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**MSc Economics**

# A Simple Model with Wealth and Frictional Labor Markets

A Master's Thesis submitted for the degree of
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supervised by
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**MSc Economics**

# Affidavit

I, Péter Pusztai

hereby declare

that I am the sole author of the present Master's Thesis,

A Simple Model with Wealth and Frictional Labor Markets

20 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

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**Abstract**

A general equilibrium model with incomplete markets and directed search is developed, in order to analyze the optimal behavior of workers and comparative statics. Workers have per-period utility of the constant absolute risk aversion (CARA) type. The only form of saving is investment in vacancy creation, which connects the interest rate and job creation through the free entry condition.

Because of the CARA utility, all unemployed workers search in the same submarket regardless of their wealth. The optimal saving decisions also make it possible to define the equilibrium solely in aggregate variables without the distribution of asset levels.

The model is calibrated and the existence and uniqueness of the steady state are shown for the specific parameter settings. The effects of a one-percent increase in productivity are examined. The most surprising result is that the interest rate decreases. This is caused by the fact that almost all of the increase in the value of a filled job is offset by the increase in the equilibrium wage and the lower job-filling probability decreases the expected return on a unit of savings and hence, by the free entry condition, the interest rate has to fall.

# 1 Introduction

In most macroeconomic models, an individual's decisions can be divided into two kinds – consumption/saving decisions and labor market decisions. I aim to develop a model with frictional labor markets, in which both can be analyzed.

There are several ways one can model the labor market. One of them is the *random search* framework.[1] Random search, in its most basic form, means that in every period, with a certain probability (or at a certain Poisson rate in continuous time), the unemployed receive a wage offer which is a random variable that follows an exogenous probability distribution. Most often, the optimal decision rule can be characterized by a *reservation wage*, meaning that every offer above a certain level will be accepted and all offers below that will be rejected. However, workers can neither influence the probability of receiving an offer nor the offer itself. In my model, I also intend to model the firms in the economy as well. In this case, the most widespread wage determination procedure is *Nash-bargaining*. This solution concept gives us a rule on how to share the surplus from the match between the worker and the firm in the form of an equilibrium wage. The worker's surplus is increasing in the wage while the firm's surplus is decreasing. However, with heterogeneity among workers with respect to their wealth, the worker's surplus would clearly depend on her wealth and hence so would the wage. My aim is to build a model with a single equilibrium wage. This could, in principle, be achieved by introducing information asymmetries, however, this would make the model intractable.

Because of this, I model the labor market as in the *directed search* framework as described in the papers of Moen (1997) and Acemoglu and Shimer (1999).[2] Directed search means that the labor market consists of several sub-markets. They make up a full partition of the possible values of variables of interest to those doing search. In the simplest form, sub-market are labeled by the wage offered. The workers looking for a job know the set of sub-markets and also the probability of finding a job in each of them and make a decision on which sub-market to search in. The probability depends on the number of *vacancies* and the number of unemployed searching in the same sub-market. Their choice is based on the trade-off between earning a higher conditional on finding a job having a higher value, but a lower probability of actually getting a job.

Workers' saving decisions and labor market conditions could be connected. Assume that vacancies are created using savings as input. Workers directly investing in vacancies would introduce idiosyncratic risk, though, since jobs are filled with a certain probability in each sub-market and the outcome is stochastic. I get rid of this idiosyncratic risk by

[1] See e.g. Gronau (1971), McCall (1970), Mortensen (1970) and Pissarides (1985).

[2] Other influential papers on the topic include Montgomery (1991), Shimer (1996), Delacroix, Shi (2006), Menzio, Shi (2010).

incorporating another agent into the model, the *holding*. The holding pays interest on the workers' savings and faces the risk on the labor market. In expectation, its profit should be zero, which determines the interest rate endogenously. This way I create a general equilibrium model, where labor market and saving decisions might be separable on the individual level, however, on the aggregate, savings affect the trade-off workers face when applying to a job. I rule out the possibility of on-the-job search, meaning that only the unemployed search.

Aiyagari (1994) analyzes a general equilibrium model with heterogeneous workers, whose initial asset level follows a given distribution.[3] Besides that, they face idiosyncratic risk to their wage, which they cannot insure against. The production side of the economy is as in a standard neoclassical model, with perfect factor markets. The author shows that the equilibrium interest rate, determined by the level of aggregate savings as a function of the interest rate and capital as a function of the interest rate derived from the marginal product and capital and aggregate savings being equal, is lower than the subjective discount rate, whereas the two are equal in the case of complete markets or, equivalently, full insurance. The article then shows that the precautionary savings motive in such models is not necessarily a product of a convex marginal utility function, but incomplete markets, borrowing limits and the general equilibrium determination of the interest rate. The precautionary savings motive, however, is not particularly strong for plausible parameter values.

In this thesis, I also develop a general equilibrium model with incomplete markets, however, the production side of the economy and the labor market are different from those in Aiyagari (1994) and so is the nature of uncertainty the workers face. Also, to a certain extent, workers can decrease the level of uncertainty through directed search, since they know the probability of finding a job in the sub-market they choose. Nevertheless, it is an interesting question, how the equilibrium interest rate is related to the subjective discount rate in this economy.

The rest of the thesis is organized as follows. Section 2 first defines the model, characterizes the optimal behavior of workers, then defines the steady state equilibrium of the economy. Section 3 discusses the results of numerical experiments. First the baseline calibration of the model is described, characterizing the steady state for multiple parameter settings. Existence and uniqueness of the steady state is also discussed for this specific calibration. This is followed by analyzing the comparative statics with respect to a one percent increase in productivity. Section 4 concludes.

[3] For other general equilibrium models with heterogeneous workers, see Bewley (1983) and Huggett (1993).

# 2 The Model

For my analysis, I use a discrete time model with three types of agents the economy.

First, there is a unit measure of *workers*, who, at any point in time, can be either employed or unemployed. Unemployed workers also search on the labor market, which, as usual in the directed search framework, consists of several sub-markets, where one-period contracts are offered. This means that a worker who finds a job becomes unemployed at the beginning of the next period. At the same time, a worker can only search in one sub-market.[4] In each of them, a job can be found with a certain probability depending on the tightness of the sub-market, i.e. the ratio between the number of vacancies and unemployed workers looking for a job. Using a standard constant-returns-to-scale matching function, job-finding probability is defined as the total number of matches divided by the number of unemployed in the sub-market. There is no possibility for the employed to do on-the-job search.[5] Workers can save or borrow at a deterministic interest rate. Interest on savings is paid by the *holding*.

