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Entanglement of massive, charged particles via a lossy conductive wire

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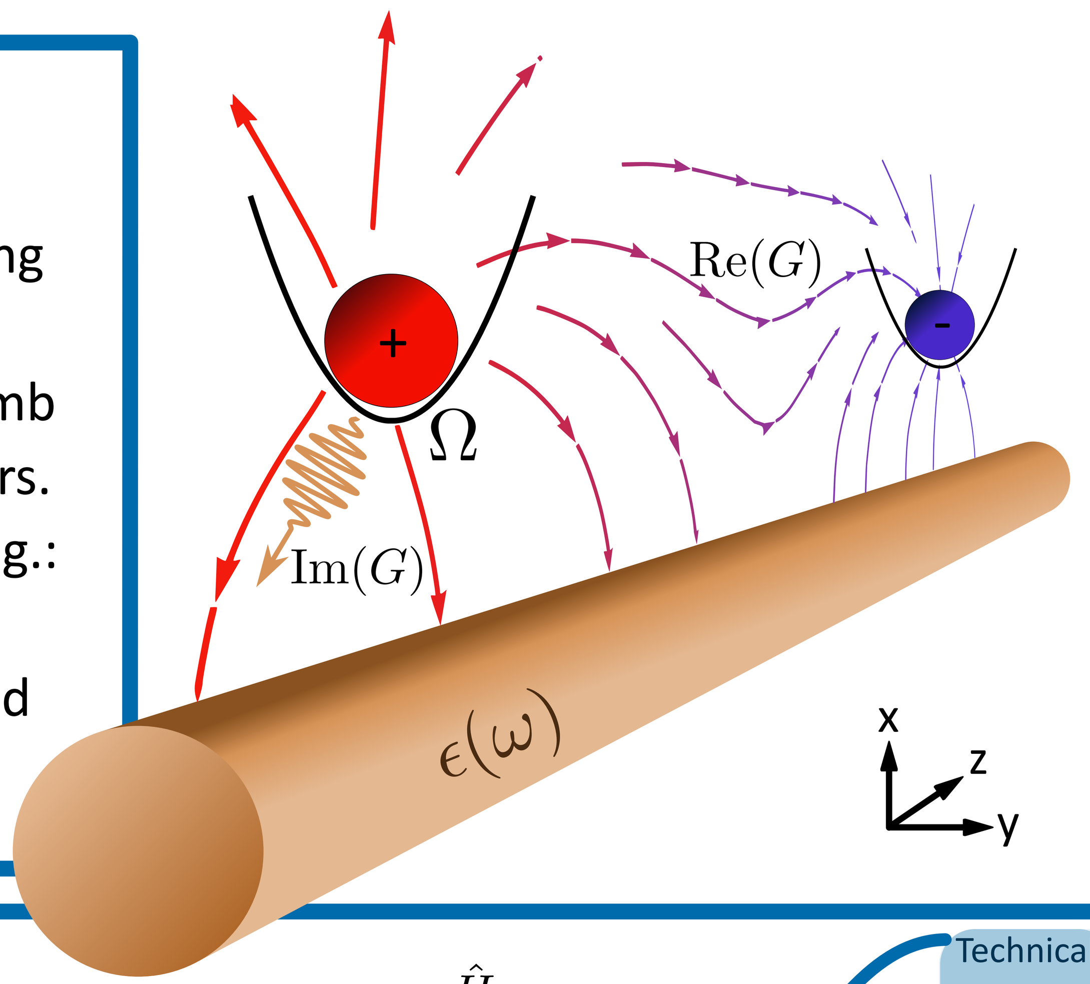
Motivation & Introduction

Gravity and the Coulomb force are both **central forces**. Therefore, generating **entanglement in mesoscopic bodies** with the latter is a first crucial step towards testing the quantumness of the former [1, 2]. In free space, Coulomb coupling decays as r^{-3} making this challenging especially with bulk resonators. Here we propose to **use a wire to enhance its range and tunability** (with e.g.: variable resistor elements). Although our results are general, we use parameters for levitated optomechanics where the preparation of entangled states would be a especially coveted milestone.



