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MSc Economics

Oligopolistic Pricing with Heterogeneous Firms and Sequential Consumer Search

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MSc Economics

Affidavit

I, Alexander Maratovich Satdarov,

hereby declare

that I am the sole author of the present Master's Thesis,

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56 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

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Abstract

Two types of stores compete by choosing prices for a homogeneous good with constant marginal costs. The first type is in charge of two prices, while the second type is a regular firm that chooses one price. Consumers search sequentially with perfect recall. Some consumers have zero search costs, and some of them get first price quotation for free then incur positive search costs for each next price drawn. We examine whether the firm of the first type can exploit its additional market power and compete for both types of consumers simultaneously by setting different prices and creating a price dispersion in equilibrium. Three types of equilibria are considered, which fully exhaust the parameter space. In all types of equilibria, the presence of the firm of the first type leads to uniformly higher profits for both firm types of firms when compared with the Stahl(1989) model in which all firms are identical. Furthermore, being a regular firm is beneficial for some parameter values in terms of higher profits.

1 Introduction

The classical model of market competition is the model of perfect competition. In this model, N firms produce a homogeneous good and compete by setting prices. Besides, there are no information frictions — every agent is perfectly informed of the actions of other agents. In other words, everybody knows everything. By definition, a market is perfectly competitive if there is no firm which can exploit some level of the market power, i.e. market participants lack the ability to manipulate the price. The underlying assumption is that there are so many firms in the market, so all of them act like price-takers. This means that if one particular firm changes the price it does not affect the market price, because this firm's market share is negligible. As a consequence, strategy does not play a role in perfectly competitive markets because prices are determined by market forces. Basically, in this model the equilibrium boils down to equating supply and demand equations as well as clearing the markets. The equilibrium outcome is the unique price for all firms, so called the law of one price. Consequently, all transactions are executed at this unique equilibrium price which is equal to marginal costs. This result is known as the *Bertrand paradox*—if there are at least two firms in the market, the competition leads to pricing at marginal costs where the profits are zero.

The model of perfect competition is a natural benchmark with which other market structures can be compared. However, this model relies on very strong assumptions. The relevant question to ask would be whether the results of the competitive model are robust to changes in the underlying assumptions. What if, for example, the assumption of perfect information is relaxed in a way that agents do not know prices. As it occurred, the results do not hold anymore. Hence, one cannot rely on conclusions from the perfect competition, and perfectly competitive markets are not a good approximation of what is going on in the real world. Furthermore, it is empirically proven that the law of one price does not hold for many markets[1]. On the contrary, there are a lot of examples[2] where markets are more or less in equilibrium but the price dispersion can be observed. Hence, there was a demand for a literature that would explain price dispersion as an equilibrium outcome. The revolutionary paper appeared in early 1960s. George Stigler's "The Economics of Information" (1961) has introduced a new branch in Economics. The basic idea is that "information is a valuable resource" and it is not necessarily publicly available. Hence, one might incorporate information

[1]We do not consider markets that are regulated where the price is set due to non-market reasons.

[2]In the Literature review section we mention papers that empirically support the argument.

frictions to understand the behavior of markets which cannot be explained by competitive equilibrium.

In particular, Stigler (1961)'s paper focuses on a field of search theory, where the markets are imperfect due to the presence of search frictions. In a nutshell, the agent (either buyer or seller) cannot make a trade immediately like in the competitive model and has to search for a trading partner. There are two main areas of research: a consumer search and a labor search. Labor search deals with agents who are looking for a job. The main research question is to explain the presence of unemployment as an equilibrium outcome. The friction in this type of models is that it takes time for potential workers to find a job because they do not immediately agree to the wage offer or they have generous outside options (e.g. unemployment benefits) etc. In this paper, we deal with consumer search. The basic idea is that consumer wants to buy a good but she is unaware of its price. Hence, she has to search for it. When the consumer visits a seller and observes its price, she has to decide whether she wants to buy the good at this price or to continue searching. If the latter happens, the consumer incurs some search costs and goes to the other seller. The same story repeats until the good is finally bought.

Consumer search theory is an interesting topic to explore because in reality homogeneous goods[3] are often sold for different prices even if markets are competitive. Furthermore, many markets have a large price dispersion. Consumer search does so in the models, i.e. the presence of search costs is the friction that generates the price dispersion as an equilibrium outcome.

Generally, the typical model of consumer search considers N firms (possibly infinite) which are competing by setting prices for a homogeneous good. This means that each firm is in charge of one price. To the best of our knowledge, little or no attention was paid to chains of stores. For example, if we consider a market for groceries, it is highly likely that the local market is covered solely by a few chains. Hence, it would be interesting to see how the presence of the firm which is in charge of more than one price affects the market outcome. Is there some additional market power the chain firm can exploit to increase its profits? Furthermore, firms compete for different types of consumers, i.e. the consumers' side of the market is modeled like in Stahl (1989) where there are two types of consumers in the market — those who have zero search costs and as a consequence are aware of all prices in the market (informed consumers) and those

[3]It is undoubtedly an issue which goods can be treated as homogeneous because even the same good delivered to different places can be thought of as two different goods due to different costs of transportation. Nevertheless, we assume that, in the first approximation, such goods can be considered as homogeneous.

who have positive search costs and are actually involved in a search process for a lower price (uninformed consumers). Since a chain firm is in charge of two prices, and there are two types of consumers, it can set different prices in its stores: one price is low and aims at informed consumers, the other price is high and meant for uninformed consumers. In order to have several prices in equilibrium, we need to generate price dispersion what is done by search friction.

To sum up, the research question is whether the chain firm, which sets two prices, can benefit from it. i.e. is it possible that this firm chooses prices in such a way that it competes for both types of consumers rather than focusing on one type?

The symmetric Nash equilibrium of the Stahl (1989) model, in which all firms use the same pricing strategy, does not apply to our model. If it were the case, the chain firm could not exploit its additional market power and price dispersion would not arise in the equilibrium. Still, in equilibrium all firms choose a price from the same distribution, But this is only the part of the strategies of firms, which is common across firms. The other part where they set prices which occur with positive probabilities (mass points) are different. This assumption allows us to have price dispersion in equilibrium which is necessary to exploit the market power of setting two prices.

The rest of the paper is organized as follows. Section 2 briefly reviews the existing literature. Section 3 describes the basic setup of the model. Section 4 justifies the choice of the first type of equilibrium and solves for it. Section 5 considers the second equilibrium which naturally appears as a consequence of the first one. A correlated equilibrium which is a mixture of the first two equilibria is presented in Section 6. Section 7 concludes with a further discussion.

2 Literature Review

Consumer search literature was a very popular topic during 1970-1980s. A vast amount of papers exploring different search protocols under different underlying assumptions were published over those two decades. Then 1990s was a period of relative calm. But with the internet started to play a significant role in our lives, the interest to the field has reappeared. There was (and still is) a demand for models to explain the behavior of firms that sell goods online. This literature review considers mostly classical papers which are eventually happened to be most influential in the consumer search and a big portion of further literature is based on these models.

Diamond (1971) considers a market where identical consumers wish to buy at most one unit of a good and have a fixed maximum willingness to pay. The search protocol is the following: consumers get a first quote for free (and this is crucial for this model[4]) and have to pay a fixed search costs for each additional quote. The distribution of prices and, hence, the equilibrium is a common knowledge. Firms compete by choosing prices. If search costs are zero, then the model has a competitive equilibrium (marginal cost pricing). But if search costs are positive (even if they are arbitrarily close to zero), then every firm will choose a monopoly price which is equal to the maximum willingness to pay. This result is known as *Diamond paradox*: no matter how many firms (at least two) operate in the market, the equilibrium outcome is monopoly pricing. Even more surprising is the fact that when the number of firms go up, the Diamond equilibrium is more likely to happen.

Although Diamond (1971) is a not a good model for price dispersion and, even worse, there is no consumer surplus (due to the unit demand and a common willingness to pay), this paper has generated a lot of interest in the search literature.

Salop and Stiglitz (1977)[5] propose a model with a continuum of consumers who have different search costs. They are drawn from some distribution and this distribution is publicly known. Furthermore, the search is non-sequential: consumers face a “all-or-nothing” decision: they either pay the costs and learn all the prices or do not pay and stick to the *ex ante* distribution. Firms face a U-shaped average search costs and there is a free entry to the market, i.e. the number of firms is endogenously defined. Depending on parameters of the model,

[4]In the equilibrium consumers get no surplus. Consequently, if the first quote is also costly, nobody will buy. The market vanishes.

[5]The model presented here is a version published later in the Handbook of Industrial Organization (see Stiglitz, 1989).

the model can have an equilibrium with the limited price dispersion[6] (there are two possible prices in equilibrium), the Bertrand competitive equilibrium, or the Diamond monopolistic equilibrium.

Varian (1980) addresses the question whether price dispersion can be explained due to sales. Firms engage in sales behavior and they price discriminate between informed and uninformed consumers. In this equilibrium, firms set a price from a smooth distribution function without mass points on the interval from the marginal cost pricing to the maximum willingness to pay.

Stahl (1989) obtains a similar result. The paper considers again two types of consumers: informed consumers, who have zero search costs, and, hence, buy at the lowest price; and uninformed consumers, who have strictly positive search costs and are engaged in a sequential consumer search. N identical firms compete by setting prices for a homogeneous good. All of them have the same constant marginal costs. The main result is that, as the share of informed consumers changes from 0 to 1, the unique symmetric Nash equilibrium price distribution changes smoothly from marginal cost pricing (the share is zero; the Bertrand paradox) to price dispersion equilibrium (the share is in the $(0;1)$ interval), to monopoly pricing (the share is 1; the Diamond paradox). Furthermore, as the number of firms goes up, the equilibrium gets closer to monopolistic.

Kohn and Shavell (1974) study a general formulation of a search problem which can be applied to various fields (e.g. consumer search, labor search, quality of nonhomogeneous goods). The aim of the paper is to provide a search literature with some general results that can be used in future research. In particular, Kohn and Shavell (1974) has proven a reservation price property (or a switchpoint level of utility as they call it in the paper), i.e. if the price is below the certain threshold, then the search stops; if the price is above, consumer continues to search.

With the appearance of the Internet, the consumer search literature has found its interests in explaining consumers' decisions in online markets. These markets can be thought of as an additional competition to regular markets. Hence, models with varying search costs are a good start to explain this phenomena. Besides, it seems logical that online markets reduce price dispersion because sitting at home and clicking on the computer is less costly than going from one shop to another. That is why it is actually surprising to see that the price dispersion is in fact significantly higher in online markets rather than offline markets (see e.g. Carlton and Chevalier, 2001).

[6] Salop and Stiglitz (1982) have a similar result with the limited price dispersion. They consider an OLG model with a possibility to storage a one unit of good until the next period. In a sense, it is similar to the decision not to buy and search for the next firm.