The holding uses the aggregate net savings of the workers to invest in vacancies in a *firm*. Vacancies are filled with a certain probability depending on the tightness of the sub-market in which they are posted. Job-filling probability is defined as the number of total matches divided by the number of vacancies in the sub-market. The holding does not post vacancies in sub-markets where there is no search in equilibrium. The holding has zero profit.[6] All workers who find a job at the firm have the same productivity.

In the remainder of this section, I define the workers' search and savings problems and characterize their solutions. It is followed by the definition of the steady state equilibrium.

## 2.1 The Workers' Problem

Unemployed workers have to make a savings and a search decision, while employed workers only have to choose their optimal level of savings. Both types discount the future at rate $\beta < 1$ and their preferences are represented by a per-period utility function of the *constant absolute risk aversion* (CARA) class, with the same parameter $\alpha > 0$. We assume that workers know the gross interest rate $R$ and the functions describing the job-

[4]Galenianos, Kircher (2009) explores the possibility of multiple job applications. Here multiple job applications would not change the nature of the workers' behavior, but it would add wage dispersion in the equilibrium.

[5]See e.g. Menzio and Shi (2010) and Shi (2009) on on-the-job search.

[6]The role of the holding is important, even though it does not face an optimization problem. If workers were to invest in vacancies directly, their returns would be subject to idiosyncratic risk because of the labor market frictions.

finding probability and sub-market tightness, $p(\theta)$ and $\theta(w)$ respectively.[7] $p(\theta)$ is strictly increasing. Let $a$ denote beginning-of-period assets, $U(a)$ the value of being unemployed with asset level $a$ and $W(a, w)$ the value of being employed with wage $w$ and asset level $a$. The total resources at a worker's disposal are her assets held at the beginning of the period and her income – wage for the employed, benefit $b$ for the unemployed, which includes value of leisure as well – in the current period. She can spend these resources on consumption, denoted by $c$, and buying assets. Hence her budget constraint implies

$$c_i = a_i + x_i - \frac{\hat{a}_i}{R} \tag{1}$$

where $x_i$ is the individual's income in the current period and $\hat{a}_i$ is next period's asset level at the beginning of the period, before wages or unemployment benefits are paid.

This leads to the following Bellman equations.

$$U(a) = \max_{w,\hat{a}_i} u(b + a - \Delta\hat{a}_i) + \beta(p(\theta(w))W(\hat{a}_i, w) + (1 - p(\theta(w)))U(\hat{a}_i)) \tag{2}$$

$$W(a, w) = \max_{\hat{a}_i} u(w + a - \Delta\hat{a}_i) + \beta U(\hat{a}_i) \tag{3}$$

where $u(c) = -e^{-\alpha c}$ and $\Delta = \frac{1}{R}$.

To solve the above optimization problems, we guess and verify. The following guesses are used.

$$U(a) = -e^{-\gamma a - C}$$
$$W(a, w) = U(a)X(w)$$
$$X(w) = e^{-\chi(w-b)-D}$$

for some real parameters $\gamma$, $\chi$, $C$ and $D$. Detailed computations can be found in the Appendix. The following characterization of the optimal wage is derived.

$$\bar{w} = \arg\min_{w} p(\theta(w))(e^{-\alpha(1-\Delta)(w-b)-\Delta\log(\bar{\psi}+1)} - 1) \tag{4}$$

where $\bar{\psi}$ is the optimal value of the objective. Given our guesses, $\bar{\psi}$ can be formulated as

$$\bar{\psi} = p(\theta(\bar{w}))(X(\bar{w}) - 1) = \frac{p(\theta(\bar{w}))W(\hat{a}_i, \bar{w}) + (1 - p(\theta(\bar{w})))U(\hat{a}_i) - U(\hat{a}_i)}{U(\hat{a}_i)} \tag{5}$$

where $\hat{a}_i$ denotes the optimal level of next period's assets and $\bar{w}$ denotes the optimal wage; hence $\bar{\psi}$ equals the difference between the expected continuation value and the value of staying unemployed for sure, $U(\hat{a})$, relative to the latter. Note that, however,

[7] Since in every sub-market one-period contracts are offered, they are labeled by only the wage for the one period.

$U(\hat{a})$ is always negative, hence $-\bar{\psi}$ should be interpreted as the *return on search* and so a lower $\bar{\psi}$ means a higher return.

Note that neither the objective function in (4), nor the set of sub-markets available to an individual depends on her level of assets. Thus, in equilibrium, all unemployed search in the same sub-market and there is only one wage in the economy.[8] This result depends heavily on the CARA utility assumption, namely – as shown in the Appendix – on the fact that the assumption allows us to write $W(a, w)$ as a product of two terms, one that is only a function of the asset level, and the other only a function of the wage. Hence the two decisions of the unemployed can be separated.[9]

Regarding the savings decisions of the unemployed and employed, the following optimal decision rules can be derived, respectively.

$$\hat{a}_i^U(a) = a + \frac{1}{\alpha(1-\Delta)}(\log\beta - \log\Delta + \log(\bar{\psi}+1)) \tag{6}$$

$$\hat{a}_i^W(a, w) = a + w - b + \frac{1}{\alpha(1-\Delta)}(\log\beta - \log\Delta + \Delta\log(\bar{\psi}+1)) \tag{7}$$

In both (6) and (7), $a$ has coefficient one. It means that a worker who has an asset level higher by one unit will, ceteris paribus, also have an asset level higher by unit next period. Equivalently, a worker only consumes a $1-\Delta$ fraction of her beginning-of-period wealth. Since the fraction $\frac{1}{\alpha(1-\Delta)}$ is positive, a higher $\beta$ or lower $\Delta$ gives an incentive for a worker to save more. A more patient worker values future consumption more and wants to save more and higher returns give a worker better trade-off between consumption today and consumption tomorrow. Note that $\log\beta - \log\Delta < 0$ holds if and only if $\beta R < 1$. The latter usually induces an increasing consumption path, because it means that a consumer gets higher return on her savings that what she loses by not consuming the amount of goods today.

