists of two submodels: a mathematical model of the mechanical subsystem and a model of the coupled electro-magnetic subsystem. Mechanical vibrations of the brake-frame are described by four degrees of freedom. Nonlinear dry friction, which is observed between the rail and the frame, is accounted for in the mechanical model. The coupled electromagnetic subsystem is modelled by a lumped parameter approach, describing the oscillating magnetic fluxes due to the mechanical motion of each electromagnet. The self-sustained vibrations are studied numerically for different vehicle velocities. The NeimarkSacker bifurcation is observed. As a result of this bifurcation, the self-sustained quasi-periodic vibrations are originated. The stickslip motions of the frame are observed, caused by the dry friction between the rail and the magnet.

References of Paper D

[1] G. Gfatter, S. Haas and G. Vohla. Schienenbremsen Track Brakes. Knorr-Bremse GmbH. Vol. 1st. Mödling: Knorr-Bremse GmbH, 2004.

[2] O. Arias-Cuevas and Z. Li. Field investigations into the performance of magnetic track brakes of an electrical multiple unit against slippery tracks. Part 1: Adhesion improvement. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 225.6 (2011), 613–636. DOI: 10.1177/0954409711401515.

[3] O. Arias-Cuevas and Z. Li. Field investigations into the performance of magnetic track brakes of an electrical multiple unit against slippery tracks. Part 2: Braking force and side effects. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 226.1 (2012), 72–94. DOI: 10.1177/0954409711408375.

[4] ERRI and B126/DT371. Braking problems Precalculating the performance of the magnetic track brake. Technical report. *European Rail Research Institute* (1999).

[5] C. Cruceanu and C. Crciun. Necessity and conditionality regarding the electromagnetic track brake-parametric study. *MATEC Web of Conferences*. Vol. 178. EDP Sciences. 2018, 06020. DOI: 10.1051/matecconf/201817806020.

[6] M. Yao and L. Wang. Analysis and simulation on the influencing factors of braking force for permanent magnetic brake. *Applied Mechanics and Materials* 278 (2013), 278–281. DOI: 10.4028/www.scientific.net/AMM.278-280.278.

[7] M. Jirout, W. Mack and P. Lugner. Non-smooth dynamics of a magnetic track brake. *Regular and Chaotic Dynamics* 14.6 (2009), 673–681. DOI: 10.1134/S1560354709060057.

[8] E. Galardi et al. Development of efficient models of Magnetic Braking Systems of railway vehicles. *International Journal of Rail Transportation* 3.2 (2015), 97–118. DOI: 10.1080/23248378.2015.1015219.

[9] D. Tippelt et al. Modelling, analysis and mitigation of self-excited vibrations of a magnetic track brake. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 236.6 (2021), 684–694. DOI: 10.1177/0954409721103.

[10] D. Tippelt et al. Analysis of self-excited vibrations of an electromagnetic track brake. *The IAVSD International Symposium on Dynamics of Vehicles on Roads and Tracks*. Springer. 2019, 442–451. DOI: 10.1007/978-3-030-38077-9_52.

[11] B. Ebner et al. Active mitigation of self-excited vibrations of a magnetic track brake. *Journal of Physics: Conference Series*. Vol. 2647. 15. IOP Publishing. 2024, 152007. DOI: 10.1088/1742-6596/2647/15/152007.

[12] M. Denny. Stick–slip motion: an important example of self-excited oscillation. *European journal of physics* 25.2 (2004), 311. DOI: 10.1088/0143-0807/25/2/018.

[13] M. Oestreich, N. Hinrichs and K. Popp. Bifurcation and stability analysis for a non-smooth friction oscillator. *Archive of Applied Mechanics* 66.5 (1996), 301–314. DOI: 10.1007/BF00795247.

[14] K. Popp and P. Stelter. Stick-slip vibrations and chaos. *Philosophical Transactions: Physical Sciences and Engineering* (1990), 89–105.

[15] J. Awrejcewicz and P. Olejnik. Stick-slip dynamics of a two-degree-of-freedom system. *International Journal of Bifurcation and Chaos* 13.04 (2003), 843–861. DOI: 10.1142/S0218127403006960.