There is a vast empirical literature on consumer search which finds evidence of a price dispersion in many markets. Baye et al. (2006)[7] have a summary of empirical papers. As they write in the paper: “The empirical evidence suggests that price dispersion in both online and offline markets is sizeable, pervasive, and persistent—and does not purely stem from subtle differences in firms’ products or services.”

Table 1: A sample of models with different setups.

Paper	Cons. info	Search protocol	Search costs	Cons. demand	No. of stores	Prod. costs
Salop and Stiglitz (1977)	Stkb	AoN	TT	UC	N*	U
Braverman (1980)	Stkb	AoN	Al+	D	N*	U
Rob (1985)	Stkb	SwR	Al	UI	∞	K
Stiglitz (1987, Appendix A)	Stkb	Seq	Al	D	N	K
Stiglitz (1987, Appendix B)	Stkb	SwR	Al	D	N	K
Burdett and Judd (1983)	Nash	Par	Hc	UC	∞	K
Axell (1977)	Nash	SwR	Al	D	∞	Co
Stahl (1989)	Nash	Seq	TT	D	N	K
Stahl (1996)	Nash	Seq	Al+	D	N	K

Key to consumer information:
Stkb—Stackelberg paradigm
Nash—Nash paradigm
Key to search protocol:
AoN—All or nothing
Seq—Sequential (without replacement)
SwR—Sequential (with replacement)
Par—Parallel Search
Key to search costs:
Al—Atomless distribution over consumers
Al+—Atomless distribution except possibly for an atom of shoppers
Hc—Homogeneous $c > 0$
TT—Two types with costs c_1 and c_2
Key to consumer demand:
UC—Unit demand up to a choke price
UI—Unit demand even at infinite prices
D—Regular downward-sloping demand
Key to number of stores:
N*—Long-run free-entry zero-profit case only
Key to production costs:
U—Identical U-shaped average costs
K—Identical constant marginal costs
Co—Convex total cost function

[7]In fact, this paper also surveys theoretical papers that has been written in consumer search.

To conclude the literature review, we want to emphasize that there is no unified approach to model consumer search. Table 1[8] shows that assumptions across the models are quite different. In the table, the Stackelberg paradigm means that consumers know not only the probability distribution, but also a market distribution, i.e. a particular market realization of prices while the Nash paradigm assumes that consumers are aware solely of the optimal price distribution.

The contribution of this paper to the literature is that it is one of the first attempts to combine consumer search with merger literature. In a sense, the model builds up on several papers. Varian (1980) used two types of consumers — with zero and positive search costs. Stahl (1989) put this framework into sequential search. Janssen et al. (2005), Janssen et al. (2011) studied different variations of Stahl (1989). In particular, Janssen et al. (2005) has shown that if the first quote is costly and firms face unit demand, then there is an equilibrium with partial consumer's participation, while Janssen et al. (2011) introduced stochastic production costs and has shown that there is an equilibrium with the reservation price property. Non (2010) considered the location choice of competing shops, i.e. they can be either in the mall or isolated. She has derived an equilibrium in which both can coexist. In a sense, one can think of a mall as a firm which sets many prices, so this model is related to ours.

[8]This table is taken from Stahl (1996).

3 Model Setup

Consider a market where $N \geq 2$ firms compete by choosing prices for a homogeneous good with constant marginal costs. Without loss of generality these costs are set to zero. Consumers have a unit demand for this good and a common value v. This value is assumed to be sufficiently large. This guarantees that search plays a role in equilibrium (non-degenerate case). Consumers search sequentially with perfect recall. Furthermore, following Stahl (1989) there are two types of consumers: a share λ of them has zero search costs, while the remaining part $(1-\lambda)$ has a positive search cost $c > 0$. Firm side is modeled as follows: one firm has two shops in the market, while all others have one shop. We will refer to the firm with two shops as a chain firm (one can think of this firm as a chain of supermarkets) and to the firms with one shop as regular. Overall, we have N shops operating in the market.

We will consider three different equilibria of this model. First of all, we start out by motivating the first type of equilibrium pricing strategies and consumers' behavior. The second and the third type of equilibria will follow from the results of the first equilibrium.

Since there two types of firms, uninformed consumers prior to search have to choose which firm they should visit first. It seems natural to assume that, since the chain firm has more market power, it will set prices in such a way that uninformed consumers are at least indifferent between either of firms. If it were not the case, then the chain firm would change its strategy to make them indifferent. On the other hand, making uninformed consumers prefer the chain firm over the regular firm would require the chain firm to price from a different (lower) price interval, but this behavior will make the model too complicated. Hence, as a starting point we assume that, before searching, uninformed consumers are indifferent between visiting either of firms first. In the first equilibrium, we have an endogenous parameter $\mu \in (0; 1)$ — the probability that uninformed consumer visits a regular firm first. All propositions in this section are derived assuming $\mu \in (0; 1)$.

Proposition 1. *In equilibrium with $\mu \in (0; 1)$, the chain firm sets a maximum price a consumer is willing to pay (i.e. the reservation price) at one of its shops.*

Proof. Denote the prices set by the chain firm p_1 and p_2. Uninformed consumers choose to visit either of the shops in the market at random, so we assume they are equally distributed among them. We have several cases to consider:

1. $p_1 = p_2 = min\{p_1, \dots, p_{N+1}\}$: The chain firm attracts all informed consumers and a fraction of uninformed consumers. Hence, uninformed con-

sumers pay the lowest price. There is a profitable deviation from this strategy. Given any strategy of the regular firm, if the chain firm raises the price in one of the shops, it still attracts all informed consumers to the other shop, but it makes more profits from the uninformed consumers since the fraction of them who visits this shop does not depend on price. This argument holds true for all prices in the support. Hence, the firm will charge the maximum price consumer is willing to pay at this firm, i.e. the reservation price.

2. $p_1 = p_2 \neq min\{p_1, \ldots, p_{N+1}\}$: The firm cannot attract informed consumers anyway, hence, it can gain profits from uninformed consumers by raising a price to the reservation price. But it doesn't raise the price in both shops, because *ex ante* the firm does not know whether the lower price it chooses will be the smallest.

3. $p_1 > p_2$ (or $p_2 > p_1$): Irrespective of whether p_2 (p_1) is the lowest price, the first (second) shop cannot attract informed consumers and, hence, it competes only for uninformed consumers. Consequently, there is no reason to price below the reservation price since, in this case, the chain firm incurs losses.

□

Proposition 1 shows us how the chain firm can exploit its additional market power. By setting a reservation price at one shop, it gains maximum profits from uninformed consumers who visits the shop anyway. Meantime, by setting a lower price in the other shop the chain firm stays in the competition for the informed consumers. The trade-off at this shop is between losing profits from uninformed consumers who are willing to pay a high price and gaining profits from informed consumers who buy at the lowest price.

Proposition 2. *In equilibrium with* $\mu \in (0; 1)$*, uninformed consumers will not search beyond the first firm.*

Proof. See Lemma 2 of Stahl (1989). □

The intuition behind Proposition 2 is that pricing above consumers' reservation price is never optimal because consumers in this case will not buy and continue to search. Hence, the price consumers see at first firm visit is always accepted and no further search arises in equilibrium.

Since price dispersion in the equilibrium is one of necessary tools for our analysis, we assume that both the second shop of the chain firm and regular firms price from some interval $[\underline{p}; \tilde{p})$[9] with a distribution function $F(p)$. But as the next proposition shows this is not enough to generate a price dispersion in equilibrium.

Proposition 3. *In equilibrium with $\mu \in (0;1)$, if all regular firms and one of the shops of the chain firm price from the interval $[\underline{p}; \tilde{p})$ and the other shop of the chain firm sets a deterministic price $\tilde{p}$, then there is no price dispersion in equilibrium.*

Proof. Indifference of uninformed consumers implies that expected prices prior to search should be the same[10]. Then

$$\frac{1}{2}\tilde{p} + \frac{1}{2}\mathbb{E}p = \mathbb{E}p$$
$$\Rightarrow \mathbb{E}p = \tilde{p},$$

where $\mathbb{E}p$ is the expected price of visiting a regular firm. This condition shows that consumers expect the price to be at the upper bound. The only way this is possible if the distribution function over the interval $[\underline{p}; \tilde{p})$ is degenerate and all the mass is on the price $\tilde{p}$. Consequently, there is no price dispersion. □

Proposition 3 concludes that we need to add something to the pricing strategies of firms to generate the price dispersion. One (and probably the only) way to do that is to introduce mass points in pricing strategies[11]. This means that firms with some probability should choose a deterministic price and with the remaining probability set a price from the interval $[\underline{p}; \tilde{p})$.

Proposition 4. *In equilibrium with $\mu \in (0;1)$, if $F(p)$ is a Nash equilibrium price distribution, then there are no mass points in it, i.e. there does not exist a price $p \in [\underline{p}; \tilde{p})$ such that firms choose it with positive probability.*

Proof. Suppose there is a price $p \in [\underline{p}; \tilde{p})$, which is chosen with positive probability in the equilibrium. Then by deviating to the price $p - \epsilon$, where ϵ is arbitrarily close to zero, the deviant loses profits of order ϵ $(\frac{1-\lambda}{N}\epsilon)$ from the uninformed consumers, but with a positive probability $(1 - F(p - \epsilon))^{N-2}$ it gains additional profits from informed consumers $(\lambda(p - \epsilon))$. Since the latter is of higher order than the former, the expected profits are higher if a firm sets a price $(p - \epsilon)$.

[9]We assume the interval to be half-open to avoid the case when several firms set the upper bound of the support and share the market profits in equal proportion.

[10]The implicit assumption here is that there is no search in equilibrium, i.e. uninformed consumers irrespective of price will buy at the first store they visit

[11]Playing mixed strategies over several intervals does not qualitatively change the result of Proposition 3. Although it might work, if the intervals are not the same for different firms, but it will make the model too complicated.

Hence, the mass point price p is not played in the equilibrium. This contradicts the assumption that this price is taken from the set of equilibrium prices. Since p is chosen arbitrarily, we can conclude that there is no price played with positive probability in the interval $[\underline{p}; \tilde{p})$, i.e there are no mass points. □

Proposition 5. *In equilibrium with $\mu \in (0;1)$, the mass point price for the store of the chain firm which plays mixed strategies should be equal to reservation price.*

Proof. If the price is higher than the reservation price, nobody will buy at it, so there is no reason to charge above the reservation price. Suppose the price is strictly below the reservation price, then the distribution function $F(p)$ over the interval $[\underline{p}; \tilde{p})$ will have a mass point at this price. By Proposition 4, it cannot be the case. Hence, the mass point price should be exactly the reservation price. □

With the Proposition 5 in hand, we have fully described the pricing strategy of the chain firm. It sets a deterministic price in the one shop and chooses between the same deterministic price and a price from the interval $[\underline{p}; \tilde{p})$ with the distribution function $F(p)$ with some complementary probabilities. However, this strategy is optimal for the chain firm, if the regular firm has a mass point as well in its pricing strategy. Otherwise, the uninformed consumers prefer to go to the regular firm first since it has a lower expected price. The next proposition shows which deterministic price the regular firm should choose as a part of its pricing behavior.

