MSc Economics

Essays on Contract Theory with Unawareness of Actions

A Master's Thesis submitted for the degree of
"Master of Science"

supervised by
Dr. Martin Meier

Franz Peter Ostrizek, Bakk. rer. soc. oec.
0903803

Vienna, June 9th 2014

Acknowledgments

I would like to thank my adviser Dr. Martin Meier for the continuous support of my research. His guidance helped me in all the time of research and writing of this thesis.

I also want to thank Dr. Klaus Ritzberger for insightful discussions and the participants of the VGSE Micro Research Seminar for helpful comments.

MSc Economics

Affidavit

I, Franz Peter Ostrizek

hereby declare

that I am the sole author of the present Master's Thesis,

Essays on Contract Theory with Unawareness of Actions

57 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

Vienna, 9.6.2014

Signature

Contents

List of Figures

Abstract

We analyze a moral hazard problem with unaware agents and limited liability. A fraction of agents is unaware of their effort choice and chooses a default effort instead. The principal cannot observe whether an agent is unaware or not. He thus wants to screen the agents using a menu of contracts. Furthermore, the principal has the option to make all agents aware of their effort choice.

The first paper treats the case of a monopolist principal. We show that under monopoly unawareness is self-reinforcing and contrast the results under limited liability with those under risk aversion. In the limited liability case, unaware agents can receive a rent and aware agents may loose from the presence of unaware agents.

The second paper studies competing principals. Under competition, the share of unaware agents does not affect the equilibrium outcome. Agents are made aware even if this decreases total output. Thus, competition between principals can decrease efficiency.

1 Introduction

Individuals only consider a small subset of all possibilities in the complex world around them. When evaluating their choices, only a few contingencies are actively taken into account. The actions considered themselves are seldomly a complete set of possible acts. Individuals are simply unaware of a large portion of the world around them, including their action possibilities. They thus miss opportunities and make decisions that look foolish ex-post. Even though rational within their mental model, individuals may be genuinely surprised by information and actions they simply never considered.

Lack of awareness is not merely a psychological phenomenon but also important in the economic sphere. A large part of research and development is not about answering known questions but considering new possibilities. The same can be said about entrepreneurial decision making. Also beyond areas associated with creativity and creation awareness and its limit are important for economic considerations. Every no-arbitrage argument requires an individual who would be aware of this arbitrage opportunity.

Economic theory has a hard time capturing (un)awareness of individuals. The standard model of decision making tacitly assumes that all decision makers have a complete description of all relevant contingencies and choices in their mind. Based on this framework virtually all of economics worked with decision makers with a possibly uncertain but complete understanding of the world around them.

A field of economics that may profit a lot from modeling unawareness is contract theory. Two challenges for this literature are explaining the widespread use of incomplete contracts and very simple contracts used in real life situations.[1] In both cases unawareness can give potential explanations. An incomplete contract may be the only contract two unaware parties can write or it may allow them to preserve the other parties unawareness.[2] Simple contractual forms may be more robust to different subsets of contingencies or actions the contracting parties are aware of. Models of contracting with unawareness can also help to provide foundations for behavioral restrictions and delusional beliefs in explicit models of reasoning and cognition.

This thesis deals with unawareness of actions in contracting problems. We analyze how the presence of a share of unaware agents affects moral hazard problems. Since the type of an agent is not observable, principals have to resort to screening agents using a menu of contracts. In some cases, they have an incentive to make agents aware of the full problem, even though this introduces an incentive compatibility constraint. Furthermore, the contract offered to aware agents will be distorted from the second

[1] For a survey of the empirical literature on contract theory and further references, see Chiappori and Salanié (2003).

[2] For an explanation along these lines, see Tirole (2009).

best contract in order to allow for screening and reduce the rent that leaks to unaware agents.

We find that the qualitative effects of adding a share of unaware agents depend on the source of the agency friction. Under limited liability, unaware agents may gain and aware agents may lose from the presence of the other type. Competition between principals can reduce efficiency because productive unaware agents are made aware too often. Once we refine the set of equilibria using lexicographic preferences, this outcome is even unique.

This master thesis is organized as follows. Section 2 gives a review of the related literature. Section 3 is a paper analyzing a moral hazard problem with a monopolist principal, limited liability and a share of unaware agents. We show that the results from the problem under limited liability are qualitatively different from the risk aversion case considered in Thadden and Zhao (2012). Our findings generalize to the case of risk averse agents under limited liability. Section 4 analyzes a similar model with competing principals. Section 5 concludes.

2 Literature Review

The literature on contract theory with unawareness bridges the gap between formal models of knowledge accommodating unawareness and behavioral contract theory. Unawareness provides a microfoundation for delusion, lack of Bayesian updating and behavioral agents. When certain contingencies or actions are not in the mind of an agent, the usual requirements of rationality do not apply. Thus models with limited awareness enables us to combine rationality within the domain of awareness with deviations from rationality founded in realistic generalizations of knowledge models. This helps to explain many economic phenomena ranging from incomplete contracts over low powered incentives to shrouded attributes.

So far the association between contract theory with unawareness and formal models of unawareness is rather loose, however. In general, unawareness is modeled in a reduced form way. We follow the same approach in the main chapters of this thesis. We model unawareness of actions by a fixed default action. Furthermore, many contributions define their own equilibrium concepts rather than resorting to general theory. Nevertheless, an understanding of the formalism behind unawareness is helpful when thinking about unawareness and even required for modeling mutual knowledge of unawareness or using solution concepts geared towards games played by unaware agents. Ad-hoc models with unawareness may violate key axioms of rationality such as introspection in hidden ways. Furthermore, these undesirable features may drive the results. It is therefore important to base applied models on sound theoretical foundations that are constructed carefully to satisfy the desiderata of unawareness and knowledge.

In this literature review we provide an overview of the theory of unawareness focusing on models in the language of set theory - a formulation more familiar to economists. We then survey the growing literature in contract theory with unawareness and discuss related papers and themes from behavioral contract theory.

Unawareness

> „[T]here are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns – there are things we do not know we don't know."
>
> - Donald Rumsfeld, former Secretary of Defense

Unawareness can be loosely defined as the lack of all positive knowledge about an event. This goes beyond uncertainty as the individual cannot reason about or asses a specific event he is unaware of in any way. Clearly, unawareness also rules out the formation of beliefs. Also, an individual cannot know that he is unaware of a specific event. It is possible, however, that an individual knows that he is unaware of some event. For example, mathematicians are unaware of the specific new proof technique that will be used in the next proof of a Millennium problem theorem. Most mathematicians are aware of the possibility, however, that such a proof will involve some new technique and may even consider it likely.[3]

Non-trivial Unawareness is Impossible in Standard State Space Models

The standard partitional model of knowledge used in economics cannot accommodate this complete absence of knowledge. This model builds upon a state space and a knowledge partition. The state space is a set of real states, that is complete and mutually exclusive descriptions of the possible objective states of the world. The partition consists of cells that contain states the agent cannot distinguish. When the agent learns about the true state of the world, all he learns is the cell of the partition the true state is in but not which of these states is the case. We call a set of states an event and say that an event E is known if all states the agent considers possible are contained in the set E. Consequently, the set of states at which an event E is known is the union of all cells of the partition that are contained in it. We well denote this event by $K(E)$. In this model, it follows that whenever an event is not known, it is known that the event is not known. Formally, $\neg K(E) \implies K(\neg K(E))$. This relation is called negative introspection. It is easy to see that negative introspection has to hold in the partitional model. The event $\neg K(E)$ is simply the union of all cells of the partition that are not fully contained in E. As $\neg K(E)$ is defined as a union of cells of the partition, it is known whenever it occurs. Even the broadest definition of unawareness requires that an agent is unaware of an event E only if he doesn't know that it is true and doesn't know that he doesn't know, that is when negative introspection fails for this event. As we have seen, unawareness even in this broad sense is ruled out in the partitional model.

The incompatibility between standard models of knowledge and unawareness runs deeper than the partitional structure. The standard state space model generalizes the partitional model by using possibility correspondences instead of a partition. A possibility correspondence $P(\omega)$ associates with every state ω a set of state that the agent considers possible if the true state is ω. The partitional model is contained in this framework if we set $P(\omega)$ to the cell of the partition containing ω, but $P(\cdot)$ can have a more general structure. An event E is known at ω if all states that are considered possible at ω are contained in E, that is if $P(\omega) \subset E$. Dekel, Lipman and Rustichini (1998) show that also this more general class of models rules out nontrivial unaware-

[3] Note that some models of unawareness have difficulties expressing statements like these, even though the concept of unawareness doesn't rule them.

ness. More specifically, it rules out that an agent is unaware of some event E while knowing some other event E'. To arrive at this result, Dekel et al. (1998) use a notion of unawareness stronger than the minimal requirement given above. They define an unawareness operator U that has to satisfy the above minimal condition and two additional requirements: First of all, an agent never knows that he is unaware of a specific event ($KU(E) = \emptyset$, KU-introspection) and secondly unawareness implies unawareness of unawareness ($U(E) \subset UU(E)$, AU-introspection). These two requirements are natural as it seems paradoxical to have any knowledge about being unaware of a specific event and have become standard in the unawareness literature.[4] They show that an agent cannot be unaware in this sense when he knows anything. The intuition behind this result is that - loosely speaking - the standard axioms of knowledge prescribe that agents have a correct and complete conceptualization of the state space in mind. But that is exactly what we want to relax when thinking about unawareness. Thus a satisfying notion of unawareness clashes with the basic axioms of knowledge such as necessitation ($K(\Omega) = \Omega$) and monotonicity ($E \subset F \implies K(E) \subset K(F)$).[5]

Extensions Allowing for Unawareness

The key step to resolving this impossibility result is a change in the state space. Not only external facts, also the awareness of the agent has to be part of the description of a state. In such a setup, the basic axioms are reinterpreted to take the limited mental model of the agent into account. This way, unawareness can be accommodated. Numerous contributions build on constructions of this sort, either explicitly or by weakening the axioms in similar ways. There are two classes of models in this literature. A large share of contributions are formulated in terms of semantics, i.e. in a way closer to mathematical logic and computer science (e.g. Fagin and Halpern (1987); Galanis (2011); Halpern (2001); Halpern and Rêgo (2008, 2009, 2013); Heifetz et al. (2008); Modica and Rustichini (1994, 1999); Sillari (2008)). Some later contributions are formulated in set theoretic terms, an approach more familiar to economists (e.g. Board and Chung (2007); Heifetz et al. (2006, 2013b)). We do not attempt a full survey of this literature. Rather we point out selected contributions and present the framework of Heifetz, Meier and Schipper (2006) in more detail in order to point out some of the issues involved in constructing set theoretic models of knowledge with unawareness.

[4] Knowing that you are unaware of some event, however, is a valid possibility. Allowing for this possibility is the goal of some extensions of standard unawareness models. See Halpern and Rêgo (2009) and Halpern and Rêgo (2013).

[5] Chen et al. (2012) show that in this class of models, AU-introspection and KU-introspection are equivalent to negative introspection. Allowing violations of negative introspection via the use of non-partitional possibility correspondences is thus not sufficient to achieve unawareness, since the very same relaxation also rules out AU-introspection and KU-introspection.

The earliest model of knowledge with unawareness was introduced to theoretical computer science by Fagin and Halpern (1987). They add an awareness operator to a semantic model of knowledge. Geanakoplos (1989) analyzes games and decision problems with non-partitional information structures. He gives sufficient conditions for no-trade and no-agreeing-to-disagree results. The first model of unawareness in the economics literature is Modica and Rustichini (1994). Modica and Rustichini (1999) consider a logical system what allows for unawareness and partitional information structures. The models developed in MR are a special case of Fagin and Halpern (1987) (Halpern (2001)). These models are geared towards representing the knowledge and awareness of a single decision maker and are unable to express knowledge about the awareness of other decision makers. In contrast to standard models of knowledge, extending models of unawareness to allow for interactive epistemology of this sort is a nontrivial task. That is because decision makers can exhibit uncertainty about each others awareness so that the ascribed possibility set of the other decision maker resides in a less rich state space.

Li (2009) and Heifetz et al. (2006) independently develop a set theoretic model of interactive unawareness. We will present the approach of Heifetz et al. (2006) in more detail. The state space $\mathscr{S}$ consists of a complete lattice of spaces S, ordered by $\preceq$ denoting their expressive power. For every pair $S \preceq S'$ there is a surjective projection $r_S^{S'}$ that maps a set in S' to its less expressive description in S. These projection maps commute. Events in this structure are not simply subsets of the state space but include all equivalent descriptions in more descriptive spaces. Formally, an event is of the form $\bigcup_{S' \succeq S} \left(r_S^{S'}\right)^{-1}(B)$ for some B in S. This set contains all the states in which the event can be expressed and is true. The negation of the event is constructed similarly as $\bigcup_{S' \succeq S} \left(r_S^{S'}\right)^{-1}(\neg B)$, that is the set of states in which the event can be expressed and is not true. It follows that for any event E of this form there may be states at which neither E nor $\neg E$ obtains. These are the states at which E cannot be expressed. This is a deviation from the “real states” assumption.