[16] U. Galvanetto, S. Bishop and L. Briseghella. Mechanical stick-slip vibrations. *International Journal of Bifurcation and Chaos* 5.03 (1995), 637–651. DOI: 10.1142/S0218127495000508.

[17] M. Bengisu and A. Akay. Stability of friction-induced vibrations in multi-degree-of-freedom systems. *Journal of Sound and Vibration* 171.4 (1994), 557–570. DOI: 10.1006/jsvi.1994.1140.

[18] R. Leine et al. Stick-slip vibrations induced by alternate friction models. *Nonlinear dynamics* 16.1 (1998), 41–54. DOI: 10.1023/A:1008289604683.

[19] B. Van de Vrande, D. Van Campen and A. De Kraker. An approximate analysis of dry-friction-induced stick-slip vibrations by a smoothing procedure. *Nonlinear Dynamics* 19.2 (1999), 159–171. DOI: 10.1023/A:1008306327781.

[20] J. Kang, C. M. Krousgrill and F. Sadeghi. Oscillation pattern of stick–slip vibrations. *International Journal of Non-Linear Mechanics* 44.7 (2009), 820–828. DOI: 10.1016/j.ijnonlinmec.2009.05.002.

[21] Z. Veraszto and G. Stepan. Nonlinear dynamics of hardware-in-the-loop experiments on stick–slip phenomena. *International Journal of Non-Linear Mechanics* 94 (2017), 380–391. DOI: 10.1016/j.ijnonlinmec.2017.01.006.

[22] D. Wei et al. Analysis of the stick-slip vibration of a new brake pad with double-layer structure in automobile brake system. *Mechanical Systems and Signal Processing* 118 (2019), 305–316. DOI: 10.1016/j.ymssp.2018.08.055.

[23] D. Wei et al. Properties of stability, bifurcation, and chaos of the tangential motion disk brake. *Journal of Sound and Vibration* 375 (2016), 353–365. DOI: 10.1016/j.jsv.2016.04.022.

[24] R. A. Ibrahim. Friction-induced vibration, chatter, squeal, and chaospart II: dynamics and modeling. *Appl. Mech. Rev.* (1994). DOI: 10.1115/1.3111080.

[25] K. Popp. Non-smooth mechanical systemsan overview. *Forschung im Ingenieurwesen* 64.9 (1998), 223–229. DOI: 10.1007/PL00010860.

[26] J. H. Argyris, S. Kelsey et al. Energy theorems and structural analysis. Vol. 60. Springer, 1960, 81. DOI: 10.1007/978-1-4899-5850-1.

[27] F. Yang, W. Zhang and J. Wang. Sliding bifurcations and chaos induced by dry friction in a braking system. *Chaos, Solitons & Fractals* 40.3 (2009), 1060–1075. DOI: 10.1016/j.chaos.2007.08.079.

[28] U. Galvanetto and S. R. Bishop. Stick-slip vibrations of a two degree-of-freedom geophysical fault model. *International Journal of Mechanical Sciences* 36.8 (1994), 683–698. DOI: 10.1016/0020-7403(94)90085-X.

[29] S. D. Sudhoff et al. Magnetic equivalent circuit modeling of induction motors. *IEEE Transactions on Energy Conversion* 22.2 (2007), 259–270. DOI: 10.1109/TEC.2006.875471.

[30] J. Guckenheimer and P. Holmes. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Vol. 42. Springer Science & Business Media, 2013. DOI: 10.1007/978-1-4612-1140-2.

[31] J. Guo et al. Double-parameter Hopf bifurcation analysis of a high-speed rail vehicle with an alternative wheel/rail contact approximation. *Vehicle System Dynamics* 61.2 (2023), 530–549. DOI: 10.1080/00423114.2022.2050770.

[32] J. Guo et al. Bifurcation analysis of a railway wheelset with nonlinear wheel–rail contact. *Nonlinear Dynamics* 104.2 (2021), 989–1005. DOI: 10.1007/s11071-021-06373-8.

[33] T. S. Parker and L. Chua. Practical numerical algorithms for chaotic systems. Springer Science & Business Media, 2012. DOI: 10.1007/978-1-4612-3486-9.