If the return on search increases – $\bar{\psi}$ decreases – both the unemployed and the employed lower their optimal level of savings. Intuitively, if an unemployed gains more by search then she is better off in the next period in expectation and a lower amount of savings is needed to achieve a certain level of utility in the next period, which makes the marginal utility of consumption relatively higher in the current period which makes it possible for her to increase her life-time utility by consuming more in the current period. The effect is stronger for the unemployed, since they face a search decision one period earlier.

[8]This result is in line with the findings of Shimer, Werning (2007), who show that in their model, which belongs in the random search framework, an individual's optimal search decision, characterized by her reservation wage, does not depend on her wealth and thus in an economy with heterogeneous agents, there is only one reservation wage.

[9]This also implies that $\bar{\psi}$ does not depend on $a$, which means that every individual has the same return on search regardless of her wealth.

The employed also save more if their wage increases or unemployment benefit decreases. The first is straightforward and the latter comes from the fact that contracts last only one period. All the employed this period will become unemployed at the beginning of next period, but a higher benefit means a lower amount of savings is needed to attain the same level consumption next period. Hence the motive to smooth consumption implies an increase in this period's consumption.

Define the aggregate asset levels as

$$A = \int a_i di \tag{8}$$

$$\hat{A} = \int \hat{a}_i di \tag{9}$$

For a given unemployment rate $u$, we can obtain the following form for next period's aggregate asset level using the optimal decision rules in (6) and (7) and the fact that, in equilibrium, there is only one wage in the economy.

$$\hat{A} = A + \frac{\log\beta - \log\Delta}{\alpha(1-\Delta)} + \frac{u\log(\bar{\psi}+1))}{\alpha(1-\Delta)} + (1-u)\Big(\bar{w} - b + \frac{\Delta\log(\bar{\psi}+1))}{\alpha(1-\Delta)}\Big) \tag{10}$$

## 2.2 The Holding and the Firm

Let $J(w)$ denote the value of a filled job in sub-market $w$. A filled job produces revenue $y$ in each sub-market. In sub-market $w$, wage $w$ is paid to the worker. After the contract expires, the job and also the vacancy disappears, hence the continuation value is zero. This implies the Bellman equation

$$J(w) = y - w \tag{11}$$

The holding invest the aggregate savings of the workers in vacancies at the firm. In order to create a vacancy, $\frac{1}{k}$ units of savings are needed. This means

$$v = \frac{\Delta\hat{A}}{k} \tag{12}$$

In sub-market $\bar{w}$, a vacancy is filled with probability $q(\theta(w))$, where the strictly decreasing function $q(\theta)$ is known. For every unit of aggregate savings, the holding pays $\frac{1}{\Delta}$ gross interest. The holding has zero profit, also known as the *free entry condition*. Intuitively, if the holding had a positive profit, another holding could enter the market and use the same technology to create vacancies and the profit of the holding would decrease. On the other hand, if the holding had a negative profit, it would exit the market. This leads to the free entry condition

$$\frac{1}{\Delta} = \frac{q(\theta(\bar{w}))(y-\bar{w})}{k} \tag{13}$$

### 2.3 Equilibrium

It has already been established, that there is only one wage in equilibrium and all unemployed workers search in the same sub-market. Thus all vacancies are created in the same sub-market as well. Also, only one sub-market tightness, job-finding and job-filling probability has to be determined in equilibrium. Since vacancy creation does not depend on the distribution of savings and the aggregate level of savings does not depend on the distribution of beginning-of-period asset levels,[10] the steady state equilibrium does not depend on these distributions either. This leads to the following definition of the steady state equilibrium.

**Definition 1.** *The* steady state equilibrium *is a 7-tuple ($u$, $A$, $\bar{w}$, $\Delta$, $v$, $\bar{\theta}$, $\bar{\psi}$), where*

*(a) $\bar{w}$ is the minimizer of the problem in (4) given $\Delta$ and functions $p(\theta)$, $\theta(w)$*

*(b) $\bar{\psi}$ is the optimal value of the objective*

*(c) $q(\theta(\bar{w}))$ is determined by (11) and (13)*

*(d) $\bar{\theta} = q^{-1}(q(\theta(\bar{w})))$*

*(e) $u = \frac{1}{1+p(\bar{\theta})}$*

*(f) $v = \bar{\theta} u$*

*(g) $A = \frac{kv}{\Delta}$ from (10)*

*(h) $\Delta$ is the solution to*

$$0 = \frac{\log\beta - \log\Delta}{\alpha(1-\Delta)} + \frac{u\log(\bar{\psi}+1))}{\alpha(1-\Delta)} + (1-u)\Big(\bar{w} - b + \frac{\Delta\log(\bar{\psi}+1))}{\alpha(1-\Delta)}\Big)$$

Definition 1 also shows how the equilibrium can be computed. Points (e) and (i) can be derived as follows. If $u'$ and $A'$ denote next period's states, then the model would imply the following laws of motion

$$u' = 1 - \bar{p}u \tag{14}$$

$$A' = \hat{A} \tag{15}$$

The economy is in steady state if the state variables do not change, i.e. $u' = u$ and $A' = A$. These two conditions are equivalent to

$$u = \frac{1}{1+\bar{p}} \tag{16}$$

$$0 = \frac{\log\beta - \log\Delta}{\alpha(1-\Delta)} + \frac{u\log(\bar{\psi}+1))}{\alpha(1-\Delta)} + (1-u)\Big(\bar{w} - b + \frac{\Delta\log(\bar{\psi}+1))}{\alpha(1-\Delta)}\Big) \tag{17}$$

[10] As can be seen from (12) and (10), respectively.

As already established, an individual's wealth and savings decision do not play a role in her search decision. However, aggregate savings are used to create vacancies, which affects sub-market tightness. Tightness, in turn, affects the trade-off the unemployed face between the higher value from working at a higher wage and the lower probability of actually finding a job in the sub-market.

On the other hand, changes on the labor market can also affect the savings decision of both the unemployed and the employed. Since the return of the holding on a unit of invested savings depends on labor market conditions – directly on the job-filling probability – they can have an effect on $\Delta$ through the free entry condition.

This interrelatedness of aggregate savings and labor market conditions allow for interesting mechanisms. Think of an increase in productivity $y$. Its direct effect is an increase in the value of a filled job. From the free entry condition, either $q(\bar{\theta})$ or $\Delta$ has to decrease (or both).