Proposition 6. *The mass point price for the regular firm should be equal to the reservation price for this firm. Furthermore, this reservation price is strictly higher than the reservation price for the chain firm.*

Proof. Suppose the mass point price is below the reservation price for the chain firm. Then the expected price prior to search is lower at the regular firm and the uninformed consumers prefer to visit a regular firm first, and, consequently, the pricing strategy of the chain firm is not optimal. If the mass point price equals the reservation price for the chain firm, let us write down indifference condition for visiting either of the firms first, assuming again there is no search in equilibrium.

$$\begin{aligned}\frac{1}{2}\tilde{p} + \frac{1}{2}\nu_M\tilde{p} + \frac{1}{2}(1-\nu_M)\mathbb{E}p &= \nu_{NM}\tilde{p} + (1-\nu_{NM})\mathbb{E}p \\ \Rightarrow \left(\frac{1}{2} + \frac{1}{2}\nu_M - \nu_{NM}\right)\tilde{p} &= \left(\frac{1}{2} + \frac{1}{2}\nu_M - \nu_{NM}\right)\mathbb{E}p \\ \Rightarrow \tilde{p} &= \mathbb{E}p,\end{aligned}$$

where ν_M is the probability that the chain firm sets a deterministic price at the second shop and ν_{NM} is the probability that the regular firm chooses a deterministic price.

Hence, we can draw the same conclusion as in the Proposition 3 that there is no price dispersion. To generate price dispersion, we should require the mass point price to be strictly higher than the reservation price for the chain firm. Since this price is anyway higher than any price of the chain firm, the regular firm cannot have profits from the informed consumers. It acquires profits solely from the uninformed consumers who visit the firm. For this reason, to gain higher profits the regular firm will choose to set a maximum price consumers are willing to pay, i.e. the reservation price. $\square$

This concludes the reasons to consider the proposed behavior of the firms as possible equilibrium strategies as well as the fact that the uninformed consumers will randomly choose which firm they will visit first.

The next step is to formally set up these strategies and consumers' behavior, and show that it is indeed an equilibrium for some parameter values.

4 Equilibrium with indifferent uninformed consumers

Firms play the following strategies: chain firm sets a deterministic price $\tilde{p}$ in the first shop, called $M1$. In the second shop, called $M2$, the pricing scheme is to set a price $\tilde{p}$ with probability ν_M, and to choose a price $p \in [\underline{p}; \tilde{p})$ from a distribution $F(p)$ with the remaining probability $(1 - \nu_M)$. $(N - 2)$ regular firms set a deterministic price $\hat{p} > \tilde{p}$ with probability ν_{NM}, and a random price $p \in [\underline{p}; \tilde{p})$ from the same distribution $F(p)$ with the remaining probability $(1 - \nu_{NM})$.

Consumers with zero search costs visit all shops in the market and buy from the one with the lowest price. They are referred to as informed consumers. The remaining fraction $(1 - \lambda)$ are uninformed consumers, who have positive search costs $c > 0$, and they are engaged in a sequential search. A fraction μ of uninformed consumers visits a regular firm first and buys from it, if the observed price p is less than or equal to $\hat{p}$, i.e. $p \leq \hat{p}$. The remaining fraction $(1 - \mu)$ visits one of the shops of the chain firm and buy from it, if the observed price p is less than or equal to $\tilde{p}$, i.e. $p \leq \tilde{p}$.

Any firm will play a mixed strategy in equilibrium, i.e. set a price from some distribution rather than a single price, if it is indifferent between these prices. This means that total profits of the firm should be the same for all possible prices. If the chain firm sets a price $p \in [\underline{p}; \tilde{p})$, then its total profits are given by

$$\pi_M(p) = \lambda p \cdot \mathbb{P}_1(\text{the lowest price}) + \frac{1-\mu}{2}(1-\lambda)p + \frac{1-\mu}{2}(1-\lambda)\tilde{p},$$

where $\mathbb{P}_1$(the lowest price) is the probability that the chain firm has set the lowest price in the market and has attracted all informed consumers. It can be calculated as follows

$$\mathbb{P}_1(\text{the lowest price}) = [(1 - F(p))(1 - \nu_{NM}) + \nu_{NM}]^{N-2}$$

The intuition behind this probability is the following: if the regular firm sets a price from an interval $[\underline{p}; \tilde{p})$, which happens with the probability $(1 - \nu_{NM})$, then the probability that the chain firm has a lower price is $(1 - F(p))$, and it does not depend on the price set by the other firm (the first term); if the regular firm sets a price $\hat{p}$, which happens with the probability ν_{NM}, then the chain firm always has a lower price (the second term).

The total profits of the chain firm are given by

$$\pi_M(p) = \lambda \left[(1 - F(p))(1 - \nu_{NM}) + \nu_{NM}\right]^{N-2} p + \frac{1-\mu}{2}(1-\lambda)p + \frac{1-\mu}{2}(1-\lambda)\tilde{p},$$

where the first term gives the profits from setting the lowest price in the market and attracting all informed consumers; the second term accounts for the profits from the uninformed consumers who visited first the shop $M2$ and bought there; and the third term is the profits from the uninformed consumers who went first to the shop $M1$ and purchased the good. The fraction $1/2$ in two last terms means that it is equally likely that the uninformed consumer, who visits the chain firm first, will be at either of the shops of it.

Besides, the chain firm's profits, if it sets price arbitrarily close to $\tilde{p}$, are

$$\lim_{p \to \tilde{p}} \pi_M(p) = \left[\lambda \nu_{NM}^{N-2} + (1-\mu)(1-\lambda)\right] \tilde{p}$$

Since in the equilibrium the chain firm is indifferent between any price in the support, it should be the case that $\pi_M(p) = \lim_{p \to \tilde{p}} \pi_M(p)$:

$$\begin{aligned} &\lambda \left[(1 - F(p))(1 - \nu_{NM}) + \nu_{NM}\right]^{N-2} p + \frac{1-\mu}{2}(1-\lambda)p \\ &\qquad + \frac{1-\mu}{2}(1-\lambda)\tilde{p} = \left[\lambda \nu_{NM}^{N-2} + (1-\mu)(1-\lambda)\right] \tilde{p} \end{aligned} \tag{1}$$

Using Equation 1, we can derive the distribution function[12] $F(p)$

$$F(p) = \frac{1}{1-\nu_{NM}} - \left[\frac{a}{p} + b\right]^{\frac{1}{N-2}}, \tag{2}$$

where $a = \frac{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}}\tilde{p}$ and $b = -\frac{\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}}$

In a similar fashion, the regular firm's total profits, if it sets a price $p \in [\underline{p}; \tilde{p})$, are

$$\pi_{NM}(p) = \lambda p \cdot \mathbb{P}_2(\text{the lowest price}) + \mu(1-\lambda)p,$$

where the first term is the profits from setting the lowest price in the market and capturing the informed consumers, and the second term is the profits from the uninformed consumers who chose to visit the regular firm first. The probability that the regular firm has chosen the lowest price in the market can be calculated as follows (for the sake of simplicity assume that we consider the regular firm 1,

[12] See appendix A.1 for a detailed derivation.

i.e. $i = 1$)

$$\mathbb{P}_2(\text{the lowest price}) = \left[(1-F(p))(1-\nu_M)+\nu_M\right]\left[(1-F(p))(1-\nu_{NM})+\nu_{NM}\right]^{N-3}$$

This is the probability that the price of the regular firm is lower than both the price of the chain firm (the term in the first square brackets) and all other $(N-3)$ regular firms (the term in the square brackets raised to the $(N-3)^{\text{rd}}$ power). Specifically, the first term: if the chain firm sets the price $\tilde{p}$, what happens with the probability ν_M, then the price of the regular firm is always lower, and if the price is chosen from the interval $[\underline{p};\tilde{p})$, what happens with the probability $(1-\nu_M)$, then the probability that the price of the regular firm is lower is $(1-F(p))$; the second term: with the probability ν_{NM} the other regular shop has set a price $\hat{p}$ which is always higher than any price p, and with the remaining probability $(1-\nu_{NM})$ it has chosen the price from the same interval, which means that the probability, that the price of the regular firm under consideration is lower, is $(1-F(p))$ (keep in mind that there are $(N-3)$ firms of this type in the market to which we compare the price, that is why we raise this probability to the power).

The total profits of the regular firm can be written as

$$\pi_{NM}(p) = \lambda\left[(1-F(p))(1-\nu_M)+\nu_M\right]\left[(1-F(p))(1-\nu_{NM})+\nu_{NM}\right]^{N-3}p+\mu(1-\lambda)p$$

Besides, total profits for the cases when the price is arbitrarily close to $p = \tilde{p}$ and when the price is $p = \hat{p}$ are given by

$$\lim_{p\to\tilde{p}}\pi_{NM}(p) = \lambda\tilde{p}\nu_M\nu_{NM}^{N-3} + \mu(1-\lambda)\tilde{p}$$
$$\pi_{NM}(\hat{p}) = \mu(1-\lambda)\hat{p}$$

In the equilibrium, regular firms randomize over these prices if total profits are the same for all prices, i.e. $\pi_{NM}(p) = \lim_{p\to\tilde{p}}\pi_{NM}(p)$ and $\lim_{p\to\tilde{p}}\pi_{NM}(p) = \pi_{NM}(\hat{p})$

$$\begin{aligned}\lambda\left[(1-F(p))(1-\nu_M)+\nu_M\right]\left[(1-F(p))(1-\nu_{NM})+\nu_{NM}\right]^{N-3}p&\\ +\mu(1-\lambda)p = \lambda\nu_M\nu_{NM}^{N-3}\tilde{p}+\mu(1-\lambda)\tilde{p}&\end{aligned} \tag{3}$$

$$\lambda\nu_M\nu_{NM}^{N-3}\tilde{p}+\mu(1-\lambda)\tilde{p} = \mu(1-\lambda)\hat{p} \tag{4}$$

To ensure that both the chain firm and regular firms sample prices from the same distribution, ideally we would express the distribution function $F(p)$ from Equation 3 and set it equal the distribution function in Equation 2. Unfortunately, we cannot derive a closed-form expression from the latter. For this reason, we do the following trick: express the common term $[(1-F(p))(1-\nu_{NM})+\nu_{NM}]^{N-3}$ from the Equation 1 and the Equation 3 and set these expressions equal to each other.

$$[(1-F(p))(1-\nu_{NM})+\nu_{NM}]^{N-3} = \frac{\lambda\nu_{NM}^{N-2}\tilde{p} + \frac{(1-\mu)(1-\lambda)}{2}(\tilde{p}-p)}{(1-F(p))(1-\nu_{NM})+\nu_{NM}}$$
$$[(1-F(p))(1-\nu_{NM})+\nu_{NM}]^{N-3} = \frac{\lambda\nu_M\nu_{NM}^{N-3}\tilde{p} + \mu(1-\lambda)(\tilde{p}-p)}{(1-F(p))(1-\nu_M)+\nu_M}$$