Parameter		Value
Conductivity	σ	59.6 MS/m
Wire radius	R	1 μm
Wire distance	d	1 μm
Particle charges	q	80 e
Particle Masses	m	2×10^{-18} kg
Trapping frequency	Ω	$2\pi\times100$ kHz
Temperature	T	4.2 K

Theory

- **Quasielectrostatic Hamiltonian:** $$\hat{H} = \overbrace{\hat{H}^{(1)}_{h.osc.} + \hat{H}^{(2)}_{h.osc.} + \hat{H}^{(12)}_{Coulomb}}^{\hat{H}_S} + \hat{H}_{wire} + \overbrace{\sum_i q_i \hat{\phi}_{wire}(\hat{r}_i)}^{\hat{H}_{int}}$$

- **Green's function** solves the electrostatic boundary condition problem with wire permittivity $\epsilon(\omega) = \epsilon_0 + i\frac{\sigma}{\omega}$

$$\nabla\cdot\epsilon_0\epsilon(\mathbf{r},\omega)\nabla g(\mathbf{r},\mathbf{r}',\omega) = -\delta(\mathbf{r}-\mathbf{r}')$$ and the **Green's tensor** $$\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \approx -\frac{1}{\mu_0\omega^2}\frac{\partial}{\partial\mathbf{r}}\otimes\frac{\partial}{\partial\mathbf{r}'}g(\mathbf{r},\mathbf{r}',\omega)$$

Technical details

Macroscopic quantum electrodynamics (MQED):

Build lossy quantum field using Green's tensor on quantized fluctuating sources. [3]

$$\hat{\phi}(\mathbf{r},\omega) = \int d^3\mathbf{r}'(\nabla' g(\mathbf{r},\mathbf{r}',\omega))\hat{\mathbf{P}}_n(\mathbf{r}',\omega)$$

- Tracing out the wire, leaves us with the following **master equation:**

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\Big[\hat{H}_S + \sum_{ij}\delta\hat{H}_{ij}, \hat{\rho}\Big] + \underbrace{\sum_{i,j\in\{1,2\}}\left(-\frac{\Gamma_{ij}}{2}[\hat{x}^i,[\hat{x}^j,\hat{\rho}]] + i\mathcal{N}_{ij}(\mathcal{D}_{\hat{x}^i\hat{x}^j}\hat{\rho} - \mathcal{D}_{\hat{x}^j\hat{x}^i}\hat{\rho})\right)}_{\text{Decoherence due to wire}}$$

$$+\sum_i\left(\bar{n}\gamma\mathcal{D}_{\hat{b}_i^\dagger\hat{b}_i}\hat{\rho} + (\bar{n}+1)\gamma\mathcal{D}_{\hat{b}_i\hat{b}_i^\dagger}\hat{\rho}\right)$$

modified Hamiltonian; Lindblad dissipators; thermal damping due to other sources (i.e.: intrinsic damping of the oscillators)

Master equation rates

- $\Gamma_{ij} = 2\frac{q_iq_jx_{0,i}x_{0,j}}{\hbar}\bar{n}\,\mu_0\Omega_j^2\,\mathrm{Im}(G(\mathbf{R}_0^i,\mathbf{R}_0^j,\Omega_j)) = \bar{n}\Gamma^0_{ij}$ — losses
- $\mathcal{N}_{ij} = \frac{q_iq_jx_{0,i}x_{0,j}}{2\hbar}\mu_0\Omega_j^2\,\mathrm{Re}(G(\mathbf{R}_0^i,\mathbf{R}_0^j,\Omega_j))$ — image charge response
- $\delta\hat{H}_{ij} = \underbrace{\frac{\hbar}{4}\Gamma^0_{ij}\frac{1}{2}(\hat{x}^i\hat{p}^j + \hat{p}^j\hat{x}^i)}_{\text{squeezing term}} - \underbrace{\hbar\mathcal{N}_{ij}\hat{x}^i\hat{x}^j}_{\text{frequency shift and image-charge interaction}}$