For each individual i there is a possibility correspondence Π_i satisfying axioms that ensure the compatibility of the possibility correspondence and the lattice structure of the space. The corresponding knowledge operator K_i is defined in the usual way. Unawareness is defined by a lack of knowledge and negative introspection, i.e. $U_i(E) = \neg K_i(E) \cap \neg K_I(\neg K_i(E)$. An awareness operator A is defined as the negation of unawareness. Heifetz et al. (2006) show that these operators satisfy all desiderata of knowledge in the presence of unawareness. To see how these properties are modified to be compatible with unawareness, consider weak necessitation property. This property reads

$$A(E) = K\left(\bigcup_{S' \succeq S} \left(r_S^{S'}\right)^{-1}(S(E))\right)$$

where $S(E)$ is the least descriptive state that the event E can be fully expressed in.[6] Intuitively, this property says that the agent knows that one layer of the state space that is rich enough to express all events of which he is aware has obtained. Clearly, for full awareness and without the lattice state space this generalizes to the necessitation axiom. In the case with unawareness it allows the individual to be oblivious of some parts of the state space in a way the usual necessitation axiom does not.

The interactive nature of the model requires a complete description of the state space in a way not necessary in models of one individual only. Even states that are not part of any individuals possibility correspondences have to be modeled in order to capture one individual knowing that another is unaware of an event of which he is aware.[7]

Heifetz et al. (2008) provide a sound and complete axiom system for the model in Heifetz et al. (2006) and show that a state space constructed from maximally consistent sets of formulas naturally maps to the lattice state space under this set of axioms.

Probabilistic beliefs on these structures are considered in Heifetz et al. (2013b). They extend the notion of a common prior to this model and show that speculative trade with a common prior is possible in this framework. However a weaker no-trade theorem continues to hold, a common prior is incompatible with common certainty of strict preference to carry out a speculative trade. A consequence is that arbitrarily small transaction fees rule out speculative trade in this framework. Also a version of the no-agreeing-to-disagree theorem holds, individuals with a positive common prior cannot agree-to-disagree about the posteriors of events of which they all are aware.

Creating deviations from these two results based on unawareness may be possible if one considers not only unawareness of events but also unawareness of theorems. Galanis (2013) shows that if individuals are allowed to be unaware of the logical connections between events they are aware of, the no-trade theorem no longer holds and agents can agree to disagree. How far reaching these results are for weakened no-trade and agreeing-to-disagree theorems as in Heifetz et al. (2013b) is not immediate.

Halpern and Rêgo (2009) extend the logic of Fagin and Halpern (1987) to allow for expressing that an individual know that he is unaware of some event. Halpern and Rêgo (2013) extend this framework further to allow the agent to be uncertain whether there are events of which he is unaware. Formally, they allow quantifiers in their

[6] A semantic version of this property has been proposed by Dekel et al. (1998).

[7] For a more detailed discussion with a fully specified example, see Heifetz et al. (2006, p. 87).

language, a feature not present in most other models of unawareness.[8] Since the kind of statements about unawareness used in the applied literature often need quantifiers, these are the only theoretical models that can be directly applied to these questions. For a more complete discussion of this view and a development of a set theoretic model with quantifiers see Board and Chung (2007, 2008). Board et al. (2011) show that this approach is equivalent to a generalization of Heifetz et al. (2006) unawareness structures.

Decision Theory with Unawareness

The models discussed so far do not contain a formal decision theory. They pose interesting questions for decision theory, however, some of which have been answered by recent contributions.

Contrary to fully aware agents for whom learning means ruling out a previously conceivable state, unaware agents can be genuinely surprised. It is unclear how they should update their priors when there subjective state space is enlarged. Karni and Vierø (2013) show that under compelling axioms linking preferences at different awareness levels, agent engage in so-called "Reverse Bayesianism". Similar to Bayesian updating, the likelihood ratios between states the agent was originally ware of remains the same. Were an agent to learn that the state he is newly aware of does not obtain, the posterior arrived at via Bayesian updating would correspond to his prior before enlarging awareness.

Combining unawareness structures as in Heifetz et al. (2013b) and the Anscombe-Aumann approach to decisions under uncertainty, Schipper (2013) provides a choice based characterization of unawareness. An agent is unaware of an event E if and only if he acts as if both the event E and its complement $\neg E$ are null events. This also settles the question on the relation between unawareness and probability zero prior belief. Schipper (2014) goes further by providing preference based definitions of unawareness and knowledge based on acts in unawareness structures consisting of states of the world.[9]

Games with Unawareness

Meier and Schipper (2014) define Bayesian games with unawareness based on Heifetz et al. (2013b) unawareness structures with beliefs. They allow for unawareness of contingencies as well as for unawareness of actions. The equilibrium concept for these

[8] Another exception is Sillari (2008)

[9] States of nature give a full description of all relevant facts external to the decision maker. States of the world include the beliefs and knowledge of the decision maker in their description. Most of decision theory is built on states of nature while the literature in epistemics uses states of the world.

kind of games is an extension of an equilibrium of types to situations of limited awareness: Limited awareness types only take into account deviations and other types strategies within their mental model.

Extensive form games with unawareness are defined in Halpern and Rêgo (2014) and Heifetz et al. (2013a). The key difference between these two contributions is that Halpern and Rêgo (2014) base their construction on a standard extensive form and study an extension of Nash equilibrium while Heifetz et al. (2013a) build their framework on a tree of physical moves and study an extension of extensive form rationalizability (Pearce, 1984). Both constructions allow for awareness of unawareness.

Feinberg (2012) provides a unified definition for normal form, Bayesian, repeated and extensive form games using sets of games without unawareness. Contrary to the approaches discussed above, this construction relies on unbounded sequences of mutual views. The sequence elements describe explicitly the view of each player about the view of each player, etc. at each decision node.[10] Games created using this approach are comparatively large formal objects, but it allows for a common approach to all forms of games.

Grant and Quiggin (2013) construct a semantic model for extensive form games with unawareness to investigate whether individuals can believe that there is a proposition they are unaware of. They demonstrate that this is not the case, even though individuals experience that they become aware of new propositions. They develop a notion of induction reasoning to formalize the idea that individuals should learn from these experiences and consider future surprises possible.

Nielsen and Sebald (2011) extend psychological games to allow for unawareness. Psychological games are games in which payoffs depend not only on the outcome of the game but directly on the players' beliefs about strategies as well as higher order beliefs. This approach can be used to model reciprocity or guilt. As unawareness restricts the inferences of players, it can have important effects on the inferred beliefs and thus payoffs in psychological games.

Applications of Games with Unawareness

So far very little applied work is based on the explicit theoretical models of games with unawareness discussed in the previous subsection. A notable exception is Schipper and Woo (2014). They study how electoral campaigns can affect voting behavior if voters have limited awareness of political issues. If there is sufficient political competition, political campaigns can use micro targeting and voters have full reasoning abilities, the

[10] Note that the same information is contained in condensed form in the above constructions. The differences is similar to the difference between a type approach to uncertainty in games and an explicit construction of infinite belief hirachies.

outcome is the same as in the full awareness case. If these conditions are not satisfied, the effect of unawareness depends on the form of election and may be remedied by dirty campaigning.

Contributions to contract theory will be surveyed in a later subsection, even if the build on the same theoretical framework.

Applications to General Equilibrium Theory

Modica et al. (1998) consider a pure-exchange economy with unawareness. Due to unawareness an equilibrium might not exist even in the case of complete markets. They show existence for the case that there all agents are aware of sufficiently many states. A key feature of the model is that agents may go bankrupt in unforeseen states. Since agents do not optimize for these states, we can accommodate bankruptcy in an equilibrium without a short selling constraint. Such an outcome is impossible if we interpret unawareness as probability zero belief, since agents would demand infinitely large short positions in all contingent commodities for such a state.

Kawamura (2005) extends the model to allow for entrepreneurial production and proves existence for the case of decreasing marginal returns. He provides an example to demonstrate that unawareness may be without consequence for the allocation even in a setting with investment decisions.

Contract Theory and Unawareness

Contract theory with unawareness can be seen as a subfield of behavioral contract theory. Unawareness provides microfoundations for a lack of Bayesian updating, wrong priors and behavioral agents. Notions of unawareness have been used in behavioral contract theory without direct reference to theoretical models of unawareness. This may be problematic since ad-hoc models are seldomly checked for necessary conditions of consistent reasoning with unawareness such as KU-introspection.

In the following we will provide a selective survey of relevant contributions from behavioral contract theory and full survey of the small but growing literature on contract theory with unawareness. We will also cover some contributions on key topics of contract theory that can be reinterpreted or explained in terms of unawareness.

Wrong Priors, Overconfidence and Reduced Form Unawareness

The literature on overconfidence and wrong priors analyzes situations in which individuals have distorted beliefs about their own abilities or relevant contingencies. Often these papers include agents that agree to disagree or fail to update their beliefs based on available information. Unawareness provides a foundation in explicit models of knowledge for this kind of behavior. In addition, there is a strong connection between unawareness of contingencies and overconfident beliefs that those contingencies will not realize. Often models with wrong beliefs can be recast as models with unawareness of some contingency.

Overconfidence in a moral hazard framework is studied by de la Rosa (2011). The effect on the shape of the incentive contract depends on the shape of overconfidence. On the one hand lower powered incentives are sufficient to induce a given level of effort since the agent overestimates the marginal effect of effort. On the other hand higher powered incentives are cheaper for the principal since the agent overestimates the likelihood of good realizations.

Meza and Southey (1996) show that overconfident individuals are more likely to obtain collateralized loans for investment projects even in the presence of a fully rational bank. Contrary to standard models of lending with agency frictions, there is too much lending in this model. They conclude that this mechanism can account for the characteristics of small businesses, including high failure rates since overconfident entrepreneurs are more likely to start a project. A similar model with an amplification effect could be written in terms of unawareness. Individuals that are unaware of some contingency fatal for their future business are more likely to enter and simultaneously less prepared for this contingency.

When buying a base good such as a printer, some consumers might not think about the price of add-ons such as ink cartridges. This behavior is formalized in Gabaix and Laibson (2006). They show that it might not be profitable for competitors to educate consumers about the other firms product characteristics even if this is costless. That is because the intransparent firm can offer a better deal for informed consumers. Therefore newly informed consumers will not defect to the other firm.

Eliaz and Spiegler (2006) analyze a market with consumer that are unaware of their future preference change. Consider a two period model. When consumers buy a product, preferences change for the second period. The degree of naivete of a consumer is given by the probability he ascribes to his preferences remaining unchanged. The firm faces a distribution of consumers with unknown naivete and tries to screen optimally. They find that sufficiently sophisticated consumers pool into an optimal contract while naive consumers choose exploitative contracts. Profits from a given consumer are increasing in his naivete.

Incomplete Contracts

Incomplete contracts are a recurring theme of the contracting literature. Loosely speaking, an incomplete contract is a contact that does not condition on some relevant contingency. A common explanation for incomplete contracts are indescribable or unforeseeable contingencies. This argument has been criticized by Maskin (2002) and Maskin and Tirole (1999) who show that indescribable or unforeseeable and indescribable contingencies cannot justify incomplete contracts. As long as payoffs are foreseeable in principle, the contracting parties can contract around the unforeseeability or indescribability.

Unforeseeability in the sense of unawareness can justify incomplete contracts, however (Tirole, 2009). Once conditioning creates awareness, it may be profitable not to mention a certain contingency. The other contracting party then fails to take the low implied payoffs in this state into account and evaluates the contact offer to favorably.

Intrinsic Motivation

It is often claimed that monetary incentives crowd out intrinsic motivation. Bénabou and Tirole (2003) provide a model explaining this behavior based on the interaction between a fully rational principal and a fully rational agent. If the principal has private information about the agents ability, the incentive pay transmits information. This mechanism can explain why incentives work as long as they are in place but have a discouraging effect for future behavior. Furthermore, the same mechanism can account for the observation that forbidden fruit are the most attractive.

Crowding out of intrinsic motivation can be recast as the creation of awareness. In this interpretation, intrinsic motivation is simply an unusually high effort that is exerted not because of conscious optimization but rather as a default action. Once incentives come into play, individuals become more aware of their options and their consequences. This effect may weaken the effect of incentives and persists beyond the period in which incentives are provided. Bans also focus individuals attention. It may be possible to recast the attractiveness of forbidden fruit as a story about unawareness.

Contract Theory with Unawareness

The papers discussed in this subsection explicitly discuss unawareness in contract theoretic settings. Some of the define solution concepts taking into account the agents limited awareness or relate to the theoretical literature on unawareness.

Zhao (2008) considers an abstract moral hazard problem in which both the principal and the agent may be unaware of some of their own and the other parties actions. He shows that increased awareness is not necessarily beneficial. Awareness of an additional own action may induce a player to take this action which turns out to give low payoffs because of an unforeseen action by the other party. Thus increased awareness of own actions is unambiguously beneficial only if the individual is aware of all the other player actions of which he is aware. Similarly, increased awareness about the other players action may be harmful.