Hence,

$$\frac{\lambda\nu_{NM}^{N-2}\tilde{p} + \frac{(1-\mu)(1-\lambda)}{2}(\tilde{p}-p)}{(1-F(p))(1-\nu_{NM})+\nu_{NM}} = \frac{\lambda\nu_M\nu_{NM}^{N-3}\tilde{p} + \mu(1-\lambda)(\tilde{p}-p)}{(1-F(p))(1-\nu_M)+\nu_M} \quad \forall p \in [\underline{p};\tilde{p}) \tag{5}$$

Since this equality has to hold $\forall p \in [\underline{p};\tilde{p})$, in particular, it is true for $p = \underline{p}$. Equation 2 and the fact that $F(\underline{p}) = 0$ allow us to derive an expression[13] for $\underline{p}$

$$\underline{p} = \frac{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}\tilde{p} < \tilde{p}, \text{ since } \nu_{NM}^{N-2} < 1 \tag{6}$$

Plugging Equation 6 into Equation 5 and using $F(\underline{p}) = 0$ again, we derive a condition which ensures that the distribution function of prices is the same for all firms[14]

$$\mu(1-\lambda) = \frac{2\lambda(\nu_{NM}^{N-2} - \nu_M\nu_{NM}^{N-3}) + (1-\lambda)(1-\nu_M\nu_{NM}^{N-3})}{2(1-\nu_{NM}^{N-2}) + (1-\nu_M\nu_{NM}^{N-3})} \tag{7}$$

Using Equation 4, we can express $\hat{p}$ in terms of $\tilde{p}$

$$\hat{p} = \left[1 + \frac{\lambda\nu_M\nu_{NM}^{N-3}}{\mu(1-\lambda)}\right]\tilde{p} \tag{8}$$

A few restrictions should be imposed to make the proposed behavior of uninformed consumers optimal. First, prior to search they should be indifferent between visiting a chain firm first and visiting a regular firm first, i.e. the expected price should be the same

$$\frac{1}{2}(1+\nu_M)\tilde{p} + \frac{1}{2}(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p) = \nu_{NM}\hat{p} + (1-\nu_{NM})\int_{\underline{p}}^{\tilde{p}} p\,dF(p) \tag{9}$$

[13]See appendix A.2 for a detailed derivation.
[14]See appendix A.3 for a detailed derivation.

The left-hand side is the expected price a consumer will get if she visits the chain firm first. To be more specific, with probability $1/2$ (it is equally likely to end up at either of the chain firm's shops) she goes to the shop $M1$ and observes a price $\tilde{p}$; with probability $\nu_M/2$ she goes to the shop $M2$ which has set the high price $\tilde{p}$; and with the remaining probability $(1-\nu_M)/2$ she is at the shop $M2$ which has picked a price from the interval $[\underline{p};\tilde{p})$. The right-hand side is the expected price if consumer visits the regular firm first, and the intuition is the same as above.

A second restriction is when the uninformed consumer chooses the regular firm first and observes the highest price $\hat{p}$, she should be indifferent between buying and continuing to search, i.e. this price should be equal to the expected price of visiting another store plus search costs $s > 0$ a consumer has to spend to go to the other firm

$$\hat{p} = \left(1 - \mu\frac{N-3}{N-2}\right)\left(\frac{1}{2}(1+\nu_M)\tilde{p} + \frac{1}{2}(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)\right) + \mu\frac{N-3}{N-2}\left(\nu_{NM}\hat{p} + (1-\nu_{NM})\int_{\underline{p}}^{\tilde{p}} p\,dF(p)\right) + s \tag{10}$$

Note that the probabilities of being at the chain firm and at the regular firm were updated using Bayes' rule. It is done because the consumer has attended one regular firm and at this point there are only $(N-2)$ firms left including $(N-3)$ regular firms and one shop of the chain firm.

The third restriction is, when observing a price $\tilde{p}$ at the chain firm, the uninformed consumer should be indifferent between buying and continuing to search. In this case, it means that she goes to the other shop of the chain firm because it has a lower expected price[15]

$$\tilde{p} = \frac{\nu_M}{1+\nu_M}\tilde{p} + \frac{1}{1+\nu_M}\left(\nu_M\tilde{p} + (1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)\right) + s$$

[15] Formal proof will be given for the case of $N = 3$. The intuition is the following: after observing the highest possible price the chain firm can possibly set, the consumer updates her probabilities. Intuitively, she puts more weight on having visited the shop $M1$ which always sets price $\tilde{p}$. Since *a priori* uninformed consumers are indifferent between visiting chain firm or regular firm (Equation 4), after visiting one of the chain firm she prefers to go to the other shop of the chain firm because there is more information about it.

Rearranging the last equation gives us the following expression

$$\tilde{p} = \frac{1+\nu_M}{1-\nu_M}s + \int_{\underline{p}}^{\tilde{p}} p\, dF(p) \tag{11}$$

The only thing left is to calculate the integral[16] $\int_{\underline{p}}^{\tilde{p}} p\, dF(p)$.

$$\int_{\underline{p}}^{\tilde{p}} p\, dF(p) = g(\nu_{NM}, \mu; \lambda, N)\tilde{p} \equiv g\tilde{p} \tag{12}$$

This integral is a function of variables ν_{NM}, μ, $\tilde{p}$ and parameters λ, N.

We have five equations, namely Equation 7, Equation 8, Equation 9, Equation 10, and Equation 11, and five unknowns, namely two prices $\tilde{p}$ and $\hat{p}$, and probabilities ν_M, ν_{NM}, and μ. Hence, we can solve this system numerically to find an equilibrium.

Since numerical solution requires an initial guess of the solution, we should reduce our system (otherwise, we have to guess a five-dimensional vector with two unbounded parameters i.e. prices). Equation 7 gives us μ as a function of two other probabilities ν_M and ν_{NM}. Equation 11 together with Equation 12 allows us to express $\tilde{p}$ in terms of ν_M and ν_{NM} as well. Finally, Equation 8 describes $\hat{p}$ as a function of ν_M and ν_{NM}. Hence, we have reduced our system to the system of two equations with two unknowns. Furthermore, both of them are bounded between zero and one.

$$\begin{aligned}
\mu &= \frac{2\lambda(\nu_{NM}^{N-2} - \nu_M \nu_{NM}^{N-3}) + (1-\lambda)(1-\nu_M \nu_{NM}^{N-3})}{(1-\lambda)\left(2(1-\nu_{NM}^{N-2}) + (1-\nu_M \nu_{NM}^{N-3})\right)} \\
\tilde{p} &= \frac{1+\nu_M}{1-\nu_M}s + g\tilde{p} \Rightarrow \tilde{p} = \frac{\frac{1+\nu_M}{1-\nu_M}s}{1-g} \\
\hat{p} &= \left[1 + \frac{\lambda \nu_M \nu_{NM}^{N-3}}{\mu(1-\lambda)}\right] \frac{\frac{1+\nu_M}{1-\nu_M}s}{1-g}
\end{aligned}$$

Now we can use these expressions to plug them to Equation 9 and Equation 10 and solve numerically for ν_{NM} and ν_M.

We will solve for the first equilibrium numerically for the case $N = 3$, i.e. there is one chain firm and one regular firm. First, we rewrite relevant equations for this case. Namely, we plug $N = 3$ to Equation 7, Equation 8, Equation 9,

[16] See appendix A.4 for a detailed derivation.

Equation 10, and Equation 11.

$$\mu(1-\lambda) = \frac{\lambda(\nu_{NM}-\nu_M)+\frac{(1-\lambda)}{2}(1-\nu_M)}{(1-\nu_{NM})+\frac{1-\nu_M}{2}} \tag{13}$$

$$\hat{p} = \frac{\mu(1-\lambda)+\lambda\nu_M}{\mu(1-\lambda)}\tilde{p} \tag{14}$$

$$\frac{1}{2}(1+\nu_M)\tilde{p} + \frac{1}{2}(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p) = \nu_{NM}\hat{p}+(1-\nu_{NM})\int_{\underline{p}}^{\tilde{p}} p\,dF(p) \tag{15}$$

$$\hat{p} = \frac{1}{2}(1+\nu_M)\tilde{p}+\frac{1}{2}(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)+s \tag{16}$$

$$\tilde{p} = \frac{1+\nu_M}{1-\nu_M}s+\int_{\underline{p}}^{\tilde{p}} p\,dF(p) \tag{17}$$

Before proceeding, we should formally prove that Equation 17 is correct, i.e. uninformed consumers do prefer to visit the other shop of the chain firm if they observe a price $\tilde{p}$ at the chain firm. This means that $\mathbb{E}_M(p|p_1=\tilde{p}) < \mathbb{E}_{NM}(p|p_1=\tilde{p}) = \mathbb{E}_{NM}(p)$, where p_1 is the price observed at the chain firm.

Proposition 7. *In equilibrium with $\mu \in (0;1)$ and $N=3$, after visiting the chain firm and observing a price $\tilde{p}$, uninformed consumers prefer to go to the other shop of the chain firm over the regular firm.*

Proof. We have to show that $\mathbb{E}_M(p|p_1=\tilde{p}) < \mathbb{E}_{NM}(p)$.

$$\frac{\nu_M}{1+\nu_M}\tilde{p}+\frac{1}{1+\nu_M}\left(\nu_M\tilde{p}+(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)\right)+s < \nu_{NM}\hat{p}+(1-\nu_{NM})\int_{\underline{p}}^{\tilde{p}} p\,dF(p)+s \tag{18}$$

Using Equation 15, we can substitute for the right-hand side of inequality.

$$\frac{\nu_M}{1+\nu_M}\tilde{p}+\frac{1}{1+\nu_M}\left(\nu_M\tilde{p}+(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)\right) < \frac{1}{2}(1+\nu_M)\tilde{p}+\frac{1}{2}(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)$$

$$\Rightarrow \left(\frac{2\nu_M}{1+\nu_M}-\frac{1+\nu_M}{2}\right)\tilde{p} < \left(\frac{1-\nu_M}{2}-\frac{1-\nu_M}{1+\nu_M}\right)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)$$

$$\Rightarrow (4\nu_M - (1+\nu_M)^2)\tilde{p} < (1 - \nu_M^2 - 2(1-\nu_M)) \int_{\underline{p}}^{\tilde{p}} p\, dF(p)$$

$$\Rightarrow -(1-\nu_M)^2 \tilde{p} < -(1-\nu_M)^2 \int_{\underline{p}}^{\tilde{p}} p\, dF(p)$$

$$\tilde{p} > \int_{\underline{p}}^{\tilde{p}} p\, dF(p)$$

The worst case scenario, when the distribution is degenerate and all the mass is on $\tilde{p}$, makes these expressions equal. Since we assume that there is price dispersion in equilibrium and, hence, the distribution $F(p)$ is non-degenerate, this inequality holds. Consequently, uninformed consumers indeed prefer to visit the other store of the chain firm after observing price $\tilde{p}$ at the chain firm. □