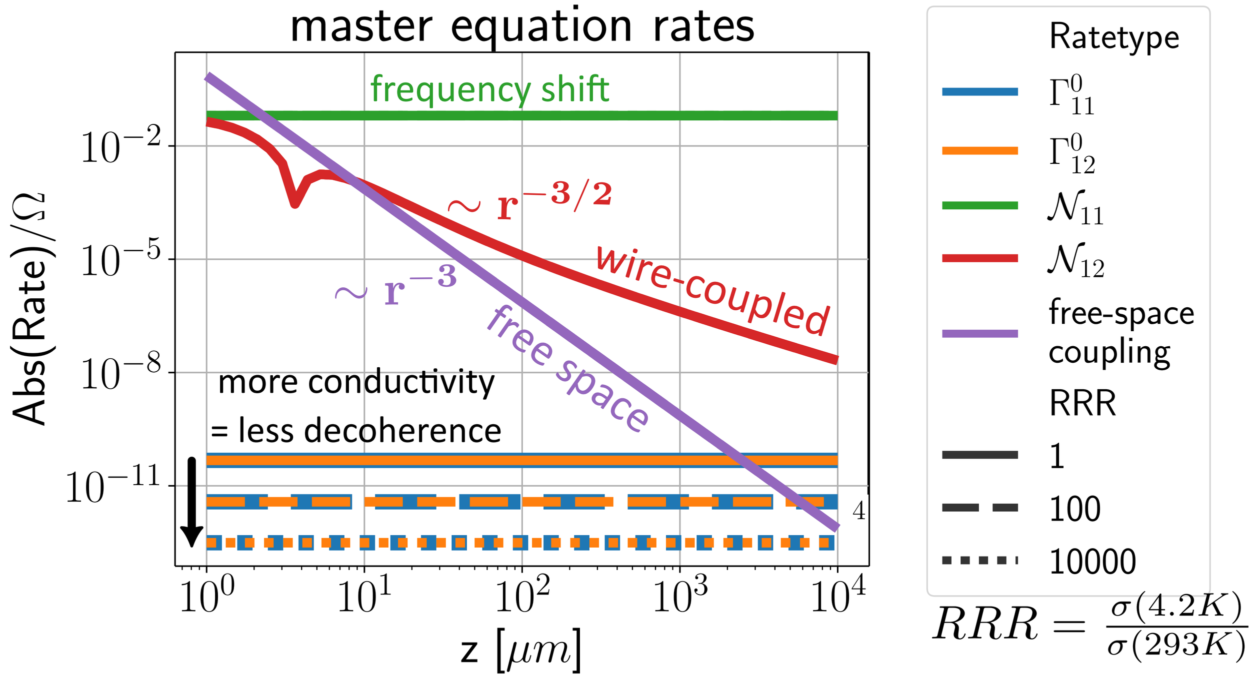
Entanglement

- Quadratic master equation implies **Gaussian** evolution.
- For Gaussian states, the **Simon-Duan criterion** gives a **necessary and sufficient** condition for entanglement [4].
- $$C_{jk} = \frac{1}{2}\langle(S_j - \langle S_j\rangle)(S_k - \langle S_k\rangle) + (S_k - \langle S_k\rangle)(S_j - \langle S_j\rangle)\rangle,$$ where $\vec{S} = (\hat{x}_1,\hat{p}_1,\hat{x}_2,\hat{p}_2)$

Entanglement $\Leftrightarrow$ $\nu < 1/2$, (or $\mathcal{E}_N = -\log(2\nu) > 0$) where ν is the symplectic eigenvalue of C_{jk}.