Less abstract versions of the moral hazard problem with unawareness of actions are analyzed by Thadden and Zhao (2012, 2013). Thadden and Zhao (2012) discuss the trade-off between making agents aware and thus introducing an agency friction and leaving them unaware at a possibly inefficient default action. They find that unawareness is self-reinforcing. The higher the share of unaware agents, the larger the parameter space in which they will be kept unaware by the principal. For a more detailed discussion see Section 3.

Thadden and Zhao (2013) consider a two dimensional effort choice model. They analyze the case of linear contracts with CARA utility and additive normally distributed noise analytically. They show that the incentives for aware agents can be distorted in either way due to the presence of unaware agents.

Unawareness of contingencies in an insurance problem is analyzed in Filiz-Ozbay (2012). Insurers have superior awareness regarding possible states. They can make agents aware of those contingencies, but agents then take (the lack of) insurance payments in those states into account. It is shown that competition between insurers promotes awareness.

Auster (2013) considers a moral hazard problem with unawareness of contingencies. The principal is aware of all contingencies and has the option to make the agent aware. Since conditional payments act only as incentives if the agent is aware of them, the principal would like to condition on all informative contingencies if this were costless (Hölmstrom, 1979). Unawareness allows the principal to pay minimal transfers in these contingencies without affecting the agents assessment of his expected utility. Thus there is a cost for each signal that depends on the likelihood that the contingency occurs. The principal reveals contingencies that are unlikely and strong signals of effort. Competition among principals may not increase awareness in this setting. The intuition is the same as in Gabaix and Laibson (2006): agents with higher awareness can exploit the original offer and do not defect to the principal that made them aware, thus it is not profitable in the first place.

Ozbay (2006) discusses a sender receiver game with unawareness. The decision maker is unaware of some possible contingencies while the sender has full awareness. The paper also considers a refinement called "reasoning" similar to forward inductive reasoning. Intuitively, the refinement prescribes that if the announcer sends a message, this is meant to change the decision makers behavior. Even under common interest, it can be optimal not to reveal all contingencies since announcements of contingencies are used as a communication device.

Chen and Zhao (2013) discuss solution concepts for principal agent problems with unawareness. They propose a justifiability constraints on contracts. Agents reject contracts that are not optimal for the principal from their point of view for fear of exploitation.

3 Unawareness with Limited Liability

In this paper we consider unawareness of actions in an optimal contract problem. We say an agent is unaware of one of his actions, if he does not consider this action in his choice. This is modeled by a smaller choice set in the agents optimization problem. In our setup, a monopolist principal faces an agent who may or may not be unaware of his effort choice in a standard moral hazard problem. The principal cannot observe if the agent is unaware or not. To take advantage of the two different types of agents, he wants to screen them by offering a menu of different incentive contracts that are then used in the second stage (moral hazard). This way he can give the right incentives to the agent with the full choice set while abusing the fact that some agents have only a restricted choice set in the effort choice problem.

The setup is similar to a screening before moral hazard problem. The key difference is that the agents differ in their action set but not the parameters of the problem. Furthermore, the principal can “make an agent aware”. By this we mean that the principal can extend the agents choice set by giving instructions in the contract. Thus there is an additional set of questions: Not only how does the principal screen, but does he want to screen at all or rather homogenize the population instead?

The problem of moral hazard with unawareness of actions has already been analyzed in Thadden and Zhao (2012). They use a setup with one type of unaware agent with only one effort level to obtain strong conclusions. First of all, aware agent gain from the presence of unaware agents in the population. Secondly, unawareness is self reinforcing in the sense that the subset of the parameter space in which initially unaware agents are made aware is decreasing in the share of unaware agents. Finally, unaware agents rely on the presence of sufficiently many other unaware agents to remain unaware. Formally, for every effort level there exists a share of unaware agents below which agents will be made aware by the principal.

We show that the qualitative predictions of unawareness in moral hazard problems are different from these findings in the case with binding limited liability constraints. Concretely, we find that aware agents can loose from the presence of unaware agents and that unaware agents with some restricted action sets can remain unaware even if their share is arbitrarily small. The stark difference between our findings and those of Thadden and Zhao (2012) in the case of risk aversion are due to the presence of a limited liability rent. This rent can be used to screen the agents in the sorting problem while still satisfying their participation constraints at no cost. This is impossible in the setup without limited liability, so that an additional rent has the be introduced to sort the agents.

The organization of the paper is as follows. Subsection 3.1 introduces the problem formally. We analyze the case with binding limited liability constraints in Subsection 3.2. We compare the results to the case without binding limited liability in Subsection 3.3 and study how they generalize to the case combining risk aversion with binding limited liability constraints. In Subsection 3.4 we discuss the interpretation and some limitations of our approach. Subsection 3.5 concludes.

3.1 The General Setup

Consider first the second stage, a general moral hazard problem. The monopolist principal faces an agent who can choose an unobservable effort level $e \in [0,1]$ at a cost $c(e)$. Depending on the effort level, an output level $y \in Y \subset \mathbb{R}$ is realized according to a cumulative distribution function $F(y|e)$. The principal observes only this output. A contract is a function from the realized output level to the share of output appropriated by the principal and the agent, respectively. As we will assume monotonic target functions throughout, we can represent a contract by the transfers $t(y)$ from the principal to the agent. A menu of contracts is a set of contracts from which the agent can choose. The timing of the game is as follows: The principal offers a menu of contracts. Based on this menu, the agent chooses if he[11] wants to participate and in case he participates the contract. He then chooses the unobservable action level. In the end, output is realized and the transfers specified by the chosen contract are made. We assume that the principal's profit and the agent's utility is linear in output and transfers whenever necessary for the argument. Furthermore, the agent is protected by a limited liability constraint. The transfer is not allowed to go below m for any level of realized output. The outside option of the agent gives a utility level of $\bar{u}$ irrespective of his awareness.

There are two types of agents in the economy, one aware the other unaware of some of his actions. The principal may want to sort these two types into different contracts. More specifically, with a probability of λ the agent is unaware of his effort choice and chooses a fixed default effort $\bar{e}$ instead. The principal does not know if the agent is unaware or not, but she knows the default action and the share of unaware agents in the population. Furthermore we assume that the principal can make an agent aware before they commit to a contract. In this case, the agent learns about the whole action space, the principal cannot restrict the information to a subset.[12] As there are only two types of agents in the economy, we can assume without loss of generality that only two contracts are offered.

To proceed with our analysis, we will make some additional assumptions on the second stage moral hazard problem.

[11]We use masculine pronouns for the agent and feminin pronouns for the principal throughout.
[12]For a discussion of these assumptions, see Subsection 3.4.

Assumption 1 (Non-Degeneracy)**.** *There exists an effort level* $e \in [0,1]$ *such that* $\int y \, \mathrm{d}F(y|e) - c(e) > \bar{u}$*. Furthermore there exists an effort level* $e \in [0,1]$ *such that there exist a feasible contract that implements e from the aware agent at minimal expected transfers and this contract maximizes the principals profit* $\int y - t(y) \, \mathrm{d}F(y|e)$ *over all feasible contracts.*

This assumption means that the technology is profitable in the sense that the agent would not take the outside option if he can appropriate all of the output. Since otherwise any contract that discourages the agent from participating is optimal, this assumption is needed to make the problem interesting. The second part of the assumption states that the optimal effort level can be incentivised by an actual contract and that the set of transfers inducing this effort level has a minimum, not only a supremum. The existence of a contract that incentivizes a given effort level is not problematic since we assume that there is no upper bound on payments. The existence of an optimal contract is somewhat restrictive but clearly needed for the analysis of the problem. See Mirrlees (1999) for the classic counterexample to existence, Page (1987) for sufficient conditions and Moroni and Swinkels (2014) for further counterexamples and discussion.

Note that the optimal contract is not necessarily unique as we assume linear utility.

Assumption 2 (Regularity)**.** *The cost function* $c(e)$ *is continuously differentiable and strictly increasing. We normalize* $c(0) = 0$*. A contract* t_I *is assumed to be continuous almost everywhere.*

These are mostly technical condition needed for further analysis. Convexity of the cost function corresponds to increasing marginal costs.

Assumption 3 (Stochastic Order)**.** *For any* $e < e'$ *the distribution over outcomes* $F(y|e)$ *is first order stochastically dominated by* $F(y|e')$*.*

This is a standard assumption to ensure that higher effort induces an unambiguously better distribution over outcomes. This assumption is natural to make since we are concerned with ways of incentivizing effort and the first stage screening problem, not with conditions for the effectiveness of effort per se.

Assumption 4 (Concavity)**.** *The cost function is strictly convex, the distribution over outcomes admits a continuously differentiable density and satisfies the MLRP and CDFC, i.e.* $\mathrm{D}_y \frac{f_e(y|e)}{f(y|e)} > 0$ *and* $F_{ee}(y|e) > 0$ *for all* y, e*.*

These conditions will be assumed only in a few propositions and examples. Jointly they imply that the objective of the agent is concave for every transfer function and thus that the first order approach to the moral hazard problem is valid.[13]

[13] A rigorous proof of these assertion can be found in Rogerson (1985). Different conditions for the validity of the first order approach are given by Jewitt (1988).

3.2 Qualitative Analysis

The analysis of this contracting problem has to deal with two intertwined issues. First of all, under which conditions will a principal make an initially unaware agent aware and secondly how will the principal design contracts when facing a population comprised of unaware and aware agents.

Given that the initially unaware agent is made aware, the principal faces a standard moral hazard problem with limited liability.[14] Thus we can analyze the decision to make the unaware agent aware and the contract design for the case of possibly unaware agents separately. The general contracting problem with unawareness under limited liability assuming that the agent remains unaware reads

$$
\begin{aligned}
\max_{t_A, t_U} \quad & \lambda \int_Y y - t_U(y)\, \mathrm{d}F(y|\bar{e}) + (1-\lambda) \int_Y y - t_A(y)\, \mathrm{d}F(y|e^*) \\
\text{s.t.} \quad & t_A(y) \geq m, \quad \forall y \in \mathbb{R} \\
& t_U(y) \geq m, \quad \forall y \in \mathbb{R} \\
& e^* \in \arg\max \int_Y t_A(y)\, \mathrm{d}F(y|e) - c(e) \\
& \int_Y t_A(y)\, \mathrm{d}F(y|e^*) - c(e^*) \geq \bar{u} \\
& \int_Y t_U(y)\, \mathrm{d}F(y|\bar{e}) - c(\bar{e}) \geq \bar{u} \\
& \int_Y t_A(y)\, \mathrm{d}F(y|e^*) - c(e^*) \geq \max \int_Y t_U(y)\, \mathrm{d}F(y|e) - c(e) \\
& \int_Y t_U(y)\, \mathrm{d}F(y|\bar{e}) - c(\bar{e}) \geq \int_Y t_A(y)\, \mathrm{d}F(y|\bar{e}) - c(\bar{e})
\end{aligned}
$$

Already from the general program, we see the key tradeoffs when designing contracts for possibly unaware agents. First of all, unaware agents do not optimize their effort but exert a default effort level instead. This means that there is no incentive compatibility constraint. This makes the agent cheaper for the principal, since there is no incentive rent. On the downside, the default action level may be very inefficient. This tradeoff will be handled by the decision to make the agent unaware. Secondly, the rent for the unaware agent has to be at least as big as the rent the aware agent would get at the default action level in order to satisfy the sorting constraints. This creates a tradeoff between the rent handed over to the unaware agent and the shape of incentives to the aware agent. In the optimum, the rent at the default action level will be lower compared

[14] We do not analyze this problem in any detail. For an exposition of the solution of this problem, see Bolton and Dewatripont (2004) and Laffont and Martimort (2002). For a treatment of the general case see Jewitt et al. (2008).

to their level for $\lambda = 0$. This affects the strength and costs of incentives for the aware agent.

In this general form, the condition to make the unaware agent aware are very simple. Let us denote the expected profit from the above contracting problem by $\Pi(\lambda, \bar{e})$. Clearly, the contracting problem after the agents have been made aware coincides with $\Pi(0, \bar{e})$ for all $\bar{e}$. We obtain the following simple Lemma:

Lemma 1. *The profit function for fixed awareness satisfies*

$$\Pi(\lambda, e) \leq \lambda \Pi(1, \bar{e}) + (1 - \lambda)\Pi(0, \bar{e}).$$

Proof. The contracts of the joint problem are feasible in the corner problems. Furthermore profit is linear in the two components. $\square$

Thus the principal makes the agents aware if and only if

$$\Pi(0, \bar{e}) \geq \Pi(\lambda, \bar{e}).$$

The solution of the problem without unaware agents is an important benchmark case. Many results about the problem with a share of unaware agents will be framed in terms of its solution. We denote by e_{FB} the solution to the problem without an agency friction, i.e. e_{FB} maximizes net output. Formally, $e_{FB} = \arg\max \int_Y y \, dF(y|e) - c(e)$. Furthermore we denote by e^* the optimal effort level chosen by the principal with an agency friction and by t^* a contract implementing this effort at minimal expected transfers.