If $q(\bar{\theta})$ decreases, it means that $\bar{\theta}$ increases as $q(\cdot)$ is decreasing. This gives the unemployed a more favorable trade-off in their search decision, which likely results in an increase in the equilibrium wage. This, however, as a secondary effect, counteracts the increase in the value of a filled job and hence the secondary effect in all variables comes with the opposite sign.

If $\Delta$ decreases, workers have an incentive to save more. More savings results in more vacancies and, since the unemployment rate is predetermined, a higher sub-market tightness. This, following the previous argument, likely results in a higher wage and a lower $q(\bar{\theta})$. However, that has a secondary negative effect on the returns of the holding and hence $\Delta$ has to increase.

The net effect of the above mechanisms is unclear. In the next section, I examine the comparative statics in the model with respect to an increase in productivity in order to find out which mechanism dominates for a reasonable set of parameters.

# 3 Numerical Experiments

In this section, I first discuss the calibration of the model and the steady state values for this parameter setting. Then I proceed by examining the differences between the new steady states and the baseline after an increase in productivity.

## 3.1 Baseline Calibration

One period corresponds to one quarter. In the model $p(\theta)$ and $q(\theta)$ assumed to be given. I will derive them from an assumption on the *matching function*. Let $M$ denote the number of total matches and assume

$$M(u, v) = \frac{uv}{(u^\lambda + v^\lambda)^{1/\lambda}} \tag{18}$$

as introduced in Den Haan, Ramey, Watson (2000), which implies

$$p(\theta) = \frac{M(u,v)}{u} = \frac{\theta}{(1+\theta^{\lambda})^{1/\lambda}} \tag{19}$$

$$q(\theta) = \frac{M(u,v)}{v} = \frac{1}{(1+\theta^{\lambda})^{1/\lambda}} \tag{20}$$

Note that $p : \mathbb{R}_+ \to [0,1)$ and $q : \mathbb{R}_+ \to [0,1)$ and $p(\cdot)$ is strictly increasing, while $q(\cdot)$ is strictly decreasing as expected.

Now the following parameters of the model have to be pinned down: $\beta$, $y$, $b$, $\alpha$, $k$, $\lambda$. In the baseline model I use $\beta = 0.95$ for the subjective discount rate. The productivity parameter is normalized to $y = 1$ as usual. $b = 0.4$ is a usual calibration in the literature.[11] For $\lambda$, I use the value in the original paper (Den Haan, Ramey, Watson (2000)), $\lambda = 1.27$. I calibrate $k$ such that the savings rate in the economy[12] is around 20% and $k = 0.12$ is the right parameter value. For $\alpha$, I will repeat the calculations for $\alpha = 10$, $\alpha = 20$, $\alpha = 40$ and $\alpha = 100$, the values used in Acemoglu, Shimer (1999), excluding the $\alpha = 0$ limit case.

For the workers' problem, $\theta(w)$ is also assumed to be known. This function is determined by the free entry (13) and the assumption for $q(\cdot)$ (20), i.e.

$$\theta(w) = \left( \left( \frac{(y-w)\Delta}{k} \right)^{\lambda} - 1 \right)^{1/\lambda} \tag{21}$$

Note that for $w > y - \frac{k}{\Delta}$ the function is not defined. Also, I exclude $\Delta < \frac{k}{y-b}$, since for such values all sub-markets other than $b$ would be ruled out.

The first question is whether the steady state exists. Given $\bar{w}$ and $\Delta$, one can easily solve for the steady state. Also, if $\bar{w}$ and $\Delta$ are uniquely determined, the steady state is unique. To check this, we have to check the optimization problem in (4) and (17) – using (16) for the steady state unemployment rate. These two give us two $(w, \Delta)$ loci, shown in Figure 1. The "FOC" curve refers to the solutions (4) for the given $\Delta$. However, there are values for $\Delta$ for which no $w \in [b, y - k/\Delta]$ solves (17). For such values, the "dA" curve shows $w = 0$. As we can see, there is only one point where the two loci touch, which means that there is a unique steady state. For Figure 1, $\alpha = 10$ was chosen, but there is no difference in the existence and uniqueness for the higher $\alpha$ values.

The baseline steady state values for the four parameter settings, to which changes will be compared when discussing comparative statics, are shown in Table 1.

$\bar{\psi}$ is negative for all four parameter settings, which means that there *is* a gain from search for the unemployed. Also, $\Delta > \beta$ or, equivalently $\beta R < 1$ in all four cases. This is in line with the results of Aiyagari (1994), who finds that in a general equilibrium model

[11] See e.g. Hornstein, Krusell, Violante (2011).

[12] $\frac{\Delta \hat{A}}{y v \bar{q}} = \frac{k}{y \bar{q}}$ from (12).

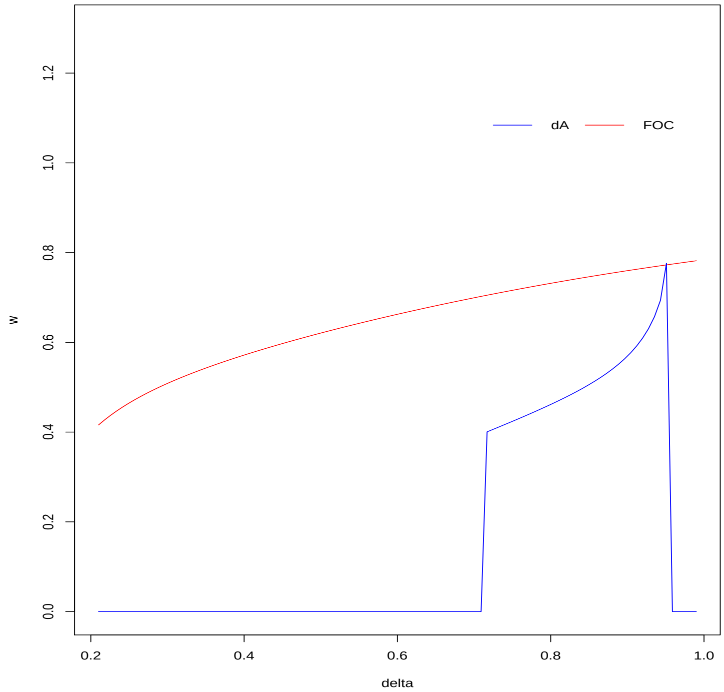


Figure 1: The $(w, \Delta)$ loci from the optimal wage decision and the no change in asset levels. The "FOC" curve describes the locus given by the optimization problem in (4) and the "dA" curve shows the locus given by the no change in asset levels in the steady state. As can be seen, the two curves only intersect once, hence the equilibrium exists and it is unique.