[34] K. Avramov. Analysis of forced vibrations by nonlinear modes. *Nonlinear Dynamics* 53.1 (2008), 117–127. DOI: 10.1007/s11071-007-9300-8.

[35] K. Avramov. Non-linear beam oscillations excited by lateral force at combination resonance. *Journal of sound and vibration* 257.2 (2002), 337–359. DOI: 10.1006/jsvi.2002.5043.

[36] K. Avramov and O. Gendelman. Interaction of elastic system with snap-through vibration absorber. *International Journal of Non-Linear Mechanics* 44.1 (2009), 81–89. DOI: 10.1016/j.ijnonlinmec.2008.09.004.

[37] K. Avramov and E. Strel'Nikova. Chaotic oscillations of plates interacting on both sides with a fluid flow. *International Applied Mechanics* 50.3 (2014), 303–310.

Paper E

Stability behaviour of a basic magnetic track brake model: Influences of system parameters and motion-induced eddy currents

Bernhard Ebner, Manfred Plöchl, Johannes Edelmann
Nonlinear Dynamics, 2025
https://doi.org/10.1007/s11071-025-11574-6

Keywords: Electromagnetic track brake, Self-excited vibration, Poincare sections, Nonlinear normal mode, Bifurcation, Electromagneticmechanical coupling

Abstract: Insights into the vibrational behaviour of emergency braking systems of railway vehicles, such as the electromagnetic track brake, are indispensable for short stopping distances and safe operation. Moreover, they are fundamental for the development of lightweight railway vehicles. Hence, the nonlinear stability behaviour of the electromagnetic track brake is examined within this paper by applying bifurcation analysis and numerical continuation methods to a newly developed minimal model. The proposed electromagnetic-mechanical model accounts for a segmented brake magnet, incorporating the frictional contact of individual brake elements and a velocity-dependent attraction force resulting from motion-induced eddy currents, revealed by experiments. Two super-critical Hopf bifurcations give rise to periodic orbits of pure slip motions for varying vehicle velocities, compliant with observations from measurements. Further, a second branch of stable periodic orbits with large amplitudes at a similar oscillatory frequency exhibiting stick-slip motions is found. By modifying the friction characteristics, the coexisting periodic attractors converge, resulting in a branching point that marks the onset of a parameter region where only large-amplitude periodic orbits with stick-slip motions are stable. Influences of main design and environmental parameters are studied.

References of Paper E

[1] J. Winnett et al. Development of a very light rail vehicle. *Proceedings of the Institution of Civil Engineers-Transport*. Vol. 170. 4. Thomas Telford Ltd. 2017, 231–242. DOI: 10.1680/jtran.16.00038.

[2] C. Cruceanu and C. Crciun. Necessity and conditionality regarding the electromagnetic track brake-parametric study. *MATEC Web of Conferences*. Vol. 178. EDP Sciences. 2018, 06020. DOI: 10.1051/matecconf/201817806020.

[3] International Union of Railways. UIC-Kodex 546, Vorschriften für den Bau der verschiedenen Bremsteile - Hochleistungsbremsen für Personenzüge. *UIC-Kodex 546* (2014), 8.

[4] O. Arias-Cuevas and Z. Li. Field investigations into the performance of magnetic track brakes of an electrical multiple unit against slippery tracks. Part 1: Adhesion improvement. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 225.6 (2011), 613–636. DOI: 10.1177/0954409711401515.

[5] O. Arias-Cuevas and Z. Li. Field investigations into the performance of magnetic track brakes of an electrical multiple unit against slippery tracks. Part 2: Braking force and side effects. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 226.1 (2012), 72–94. DOI: 10.1177/0954409711408375.

[6] M. Fischer et al. Investigations of Degraded Adhesion Conditions and Interrelated Methods for Improving Braking Performance using the Advanced TrainLab (aTL). *EuroBrake 2023*. FISITA. 2023.

[7] R. Rathammer. Mehrkörpersimulation einer Magnetschienenbremse in Tiefaufhängung. MA thesis. TU Wien, 2000. DOI: 20.500.12708/182348.

[8] E. Galardi et al. Development of efficient models of Magnetic Braking Systems of railway vehicles. *International Journal of Rail Transportation* 3.2 (2015), 97–118. DOI: 10.1080/23248378.2015.1015219.