Making Agents Aware: Boundaries

An important decision for the principal is whether to make unaware agents aware of their effort choice possibilities. Making unaware agents aware introduces an incentive constraint and is thus costly but also enables the principal to influence the agents effort choice. The optimal decision may depend on λ since the relative costs of changing the contract offered to aware agents varies with λ. At a high share of unawares, it is profitable to change the contract offered to the aware agents in order to take advantage of unaware agents, thereby making it profitable to keep them unaware. At a high λ, the contract offered to aware agents stays essentially fixed and may force the principal to grant a rent to the unaware agents. Let $U(\lambda) \subseteq [0, 1]$ denote the set of default efforts for which the unaware agents remain unaware. We obtain the following result:

Proposition 1. *The set of default actions for which unaware agents remain unaware $U(\lambda)$ is increasing in λ with respect to set inclusion. This conclusion is robust to the introduction of risk aversion.*

Proof. Consider an arbitrary $e \in U(\lambda)$. By assumption the profit from the optimal contract for this problem is greater than the profit after making all agents aware $\Pi(0,\bar{e})$. Clearly the same set of contracts (t_A, t_U) is feasible in the problem for any other λ. Since

$$\begin{aligned}\Pi(0,\bar{e}) \leq \Pi(\lambda,\bar{e}) &= \lambda \int_Y y - t_U(y)\,\mathrm{d}F(y|\bar{e}) + (1-\lambda)\int_Y y - t_A(y)\,\mathrm{d}F(y|e^*)\\ &\leq \lambda \int_Y y - t_U(y)\,\mathrm{d}F(y|\bar{e}) + (1-\lambda)\Pi(0,\bar{e})\end{aligned}$$

we know that the profit obtained from an unaware agent under these contracts is higher than the profit obtainable from an aware agent.

Consider a $\tilde{\lambda} > \lambda$. We know that

$$\begin{aligned}\Pi(\tilde{\lambda},\bar{e}) &\geq \tilde{\lambda} \int_Y y - t_U(y)\,\mathrm{d}F(y|\bar{e}) + (1-\tilde{\lambda})\int_Y y - t_A(y)\,\mathrm{d}F(y|e^*)\\ &\geq \lambda \int_Y y - t_U(y)\,\mathrm{d}F(y|\bar{e}) + (1-\lambda)\int_Y y - t_A(y)\,\mathrm{d}F(y|e^*)\\ &= \Pi(\lambda,\bar{e}) \geq \Pi(0,\bar{e})\end{aligned}$$

and thus $e \in U(\tilde{\lambda})$.

Note that we never used the shape of the utility function in the proof, thus the result generalizes to the case of risk aversion. □

Corollary. *Expected profits $\Pi(\lambda,\bar{e})$ are increasing in the share of unaware agents λ.*

Since the principal can always make unaware agents aware of their effort choice, she never looses from their presence. Clearly, it is only profitable to keep agents unaware, if the profit that is compatible with the constraints of the contracting problem is higher than the highest profit obtained from aware agents at least for some contract offered to the aware agent. In fact, this characterizes the set of unawares that are made aware irrespective of λ. Once the profit from unaware agents under a contract geared towards exploiting unawares is higher than the maximal profit from an aware agent, offering such a contract is profit enhancing for a suitably high share of unaware agents.

Proposition 2. *An unaware agent with default action level $\bar{e}$ is made aware irrespective of the share of unaware agents λ if and only if the maximal profit from an unaware agent with default action level $\bar{e}$ given participation and limited liability constraint is strictly smaller than the profit from the problem without unawareness. Formally*

$$\Pi(1,\bar{e}) \leq \Pi(0,\bar{e}) \iff \Pi(\lambda,\bar{e}) \leq \Pi(0,\bar{e}) \qquad \forall \lambda \in (0,1)$$

Furthermore, this conclusion is independent of risk aversion.

Proof. $\Longrightarrow$: We know that $\Pi(\lambda, e) \leq \lambda\Pi(1, \bar{e}) + (1-\lambda)\Pi(0, \bar{e})$. The desired result follows immediately.

$\Longleftarrow$: Suppose $\Pi(1, \bar{e}) > \Pi(0, \bar{e})$. Then there exists a $\tilde{\lambda}$ such that $\tilde{\lambda}\Pi(1, \bar{e}) + (1 - \tilde{\lambda})\eta > \Pi(0, \bar{e})$, where η denotes the loss from an aware agent abusing the contract implementing a profit of$\Pi(1, \bar{e})$ from the unaware. This bound η exists at least for one of these contracts, since payoffs at arbitrary high realizations can be capped and redistributed to low realizations without affecting the rent of the unaware. Since the optimal set of contracts for $\tilde{\lambda}$ performs better than this combination, $\Pi(\tilde{\lambda}, \bar{e}) > \Pi(0, \bar{e})$.

Note that we never used the shape of the utility function in the proof, thus the result generalizes to the case of risk aversion. $\square$

Note that in order to consider this proposition a full characterization of the set of default action levels for which unaware agents are made aware, we have to assume that an indifferent principal makes unaware agents aware.

Another set of interest is the set of default action levels at which unaware agents stay unaware irrespective of their share in the population.

Proposition 3. *Consider the set of efforts that create higher net output than e^**

$$N = \{e \in [0,1] : \int y \, \mathrm{d}F(y|e^*) - c(e^*) \leq \int_Y y \, \mathrm{d}F(y|e) - c(e)\}$$

and the set of efforts at which the baseline contract gives a non-negative net rent

$$U = \{e \in [0,1] : \int_Y t^*(y) \, \mathrm{d}F(y|e) - c(e) \geq \bar{u}\}$$

If the default action level $\bar{e}$ is in $N \cap U$, unaware agents are kept unaware at every λ.

This result generalizes to the risk aversion case if expected net transfers are higher than at $e^*$for all $\bar{e} \in N$.

$$\int_Y t^*(y) \, \mathrm{d}F(y|e^*) - c(e^*) \geq \int_Y t^*(y) \, \mathrm{d}F(y|\bar{e}) - c(\bar{e}) \qquad \forall \bar{e} \in N$$

Proof. We will show that even with the default contract for the aware, the profit from the unaware is higher than that from an aware agent at the optimal contract. First note that the rent of the aware agent under the default contract t^* is lower at $\bar{e}$ than at e^* by the incentive compatibility constraint. Thus

$$\int_Y t^*(y) \, \mathrm{d}F(y|e^*) - c(e^*) \geq \int_Y t^*(y) \, \mathrm{d}F(y|\bar{e}) - c(\bar{e})$$

Furthermore, by the definition of N we know that

$$\int_Y y\,\mathrm{d}F(y|e^*) - c(e^*) \leq \int_Y y\,\mathrm{d}F(y|\bar{e}) - c(\bar{e})$$

Subtracting the first inequality from the second gives

$$\int_Y y - t^*(y)\,\mathrm{d}F(y|e^*) \leq \int_Y y - t^*(y)\,\mathrm{d}F(y|\bar{e})$$

This shows that the profit from an unaware is greater than the profit from an aware even without distorting the contract of the aware. Thus there is no incentive to make the unaware aware. This contract is feasible since we assumed that $e \in U$. The extension to the risk aversion case is immediate. □

In case net output $\mathbb{E}_{y|e}y - c(e)$ is concave in effort, e.g. under Assumption 4, the set $N \cap U$ contains at least a half closed interval around e^* if the limited liability constraint is binding in the aware problem.

A sufficient condition for keeping an agent unaware is that the profit for the principal from an unaware agent is larger than the profit from an aware agent under the contract t^*. Thus the set of default action levels at which initially unaware agents remain unaware is in general larger than $N \cap U$. First of all the set of default efforts for which the agent remains unaware irrespective of the share of unaware agent in the population, $U(0)$ can be strictly greater than $N \cap U$. Even though net output is lower at these effort levels, also the rent to the agent is lower in case of unawareness so the change in the difference is not determined. Secondly, for interior λ we can have a larger set of default action levels at which the agent remains unaware since $U(\lambda) \supseteq U(0)$.

Since we consider a model with linear utility, net output equals total surplus. A monopolist principal thus keeps an agent unaware if this increases total surplus and satisfying the participation constraint is not an issue. An agent may remain unaware, however, even if making him aware would increase total surplus.

Proposition 4. *Consider an unaware agent with default action level $\bar{e} < e^*$. Assume that the derivative of expected transfers with respect to effort $\mathrm{D}_e\mathbb{E}(t^*(y)|e)$ exists and is decreasing in e (Assumption 4). Then there exists a $\bar{\lambda}$ such that he is made aware by the principal for all $\lambda \in [0, \bar{\lambda}]$.*

Proof. Consider the contract $t_\alpha = m + \alpha(t^*(y) - m)$. The first order condition for the agent reads

$$\alpha \mathrm{D}_e\mathbb{E}(t^*(y)|e) = c_e(e)$$

Since $c_{ee} > 0$ there exists an α such that this condition is satisfied for $\bar{e}$, so this contract implements $\bar{e}$ by concavity given that the participation constraint is satisfied. If participation is not satisfied, this can be done by an additive shift that leaves incentives unaffected. Clearly, the expected transfer under $t_\alpha$for $\bar{e}$ is lower than for t^* under $\bar{e}$ or t^* violates the participation constraint. This implies, that the optimal contract implementing any $e < e^*$ has lower expected transfers at this effort level than the cheapest contract that satisfies all the constraints given t^*. Since (t^*, e^*) is profit maximizing for the principal, the profit of an unaware agent gaining a rent based on t^*at $\bar{e}$ is strictly lower that the profit from an aware agent acting according to (t^*, e^*). For suitably small λ, any gains from changing the contract to decrease the rent for the unaware are outweighed by decreases in profit from awares. It is thus profitable to make all agents aware. □

In conjunction with Proposition 4 we can see that e^* has to be the greatest lower bound of the set $U(0)$. It is only profitable to leave agents unaware that exert an effort that is lower than the second best effort for reasons of sorting. Note that this proposition depends on the validity of the first order approach for the second stage moral hazard problem.

The Contracts

Since the contract for the unaware agent has to satisfy the participation constraint and the sorting constraint, the rent at the default action level $\bar{e}$ has a lower bound. By linear utility, this rent is a sufficient statistic for the costs of the contract to the unaware agent. If this bound is attainable, this gives a characterization of the cost minimizing contract for the unaware agent given a fixed contract for the aware agent for every default action level $\bar{e}$. We can thus determine one set of optimal contracts for the unaware agents depending on the contract given to the aware agents. More formally:

Proposition 5. *For every contract t_A for the aware agent that implements a non zero effort, there is a feasible and optimal contract for the unaware agent with expected transfers $\int_{-\infty}^{\infty} t_U(y)\,\mathrm{d}F(y|\bar{e}) = c(\bar{e}) + \max\{\bar{u}, u(t_A, \bar{e})\}$ for default action levels satisfying $\bar{e} < e^*$and*

$$\int_Y y\,\mathrm{d}F(y|\bar{e}) - c(\bar{e}) - \max\{\bar{u}, u(t_A, \bar{e})\} \geq \int_Y y - t_A(y)\,\mathrm{d}F(y|e^*)$$

Proof. Fix a contract t_A for the aware agent. Let $u(t_A, e)$ denote the utility level attained from effort e under contract t_A. Clearly

$$\int_Y t_U(y)\,\mathrm{d}F(y|\bar{e}) - c(\bar{e}) \geq \max\{\bar{u}, u(t_A, \bar{e})\}$$

by the participation and sorting constraints of the unaware agent. We will show that this bound can be attained by a contract.

First we will deal with the case where $\max\{\bar{u}, u(t_A, \bar{e})\} = u(t_A, \bar{e})$. The the contract $t_U = t_A$ attains this bound. Clearly, all other constraints are satisfied as well. Since we are in linear (=transferable) utility world, the rent to the agent is a sufficient statistic for the cost of the transfer to the principal.

Now consider the case where $\max\{\bar{u}, u(t_A, \bar{e})\} = \bar{u}$. The existence of such a contract follows from the proof of Proposition 4. □

When the unaware agent gets a positive rent, there is no need for a dedicated contract for the unaware agent in the linear utility case. Note however that the contract for the unaware agents is in general not unique with linear utility. All contracts that attain the lower bound and satisfy the limited liability constraint are optimal.