| | | Values of $\alpha$ | | | |
|---|---|---|---|---|---|
| | | 10 | 20 | 40 | 100 |
| $\Delta$ | objective discount factor | 0.951 | 0.9535 | 0.96 | 0.9745 |
| $\bar{w}$ | wage | 0.7725 | 0.7697 | 0.7658 | 0.761 |
| $R$ | gross interest rate | 1.0515 | 1.0488 | 1.0417 | 1.0261 |
| $u$ | unemployment rate | 0.6235 | 0.6205 | 0.6158 | 0.6091 |
| $\bar{\theta}$ | sub-market tightness | 1.0883 | 1.1191 | 1.1689 | 1.2459 |
| $A$ | aggregate asset level | 0.0856 | 0.0874 | 0.09 | 0.0934 |
| $p(\bar{\theta})$ | job-finding probability | 0.6037 | 0.6117 | 0.624 | 0.6418 |
| $q(\bar{\theta})$ | job-filling probability | 0.5548 | 0.5466 | 0.5338 | 0.5151 |
| $\bar{\psi}$ | - return on search | -0.0667 | -0.1212 | -0.1956 | -0.2863 |
| $v$ | measure of vacancies | 0.6786 | 0.6944 | 0.7198 | 0.7589 |

Table 1: Baseline steady state values for the different values of $\alpha$

with incomplete markets, the equilibrium interest rate is smaller than the subjective discount rate, which is equivalent to $R < \frac{1}{\beta}$.

We can observe that all variables either decrease or increase with higher $\alpha$ values. A higher $\alpha$ means a higher level of risk-aversion. The uncertainty workers face in this economy comes from the fact that finding a job is a random event. Intuitively, a more risk-averse individual would try to decrease this uncertainty if possible. When the unemployed make their decision on which sub-market to search in, they face a trade-off between achieving higher utility conditional on finding a job, by searching in a sub-market with a higher wage, but getting the pursued job with a lower probability. For a more risk-averse individual, the probability of finding a job is more important than for one who is less risk-averse. Hence as $\alpha$ increases, the unemployed have an incentive to search in a sub-market where the job-finding probability is higher, which means a higher sub-market tightness and lower wage. A higher job-finding probability means lower unemployment and a higher sub-market tightness means a lower job-filling probability. The number of vacancies is the product of sub-market tightness and the unemployment rate. Since the first increases while the latter decreases, the net effect is unclear in general. For the functional forms specified at the beginning of the section, these changes result in an increase in vacancies.[13] Intuitively, $\bar{\psi}$ should decrease as from (4), it is the fixed point of the function

$$f(x) \equiv p(\theta(\bar{w}))(e^{-\alpha(1-\Delta)(\bar{w}-b)-\Delta\log(x+1)} - 1) \tag{22}$$

Since $-\alpha(1-\Delta)(\bar{w}-b)$ is negative and decreased and $\bar{\psi}$ was very close to zero, the term in parenthesis should be negative and have increased in absolute value. Since $p(\bar{\theta})$ increased, it further decreases the value of $f(x)$ at least for $x$'s close to the old steady state. Clearly $f(x) - x$ is decreasing in $x$. If a decreasing function is shifted downwards, the new root should be lower.

A similar analysis can be done for the right-hand side of (17). Clearly, it is decreasing in $\Delta$. The shift is not obvious, though. The first two terms decreased for every value of $\Delta$, but $1-u$ increased while $\bar{w} - b - \frac{\Delta\log(\bar{\psi}+1)}{\alpha(1-\Delta)}$ decreased. The net effect is shown in Figure 2.[14]

We can see that for $\Delta$ higher than a certain value, the curves are shifted upwards. Also, that value is smaller than $\beta$ and thus also smaller than the roots. So on the relevant part of the domain, the shift is upwards. Hence a higher $\Delta$ is required for the level of assets to remain unchanged.

[13] These calculations can be found in the Appendix.

[14] For all other variables, the appropriate steady state levels have been plugged in.

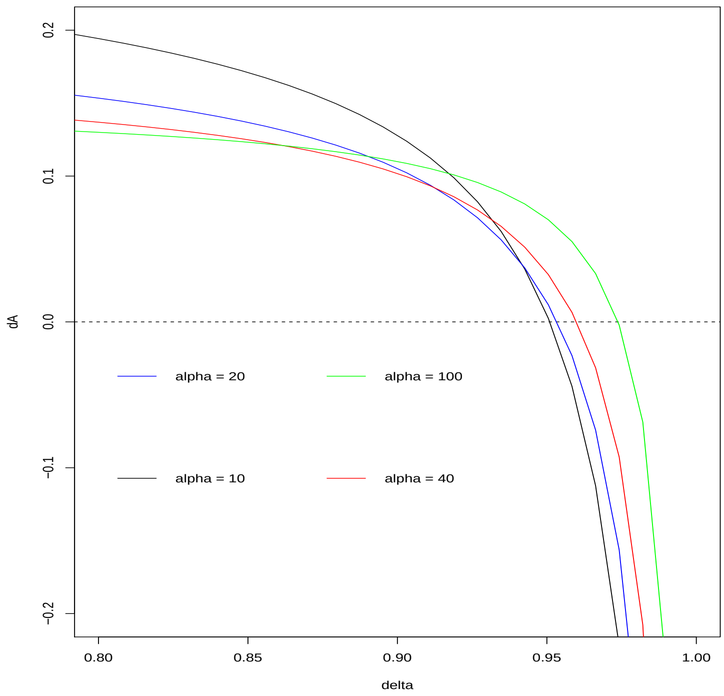


Figure 2: The the change in assets as only a function of delta. The horizontal dashed line is at the value 0. For $\Delta$ greater than a certain value – which is smaller than the old root – the shift in all curves is upward. Hence the new roots have to be higher and thus as $\alpha$ increases, so does $\Delta$.

From (12) and $\hat{A} = A$ in steady state, we get

$$A = \frac{vk}{\Delta} \tag{23}$$

Since $v$ and $\Delta$ both increased, in general, the effect on $A$ is unclear. The desired labor market conditions require more vacancies and thus more savings, however, the lower return serves as an incentive to save less. For the given calibration, the net effect is an increase in the asset level.