Variable	Description
$i=1,2$	Particle index
$\mathbf{r},\mathbf{r}'$	Position vectors
$\hat{x}_i,\hat{p}_i$	Position/ momentum quadratures
$\hat{b}_i,\hat{b}_i^\dagger$	Corresponding Ladder operators
$\bar{n}$	Mean BE-occupation number
γ	Thermal damping rate
$x_{0,i}$	zero-point motion
q_i	charge
R_0^i	Equillibrium position (with wire)

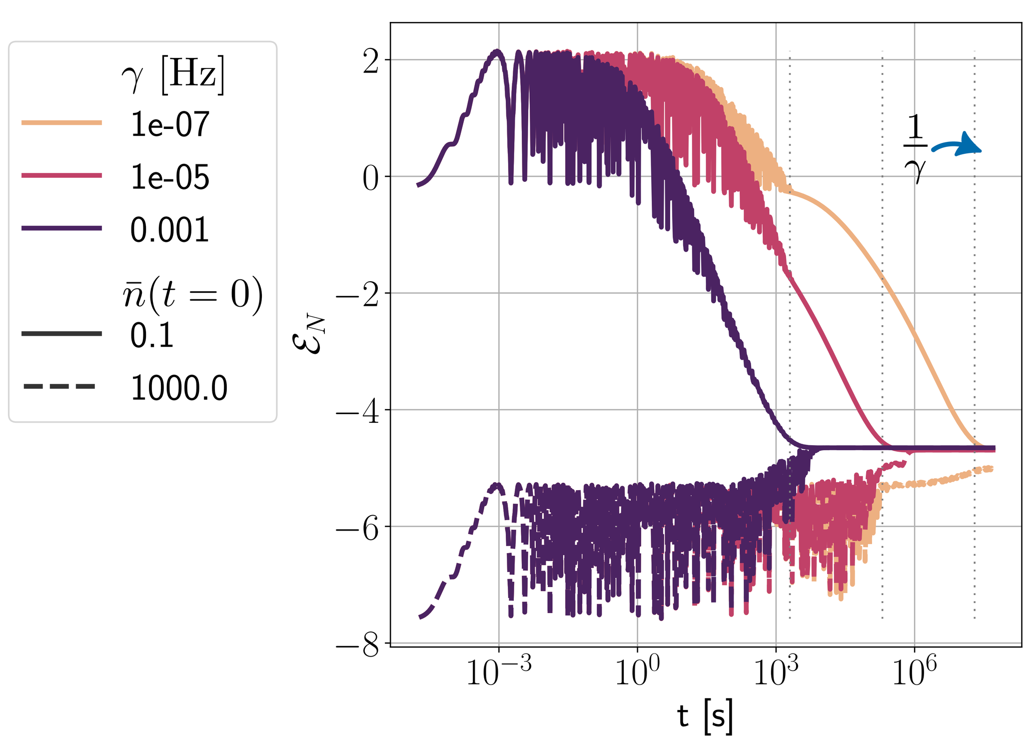
Distance scaling of master equation rates



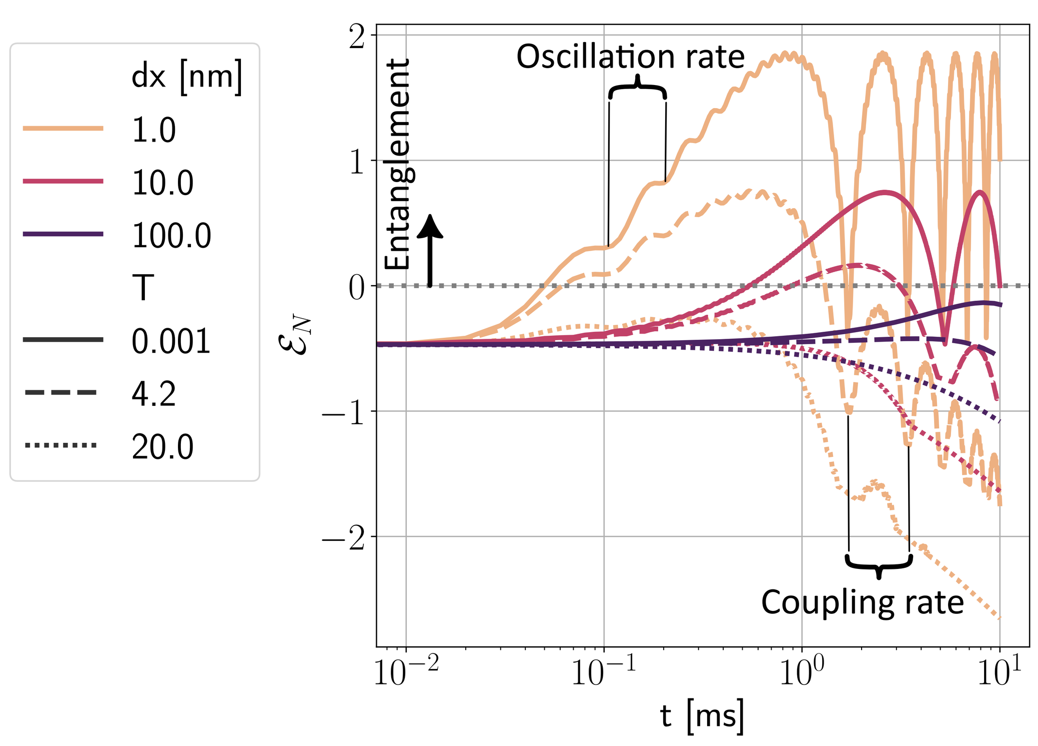
$RRR = \frac{\sigma(4.2K)}{\sigma(293K)}$