Let us now consider the generally case in which unaware agents remain unaware if Proposition 5 is applicable. The principal then solves the following problem

$$\begin{aligned}
\max_{t_A, t_U} \quad & \lambda \left(\int_Y y \, \mathrm{d}F(y|\bar{e}) - \max\{ \int_Y t(y) \, \mathrm{d}F(y|\bar{e}), \bar{u} + c(\bar{e}) \} \right) + (1-\lambda) \int_Y y - t(y) \, \mathrm{d}F(y|e^*) \\
\text{s.t.} \quad & t_A(y) \geq l, \quad \forall y \in \mathbb{R} \\
& e^* \in \arg\max \int_Y t(y) \, \mathrm{d}F(y|e) - c(e) \\
& \int_Y t(y) \, \mathrm{d}F(y|e^*) - c(e^*) \geq \bar{u}
\end{aligned}$$

In the general case, the effort level will be distorted away from the second best level. The direction of this distortion is unclear in general, since both increased as well as decreased effort may be incentivized using a contract with lower expected transfers at $\bar{e}$. We can express the regions where this contract will be unchanged based on rents and the region where unaware agents will be made aware for sure.

Proposition 6. *The contract t^* is optimal for aware agents for an arbitrary share of unaware agents λ if and only if one of the two following conditions is satisfied.*

1. *It is profitable to make unaware agents aware for every λ*

$$\int_Y y - t^*(y) \, \mathrm{d}F(y|e^*) \geq \int_Y y \, \mathrm{d}F(y|\bar{e}) - \bar{u} - c(\bar{e})$$

2. *There is no rent for unaware agents under t^**

$$u(t^*, \bar{e}) \leq \bar{u}$$

Proof. The first condition implies that unaware agents will be made aware irrespective of λ. The second condition implies that the rent granted to unaware agents is minimal. In either case there is no incentive to change the aware contract from its optimal shape. Conversely, consider an situation in which the unaware agent gets a positive rent and it is profitable to keep him unaware under the assumption that he gets no rent. Then for high λ the rent given to unawares due to the sorting constraint under t^* exceeds the expected gains from incentivizing the aware agents. By decreasing the transfers at high payoffs, the principal can decrease the rent for the unaware agent and decrease the effort from the aware agent which is locally profitable for suitable lambda. □

The rent granted to an unaware agent conditional on the agent remaining unaware is decreasing in the share of unaware agents in the population λ. This can be seen directly from the optimization problem. The effect of making an agent aware on his rent is ambiguous. Thus the total effect of an increased share of unaware agents is undetermined. The change in the rent of awares is ambiguous.

3.3 Contrasting Risk Aversion with Limited Liability

In the risk aversion case, the sorting constraint from unawares to awares is irrelevant, since aware agents are always held to their participation constraint in the baseline scenario without unaware agents. Thus since the sorting constraint from awares to unawares is driving the analysis.

Under risk aversion without binding limited liability constraints, Thadden and Zhao (2012) obtain the following characteristics of the solution:

- Unawareness is self-reinforcing.
- Unawareness only persists if it is common enough: For every default action level $\bar{e} \neq e^*$, there exists a λ such that agents are made aware for every share lower than λ.
- Aware agents gain from the presence of unaware agents.

Only the first carries over to the case of limited liability with linear utility. This is because binding limited liability constraints create a rent that can be used in sorting. These results of Thadden and Zhao (2012) are based on the need to introduce a rent to aware agents for the sorting problem.

First of all, giving a rent to aware agents in order to gain from unaware agents is only profit enhancing if there are sufficiently many unaware agents. This is no longer true with limited liability. Since there is a rent for aware agents, unaware agents with suitably close default action levels also get a rent from the default contract by continuity. Thus there is no need to modify the baseline contract in order to satisfy the constraints of the sorting problem. For some of those, it might also be profitable to keep them unaware, even with this suboptimal contract as shown in Proposition 3.

Secondly, granting aware agents an additional rent is clearly beneficial for them. As aware agents have nothing to lose in the risk aversion case - they are pushed down towards their participation constraint in the baseline solution - the effect is unambiguous. In the limited liability case, the principal has to pay for stronger incentives with a higher rent. Unaware agents participate in that rent, it leaks to them through the sorting constraint without providing any benefits to the principal. Thus it may be profitable to provide low powered incentives also for aware agent which in turn lowers their rent.

In short, we find:

- Unawareness is self-reinforcing
- Unawareness can persist for an arbitrarily low share of initially unaware agents λ for set of default actions with strictly positive measure
- Aware agents may loose from the presence of unaware agents

The relevant effects are already apparent in a simple two output example:

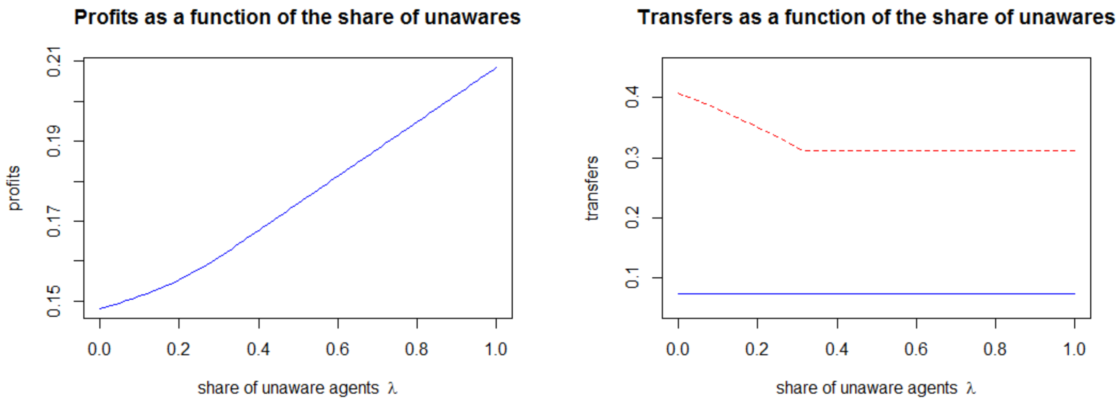
Example. Consider the output space $Y = \{0,1\}$, with probability distribution $p(1) = e$ and cost function $c(e) = e^3$. Furthermore let $\bar{u} = \frac{1}{9}$ be the reservation utility level and $m = \frac{2}{27}$ the limited liability constraint. We denote the contract by the base payment in case of $y = 0$, $\underline{t}$ and the bonus, Δt. It is easy to show that this problem is amenable to the first order approach. The first order condition of the agent reads

$$\Delta t = 3e^2$$

Clearly, there is no incentive to increase the base payment $\underline{t}$ above the limited liability bound with only aware agents. The solution to this base line problem is

$$\begin{aligned} \underline{t}^* &= \frac{2}{27} \\ \Delta t^* &= \frac{1}{3} \\ \Pi(0,\cdot) &= \frac{4}{27} \end{aligned}$$

Consider the problem with unaware agents with default action level $\bar{e} = \frac{10}{27}$. The rent to the agent given the base line contract $(\underline{t}^*, \Delta t^*)$ is $\underline{t}^* + \Delta t^* \cdot \bar{e} - \bar{e}^3 \approx \frac{4}{27} > \frac{1}{9}$. Furthermore net output is larger, so $\bar{e} \in N \cap U$. Figure 1 gives the profit under the optimal incentive contract and the optimal transfer payments as a function of the share



(a) Profit

(b) The optimal contract

Figure 1: The Two-Output Example: Numerical Results

of unaware agents λ.[15] We see that it is optimal to keep the agent unaware for every λ. Furthermore the rent given to initially aware agents in decreasing in λ since optimal incentives are weaker the more unaware agents there are in the population. For a high share of unaware agents the strength of incentives stops to decrease since the participation constraint of the unaware agents becomes binding.

Combining Risk Aversion and Limited Liability

In the previous sections we analyzed mainly the case of linear utility. This is both due to tractability and in order to contrast the effects of unawareness under risk aversion and limited liability. In this subsection, we consider the case of risk aversion with limited liability. The qualitative predictions are robust to the introduction of risk aversion.

For this subsection, we assume that every agent evaluates monetary payoffs according to a twice continuously differentiable utility function $u(x)$. Utility is increasing in transfers ($u' > 0$) and weakly convex ($u'' \leq 0$).

First, let us point out a crucial difference between linear and concave utility. As argued above, with linear utility the contract of the unaware agent can be indeterminate. This changes once we introduce risk aversion. Furthermore, there is no point to introduce an additional contract for unaware agents in the case of linear utility as long as the sorting constraint from unaware to aware agents is binding. That is true because the pooling outcome already implements the minimal rent for unaware agents. Even with low degrees of risk aversion, it is now profitable to sort aware and unaware agents even when the contract given to the aware agents determines the rent of the unaware. This is because any given level of utility of the unaware agent can be realized more cheaply by

[15]The code generating the figures can be found in Appendix A.

a contract that is less risky for the unaware agent than the awares contract. This makes the sorting constraint from awares to unawares binding in many cases. Formally,

Proposition 7. *Suppose the unaware agent has a strictly concave utility function. Consider a pair of contracts (t_A, t_U). Either t_U is a constant or the sorting constraint from aware to unaware agents is binding.*

Proof. Suppose t_U is not a constant. Then we can replace the transfer with a mixture with its certainty equivalent. This leaves the expected utility of the agent unaffected but decreases expected transfers. Thus this would be profitable. The only constraint that is tightened by this change is the sorting constraint from awares to unawares. Thus if this constraint is slack there is such a profitable mixture, establishing that t_U is not optimal. □

As discussed above, either the participation or the sorting constraint from unaware to aware agents has to be binding in order to set a lower bound to the utility of the unaware type. This proposition shows that for strictly risk averse agents, also the second sorting constraint has to be binding unless the transfer scheme is risk less. As in the linear utility case, a constant transfer scheme is attractive to the aware agent since he can free ride without exerting any effort. What is new however, is that there is an incentive for the principal to provide as risk less contracts as possible to the unaware agent. Even very low levels of risk aversion collapse the multiplicity of optimal contracts that we observe in the linear utility case for the unaware agent. Thadden and Zhao (2012) find that unaware agents always bear some risk in the case without limited liability. That is because the sorting constraint from aware to unaware agents is always binding. Since incentivizing the aware agent is costly in terms of rents, the sorting constraint is not always binding if the limited liability rent is sufficiently high and risk aversion sufficiently low. A risk less transfer scheme can therefore be optimal due to limited liability.

Proposition 1 (Monotonicity of $U(\lambda)$) is robust to the introduction of risk aversion. As mentioned above, the proof of this proposition is based on the fact that the constraints of the problem do not depend on λ and is thus independent of the shape of the utility function.

Even though Proposition 3 does not generalize directly, the qualitative prediction does generalize. Typically there is a set of effort levels that give the principal a higher profit given t^* than the chosen effort e^*. If the problem satisfies the first order approach, this has to be the case. Since the slope of expected output with respect to effort is larger than the slope of the cost function which equals the slope of expected transfers by the incentive compatibility constraint, the principals profit is locally increasing in effort at the implemented effort e^*. Furthermore binding limited liability constraint imply a positive limited liability rent. Given continuity, there is an open set around e^* at which the participation constraint is slack for an unaware agent invading the aware agents contract. Thus there is an open set of positive measure at which unaware agents are kept unaware irrespective of their share in the population λ also in the case combining limited liability and risk aversion as long as the limited liability constraint is binding in the optimal contract of the aware-only problem.

Also the logic of our third qualitative statement also generalizes to the risk aversion case. All that matters is that the presence of unaware agents may give the principal an incentive to decrease the strength of incentives which in turn decreases the limited liability rent to aware agents. It is easy to construct examples that demonstrate this possibility.

3.4 Discussion

Making Agents Aware, Timing and Commitment

In our analysis we assumed that agents can be made aware of their full choice set in the contract offer. Intuitively one can think of this process as a simple notification about the importance of certain aspects of behavior on the performance in the task. Reading this announcement, agents reconsider their choices and gain awareness of their full action set.

We assumed throughout that the principal can make agents aware of their full action set only and cannot choose subsets thereof. This is a natural assumption to make if we think of dimensions of action, such as adherence to a dress code or diligence in securing cargo. Whenever a specific aspect is pointed out by the principal, unaware agents have an aha-moment about the whole dimension of effort.

The timing of making agents aware is without loss of generality. Even if principals could make agents aware after they committed to a contract, the equilibrium would not change. This is because the newly aware agent solves the same optimization problem under the same constraints as the ex-ante aware agent. Therefore the optimal contract for these two agents is the same and there is no incentive for the principal to lock the initially unaware agent into an unfavorable contract.

Unaware Agents vs. Behavioral Agents

A common critique of considering unawareness of actions is that is equivalent to models with a share of behavioral agents. This reinterpretation can be problematic for some aspects, however. Agents with unawareness of action are equivalent with behavioral agents only as long as they are kept unaware. Introducing the possibility to turn behavioral agents into fully rational agents seems rather implausible without thinking in terms of unawareness. This problem would be especially pronounced in our setup, where making an agent aware of his full action choice is done costlessly in a simple contract offer.

It is correct that unawareness of actions can be reinterpreted as behavioral agents, particularly if agents are not made aware. We would argue that unawareness of actions thereby provides a microfoundation based on rigorous models of knowledge and cognition for behavioral agents. This makes the investigation of models with unawareness of actions more relevant rather than redundant.