### 3.2 Comparative Statics

After discussing the calibration and steady state values for different parameter settings, I examine the effect of a 1% change in productivity $y$. Table 2 shows percentage changes in the ten variables for the different values of $\alpha$.

The increase in $\Delta$ or, equivalently, the decrease in $R$ is surprising at first sight. In most macroeconomic models, a positive productivity shock results in a higher interest rate, mostly due to the fact that it directly raises the marginal product of capital for a fixed capital-to-labor ratio. Of course, the two results cannot be compared so easily.

| | | Values of $\alpha$ | | | |
|---|---|---|---|---|---|
| | | 10 | 20 | 40 | 100 |
| $\Delta$ | objective discount factor | 0.0037 | 0.0117 | 0.0254 | 0.0309 |
| $\bar{w}$ | wage | 1.1048 | 1.0987 | 1.0929 | 1.0859 |
| $R$ | gross interest rate | $-0.0037$ | $-0.0117$ | $-0.0254$ | $-0.0309$ |
| $u$ | unemployment rate | $-0.2167$ | $-0.2219$ | $-0.2249$ | $-0.2214$ |
| $\bar{\theta}$ | sub-market tightness | 1.2282 | 1.2717 | 1.3127 | 1.3297 |
| $A$ | aggregate asset level | 1.0051 | 1.0351 | 1.0592 | 1.074 |
| $p(\bar{\theta})$ | job-finding probability | 0.5769 | 0.586 | 0.5867 | 0.5677 |
| $q(\bar{\theta})$ | job-filling probability | $-0.6434$ | $-0.6771$ | $-0.7166$ | $-0.7519$ |
| $\bar{\psi}$ | - return on search | $-2.4861$ | $-2.2369$ | $-1.7927$ | $-1.2045$ |
| $v$ | measure of vacancies | 1.0088 | 1.047 | 1.0848 | 1.1053 |

Table 2: Percentage changes following a 1% increase in productivity

Here, we are looking at permanent changes and differences in steady states, whereas in new-classical models the aim is to see what dynamics a one-period shock induces in variables of interest. Still, the drop in returns seems counter-intuitive, even though the change is small.

First, the productivity increase only has a direct effect on the $\theta(w)$ function, namely that it is shifted upwards. This means that the unemployed face better match-ups when making their search decision. It is now possible for them to search in a sub-market with a higher wage which also has a higher job-finding probability. And, indeed, both $\bar{w}$ and $\bar{\theta}$ increase. This clearly increases the return on search. Since the job-finding probability is increasing and the job-filling probability is decreasing in sub-market tightness, the first increases while the latter decreases. Unemployment drops since it is decreasing in the job-finding probability. Based on the analysis in the previous subsection, the number of vacancies is increasing in sub-market tightness and thus increases as well.

The wage increase and the drop in unemployment both increase the desired change in assets for any $\Delta$. However, the increased return on search decreases it. Figure 3 shows the two curves for $\alpha = 10$ with all variables other than $\Delta$ held fix.

The horizontal dashed line is at the value 0 and the vertical is at the steady state level of $\Delta$ with parameter $\alpha = 100$. Since all else held fixed, both curves are strictly decreasing in $\Delta$ and at the root the shift was still upward, the root of the new curve cannot be lower and in fact has to be greater.

From (23), since the increase in $v$ is larger (in percentage terms) than in $\Delta$, $A$ increases as well.

Another way to look at the change in $\Delta$ is through the profit of the holding. Almost all of the gain in the value of a filled job is gone because the firm has to pay a higher

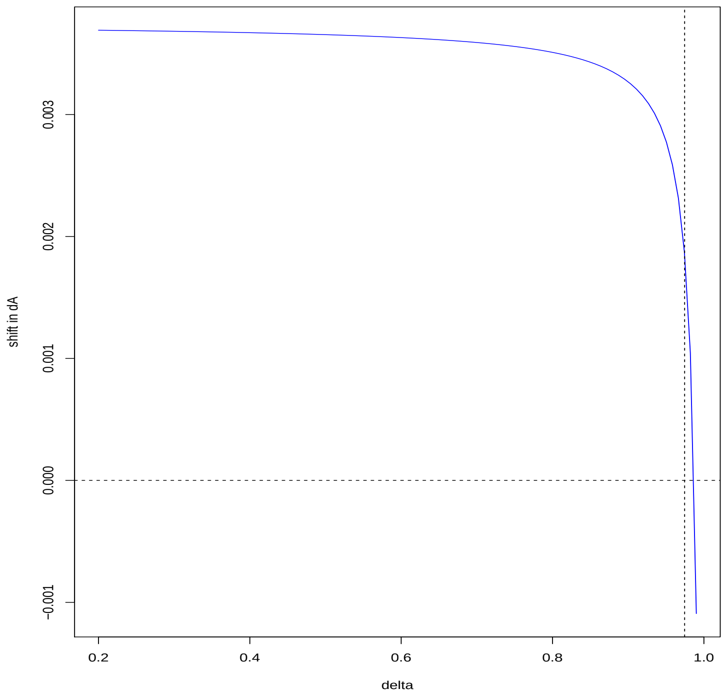


Figure 3: The shift in the change of asset levels all variables other than $\Delta$ held fixed. The horizontal dashed line is at the value 0 and the vertical one is at the steady state level of $\Delta$ with $y = 1$ and $\alpha = 100$. Since at the root the shift is upward, the new root has to be higher.

wage to its employees. Note that $y$ increases by 1%, but $\bar{w}$ increases by even more. Almost all of the gain goes to the workers. On the other hand, sub-market tightness also increases, which lowers the probability of filling a job. The expected return on a unit of savings thus becomes lower and hence for the holding to not quit, the price it has to pay, i.e. the gross interest rate, has to go down.

# 4 Conclusion

In my thesis, I develop a general equilibrium model with incomplete markets and directed search on the labor market, without on-the-job search. The workers earn a deterministic return on their savings paid by the holding, who invests savings into vacancies. Thus the holding takes on the risk from the workers. The holding has a zero-profit (free entry) condition, which makes the endogenous determination of the interest rate possible. My main questions were the optimal behavior of workers, the steady state interest rate and the effects of a one percent increase in productivity on the steady state.