Besides, the integral $\int_{\underline{p}}^{\tilde{p}} p\, dF(p)$ can be analytically calculated. Using Equation 3 for $N = 3$, we can explicitly derive a distribution function $F(p)$ and, hence, an expression for $\underline{p}$.

$$\lambda[(1 - F(p))(1-\nu_M) + \nu_M]p + \mu(1-\lambda)p = \lambda\nu_M\tilde{p} + \mu(1-\lambda)\tilde{p}$$

$$\Rightarrow F(p) = 1 - \frac{\lambda\nu_M + \mu(1-\lambda)}{\lambda(1-\nu_M)} \frac{\tilde{p} - p}{p} = 1 - c\frac{\tilde{p} - p}{p}$$

where $c = \frac{\lambda\nu_M + \mu(1-\lambda)}{\lambda(1-\nu_M)}$. Hence,

$$F(\underline{p}) = 0 = 1 - c\frac{\tilde{p} - \underline{p}}{\underline{p}}$$

$$\Rightarrow \frac{\tilde{p}}{\underline{p}} = \frac{1+c}{c}$$

Then

$$\int_{\underline{p}}^{\tilde{p}} p\, dF(p) = \int_{\underline{p}}^{\tilde{p}} p\, d\left(1 - c\frac{\tilde{p} - p}{p}\right)$$

$$= \int_{\underline{p}}^{\tilde{p}} pc\frac{\tilde{p}}{p^2}\, dp = \int_{\underline{p}}^{\tilde{p}} c\frac{\tilde{p}}{p}\, dp$$

$$= c\tilde{p}\ln(p)\Big|_{p=\underline{p}}^{p=\tilde{p}} = c\tilde{p}\ln\frac{\tilde{p}}{\underline{p}}$$
$$= \frac{\lambda\nu_M + \mu(1-\lambda)}{\lambda(1-\nu_M)}\tilde{p}\ln\left(\frac{\lambda + \mu(1-\lambda)}{\lambda\nu_M + \mu(1-\lambda)}\right)$$

Equation 13 expresses μ in terms of ν_M and ν_{NM}. Plugging the left-hand side of Equation 15 to Equation 16, we get a price $\hat{p}$ as a function of ν_M and ν_{NM}.

$$\hat{p} = \int_{\underline{p}}^{\tilde{p}} p\, dF(p) + \frac{s}{1-\nu_{NM}}$$

Now using the last expression and Equation 17, we can eliminate the integral and find a relation between $\tilde{p}$ and $\hat{p}$.

$$\hat{p} = \tilde{p} + \frac{s}{1-\nu_{NM}} - \frac{1+\nu_M}{1-\nu_M}s$$
$$\Rightarrow \hat{p} = \tilde{p} + \frac{(1-\nu_M) - (1+\nu_M)(1-\nu_{NM})}{(1-\nu_M)(1-\nu_{NM})}s$$
$$\Rightarrow \hat{p} = \tilde{p} + \frac{\nu_{NM}(1+\nu_M) - 2\nu_M}{(1-\nu_M)(1-\nu_{NM})}s \tag{19}$$

This equation together with Equation 14 pins down an expression for $\tilde{p}$ in terms of ν_M and ν_{NM}.

$$\frac{\mu(1-\lambda) + \lambda\nu_M}{\mu(1-\lambda)}\tilde{p} = \tilde{p} + \frac{\nu_{NM}(1+\nu_M) - 2\nu_M}{(1-\nu_M)(1-\nu_{NM})}s$$
$$\Rightarrow \tilde{p} = \frac{\mu(1-\lambda)[\nu_{NM}(1+\nu_M) - 2\nu_M]}{\lambda\nu_M(1-\nu_M)(1-\nu_{NM})}s \tag{20}$$

We have expressed prices $\tilde{p}$, $\hat{p}$, and probability μ in terms of probabilities ν_M and ν_{NM}. Now we can use the rest of equations to get a system of two equations with two unknowns and solve it numerically. If we plug Equation 19 to Equation 15, we get

$$[\frac{1}{2}(1+\nu_M) - \nu_{NM}]\tilde{p} = \nu_{NM}\frac{\nu_{NM}(1+\nu_M) - 2\nu_M}{(1-\nu_M)(1-\nu_{NM})}s + [\frac{1}{2}(1+\nu_M) - \nu_{NM}]\int_{\underline{p}}^{\tilde{p}} p\, dF(p)$$

Using Equation 17, we eliminate the integral from the last expression and derive the first equation relating ν_M and ν_{NM}[17].

$$\nu_{NM} = \frac{(1+\nu_M)^2}{3+\nu_M^2} \tag{21}$$

The second equation comes from Equation 17, if we plug the expression we have derived for the integral and the Equation 20.

$$\frac{\mu(1-\lambda)[\nu_{NM}(1+\nu_M)-2\nu_M]}{\lambda\nu_M(1-\nu_M)(1-\nu_{NM})}\left[1-\frac{\lambda\nu_M+\mu(1-\lambda)}{\lambda(1-\nu_M)}\tilde{p}\ln\left(\frac{\lambda+\mu(1-\lambda)}{\lambda\nu_M+\mu(1-\lambda)}\right)\right] = \frac{1+\nu_M}{1-\nu_M}$$

Finally, using expression for $\mu(1-\lambda)$ from the Equation 13, we derive the second equation for ν_M and ν_{NM}.

$$\frac{\lambda(\nu_{NM}-\nu_M)+\frac{(1-\lambda)}{2}(1-\nu_M)}{\lambda[(1-\nu_{NM})+\frac{1-\nu_M}{2}]}\left[1-\frac{\lambda\nu_M+(1-\lambda)+2\lambda\nu_{NM}}{2\lambda(1-\nu_{NM}+\frac{1-\nu_M}{2})}\ln\frac{1+2\lambda}{\lambda\nu_M+(1-\lambda)+2\lambda\nu_{NM}}\right] = \frac{\nu_M(1+\nu_M)(1-\nu_{NM})}{\nu_{NM}(1+\nu_M)-2\nu_M} \tag{22}$$

First, let us plot Equation 21 and Equation 22 in the $\nu_M - \nu_{NM}$ coordinate system for different values of λ (it varies from 0.01 to 0.99). As we can see there are two candidates for a possible solution, namely around points $(0.1; 0.4)$ and $(1; 1)$. We can drop the latter because numerical solution shows that, in fact, one of parameters exceeds one[18]. Hence, we solve numerically with the initial condition $(\nu_M = 0.1; \nu_{NM} = \nu_{NM}(0.1))$ for different values of λ[19]. The results are summarized in Table 2.

Table 2: $\nu_M - \nu_{NM}$ system solution for different values of λ

λ	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
ν_M	0.14	0.13	0.12	0.11	0.10	0.09	0.08	0.08	0.07	0.07	0.06
ν_{NM}	0.43	0.42	0.41	0.41	0.40	0.39	0.39	0.39	0.38	0.38	0.38

As we can see the solution of the $\nu_M - \nu_{NM}$ system does not vary a lot as λ changes from 0 to 1. For this reason, variation of μ in Equation 13 might mostly depend on changes in λ. To check this, let us plot Equation 13 in the $\mu - \lambda$ space for three values if $(\nu_M; \nu_{NM})$: a lower bound $(0.06; 0.38)$, an average value

[17]See appendix A.5 for a detailed derivation.

[18]Furthermore, if we take a solution in the neighborhood of the point $(1; 1)$, the value of μ is above 1 for all values of λ.

[19]The second coordinate is the value of ν_{NM} evaluated at $\nu_M = 0.1$ from Equation 21. Since functions are continuous, after the first evaluation we plug as an initial value the solution for previous iteration.

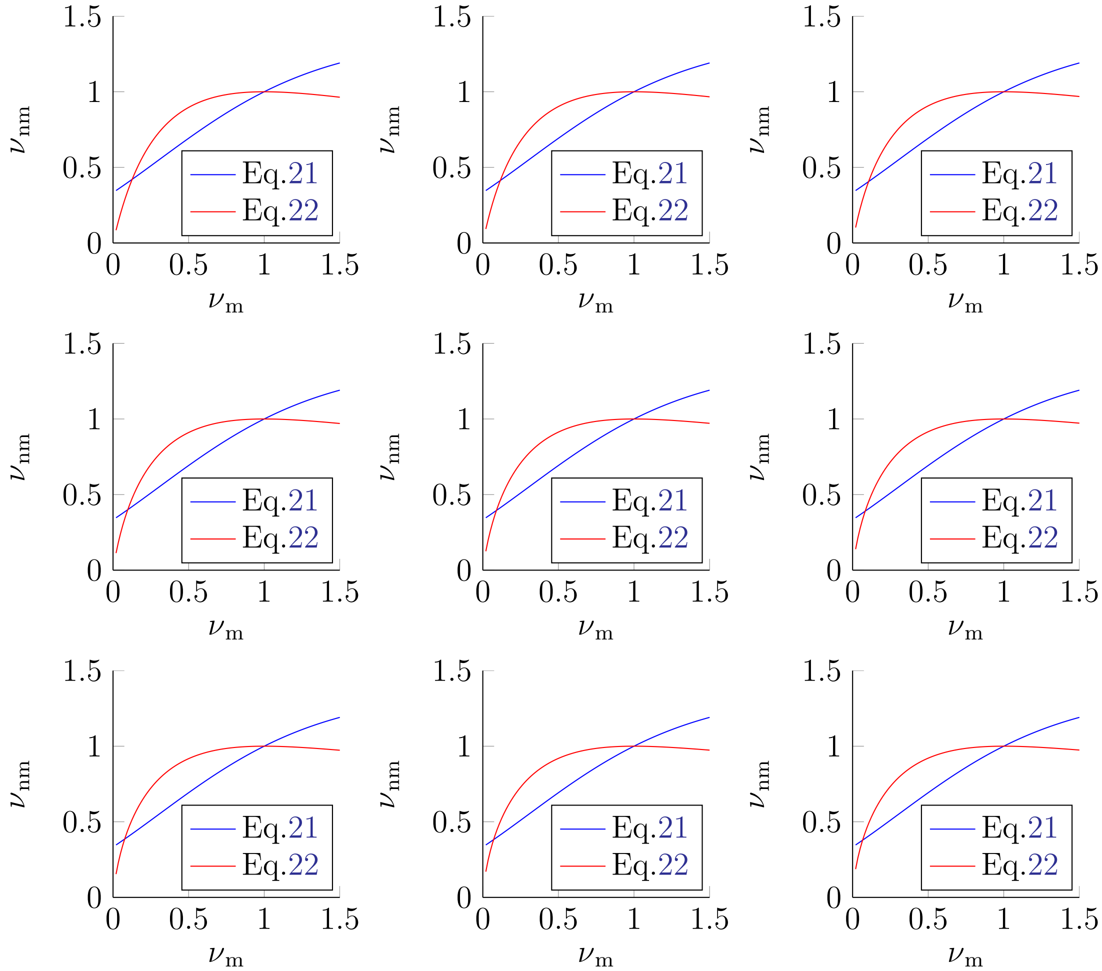

Figure 1: Graphic representation of $\nu_M - \nu_{NM}$ system for different values of λ

(0.1; 0.4), and an upper bound (0, 14; 0.43). This is a sort of confidence interval for values of μ as a function of λ.