Pre-Contractual Cognition

The set of contracts offered by the principal are typically not optimal for the principal from the unaware agents point of view. There is no reason within the unaware agents mental model for a menu of contracts at all. This can be seen as a reason for the agent to reconsider this original mental model.[16] Suppose a share of β of all unaware agents is made aware by seeing a non-rationalizable contract. The principal has three possible reactions: Either offer the optimal, not rationalizable contract for a share of $\lambda\beta$ unaware agents, offer a rationalizable contract that leaves agents unaware, or make them aware. The profit maximizing option out of these three alternatives is the outcome once we consider pre-contractual cognition.

We do not consider this problem in more detail since such cognition generally only points agents towards the observation that they are unaware of some fact. This knowledge is fully consistent with unawareness of a specific fact as discussed in the literature review section. Furthermore there can be unmodeled considerations that rationalize the contracts, such as heterogeneous preferences or abilities. An economic model is a simple representation of a complex world that offers agents many explanations for all kind of observations without necessarily making them aware of one specific fact.

A justifiability criterion demands that the contract offer has to be optimal for the principal from the perspective of the agent (Chen and Zhao, 2013). For the reasons given above, we do not think that justifiability is required in models with unawareness of actions.[17]

Sets of Default Action Levels

We only consider unaware agents with a single default action. Depending on the other assumptions of the problem, the consequences of this assumption vary. Assuming the first order approach is valid, any compact interval of effort levels reduces to two points

[16] See the discussion in (Thadden and Zhao, 2012).

[17] In models with unawareness of theorems, justifiability can be very useful by providing a bound on the admissible updated priors. See Auster (2013) and Filiz-Ozbay (2012).

of interest. The principal would never choose an interior point unless it is the second best outcome, since at interior points the problem of an unaware agent with an interval of effort choices is equivalent to an aware agent. Thus the problem is equivalent to the case of an unaware with two possible default actions. Once we relax the first order approach, the problem becomes more complicated since interior points may be of interest to the principal.

The case of a two point choice set gives the principal an additional choice and a simplified incentive compatibility constraint. The total effect on profits and contractual forms is ex-ante unclear and left for future research.

Multiple Types of Unaware Agents

Throughout the paper, we consider only two types, one unaware and one aware. This is clearly a restrictive assumption. It would be interesting to see a generalization of our analysis to the case of an arbitrary distribution of default effort levels.

3.5 Conclusions

Unawareness of action is an important characteristic of real world contracting situations. In many cases agents do not have a complete mental model of all relevant contingencies and all possible actions in mind when making their decisions. Principals may be able to exploit this fact to increase their profit. Also they might be able to make agents aware of possibilities strategically.

In this paper we investigate unawareness of action in a moral hazard framework with limited liability and the corresponding sorting problem. A potentially unaware agent engages in a standard moral hazard problem with a monopolist principal. Since the principal does not know whether the agent is unaware or not, she may want to screen the two types of agents using a menu of contracts. The principal also has the option to make agents aware of their full choice set.

We give a detailed description of the characteristics of the set of default action levels $\bar{e}$ and shares of unaware agents λ at which ex-ante unaware agents remain unaware. In particular this set is increasing in the share of unaware agents in the population. Unawareness is self-reinforcing in this sense. Furthermore we contrast our results with the qualitative statements made by Thadden and Zhao (2012) about the case with risk aversion without binding limited liability constraints. The characteristics of the set of default action levels at which agents are made aware and of the aware agents' rent are very different from those generated by the risk aversion case. This is because the existing limited liability rent is used in the sorting problem while in the risk aversion case such a rent has to be created. The findings from the limited liability case are robust to the addition of risk aversion.

The result that unawareness is self-reinforcing is independent of the source of the agency friction. It is tempting to give an intertemporal interpretation to this result. Over time groups of agents with many unaware agents will keep these unaware agents while groups with few unaware agents will go to full awareness. Note however that we analyzed only a static one-shot model. Investigating dynamic contracting problems with unawareness is left for future research.

4 Unawareness of Action in Competitive Equilibrium

In the previous section we considered a monopolist principal. In such a situation, the principal can use his superior awareness to extract higher profits from unaware agents. For some parameters the principal keeps agents unaware to increase his profit even if this decreases output further from the second best level. In this section we study how the introduction of competition between principals into the model of Section 3 changes the conditions under which unaware agents are made aware.

In many settings, competition increases the efficiency of an allocation. Even though this does not necessarily extent to cases with agency frictions, there is reason to believe that inefficient exploitation of unaware agents cannot persist in equilibrium. The intuition is that another principal could make these agents aware and offer a contract similar to the optimal contract for aware agents to gain positive profits. We show that this intuition doesn't go through in every case. The reason is that the agent may want to stay at the original principal even after he is made aware. Thus the other principal has no incentive to make him aware in the first place. This mechanism is similar to Gabaix and Laibson (2006), who find that shrouded attributes persist even if unaware consumers can be costlessly educated by competitors.

Since making agents aware is public in our setup, it is always an equilibrium that all principals make agents aware and offer the contract optimal in the case of only aware agents. Once we use an intuitively appealing refinement, this outcome is the only remaining equilibrium outcome for a large set of parameters. The refinement is achieved by endowing principals with lexicographic preferences in their own profits and decreasing in their competitors profits. In this setup also deviations that do not increase own profits but hurt competitors are viable deviations. Once a contract introduces subsidies, it is vulnerable to a malicious deviation. As these equiolibria are the only alternative to making agents aware for many parameter combinations, only equilibria making all agents aware can persist in these cases.

This paper is organized as follows. Subsection 4.1 introduces the competitive contracting problem. We discuss cross subsidies and the inability of competing principals to attract agents off cross subsidizing firms in Subsection 4.2. The results on persistence of unawareness in equilibrium are presented in Subsection 4.3. We discuss our modeling choices and their interpretation in Subsection 4.4. Subsection 4.5 concludes.

4.1 Setup[18]

We assume that there are $n \geq 2$ risk neutral principals. All of them operate the same technology $F(y|e)$, are aware of the full action space and know the share of unaware agents λ and their default effort level $\bar{e}$. The principals offer a menu of exclusive and binding contracts (t_A^i, t_U^i, a_i), where t_I^i denotes transfers as a function of output and $a_i \in \{0,1\}$ is a binary variable indicating whether this contract offer makes the agents aware or not. The principals move simultaneously, then the agent decides for a contract. An agent can choose one contract out of the menu or his outside option $\bar{u}$. We make no specific assumptions on how agents split once principals offer equivalent contracts.

We assume that "making an agent aware" is already done in the contract offer. Formally, a contract is thus a tuple of a transfer scheme and a binary variable denoting whether or not a contract makes the agent aware. Intuitively, a contract offer can induce awareness of his choice set in an agent via a very detailed description of the agents options or prescribed actions. Furthermore, making an agent aware is a public action that affects the whole pool of agents, not only the subset choosing the contract of a specific principal. Thus competing principals poison the common well if this is profitable for them. This assumption is natural if we think of settings in which making an agent aware of his choice set is possible via a simple reminder that can be put in a job advertisement. It is less natural in settings where increasing awareness this requires lengthy training.

4.2 Cross-Subsidies and Attracting "Exploited" Agents

Once again we will use the solution of the problem without unawareness as a benchmark case. For this problem, there exists a symmetric equilibrium that is the solution to the dual problem

$$\begin{aligned} \max_t \max_e \quad & \int_Y t(y)\,\mathrm{d}F(y|e) - c(e) \\ \text{s.t.} \quad & \int_Y \int y - t(y)\,\mathrm{d}F(y|e) \geq 0 \\ & t(y) \geq m \qquad \forall y \in Y \end{aligned}$$

In this problem, the principal maximizes the agents utility subject to incentive compatibility and a non-negative profit constraint. We assume that a solution for this problem

[18] The setup considered in this section is the same as the one discussed in Subsection 3.1 unless explicitly stated.

exists and is unique.[19] Denote this contract by t^* and the optimal effort level by e^*. We will assume that e^* is unique. Clearly, if this contract is offered by all principals, no principal can profitable deviate. Thus, all principals offering t^* is an equilibrium. Conversely in every equilibrium, principals have to obtain zero expected profits.

Note that even though competition tends to push the optimal effort closer to the first best solution in the moral hazard framework with limited liability, the first best effort may not be achieved with suitably strict limited liability constraints. We will assume that this is the case throughout our analysis.

It is also helpful to consider the problem only with unaware agents. Assuming agents remain unaware every contract that satisfies the zero profit condition, i.e. every contract that hands the output over to the agent, is an equilibrium contract. Agents remain aware in some equilibrium if and only if net output at the default action level $\bar{e}$ is higher than net output at e^*. Thus in this case, agents are made aware if it is efficient to do so.

Let us now turn to the problem with a fixed and known share λ of unaware agents and the issue of cross-subsidies. Clearly also in the problem with two types of agents, expected profits have to be zero. But this condition doesn't have to be satisfied point-wise, i.e. for aware and unaware agents separately. We call a contract where this is not the case *cross-subsidizing* since funds are transferred from one type of agent to another. It is possible to have such cross-subsidies from unaware to aware agents, but not vice versa.

Proposition 8. *There cannot be an equilibrium contract in which aware agents cross subsidize unaware agents, i.e.* $\int_Y t_A(y)\,\mathrm{d}F(y|e_A) \geq \int_Y y\,\mathrm{d}F(y|e_A)$.

Proof. Suppose there is such a contract. Since expected output is higher than expected transfers, net output from aware agents has to be higher than net output from unaware agents.

$$\int_Y y\,\mathrm{d}F(y|e_A) - c(e_A) > \int_Y t_A\,\mathrm{d}F(y|e_A) - c(e_A) \geq \int_Y t_U\,\mathrm{d}F(y|\bar{e}) - c(\bar{e}) > \int_Y y\,\mathrm{d}F(y|\bar{e}) - c(\bar{e})$$

The second inequality follows from the sorting constraint from aware to unaware agents. The other two are simply restatements of the assumption of cross-subsidies. Furthermore, the total rent to aware agents has to be lower than under t^* since it is maximal under this contract. Thus expected net output is lower than under the baseline contract t^*. Therefore a principal offering ε less than (t^*, t^*) and making the agents aware can attract at least the aware agents in this contract. □

[19] Uniqueness is not essential for the analysis but simplifies the notation. All results and claims can be extended easily to the case of multiple solutions.

Conversely, consider the case in which unaware agents subsidize aware agents. There may not be a profitable deviation. Making the agents aware is not a profitable deviation in general, since the maximum rent from the unaware contract may be higher than the maximal rent from t^* while still being lower than the subsidized rent from t_A. Formally,

Proposition 9. *There can be a cross subsidy from unaware to aware agents only if the sorting constraint from aware to unaware agents is binding.*

Proof. Suppose the sorting constraint is not binding and the pair of transfer schemes (t_U, t_A) with cross subsidies is taken by a positive share of both unaware and aware agents. Then we can design a pair of contracts that has lower expected transfers for the aware agent and higher expected transfers for the unaware and satisfies all constraints of the problem. This contract can be designed as to give positive profits to the principal. That is the case because expected transfers in the initial contract are strictly lower than expected output from the unaware agent and only unaware agents are attracted to this kind of contract by construction. Thus there is a profitable deviation. $\square$

We can demonstrate by a simple example that cross subsidizing contracts can actually exists in equilibrium.

Example. [20]Consider the example from above, that is a two-output case with $Y = \{0,1\}$, probability distribution $p(1) = e$ and cost function $c(e) = e^3$. Furthermore let $\bar{u} = \frac{1}{9}$ be the reservation utility level and $m = \frac{2}{27}$ the limited liability constraint. Since a higher bonus payment increases the agents effort and thereby increases his rent over proportionally, the limited liability constraint is also binding under competition.

It is easy to verify that in the unique equilibrium we have $\Delta t \approx 0.86$ and agents exert an effort of $e^* \approx 0.53$. This contract leads to net output of $\mathbb{E}y - c(e^*) = e^* - {e^*}^3 \approx 0.38$ and this is equal to expected transfers minus cost.

Let us now introduce a share of $\lambda = 0.2$ of unaware agents with default action level $\bar{e} = 0.6$. Note that this effort level does produce higher net output than the one induces by the optimal contract in the problem without unawareness. The transfer maximal contract that leaves agents unaware has a bonus payment of $\Delta t = 0.87$. Thus it is closer to the first best outcome. This contract also leads to higher net output. The rent of the unaware agent is lower than that of the aware agent in this cross-subsidizing contract as well as in the baseline contract $\mathbb{E}_{\bar{e}}t - \bar{e}^3 = 0.377 < 0.38$. Nevertheless this contract is an equilibrium contract since making the agents aware yields zero profit. An aware agent would choose to participate in this contract to the best contract he can get from a principal taking into account that all agents are aware.

[20]All computations for this and the following example can be found in Appendix A.

Similar to the mechanism in Gabaix and Laibson (2006) it is not profitable to make the agents aware even though they get less than their output. That is because an aware agent would choose the original contract over any contract that can be profitably offered to aware agents without cross-subsidies. In the above example, the cross-subsidizing outcome is constrained efficient. One can also construct examples in which the unaware agent is better of in the "exploitative" cross-subsidization equilibrium compared to the case in which he is made aware.