I characterized the optimal saving and search decisions. I showed that, in equilibrium, all the unemployed search in the same sub-market regardless of their wealth. Both types' optimal saving decision is linear in their wealth and have the same coefficient, hence the equilibrium can be defined in aggregate variables only, without the distribution of asset levels.

I calibrated the model using the matching function proposed by Den Haan, Ramey and Watson (2000). I used parameter values for the subjective discount rate, unemployment benefit and the parameter of the matching function, normalized productivity to one, used four different values for absolute risk aversion as in Acemoglu and Shimer (1999) and calibrated the vacancy creation cost to match a 20% savings rate in the steady state. I found that the steady state exists and is unique for these parameter settings and that the steady state values of several variables, namely sub-market tightness, level of assets, job-finding probability, return on search and the number of vacancies, increase as absolute risk aversion increases, while the interest rate, unemployment rate, job-filling probability and the optimal wage decrease.

Then I analyzed the effects of a one percent increase in productivity. I found that the steady state values of the wage, sub-market tightness, job-finding probability, level of assets, number of vacancies and the return on search increase and the job-filling probability, unemployment rate and surprisingly the interest rate decrease. The explanation for the latter is that by the increase in productivity, workers face a better trade-off between wages and job-finding probabilities and hence choose a higher wage, which takes away almost all of the increase in the value of a filled job. The lower job-filling probability then lowers the return on a unit of savings and hence its price, the interest rate has to fall from the zero-profit condition.

The possibility of introducing exogenous separation and on-the-job search into the model would enable us to take the model directly to the data. Also, it would be interesting to examine how the above two features alter the comparative statics. The full dynamics of the model could prove to be an interesting topic for future research, as well.

## References


[1] Acemoglu, Daron and Shimer, Robert (1999): Efficient Unemployment Insurance, The Journal of Political Economy, Vol. 107, No. 5 (Oct., 1999), pp. 893-928

[2] Aiyagari, S. Rao (1994): Uninsured Idiosyncratic Risk and Aggregate Saving, The Quarterly Journal of Economics, Vol. 109, No. 3 (Aug., 1994), pp. 659-684

[3] Bewley, Truman (1983): A Difficulty with the Optimum Amount of Money, Econometrica, Vol. 51, No. 5 (Sep., 1983), pp. 1485-1504

[4] Den Haan, Wouter J. and Ramey, Garey and Watson, Joel (2000): Job Destruction and Propagation of Shocks, American Economic Review, American Economic Association, vol. 90(3), pp. 482-498, June

[5] Delacroix, Alain and Shi, Shouyong (2006): Directed Search on the Job and the Wage Ladder, International Economic Review, Vol. 47, No. 2, pp. 651-699

[6] Galenianos, Manolis and Kircher, Philipp (2009): Directed Search with Multiple Job Applications, Journal of Economic Theory, Vol. 144, No. 2, pp. 445-471

[7] Gronau, Reuben (1971): Information and Frictional Unemployment, The American Economic Review, Vol. 61, No. 3, Part 1 (Jun., 1971), pp. 290-301

[8] Hornstein, Andreas and Krusell, Per and Violante, Giovanni L. (2011): Frictional Wage Dispersion in Search Models: A Quantitative Assessment, American Economic Review, American Economic Association, vol. 101(7), pp. 2873-98

[9] Huggett, Mark (1993): The Risk-free Rate in Heterogeneous-agent Incomplete-insurance Economies, Journal of Economic Dynamics and Control, Vol. 17, No. 5-6, pp. 953-969

[10] McCall, John J. (1970): Economics of Information and Job Search, The Quarterly Journal of Economics, Vol. 84, No. 1 (Feb., 1970), pp. 113-126

[11] Menzio, Guido and Shi, Shouyong (2010): Block Recursive Equilibria For Stochastic Models of Search on the Job, Journal of Economic Theory, vol. 145 (2010), pp. 1453-1494

[12] Moen, Espen R. (1997): Competitive Search Equilibrium, Journal of Political Economy, University of Chicago Press, vol. 105(2), pp. 385-411, April

[13] Montgomery, James D. (1991): Equilibrium Wage Dispersion and Interindustry Wage Differentials, The Quarterly Journal of Economics, Vol. 106, No. 1 (Feb., 1991), pp. 163-179

[14] Mortensen, Dale T. (1970): Job Search, the Duration of Unemployment, and the Phillips Curve, The American Economic Review, Vol. 60, No. 5 (Dec., 1970), pp. 847-862

[15] Pissarides, Christopher A. (1985): Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages, The American Economic Review, Vol. 75, No. 4 (Sep., 1985), pp. 676-690

[16] Shi, Shouyong (2009): Dynamic Search For Equilibrium Wage-Tenure Contracts, Econometrica, Vol. 77, No. 2, pp. 561-584

[17] Shimer, Robert (1996): Essays in Search Theory, PhD dissertation, MIT.

[18] Shimer, Robert and Werning, Ivan (2007): Reservation Wages and Unemployment Insurance, The Quarterly Journal of Economics, MIT Press, vol. 122(3), pp. 1145-1185, 08

# Appendix A

## Guess and verify

For the sake of convenience, I repeat the Bellman equations of the workers and the guesses.

$$U(a) = \max_{w,\hat{a}_i} u(b + a - \Delta\hat{a}_i) + \beta(p(\theta(w))W(\hat{a}_i, w) + (1 - p(\theta(w)))U(\hat{a}_i)) \tag{i}$$

$$W(a, w) = \max_{\hat{a}_i} u(w + a - \Delta\hat{a}_i) + \beta U(\hat{a}_i) \tag{ii}$$

where $u(c) = -e^{-\alpha c}$.

Guesses: $U(a) = -e^{-\gamma a - C}$, $W(a, w) = U(a)X(w)$, where $X(w) = e^{-\chi(w-b)-D}$.