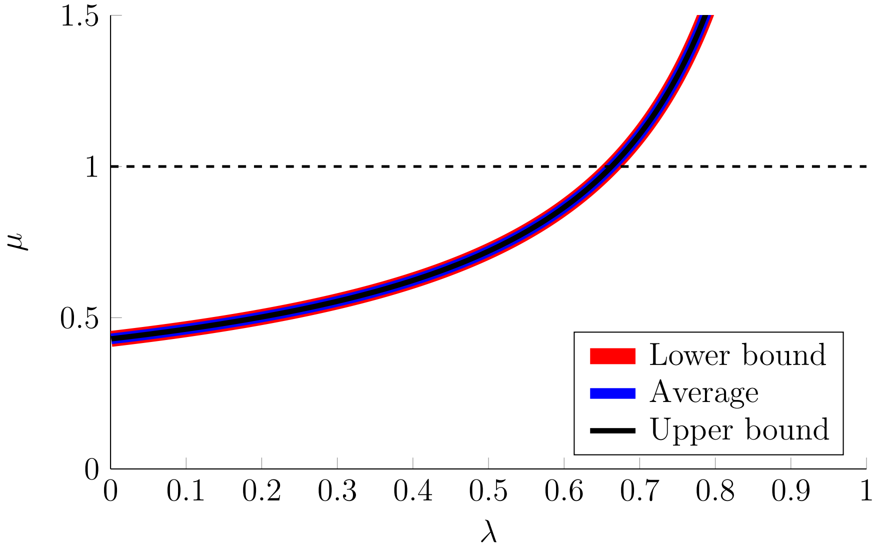

Figure 2: μ as a function of λ

Figure 2 shows that μ is an increasing function of λ and it hits the upper bound of 1 at approximately $\lambda = 2/3$. Table 3 shows the exact value of λ at which the equilibrium is not applicable anymore.

Table 3: Values of μ for some values of λ

λ	0.66	0.661	0.662	0.663	0.664	0.665	0.666	0.667	0.67	0.67	0.67
μ	0.987	0.989	0.992	0.994	0.997	0.999	1.002	1.004	1.007	1.010	1.012

Finally, we calculate equilibrium values for $0 < \lambda < 0.665$.

Table 4: Equilibrium values for $0 < \mu < 1$ ($s = 0.01$).

Heterogeneous firms model										Stahl model	
λ	ν_M	ν_{NM}	μ	$\underline{p}$	$\tilde{p}$	$\hat{p}$	$\mathbb{E}p$	π_M	π_{NM}	ρ	π
0.01	0.1383	0.4291	0.4331	1.3099	1.3361	1.3404	1.3304	0.7557	0.5747	1.0079	0.3326
0.1	0.1266	0.4208	0.4621	0.1192	0.1435	0.1479	0.1379	0.0755	0.0615	0.1074	0.0322
0.2	0.1155	0.4129	0.5019	0.0543	0.0769	0.0813	0.0713	0.0370	0.0326	0.0569	0.0152
0.3	0.1059	0.4062	0.5531	0.0332	0.0545	0.0589	0.0489	0.0237	0.0228	0.0365	0.0081
0.33	0.1033	0.4043	0.5715	0.0295	0.0504	0.0548	0.0448	0.0212	0.0210	0.0355	0.0078
0.34	0.1024	0.4037	0.5779	0.0284	0.0492	0.0536	0.0436	0.02044	0.02046	0.0308	0.0062
0.4	0.0975	0.4002	0.6214	0.0230	0.0431	0.0476	0.0376	0.0167	0.0178	0.0253	0.0042
0.5	0.0900	0.3950	0.7169	0.0170	0.0362	0.0408	0.0308	0.0123	0.0146	0.0214	0.0029
0.6	0.0832	0.3902	0.8602	0.0132	0.0316	0.0362	0.0262	0.0092	0.0124	0.0192	0.0021
0.6652	0.0791	0.3874	0.9998	0.0113	0.0293	0.0339	0.0239	0.0075	0.0113	0.0194	0.0022
0.67	0.0789	0.3872	1.0123	0.0112	0.0291	0.0337	0.0237	0.0074	0.0113	0.0357	0.0039

Notes: Stahl model is referred to the paper "Oligopolistic Pricing with Sequential Consumer Search" (Stahl, 1989); ρ is the reservation price.

Table 4, Table 5, Table 6 summarize calculations for the cases when search costs are equal $0.01, 0.05$, and 0.3 respectively. We can see that all endogenous probabilities do not depend on search costs — for a given λ, they are the same for different search costs. Besides, randomizing probabilities ν_M and ν_{NM} slightly decrease as λ goes up. Hence, the latter is the driving mechanism that changes the probability μ of visiting the regular firm first. The intuition is the following: as λ goes up, price level goes down and, hence, relative price dispersion increases. Uninformed consumers do not want to buy at a high price $\tilde{p}$ what happens with the probability $\frac{1}{2}(1+\nu_M) > \frac{1}{2}$ and prefer to go to the regular firm first where they will see a low price with the probability $(1-\nu_{NM}) > \frac{1}{2}$. Finally, search costs shift prices upwards, i.e. the higher search costs are, the higher equilibrium prices are. This is straightforward, because high search costs reduces incentives to search and firms taking this into account raise prices accordingly.

Table 5: Equilibrium values for $0 < \mu < 1$ ($s = 0.05$).

Heterogeneous firms model										Stahl model	
λ	ν_M	ν_{NM}	μ	$\underline{p}$	$\tilde{p}$	$\hat{p}$	$\mathbb{E}p$	π_M	π_{NM}	ρ	π
0.01	0.1383	0.4291	0.4331	6.5494	6.6806	6.7022	6.6522	3.7783	2.8734	5.0397	1.6631
0.1	0.1266	0.4208	0.4621	0.5960	0.7175	0.7393	0.6893	0.3775	0.3074	0.5371	0.1611
0.2	0.1155	0.4129	0.5019	0.2713	0.3844	0.4065	0.3565	0.1849	0.1632	0.2843	0.0758
0.3	0.1059	0.4062	0.5531	0.1660	0.2723	0.2947	0.2447	0.1184	0.1141	0.1984	0.0463
0.33	0.1033	0.4043	0.5715	0.1473	0.2518	0.2742	0.2242	0.1059	0.1050	0.1824	0.0407
0.34	0.1024	0.4037	0.5779	0.1418	0.2458	0.2682	0.2182	0.1022	0.1023	0.1777	0.0391
0.4	0.0975	0.4002	0.6214	0.1149	0.2157	0.2382	0.1882	0.0835	0.0888	0.1541	0.0308
0.5	0.0900	0.3950	0.7169	0.0852	0.1812	0.2040	0.1540	0.0614	0.0731	0.1265	0.0211
0.6	0.0832	0.3902	0.8602	0.0659	0.1579	0.1809	0.1309	0.0458	0.0622	0.1070	0.0143
0.6652	0.0791	0.3874	0.9998	0.0567	0.1464	0.1694	0.1194	0.0377	0.0567	0.0962	0.0106
0.67	0.0789	0.3872	1.0123	0.0561	0.1456	0.1686	0.1186	0.0372	0.0563	0.0969	0.0108

Notes: Stahl model is referred to the paper "Oligopolistic Pricing with Sequential Consumer Search" (Stahl, 1989); ρ is the reservation price.

Table 4, Table 5, Table 6 also compare profits of firms to profits in Stahl (1989) model for the case of $N = 3$ (in the notation of the paper) firms. We can see that both regular firm and chain firm benefit from the fat that the latter is in charge of two prices. Note that profits of the chain firm are always more than twice as higher as profits of the firm in the Stahl (1989) model. This implies that this equilibrium is beneficial for the chain firm. If it were not the case and profits were just a bit higher, then the chain firm would have split into two separate firms and would have played the Stahl symmetric Nash equilibrium.

The rise in profits can be explained as follows: since the number of firms changes from $N = 3$ to $N = 2$, there is less competition in the market and this leads to uniformly higher prices.

Finally, we should emphasize that the chain firm does not always have higher profits in comparison to the regular firm. It is the case for $0 < \lambda < 0.33$. For values of $\lambda \in (0.34; 0.665)$, exactly the opposite holds — the regular firm makes higher profits.

Table 6: Equilibrium values for $0 < \mu < 1$ ($s = 0.3$).

	Heterogeneous firms model									Stahl model	
λ	ν_M	ν_{NM}	μ	$\underline{p}$	$\tilde{p}$	$\hat{p}$	$\mathbb{E}p$	π_M	π_{NM}	ρ	π
0.01	0.1383	0.4291	0.4331	39.297	40.084	40.213	39.913	22.670	17.241	30.238	9.979
0.1	0.1266	0.4208	0.4621	3.5758	4.3047	4.4357	4.1357	2.2653	1.8446	3.2223	0.9667
0.2	0.1155	0.4129	0.5019	1.6281	2.3063	2.4390	2.1390	1.1095	0.9793	1.7059	0.4549
0.3	0.1059	0.4062	0.5531	0.9962	1.6341	1.7682	1.4682	0.7103	0.6846	1.1901	0.2777
0.33	0.1033	0.4043	0.5715	0.8837	1.5109	1.6454	1.3454	0.6354	0.6300	1.0945	0.2444
0.34	0.1024	0.4037	0.5779	0.8508	1.4746	1.6092	1.3092	0.6132	0.6138	1.0662	0.2346
0.4	0.0975	0.4002	0.6214	0.6896	1.2941	1.4294	1.1294	0.5011	0.5329	0.9245	0.1849
0.5	0.0900	0.3950	0.7169	0.5110	1.0874	1.2239	0.9239	0.3686	0.4387	0.7587	0.1265
0.6	0.0832	0.3902	0.8602	0.3955	0.9476	1.0851	0.7851	0.2748	0.3734	0.6422	0.0856
0.6652	0.0791	0.3874	0.9998	0.3402	0.8782	1.0164	0.7164	0.2264	0.3402	0.5816	0.0649
0.67	0.0789	0.3872	1.0123	0.3366	0.8736	1.0118	0.7118	0.2231	0.3380	0.5775	0.0635

Notes: Stahl model is referred to the paper "Oligopolistic Pricing with Sequential Consumer Search" (Stahl, 1989); ρ is the reservation price.