Example. Consider again the above example with a tighter limited liability constraint $m' = \frac{5}{27}$ and a share of unaware agents $\lambda = 0.5$ with default action level $\bar{e} = 0.45$. It is easy to show that the rents in the optimal contracts are as follows

$$\begin{aligned} \mathbb{E}_{e^*} t^* - {e^*}^3 &= 0.3544 \\ \mathbb{E}_{e_A} t - e_A^3 &= 0.3571 \\ \mathbb{E}_{\bar{e}} t - \bar{e}^3 &= 0.3570 \end{aligned}$$

Thus the unaware agent gets less than his output in the cross-subsidizing contract, but his rent is higher than in the best contract after he was made aware. Making agents aware is again not profitable since they would remain at the principal offering the cross-subsidizing contract.

4.3 Competition and Awareness

As it is typical for a game with simultaneous moves, also this problem may have multiple equilibria. There exists one very simple equilibrium for every $\bar{e}$ and λ, in which unaware agents are made aware.

Proposition 10. *There is a symmetric equilibrium in which all principals make agents aware in the contract offers and the offered transfers are (t^*, t^*) for all i.*

Proof. A single deviation doesn't change the level of awareness. Thus a single deviation principal faces a problem that is equivalent to the contracting problem with competing principals without unawareness. Thus (t^*, t^*) is a best response. □

Depending on the number of principals there can be several equilibria that have the same outcome as the one described in Proposition 10. All principals have zero profits in equilibrium. Thus any contract that gives non-negative profits can be part of an equilibrium in which all agents are aware and at least two principals offer the baseline contract. This is the only contract that will be chosen in these equilibria.

As discussed in the previous subsection, there can be equilibria in which agents remain unaware and there are cross-subsidies from unaware to aware agents. As we are interested in the persistence of unawareness under competition, we would like to have a unique prediction based on the parameters of the problem. To get such a prediction, we have to refine the set of equilibria.

Lexicographic Preferences

Suppose that the principal not only maximizes his own profits but has lexicographic preferences in his own profit and negative of the sum of other principals profits. This can be motivated by long run competition between these principals, such that it would be desirable to put competitors out of business. Clearly, cross-subsidizing equilibria no longer exist in the presence of these lexicographic preferences. Even though making agents aware does not increase profits, it leads to negative profits for the principal offering the cross-subsidizing contract and is therefore desirable.

Proposition 11. *For the case with lexicographic preferences and default action levels $\bar{e}$ such that net output is strictly higher than under the baseline contract, there is a unique equilibrium outcome in which all agents are made aware and choose the contract t^*.*

Proof. Suppose the agent remains aware and there is a set of transfers (t_U, t_A) chosen in equilibrium that is not cross subsidizing. The contract has to give a higher rent to unaware agents than to aware agents under the baseline contract. But this is impossible by the sorting constraint and the nonnegative profit condition. Therefore every contract has to be cross-subsidizing. Making all agents aware and offering (t^*, t^*) is lexicographically preferred to a cross-subsidizing contract. Thus there can be no such equilibrium.

Assume agents are made aware, then t^* is the only chosen contract by construction. □

In case of extreme competition induced by these kind of preferences, the most productive unawares cannot stay unaware. The high level of output from these unawares will be used to increase the strength of incentive to aware agents by principals in any equilibrium. This means that aware agents get a rent higher than their output. This makes principals offering this contract vulnerable to predatory creation of awareness by their competitors. There are only two types of equilibria under this refinement.

Proposition 12. *For lexicographic preferences, there are only two equilibrium outcomes*

1. *All principals make agents aware and the only accepted contract is t^*.*
2. *Agents remain unaware and the only contract accepted by aware agents is t^*.*

Proof. The first class of equilibria is the only in which agents are made aware. The agents awareness cannot be altered by deviations, there is no profitable deviation and the other principals profit is zero irrespective of deviations. Thus these equilibria are robust to the introduction of lexicographic preferences.

Suppose there is a default action level $\bar{e}$ such that net output is lower than under e^*, higher than expected net transfers under t^* at $\bar{e}$ and there is a contract t_U that equalizes expected net transfers and expected net output at $\bar{e}$ while satisfying the sorting constraints. Formally

$$\int_Y y \,\mathrm{d}F(y|e^*) - c(e^*) \geq \int_Y y \,\mathrm{d}F(y|\bar{e}) - c(\bar{e})$$

$$\int_Y y \,\mathrm{d}F(y|\bar{e}) - c(\bar{e}) \geq \int_Y t^*(y) \,\mathrm{d}F(y|\bar{e}) - c(\bar{e})$$

$$\int_Y t_U(y) \,\mathrm{d}F(y|\bar{e}) = \int_Y y \,\mathrm{d}F(y|\bar{e})$$

$$\int_Y y \,\mathrm{d}F(y|e^*) - c(e^*) \geq \max_e \int_Y t_U(y) \,\mathrm{d}F(y|e) - c(e)$$

The second class of equilibria exists if and only if these conditions are met. Clearly, making agents aware also gives no profits and does not harm competitors. Since this contract transfers the full possible output given unawareness, there is no profitable deviation keeping agents unaware.

The second class of contracts is the only without cross-subsidization. Contracts that promise aware agents a rent higher than the rent from t^* have to be cross subsidizing. Contracts that promise less are never part of an equilibrium since making agents aware and offering $(t^*_\varepsilon, t^*_\varepsilon)$ is a profitable deviation for suitable ε. □

Agents remain unaware under extreme competition only if keeping them unaware is inefficient from a total net output perspective. If the other necessary conditions of preserving unawareness can be met depends on the parameters of the problem. Most notably the space of contracts has to be rich enough. There has to be a contract that matches the unaware agents expected output while satisfying the sorting condition. Furthermore the curvature of the transfer profile induced by t^* has to be sufficiently high to allow for slackness of the sorting constraint from unawares to awares.

A key difference between competition and the monopoly case analyzed in Section 3 is that under competition, the set of equilibrium outcomes is independant of the share of unaware agents λ. This only holds under extreme competition since in this case linking the contracts offered to aware and unaware agents in any way opens principals to predetory creation of awareness. Even though we cannot give a full analysis in the general case, the competitive case appears to be less efficient than the monopoly case. For an equivalent base line effort e^* unaware agents that produce more are made aware under competition, while they generally stay unaware in the monopoly case. This comparison is imperfect, since baseline effort is different in the two cases for the same set of parameters of the problem.

4.4 Discussion

Many issues discussed in Subsection 3.4 for the monopoly case remain valid under competition. We will not reproduce these discussions here but focus on questions exclusive to the case of competition as analyzed in this section.

Public vs. Private Announcement of the Action Set and its Timing

In the case of competing principals, there are several natural and mutually exclusive possibilities for modeling the induction of awareness. First of all, the announcement could be made affecting all agents before the contracts are accepted. This is the case in our model and corresponds to a simple notice in a public job advertisement. Secondly, there could be a private announcement after the agent has accepted the contract. This assumption is most compelling if we think about firm specific awareness induced by potentially costly training. Finally, private announcements could be made before the contract is binding. This kind of assumption is only reasonable if the relevant dimension of action is firm specific. The other combination are less compelling. It seems unreasonable that a principal makes all agents in the population aware but waits until they are committed into contracts.

As opposed to the monopoly case, the timing of inducing awareness is crucial in the setting with competing principals. Given the power to make agents aware after they are committed to a contract gives the principal some monopoly power. Public announcements are also an important prerequisite of our results. One can think of the assumption that agents are made aware publicly as a result of fierce competition. Once it is profitable to make agents willing to choose jobs at other firms aware, firms may be able to find ways to achieve this. Our assumption is a simple short cut for this outcome. This is of course not plausible if the possible actions are firm specific and possibly even unknown to competitors.

If making an agent aware is only possible for agents who have already accepted a contract, the problem changes completely. Once agents are made aware, the principal no longer competes with others. We thus move away from a fully competitive setting into a setting in which principals try to attract unaware agents in order to lock them into their firm. It would be interesting to analyze how competition for unaware agents over transfers at the default action level in the primary market and the incentive to maximize expected profit at the effort that is implemented ex-post interact. This question is beyond the scope of the paper.

The possibility to make the agents at other firms aware is essential for our result. If it is impossible to break cross-subsidizing equilibria - which is the case without this option, we cannot get the unique awareness outcome. Still, private announcements also eliminate the trivial awareness equilibrium in some cases since a principal can unilaterally keep his agents unaware in this case. The full analysis of the set of equilibria with private announcements is left for future research.

Refinements

The contracting problem we consider can be seen as a extensive form game with complete but imperfect information. The equilibrium concept we use without formal definition throughout the paper is a subgame perfect equilibrium in one of the equivalent representations of the game. We further refine the set of equilibria by introducing lexicographic preferences for the principal. It is easy to see that lexicographic preferences do not introduce new equilibria as deviations that are profitable in the first coordinate remain profitable.

The same result cannot be achieved by using other refinement concepts. First of all, refinements based on small deviations in strategies are not defined for our game as long as the outcome space is infinite. An infinite outcome space makes the strategy set infinite dimensional which creates problem when selecting a topology. Even for a finite outcome space these concepts do not rule out exploitation equilibria for generic contracting problems. That is because the indifference between exploitation contracts and making agents aware is not broken by perturbations.

4.5 Conclusions

We analyze how unawareness affects the solution to a moral hazard problem with competing principals. To refine the set of equilibria we assume that principals have lexicographic preferences in their own profits and the negative sum of their competitors profits. In this case of extreme competition we find that there are only two possible outcomes. Either agents are made aware or agents remain unaware with contracts that are not interrelated. This is because any relation between the two contracts opens up the equilibrium to a predatory deviation - one that does not increase the deviating principals profits but decreases that of the other principals.

This outcome appears to be wasteful. Unaware agents with productive default actions are made aware in the unique outcome. Not only is net output reduced, but also the rent of unawares is lower than under the case of interrelated contracts. While these contracts may be “exploitative” in the sense that unaware agents get less than their full output and thus cross-subsidize aware agents, if given the choice between getting aware and receiving the baseline contract and staying unaware and being “exploited”, unaware agents would choose the latter. Furthermore, it is not assured that unaware agents with relatively unproductive default effort are made aware in any equilibrium. To the contrary, these effort levels are the only for which an equilibrium exists in which agents remain unaware.

Whether competition promotes awareness is unclear. In both the competitive and the monopoly outcome there are sets of default effort levels for which unaware agents are made aware in any equilibrium. Competition promotes awareness in the sense that there always is an equilibrium in which agents are made aware, irrespective of the default effort level $\bar{e}$ and the share of unaware agents λ. On the other hand, depending on equilibrium selection, unaware agents with low default efforts may remain unaware in the competitive outcome even though they are made aware by the monopolist irrespective of the share of unaware agents.

5 Conclusions and Outlook

Unawareness of actions has rich interactions with other properties of moral hazard problems. In a monopoly the source of the agency friction and the share of unaware agents are two key determinants of the outcome. Under competition, by contrast, the outcome is independent of the share of unaware agents if we use a simple and compelling refinement.

So far the exposition and discussion was quite abstract. Even though terms are framed as effort and output, we did not discuss concrete examples or organize the flow of the argument around the explanation of a specific phenomenon. We fill this gap in the conclusion.

Firms in the same industry can have very different corporate cultures. Some have a very competitive climate with performance based pay while others cultivate a more relaxed atmosphere. Clearly this can be explained by ex-ante heterogeneity, but how can we make these differences an outcome of the model instead of putting them into the assumptions? The model of Section 4 has a set of equilibria that feature endogenous symmetry breaking. When the contract space is rich enough and unaware agents are not too productive, the second class of equilibria described in Proposition 12 exists. In this class we can have equilibria in which some firms only cater to aware agents using high-powered incentives while other firms attract only unaware agents. If we assume risk aversion, these contracts will be less risky and offer weaker incentives. Thus, even though firms are symmetric ex-ante, we can observe heterogeneous contracts and work environments ex-post.

An interpretation of the now classic daycare study (Gneezy and Rustichini, 2000) based on unawareness of actions gives interesting welfare implications. The situation closely resembles a moral hazard problem. Parents exert unobservable effort to come on time and a stochastic delay occurs. But the classic sources of agency frictions seem to be missing. As it is possible to impose fines, limited liability constraints don't seem to be binding. Given that day care centers are not diversivied enterprises, a systematic difference in risk attitudes is possible but far from necessary. Suppose that parents were unaware of their effort choice at first but became aware due to the introduction of fines. In an idealized model, parents now optimize given the fines and since no source of agency frictions is apparent an efficiently set fine[21] can achieve the first best outcome and improve upon the ex-ante outcome.