First, I solve for the problem of the unemployed in (i) for a given $\Delta$ and differentiable $p(\theta)$, $\theta(w)$ functions. Rewrite (i) as

$$U(a) = \max_{w,\hat{a}_i} u(b + a - \Delta\hat{a}_i) + \beta(\psi(w) + 1)U(\hat{a}_i)$$

where $\psi(w) = p(\theta(w))(X(w) - 1)$. Plugging in our guess for $U(\cdot)$ and the CARA utility function, I get

$$U(a) = \max_{w,\hat{a}_i} -e^{-\alpha(b+a-\Delta\hat{a}_i)} - \beta(\psi(w) + 1)e^{-\gamma(\hat{a}_i)-C} \tag{iii}$$

From (iii) the FOC with respect to $\hat{a}_i$ is (iv) or, equivalently, (v)

$$-\alpha\Delta e^{-\alpha(b+a-\Delta\hat{a}_i)} - \gamma\beta(\psi(\bar{w}) + 1)e^{-\gamma(\hat{a}_i)-C} = 0 \tag{iv}$$

$$e^{-\alpha(b+a-\Delta\hat{a}_i)} = \frac{\gamma\beta}{\alpha\Delta}(\psi(\bar{w}) + 1)e^{-\gamma(\hat{a}_i)-C} \tag{v}$$

where $\bar{w}$ is the optimally chosen $w$, satisfying $\bar{w} = \arg\min_w p(\theta(w))(X(w) - 1)$.

Note that $U(\hat{a}_i)$ is always negative, hence the minimization in order to maximize the continuation value. Also, note that neither the objective function, nor the set of sub-markets depends on the level of assets held by the worker. Hence all unemployed will search in the same sub-market in equilibrium.

Let $\bar{\psi}$ denote $\psi(\bar{w})$.

From (iii), (v) and the guess for $U(\cdot)$, I get

$$-e^{-\gamma a - C} = -\beta\Big(1 + \frac{\gamma}{\alpha\Delta}\Big)(\bar{\psi} + 1)e^{-\gamma\hat{a}_i - C}$$

which implies

$$\hat{a}_i^U(a) = a + \frac{1}{\gamma}\Big(\log\beta + \log\Big(1 + \frac{\gamma}{\alpha\Delta}\Big) + \log(\bar{\psi} + 1)\Big) \tag{vi}$$

To obtain $\gamma$ and $C$, take the natural logarithm of both sides of (v) and plug in the optimal policy function obtained in (vi). This yields

$$-\alpha b - \alpha a + \alpha\Delta\Big(a + \frac{1}{\gamma}\Big(\log\beta + \log\Big(1 + \frac{\gamma}{\alpha\Delta}\Big) + \log(\bar{\psi}+1)\Big)\Big) =$$
$$= \log(\gamma\beta) - \log(\alpha\Delta) + \log(\bar{\psi}+1) - \gamma a - \Big(\log\beta + \log\Big(1 + \frac{\gamma}{\alpha\Delta}\Big) + \log(\bar{\psi}+1)\Big)\Big) - C \tag{vii}$$

Since (vii) has to hold for all $a \in \mathbb{R}$, by collecting terms, we arrive at

$$\gamma = \alpha(1-\Delta) \tag{viii}$$
$$C = \alpha b + \frac{\Delta}{1-\Delta}(\log\Delta - \log\beta - \log(\bar{\psi}+1)) + \log(1-\Delta) \tag{ix}$$

Next, I solve the optimization problem of the employed characterized by (ii).

Using the guess for $U(\cdot)$ the following first-order condition can be obtained for the optimal savings decision of the employed:

$$e^{-\alpha(w+a-\Delta\hat{a}_i)} = \frac{\gamma\beta}{\alpha\Delta}e^{-\gamma\hat{a}_i - C} \tag{x}$$

From (ii), using (x) and the guess for $W(a,w)$, one can obtain

$$e^{-\gamma a - C}e^{-\chi(w-b)-D} = \beta\Big(1 + \frac{\gamma}{\alpha\Delta}\Big)e^{-\gamma\hat{a}_i - C}$$

Taking logs and rearranging terms, we arrive at the optimal policy function of the employed

$$\hat{a}_i^W(a,w) = a + \frac{1}{\gamma}\Big(\log\beta + \log\Big(1 + \frac{\gamma}{\alpha\Delta}\Big) + \chi(w-b) + D\Big) \tag{xi}$$

From (x), using (xi), we get

$$-\alpha w - \alpha a + \alpha\Delta\Big(a + \frac{1}{\gamma}\Big(\log\beta + \log\Big(1 + \frac{\gamma}{\alpha\Delta}\Big) + \chi(w-b) + D\Big)\Big) =$$
$$= \log(\gamma\beta) - \log(\alpha\Delta) - \gamma a - \Big(\log\beta + \log\Big(1 + \frac{\gamma}{\alpha\Delta}\Big) + \chi(w-b) + D\Big)\Big) - C \tag{xii}$$

Since (xii) has to hold for all $a \in \mathbb{R}$ and $(w-b) \in \mathbb{R}_+$, by using (ix) and collecting terms, one can arrive at

$$\gamma = \alpha(1-\Delta) \tag{xiii}$$
$$\chi = \alpha(1-\Delta) \tag{xiv}$$
$$D = \Delta\log(\bar{\psi}+1) \tag{xv}$$

Plugging (viii), (ix) and (xiii)-(xv) into (vi) and (xi), we arrive at the the optimal policy functions

$$\hat{a}_i^U(a) = a + \frac{1}{\alpha(1-\Delta)}(\log\beta - \log\Delta + \log(\bar{\psi}+1)) \tag{xvi}$$
$$\hat{a}_i^W(a,w) = a + w - b + \frac{1}{\alpha(1-\Delta)}(\log\beta - \log\Delta + \Delta\log(\bar{\psi}+1)) \tag{xvii}$$

and the optimality condition $\bar{w} = \arg\min_w p(\theta(w))(e^{-\alpha(1-\Delta)(w-b)-\Delta\log(\bar{\psi}+1)} - 1)$.

## Change in the number of vacancies

By definition,

$$v = \bar{\theta}u = \frac{\bar{\theta}}{1 + p(\bar{\theta})}$$

which leads to

$$\frac{\partial v}{\partial \bar{\theta}} = \frac{1 + p(\bar{\theta}) - \bar{\theta}p'(\bar{\theta})}{(1 + p(\bar{\theta}))^2}$$

From (19),

$$p'(\bar{\theta}) = (1 + \bar{\theta}^{\lambda})^{-1/\lambda - 1}$$

and hence

$$1 + p(\bar{\theta}) - \bar{\theta}p'(\bar{\theta}) = 1 + \bar{\theta}((1 + \bar{\theta}^{\lambda})^{-1/\lambda} - (1 + \bar{\theta}^{\lambda})^{-1/\lambda - 1}) > 0$$

which implies $\frac{\partial v}{\partial \bar{\theta}} > 0$.