[9] M. Jirout, W. Mack and P. Lugner. Non-smooth dynamics of a magnetic track brake. *Regular and Chaotic Dynamics* 14.6 (2009), 673–681. DOI: 10.1134/S1560354709060057.

[10] E. Kocbay et al. Efficient and simplified numerical contact model for the braking simulation of a magnetic track brake. *Meccanica* 60.2 (2025), 195–216. DOI: 10.1007/s11012-024-01926-8.

[11] D. Tippelt et al. Analysis of self-excited vibrations of an electromagnetic track brake. *The IAVSD International Symposium on Dynamics of Vehicles on Roads and Tracks*. Springer. 2019, 442–451. DOI: 10.1007/978-3-030-38077-9_52.

[12] D. Tippelt et al. Modelling, analysis and mitigation of self-excited vibrations of a magnetic track brake. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 236.6 (2021), 684–694. DOI: 10.1177/0954409721103.

[13] D. Tippelt. Self-excited vibrations of magnetic track brakes: Modelling, analysis and mitigation. PhD thesis. TU Wien, 2022. DOI: 10.34726/hss.2022.33382.

[14] B. Ebner et al. Active mitigation of self-excited vibrations of a magnetic track brake. *Journal of Physics: Conference Series*. Vol. 2647. 15. IOP Publishing. 2024, 152007. DOI: 10.1088/1742-6596/2647/15/152007.

[15] B. Ebner, J. Edelmann and M. Plöchl. System analysis and active vibration control of a simplified electromagnetic track brake model. *Journal of Sound and Vibration* (2025), 119307. DOI: 10.1016/j.jsv.2025.119307.

[16] K. Avramov et al. Self-sustained oscillations of a magnetic track brake frame. *Nonlinear Dynamics* 113.10 (2025), 11121–11142. DOI: 10.1007/s11071-024-10643-6.

[17] D. S. Weaver and S. Ziada. A Theoretical Model for Self-Excited Vibrations in Hydraulic Gates, Valves and Seals. *Journal of Pressure Vessel Technology* 102.2 (1980), 146–151. DOI: 10.1115/1.3263313.

[18] M. Oestreich, N. Hinrichs and K. Popp. Bifurcation and stability analysis for a non-smooth friction oscillator. *Archive of Applied Mechanics* 66.5 (1996), 301–314. DOI: 10.1007/BF00795247.

[19] K. Popp and M. Rudolph. Vibration control to avoid stick-slip motion. *Journal of Vibration and Control* 10.11 (2004), 1585–1600. DOI: 10.1177/1077546304042.

[20] Z. Zhang et al. Analysis of friction-induced vibration in a propeller–shaft system with consideration of bearing–shaft friction. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 228.8 (2014), 1311–1328. DOI: 10.1177/0954406213508386.

[21] W. Qin et al. Self-excited vibration of a flexibly supported shafting system induced by friction. *Journal of Vibration and Acoustics* 139.2 (2017), 021004. DOI: 10.1115/1.4035203.

[22] K. Popp. Modelling and control of friction-induced vibrations. *Mathematical and Computer Modelling of Dynamical Systems* 11.3 (2005), 345–369. DOI: 10.1080/13873950500076131.

[23] J. P. Den Hartog. Mechanical vibrations. Vol. 4. Dover Publications, New York, 1985.

[24] M. Kröger, M. Neubauer and K. Popp. Experimental investigation on the avoidance of self-excited vibrations. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 366.1866 (2008), 785–810. DOI: 10.1098/rsta.2007.2127.

[25] T. Butlin and J. Woodhouse. Friction-induced vibration: Quantifying sensitivity and uncertainty. *Journal of Sound and Vibration* 329.5 (2010), 509–526. DOI: 10.1016/j.jsv.2009.09.026.

[26] Z.-X. Li, Q.-J. Cao and L. Alain. Complex dynamics of an archetypal self-excited SD oscillator driven by moving belt friction. *Chinese Physics B* 25.1 (2015), 010502. DOI: 10.1088/1674-1056/25/1/010502.