To gain a more complete understanding of the interaction between unawareness of action and moral hazard it would be beneficial to relax some assumptions we made throughout the analysis. Most notably the assumption that there is only one default

[21] Due to increasing returns in day care at the relevant scale, this wouldn't be a fixed fine but a non-linear tariff.

effort level in the population is very restrictive and it would be reassuring to know that the general predictions made about unawareness at a certain default action level go through in a richer setting.

We find that unawareness is self-reinforcing. It is tempting to give this statement a dynamic interpretation. This is more than a static model can support, however. In a fully dynamic setting, aware agents may have the option to signal their awareness by a non-stationary effort choice. This is only one of many possible complications. A full investigation of the persistence of unawareness has to be based on the study of a fully dynamic model.

References

Auster, Sarah, "Asymmetric awareness and moral hazard," *Games and Economic Behavior*, November 2013, *82*, 503–521.

Bénabou, Roland and Jean Tirole, "Intrinsic and Extrinsic Motivation," *The Review of Economic Studies*, July 2003, *70* (3), 489–520.

Board, Oliver J. and Kim-Sau Chung, "Object-Based Unawareness: Axioms," Technical Report, University of Minnesota 2007.

_ and _ , "Object-based unawareness: Theory and applications," Technical Report, University of Minnesota 2008.

_ , _ , and Burkhard C. Schipper, "Two models of unawareness: comparing the object-based and the subjective-state-space approaches," *Synthese*, April 2011, *179* (1), 13–34.

Bolton, Patrick and Mathias Dewatripont, *Contract Theory*, Cambridge, Mass: The MIT Press, December 2004.

Chen, Yi-Chun, Jeffrey C. Ely, and Xiao Luo, "Note on unawareness: Negative Introspection versus AU Introspection (and KU Introspection)," *International Journal of Game Theory*, May 2012, *41* (2), 325–329.

Chen, Ying-Ju and Xiaojian Zhao, "Solution Concepts of Principal-Agent Models with Unawareness of Actions," *Games*, August 2013, *4* (3), 508–531.

Chiappori, Pierre-Andre and Bernard Salanié, "Testing contract theory: A survey of some recent work," in Mathias Dewatripont, Lars Peter Hansen, and Stephen Turnovsky, eds., *Advances in Economics and Econometrics*, Vol. 1 2003, p. 115–149.

de la Rosa, Leonidas Enrique, "Overconfidence and moral hazard," *Games and Economic Behavior*, November 2011, *73* (2), 429–451.

de Meza, David and Clive Southey, "The Borrower's Curse: Optimism, Finance and Entrepreneurship," *The Economic Journal*, March 1996, *106* (435), 375–386.

Dekel, Eddie, Barton L. Lipman, and Aldo Rustichini, "Standard State-Space Models Preclude Unawareness," *Econometrica*, January 1998, *66* (1), 159–173.

Eliaz, Kfir and Ran Spiegler, "Contracting with Diversely Naive Agents," *The Review of Economic Studies*, July 2006, *73* (3), 689–714.

Fagin, Ronald and Joseph Y. Halpern, "Belief, awareness, and limited reasoning," *Artificial Intelligence*, December 1987, *34* (1), 39–76.

Feinberg, Yossi, "Games with unawareness," *Stanford GSB, Palo Alto*, 2012.

Filiz-Ozbay, Emel, "Incorporating unawareness into contract theory," *Games and Economic Behavior*, September 2012, *76* (1), 181–194.

Gabaix, Xavier and David Laibson, "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets," *The Quarterly Journal of Economics*, May 2006, *121* (2), 505–540.

Galanis, Spyros, "Syntactic foundations for unawareness of theorems," *Theory and Decision*, October 2011, *71* (4), 593–614.

_ , "Unawareness of theorems," *Economic Theory*, January 2013, *52* (1), 41–73.

Geanakoplos, John, "Game Theory Without Partitions, and Applications to Speculation and Consensus," *Cowles Foundation Discussion Paper*, 1989.

Gneezy, Uri and Aldo Rustichini, "A Fine Is a Price," *The Journal of Legal Studies*, January 2000, *29* (1), 1–17.

Grant, Simon and John Quiggin, "Inductive reasoning about unawareness," *Economic Theory*, November 2013, *54* (3), 717–755.

Halpern, Joseph Y., "Alternative Semantics for Unawareness," *Games and Economic Behavior*, November 2001, *37* (2), 321–339.

_ **and Leandro C. Rêgo**, "Interactive unawareness revisited," *Games and Economic Behavior*, January 2008, *62* (1), 232–262.

_ **and** _ , "Reasoning about knowledge of unawareness," *Games and Economic Behavior*, November 2009, *67* (2), 503–525.

_ **and** _ , "Reasoning about knowledge of unawareness revisited," *Mathematical Social Sciences*, March 2013, *65* (2), 73–84.

_ **and** _ , "Extensive games with possibly unaware players," *Mathematical Social Sciences*, 2014.

Heifetz, Aviad, Martin Meier, and Burkhard C. Schipper, "Interactive unawareness," *Journal of Economic Theory*, September 2006, *130* (1), 78–94.

_ **,** _ **, and** _ , "A canonical model for interactive unawareness," *Games and Economic Behavior*, January 2008, *62* (1), 304–324.

— , — , and — , "Dynamic unawareness and rationalizable behavior," *Games and Economic Behavior*, September 2013, *81*, 50–68.

— , — , and — , "Unawareness, beliefs, and speculative trade," *Games and Economic Behavior*, January 2013, *77* (1), 100–121.

Hölmstrom, Bengt, "Moral Hazard and Observability," *The Bell Journal of Economics*, April 1979, *10* (1), 74–91.

Jewitt, Ian, "Justifying the First-Order Approach to Principal-Agent Problems," *Econometrica*, September 1988, *56* (5), 1177–1190.

— **, Ohad Kadan, and Jeroen M. Swinkels**, "Moral hazard with bounded payments," *Journal of Economic Theory*, November 2008, *143* (1), 59–82.

Karni, Edi and Marie-Louise Vierø, ""Reverse Bayesianism": A Choice-Based Theory of Growing Awareness," *American Economic Review*, December 2013, *103* (7), 2790–2810.

Kawamura, Enrique, "Competitive equilibrium with unawareness in economies with production," *Journal of Economic Theory*, April 2005, *121* (2), 167–191.

Laffont, Jean-Jacques and David Martimort, *The Theory of Incentives: The Principal-Agent Model*, Princeton, N.J: Princeton University Press, January 2002.

Li, Jing, "Information structures with unawareness," *Journal of Economic Theory*, May 2009, *144* (3), 977–993.

Maskin, Eric, "On indescribable contingencies and incomplete contracts," *European Economic Review*, May 2002, *46* (4–5), 725–733.

— **and Jean Tirole**, "Unforeseen Contingencies and Incomplete Contracts," *The Review of Economic Studies*, January 1999, *66* (1), 83–114.

Meier, Martin and Burkhard C. Schipper, "Bayesian games with unawareness and unawareness perfection," *Economic Theory*, June 2014, *56* (2), 219–249.

Mirrlees, J. A., "The Theory of Moral Hazard and Unobservable Behaviour: Part I," *The Review of Economic Studies*, January 1999, *66* (1), 3–21.

Modica, Salvatore, Aldo Rustichini, and J.-Marc Tallon, "Unawareness and bankruptcy: A general equilibrium model," *Economic Theory*, August 1998, *12* (2), 259–292.

_ and _ , "Awareness and partitional information structures," *Theory and Decision*, July 1994, *37* (1), 107–124.

_ and _ , "Unawareness and Partitional Information Structures," *Games and Economic Behavior*, May 1999, *27* (2), 265–298.

Moroni, Sofia and Jeroen Swinkels, "Existence and non-existence in the moral hazard problem," *Journal of Economic Theory*, March 2014, *150*, 668–682.

Nielsen, Carsten S. and Alexander Sebald, "Unawareness in dynamic psychological games," *University of Copenhagen, Copenhagen*, 2011.

Ozbay, Erkut Y., "Unawareness and Strategic Announcements in Games with Uncertainty," 2006.

Page, Frank H., "The existence of optimal contracts in the principal-agent model," *Journal of Mathematical Economics*, 1987, *16* (2), 157–167.

Pearce, David G., "Rationalizable Strategic Behavior and the Problem of Perfection," *Econometrica*, July 1984, *52* (4), 1029–1050.

Rogerson, William P., "The First-Order Approach to Principal-Agent Problems," *Econometrica*, November 1985, *53* (6), 1357–1367.

Schipper, Burkhard C., "Awareness-dependent subjective expected utility," *International Journal of Game Theory*, August 2013, *42* (3), 725–753.

_ , "Preference-based unawareness," *Mathematical Social Sciences*, 2014.

_ and Hee Yeul Woo, "Political Awareness, Microtargeting of Voters, and Negative Electoral Campaigning," Technical Report March 2014.

Sillari, Giacomo, "Quantified Logic of Awareness and Impossible Possible Worlds," *The Review of Symbolic Logic*, 2008, *1* (04), 514–529.

Tirole, Jean, "Cognition and Incomplete Contracts," *American Economic Review*, February 2009, *99* (1), 265–294.

von Thadden, Ernst-Ludwig and Xiaojian Zhao, "Incentives for Unaware Agents," *The Review of Economic Studies*, July 2012, *79* (3), 1151–1174.

_ and _ , "Multi-task agency with unawareness," *Theory and Decision*, September 2013.

Zhao, Xiaojian, "Moral Hazard With Unawareness," *Rationality and Society*, November 2008, *20* (4), 471–496.

A R Codes for Graphs and Numerical Examples

Graphs

```
## Plots for monopoly section


require('Rsolnp')

profit  <- function(contr,lambda,ebar){
  if((contr[2]-contr[1])>0){
    -(lambda*(1-(contr[2]-contr[1]))*ebar+
      (1-lambda)*(1-(contr[2]-contr[1]))*sqrt(()contr[2]-contr[1])/3)
      -contr[1])
  } else
  -(lambda*(1-(contr[2]-contr[1]))*ebar-contr[1])
}

util <- function(contr,lambda,ebar){
  if((contr[2]-contr[1])>0){
  f1 <- contr[1]+
    (contr[2]-contr[1])*sqrt((contr[2]-contr[1])/3)
    -sqrt((contr[2]-contr[1])/3)^3
  } else {
  f1 <- contr[1]
  }
  f2 <- contr[1]+(contr[2]-contr[1])*ebar-ebar^3
  c(f1,f2)
}


opt.contr <- Vectorize(function(l){
  solnp(pars=c(0.08,0.4),fun=profit, l=l,ebar=10/27,
        ineqfun=util, ineqLB=c(1/9,1/9), ineqUB=c(1000,1000),
        LB=c(2/27,2/27), control=c(rho=0.6))$pars
})
```

```r
lgrid  <- seq(from=0, to=1, by=0.01)
optimizer  <- opt.contr(lgrid)
vec.profit  <- Vectorize(function(x1,x2,y,z){profit(c(x1,x2),y,z)})
optimum <- -vec.profit(optimizer[1,],optimizer[2,],lgrid,10/27)

plot(lgrid,optimum,type='l',
     xlab=expression("share of unaware agents" ~~ lambda),
     col='blue', ylab='profits',
     main='Profits as a function of the share of unawares')
matplot(lgrid,t(as.matrix(optimizer)), type='l',
        col=c('blue','red'), ylim=c(0.05,0.45),
        xlab=expression("share of unaware agents" ~~ lambda),
        ylab='transfers',
        main='Transfers as a function of the share of unawares')
```

Numerical Example

```r
## Computations for Competiton Section

cost <- function(x) {
    x^3
}
bb <- function(t, lambda, ebar, m) {
    lambda * ebar * (1 - t) + (1 - lambda) * sqrt(t/3) * (1 - t) - m
}


#### Example 1 baseline

lambda <- 0
ebar <- 0
m <- 2/27

tstar_b <- uniroot(bb, lambda = lambda, ebar = ebar, m = m, interval = c(0.4,
    1))$root
estar_b <- sqrt(tstar_b/3)
rent_a_b <- m + estar_b * tstar_b - cost(estar_b)
```

```r
# treatment

lambda <- 0.2
ebar <- 0.6
m <- 2/27

tstar <- uniroot(bb, lambda = lambda, ebar = ebar, m = m, interval = c(0.4,
    1))$root
estar <- sqrt(tstar/3)
rent_a <- m + estar * tstar - cost(estar)
rent_u <- m + ebar * tstar - cost(ebar)

### Example 2 baseline

lambda <- 0
ebar <- 0
m <- 5/27

tstar_b <- uniroot(bb, lambda = lambda, ebar = ebar, m = m, interval = c(0.4,
    1))$root
estar_b <- sqrt(tstar_b/3)
rent_a_b <- m + estar_b * tstar_b - cost(estar_b)

# treatment

lambda <- 0.5
ebar <- 0.45
m <- 5/27

tstar <- uniroot(bb, lambda = lambda, ebar = ebar, m = m, interval = c(0.4,
    1))$root
estar <- sqrt(tstar/3)
rent_a <- m + estar * tstar - cost(estar)
rent_u <- m + ebar * tstar - cost(ebar)
```