Quantum Theory of Cathodoluminescence and Electron-Photon Entanglement from Dielectric Slabs

S. Beltrán-Romero[† 1,2], I. C. Bicket[1,2], P. Haslinger[1,2], D. Rätzel[1,2,3,4]

[1]*Vienna Center for Quantum Science and Technology, TU Wien, Atominstitut, Vienna, Austria*

[2]*University Service Centre for Transmission Electron Microscopy, TU Wien, Vienna, Austria*

[3]*Department of Physics and Astronomy, University College London, Gower Street, WC1E 6BT London, United Kingdom*

[4]*ZARM, Unversität Bremen, Bremen, Germany*

Coherent cathodoluminescence, the phase-matched light emission from free electrons interacting with nanophotonic environments, natively generates bipartite entangled electron-photon states spanning both continuous variables (CV) [1] and discrete degrees of freedom [2]. In current free-electron quantum optics experiments, the inability to fully resolve continuous emission momenta during coincidence detection restricts our exact kinematic knowledge. Because practical measurements average over these unobserved spatial variables, underlying correlations are masked, preventing the restoration of full quantum coherence. Previous work has modeled the pair state for Cherenkov radiation in bulk media, showing that coupling to the final electron's orbital angular momentum (OAM) offers a pathway to create entangled pairs controlled by the shape of the incident electron [3].

In contrast, current free-electron quantum optics experiments operate at material interfaces [1, 2], harnessing coherent cathodoluminescence driven by transition radiation. To establish an exact quantum framework for these boundary-driven setups, we derive the bipartite electron-photon state produced across a finite dielectric slab. In the CV regime, we utilize the Wigner quasi-probability distribution and Logarithmic Negativity to rigorously map momentum-position correlations, revealing how the slab's macroscopic Fabry-Pérot cavity resonances manifest as distinct interference fringes. Transitioning to the discrete cylindrical basis, we identify the generation of high-dimensional OAM entanglement, whose correlations are dictated by the incident wavepacket's transverse symmetry and shape. Beyond OAM, this framework also allows us to isolate and engineer discrete polarization entanglement: we observe that breaking the system's cylindrical symmetry via oblique electron incidence yields a superposition of *p*- and *s*-polarized radiation fundamentally correlated with the electron's transverse momentum. This kinematic coupling allows the joint emission to be modeled as a bipartite qubit Bell state, whose relative phase can be actively tuned by leveraging the crystal's inherent birefringence. Ultimately, our exact analytical framework enables concrete protocols for deterministic state preparation and robust certification via mutually unbiased bases (MUBs) [4], providing a theoretical foundation for observing and engineering bipartite entanglement in modern nanophotonic experiments.

References

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