Results

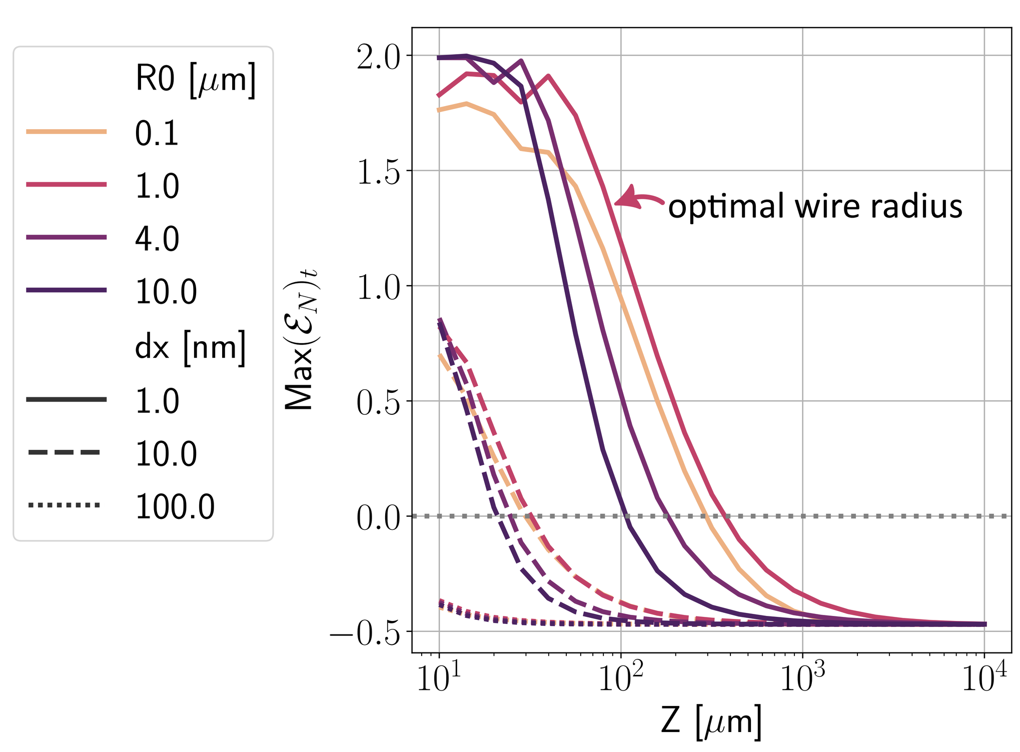
Long time behaviour of log-negativity



Short time behaviour of log-negativity



Maximal log-negativity over distance



Conclusion & Outlook

- The **decoherence rate Γ** due to the wire can be made **smaller than the wire mediated coupling rate $\mathcal{N}$**, and when it is also smaller than the thermal decoherence transient entanglement is observed even for finite wire temperatures
- The wire helps with entanglement and increases the range of the interaction.
- In the future, circuitry could be refined for entanglement over longer range.

References

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Contact & Funding

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