Although from a mathematical point of view, the solution exists for $0 < \lambda < 0.665$, this is not the case from game theoretic point of view. For high values of λ, i.e. when the share of informed consumers is high, the chain firm has a profitable deviation. It does not want to compete for both types of consumers. The chain firm focuses on in formed consumers. The deviation is the following: suppose the chain firm sets in both shops the same price $(\hat{p} - \epsilon)$. Then it can only attract informed consumers when the regular firm chooses the high price $\hat{p}$. The profits of the chain firm are given by

$$\lim_{\epsilon \to 0} \pi_M^{dev} = \lambda \nu_{NM} \hat{p} \tag{23}$$

Table 7 calculates profits for both strategies. We can see that for $\lambda > 0.662$, chain firm can profitably deviate by playing a pure strategy.

Proposition 8. *For the case $\mu \in (0; 1)$ and $N = 3$, the proposed strategy of the chain firm is not an equilibrium strategy for* $0.622 < \lambda < 0.665$.

Proof. See argument above and Table 7. □

All in all, first equilibrium is summarized in the following proposition.

Claim 1. *Given $\mu \in (0; 1)$ and $N = 3$, for $0 < \lambda < 0.6$ and $\forall s > 0$ the equilibrium in which uninformed consumers are indifferent between visiting either*

Table 7: Profitable deviation.

						$S = 0.01$					
λ	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.622	0.623	0.624	0.6652
π_M	0.75566	0.07551	0.03698	0.02368	0.01670	0.01229	0.00916	0.00859	0.00856	0.00854	0.00755
π_M^{dev}	0.00575	0.00622	0.00671	0.00718	0.00763	0.00806	0.00847	0.00856	0.00856	0.00857	0.00873
$\Delta\pi$	0.74991	0.06929	0.03027	0.01649	0.00908	0.00423	0.00069	0.00003	0.00000	-0.00003	-0.00118
						$S = 0.05$					
λ	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.622	0.623	0.624	0.6652
π_M	3.77832	0.37754	0.18492	0.11838	0.08352	0.06144	0.04581	0.04293	0.04280	0.04268	0.03773
π_M^{dev}	0.02876	0.03111	0.03357	0.03591	0.03814	0.04028	0.04234	0.04279	0.04281	0.04283	0.04365
$\Delta\pi$	3.74955	0.34643	0.15134	0.08247	0.04538	0.02116	0.00346	0.00014	-0.00001	-0.00015	-0.00592
						$S = 0.3$					
λ	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.622	0.623	0.624	0.6652
π_M	22.66989	2.26526	1.10949	0.71028	0.50115	0.36864	0.27484	0.25758	0.25682	0.25606	0.22637
π_M^{dev}	0.17257	0.18666	0.20143	0.21545	0.22884	0.24169	0.25407	0.25673	0.25685	0.25697	0.26190
$\Delta\pi$	22.49732	2.07860	0.90806	0.49483	0.27230	0.12695	0.02077	0.00085	-0.00003	-0.00091	-0.03553

Notes: $\Delta\pi = \pi_M - \pi_M^{dev}$

of firms first exists. Furthermore, this equilibrium generates higher profits for both firms in comparison to Stahl (1989).

Since the outcome of this type of equilibrium is that as λ goes up, μ increases as well and reaches 1, it is naturally to consider a ”continuation” equilibrium, where uninformed consumers visit a regular firm first, i.e. $\mu = 1$. Although this might be controversial because the chain firm has more market power, we should look at this type of equilibrium and see whether it exists for values of $\lambda > 0.622$.

5 Equilibrium where uninformed consumers prefer a regular firm

In this equilibrium three types of agents, namely the chain firm, regular firms, and uninformed consumers follow slightly different strategies.

The chain firm prices as follows: the shop $M1$ chooses a deterministic price $\tilde{p}$, while the shop $M2$ randomizes over a support $[\underline{p}; \tilde{p})$ with the distribution function $F(p)$.

$(N-2)$ regular firms set a deterministic price $\tilde{p}$ with probability ν_{NM} and randomize over the support $[\underline{p}; \tilde{p})$ with the remaining probability $(1-\nu_{NM})$ from the same distribution function $F(p)$.

Uninformed consumers first search at the regular firm and buy at a price $p \leq \tilde{p}$ (which is the reservation price), i.e. $\mu = 1$ in the notation used above.

Following the same steps as in the first model, let us write down profits of the firms and a few constraints to find an equilibrium.

The profits of the chain firm, if it sets a price p, are given by

$$\pi_M(p) = \lambda \left[(1 - F_{NM}(p))(1-\nu_{NM}) + \nu_{NM}\right]^{N-2} p$$

This equation reflects the fact that the chain firm basically competes only for informed consumers. This is so, because if the uninformed consumers find it optimal to visit a regular firm first, they will do so again given they decide to continue searching.

If the chain firm chooses the price arbitrarily close upper bound of support, i.e. $p \to \tilde{p}$, then its profits are

$$\lim_{p \to \tilde{p}} \pi_M(p) = \lambda \nu_{NM}^{N-2} \tilde{p}$$

Since it should be indifferent between any price in the support, it should be the case that $\pi_M(p) = \lim\limits_{p \to \tilde{p}} \pi_M(p)$

$$\lambda \left[(1 - F_{NM}(p))(1-\nu_{NM}) + \nu_{NM}\right]^{N-2} p = \lambda \nu_{NM}^{N-2} \tilde{p}$$

Rearranging the last equation, we can derive an explicit formula for the distribution function $F_{NM}(p)$

$$F_{NM}(p) = 1 - \frac{\left(\lambda \nu_{NM}^{N-2} \frac{\tilde{p}}{p}\right)^{\frac{1}{N-2}} - \nu_{NM}}{1-\nu_{NM}} = \frac{\nu_{NM}}{1-\nu_{NM}} \left[\left(\frac{\tilde{p}}{p}\right)^{\frac{1}{N-2}} - 1\right] \tag{24}$$

Using the fact that $F_{NM}(\underline{p}) = 0$, we derive an expression for $\underline{p}$

$$\begin{aligned}
F_{NM}(\underline{p}) = 0 = \frac{\nu_{NM}}{1-\nu_{NM}} \left[\left(\frac{\tilde{p}}{\underline{p}} \right)^{\frac{1}{N-2}} - 1 \right] \\
\Rightarrow 1 - \nu_{NM} = \nu_{NM} \left(\frac{\tilde{p}}{\underline{p}} \right)^{\frac{1}{N-2}} - \nu_{NM} \\
\Rightarrow \underline{p} = \nu_{NM}^{N-2} \tilde{p} < \tilde{p}, \text{ since } \nu_{NM}^{N-2} < 1
\end{aligned} \tag{25}$$

The profits of regular firm, if it sets a price p are

$$\pi_{NM}(p) = \left\{ \lambda(1 - F_M(p)) \left[(1 - F_{NM}(p))(1 - \nu_{NM}) + \nu_{NM} \right]^{N-3} + \frac{1-\lambda}{N-2} \right\} p$$

The first term gives the profits from setting the lowest price in the market and attracting the informed consumers while the second term reflect profits from uninformed consumers who are equally likely to visit one of the regular firms.

If the regular firm chooses a price $p = \tilde{p}$, then the profits are

$$\pi_{NM}(\tilde{p}) = \frac{1-\lambda}{N-2} \tilde{p}$$

Again, it should be the case that $\pi_{NM}(p) = \pi_{NM}(\tilde{p})$

$$\left\{ \lambda(1 - F_M(p)) \left[(1 - F_{NM}(p))(1 - \nu_{NM}) + \nu_{NM} \right]^{N-3} + \frac{1-\lambda}{N-2} \right\} p = \frac{1-\lambda}{N-2} \tilde{p}$$

We can express the distribution function $F_M(p)$ from the last equality[20]

$$F_M(p) = 1 - \frac{1-\lambda}{\lambda(N-2)\nu_{NM}^{N-3}} \left(\frac{\tilde{p}}{p} \right)^{\frac{N-2}{N-3}} \left(\frac{\tilde{p}}{p} - 1 \right) \tag{26}$$

It is easy to see that $F_M(\tilde{p}) = 1$. Since we want the two distributions $F_M(p)$ and $F_{NM}(p)$ being defined over the same interval, the lower bounds should be equal as well[21]

$$\nu_{NM} = \sqrt[N-2]{\frac{1-\lambda}{\lambda(N-2) + (1-\lambda)}} \tag{27}$$

To make the proposed behavior of the uninformed consumers optimal, we have to impose a few conditions. First, when the uninformed consumers observes an upper bound price $\tilde{p}$ at the regular firm, she should be indifferent between buying and continuing to search. In this case, it means that she continues to

[20] See appendix A.6 for a detailed derivation.

[21] See appendix A.7 for a detailed derivation.

search at another regular firm until she finds one which has chosen a price strictly less than $\tilde{p}$. If there is no such firm present at the market, i.e. all regular firms happened to choose a high price $\tilde{p}$ (what happens with a non-zero probability ν_{NM}^{N-2}), the uninformed consumer goes to the chain firm. Mathematically,

$$\tilde{p} = \mathbb{E}(\text{costs of continuing to search})^{22} \tag{28}$$

Finally, uninformed consumers should really prefer to visit a regular firm first. In this equilibrium this means that $\mathbb{E}_{NM}(p) < \mathbb{E}_M(p)$. To calculate the latter we need an expression from Equation 28. The expected price[23] at the regular shop can be calculated using the definition of the expectation, namely

$$\mathbb{E}_{NM}(p) = \frac{\nu_{NM}(N-2) - \nu_{NM}^{N-3}}{N-3}\tilde{p} \tag{29}$$

Hence, the condition is the following

$$\frac{\nu_{NM}(N-2) - \nu_{NM}^{N-3}}{N-3}\tilde{p} < \mathbb{E}_M(p).$$

This concludes all conditions required to derive equilibrium values.

For the purposes of this paper, we perform numerical analysis for the case $N = 3$ shops. Plugging in $N = 3$, to equations above gives us the following

$$\pi_M = \lambda\nu_{NM}\tilde{p} \tag{30}$$

$$\pi_{NM} = (1-\lambda)\tilde{p} \tag{31}$$

$$F(p) = 1 - \frac{1-\lambda}{\lambda}\frac{\tilde{p}-p}{p} \tag{32}$$

$$\nu_{NM} = 1 - \lambda \tag{33}$$

Equation 28 can be explicitly written as

$$\tilde{p} = \frac{1}{2}\left[\int_{\underline{p}}^{\tilde{p}} p\,dF(p) + s\right] + \frac{1}{2}\left[\int_{\underline{p}}^{\tilde{p}} p\,dF(p) + 2s\right]$$

$$\Rightarrow \tilde{p} = \frac{3s}{2} + \int_{\underline{p}}^{\tilde{p}} p\,dF(p) \tag{34}$$

[22]The expression for N shops is very lengthy and we omit it here. The reason is that we cannot simply say that the price $\tilde{p}$ should be equal to the expected price at the chain firm because we have $(N-2)$ regular firm which we have to take into account.