[27] Z. Li, Q. Cao and Z. Nie. Stick-slip vibrations of a self-excited SD oscillator with Coulomb friction. *Nonlinear Dynamics* 102.3 (2020), 1419–1435. DOI: 10.1007/s11071-020-06009-3.

[28] Y. Zhu et al. The effect of dynamic normal force on the stick–slip vibration characteristics. *Nonlinear Dynamics* 110.1 (2022), 69–93. DOI: 10.1007/s11071-022-07614-0.

[29] H.-I. Won and J. Chung. Stick–slip vibration of an oscillator with damping. *Nonlinear Dynamics* 86.1 (2016), 257–267. DOI: 10.1007/s11071-016-2887-x.

[30] R. Lima and R. Sampaio. Stick–slip oscillations in a multiphysics system. *Nonlinear Dynamics* 100.3 (2020), 2215–2224. DOI: 10.1007/s11071-020-05677-5.

[31] J. Kang, C. M. Krousgrill and F. Sadeghi. Oscillation pattern of stick–slip vibrations. *International Journal of Non-Linear Mechanics* 44.7 (2009), 820–828. DOI: 10.1016/j.ijnonlinmec.2009.05.002.

[32] D. Wei et al. Properties of stability, bifurcation, and chaos of the tangential motion disk brake. *Journal of Sound and Vibration* 375 (2016), 353–365. DOI: 10.1016/j.jsv.2016.04.022.

[33] D. Wei et al. Analysis of the stick-slip vibration of a new brake pad with double-layer structure in automobile brake system. *Mechanical Systems and Signal Processing* 118 (2019), 305–316. DOI: 10.1016/j.ymssp.2018.08.055.

[34] J. Guo et al. Double-parameter Hopf bifurcation analysis of a high-speed rail vehicle with an alternative wheel/rail contact approximation. *Vehicle System Dynamics* 61.2 (2023), 530–549. DOI: 10.1080/00423114.2022.2050770.

[35] R. A. Ibrahim. Friction-induced vibration, chatter, squeal, and chaospart II: dynamics and modeling. *Appl. Mech. Rev.* (1994). DOI: 10.1115/1.3111080.

[36] K. Popp. Non-smooth mechanical systemsan overview. *Forschung im Ingenieurwesen* 64.9 (1998), 223–229. DOI: 10.1007/PL00010860.

[37] G. Liu et al. Multiphysics analysis of a hybrid suspension system for middle-low-speed maglev trains. *The European Physical Journal Applied Physics,* 90.1 (2020), 8. DOI: 10.1051/epjap/2020200015.

[38] B. Buth and B. Lu. Dynamic analysis of vehicle-guideway interaction in a maglev cargo transportation system. *ASME International Mechanical Engineering Congress and Exposition*. Vol. 11. American Society of Mechanical Engineers. 2012, 1–8. DOI: 10.1115/IMECE2012-85552.

[39] R. Darula and S. Sorokin. On non-linear dynamics of a coupled electro-mechanical system. *Nonlinear Dynamics* 70.2 (2012), 979–998. DOI: 10.1007/s11071-012-0505-0.

[40] R. Darula and S. Sorokin. Simplifications in modelling of dynamical response of coupled electro-mechanical system. *Journal of Sound and Vibration* 385 (2016), 402–414. DOI: 10.1016/j.jsv.2016.08.036.

[41] H. Troger and A. Steindl. Nonlinear stability and bifurcation theory: an introduction for engineers and applied scientists. Springer Science & Business Media, 2012. DOI: 10.1007/978-3-7091-9168-2.

[42] R. Seydel. Practical bifurcation and stability analysis. Vol. 5. Springer Science & Business Media, 2009. DOI: 10.1007/978-1-4419-1740-9.

[43] J. Guckenheimer and P. Holmes. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Vol. 42. Springer Science & Business Media, 2013. DOI: 10.1007/978-1-4612-1140-2.

[44] Y. A. Kuznetsov. Elements of applied bifurcation theory. Vol. 4. Springer, 1998, 703. DOI: 10.1007/978-3-031-22007-4.

[45] W. Govaerts et al. MATCONT: Continuation toolbox for ODEs in Matlab. Utrecht University, 2019.