[23]See appendix A.8 for a detailed derivation.

The integral $\int\limits_{\underline{p}}^{\tilde{p}} p\,dF(p)$ can be calculated by integration by parts.

$$\int\limits_{\underline{p}}^{\tilde{p}} p\,dF(p) = \frac{1-\lambda}{\lambda}\ln\left(\frac{1}{1-\lambda}\right)\tilde{p}$$

Hence, Equation 34 derives $\tilde{p}$ as a function of parameters of the model.

$$\begin{aligned} \tilde{p}\left[1-\frac{1-\lambda}{\lambda}\ln\left(\frac{1}{1-\lambda}\right)\right] &= \frac{3s}{2} \\ \Rightarrow \tilde{p} &= \frac{\frac{3s}{2}}{1-\frac{1-\lambda}{\lambda}\ln\left(\frac{1}{1-\lambda}\right)} \end{aligned} \tag{35}$$

The last condition is that uninformed consumers prefer to visit first the regular firm. It gives us a parameter range for which this equilibrium exists.

$$\begin{aligned} \nu_{NM}\tilde{p} + (1-\nu_{NM})\int\limits_{\underline{p}}^{\tilde{p}} p\,dF(p) + s &< \int\limits_{\underline{p}}^{\tilde{p}} p\,dF(p) + \frac{3s}{2} \\ \nu_{NM}\left[\tilde{p} - \int\limits_{\underline{p}}^{\tilde{p}} p\,dF(p)\right] &< \frac{s}{2} \\ \nu_{NM}\frac{3s}{2} &< \frac{S}{2} \\ \nu_{NM} &< \frac{1}{3} \\ 1-\lambda &< \frac{1}{3} \\ \lambda &> \frac{2}{3} \end{aligned}$$

Hence, this equilibrium exists if the share of informed consumers is sufficiently high.

Table 8, Table 9, Table 10 shows equilibrium values for various search costs, namely $s = 0.01, 0.05, 0.3$. Again we can observe that search costs are positively correlated with prices. Higher search costs shift price range up. Besides, we can see that expected price at the regular firm is lower than expected price at the chain firm what confirms the initial preference of uninformed consumers to visit a regular firm first. Like in the first type of equilibrium, profits are uniformly higher than in Stahl (1989) model and they decrease with an increase in λ. This is due to the fact that as mass of informed consumers gets greater, firms gradually switch towards competing for them. Hence, prices go down. Last but not least

Table 8: Equilibrium values for $\mu = 1$ ($s = 0.01$).

Heterogeneous firms model								Stahl model	
λ	ν_{NM}	$\underline{p}$	$\tilde{p}$	$\mathbb{E}p_M$	$\mathbb{E}p_{NM}$	π_1	π_2	ρ	π
0.67	0.33	0.0109	0.0330	0.0255	0.0230	0.0073	0.0109	0.0192	0.0021
0.70	0.30	0.0093	0.0310	0.0235	0.0205	0.0065	0.0093	0.0184	0.0018
0.75	0.25	0.0070	0.0279	0.0204	0.0166	0.0052	0.0070	0.0171	0.0014
0.80	0.20	0.0050	0.0251	0.0176	0.0131	0.0040	0.0050	0.0159	0.0011
0.85	0.15	0.0034	0.0225	0.0150	0.0098	0.0029	0.0034	0.0148	0.0007
0.90	0.10	0.0020	0.0202	0.0127	0.0067	0.0018	0.0020	0.0136	0.0005
0.95	0.05	0.0009	0.0178	0.0103	0.0036	0.0008	0.0009	0.0124	0.0002
0.99	0.01	0.0002	0.0157	0.0082	0.0009	0.0002	0.0002	0.0110	0.00004

Notes: Stahl model is referred to the paper "Oligopolistic Pricing with Sequential Consumer Search" (Stahl, 1989); ρ is the reservation price.

Table 9: Equilibrium values for $\mu = 1$ ($s = 0.05$).

Heterogeneous firms model								Stahl model	
λ	ν_{NM}	$\underline{p}$	$\tilde{p}$	$\mathbb{E}p_M$	$\mathbb{E}p_{NM}$	π_1	π_2	ρ	π
0.67	0.33	0.0545	0.1652	0.1277	0.1150	0.0365	0.0545	0.0962	0.0106
0.70	0.30	0.0465	0.1550	0.1175	0.1025	0.0325	0.0465	0.0921	0.0092
0.75	0.25	0.0349	0.1394	0.1019	0.0832	0.0261	0.0349	0.0857	0.0071
0.80	0.20	0.0251	0.1255	0.0880	0.0655	0.0201	0.0251	0.0797	0.0053
0.85	0.15	0.0169	0.1127	0.0752	0.0490	0.0144	0.0169	0.0739	0.0037
0.90	0.10	0.0101	0.1008	0.0633	0.0333	0.0091	0.0101	0.0681	0.0023
0.95	0.05	0.0045	0.0890	0.0515	0.0178	0.0042	0.0045	0.0618	0.0010
0.99	0.01	0.0008	0.0787	0.0412	0.0044	0.0008	0.0008	0.0548	0.0002

Notes: Stahl model is referred to the paper "Oligopolistic Pricing with Sequential Consumer Search" (Stahl, 1989); ρ is the reservation price.

Table 10: Equilibrium values for $\mu = 1$ ($s = 0.3$).

Heterogeneous firms model								Stahl model	
λ	ν_{NM}	$\underline{p}$	$\tilde{p}$	$\mathbb{E}p_M$	$\mathbb{E}p_{NM}$	π_1	π_2	ρ	π
0.67	0.33	0.3271	0.9913	0.7663	0.6898	0.2192	0.3271	0.5775	0.0635
0.70	0.30	0.2789	0.9297	0.7047	0.6147	0.1952	0.2789	0.5526	0.0553
0.75	0.25	0.2091	0.8366	0.6116	0.4991	0.1569	0.2091	0.5140	0.0428
0.80	0.20	0.1506	0.7530	0.5280	0.3930	0.1205	0.1506	0.4780	0.0319
0.85	0.15	0.1015	0.6765	0.4515	0.2940	0.0863	0.1015	0.4433	0.0222
0.90	0.10	0.0605	0.6047	0.3797	0.1997	0.0544	0.0605	0.4086	0.0136
0.95	0.05	0.0267	0.5342	0.3092	0.1067	0.0254	0.0267	0.3707	0.0062
0.99	0.01	0.0047	0.4720	0.2470	0.0265	0.0047	0.0047	0.3289	0.0011

Notes: Stahl model is referred to the paper "Oligopolistic Pricing with Sequential Consumer Search" (Stahl, 1989); ρ is the reservation price.

thing to notice is that in this equilibrium regular firm makes higher profits than chain firm.

We summarize the results in the following claim:

Claim 2. *Given $\mu = 1$ and $N = 3$, for $\lambda > \frac{2}{3}$ and $\forall s > 0$ the equilibrium in which uninformed consumers prefer to visit the regular firm first exists. Furthermore, this equilibrium generates higher profits for both firms in comparison to Stahl (1989).*

First two types of equilibria has covered most of the parameter space. The former equilibrium exists for $\lambda < 0.6$, while the latter is an equilibrium for $\lambda > 2/3$. Still, we need to propose some other equilibrium which will cover the rest. One possible candidate is to consider a mixture of first two types of equilibria. The chain firm might want to correlate its strategies. Let us consider this candidate equilibrium in details.

6 Correlated equilibrium

This equilibrium is a sort of mixture of the first two equilibria. We will refer to it as a correlated equilibrium.

In this setup the pricing scheme of the chain firm is the following: with probability α the shop $M1$ sets a deterministic price $\tilde{p}$, while the shop $M2$ does the same with the probability ν_{NM} and randomizes over the interval $[\underline{p};\tilde{p})$ with the probability $(1-\nu_{NM})$ (the distribution function is $F(p)$); with the remaining probability $(1-\alpha)$ the shop $M1$ chooses a deterministic price $\hat{p}$ and the shop $M2$ randomizes over the interval $[\underline{\hat{p}};\hat{p})$ with the distribution function $\hat{F}(p)$. The important assumption is that $\underline{\hat{p}} > \tilde{p}$, so intervals do not overlap.

$(N-2)$ regular firms set a deterministic price $\hat{p}$ with the probability ν_{NM}. With the remaining probability $(1-\nu_{NM})$ they use a mixed strategy: either with probability β they choose a price from the interval $[\underline{p};\tilde{p})$ with the distribution function $F(p)$ or with the probability $(1-\beta)$ a price is drawn from the interval $[\underline{\hat{p}};\hat{p})$ with the distribution function $\hat{F}(p)$.

The share $(1-\lambda)$ of the uninformed consumers visits one of the regular firms first with the probability μ. They buy there if the price is lower than or equal to the highest price possible in the market, i.e. $p \leq \hat{p}$. With the remaining probability $(1-\mu)$ they go first to one of the shops of the chain firm and buy there if $p \leq \tilde{p}$. If the price is high, i.e. $p > \tilde{p}$, then they continue to search and go to the regular firm.

First, let us introduce some notation

$$G(x;p) \equiv \sum_{i=0}^{x} \binom{x}{i} \nu_{NM}^{i} (1-\nu_{NM})^{x-i} \sum_{j=0}^{x-i} \beta^{x-i-j} (1-\beta)^{j} (1-F(p))^{x-i-j}$$

$$T(x) \equiv G(x; p=\underline{p}) = \sum_{i=0}^{x} \binom{x}{i} \nu_{NM}^{i} (1-\nu_{NM})^{x-i} \sum_{j=0}^{x-i} \beta^{x-i-j} (1-\beta)^{j}$$

$$\hat{G}(x;p) \equiv \sum_{i=0}^{x} \binom{x}{i} \nu_{NM}^{i} \left[(1-\nu_{NM})(1-\beta)(1-\hat{F}(p))\right]^{x-i}$$

$$\hat{T}(x) \equiv \hat{G}(x; p=\underline{\hat{p}}) = G(x; p=\tilde{p}) = \sum_{i=0}^{x} \binom{x}{i} \nu_{NM}^{i} \left[(1-\nu_{NM})(1-\beta)\right]^{x-i},$$

where $\binom{x}{i} = \dfrac{x!}{i!(x-i)!}$

As usual, we start out by writing down the profits for the chain firm. If it sets a price $p \in [\underline{p}; \tilde{p})$, then the profits are given by

$$\tilde{\pi}_M(p) = \lambda G(N-2;p)p + \frac{1-\mu}{2}(1-\lambda)p + \frac{1-\mu}{2}(1-\lambda)\tilde{p}$$

If the chain firm chooses the price arbitrarily close to the upper bound of the lower interval, i.e. $p \to \tilde{p}$, then the profits are

$$\lim_{p\to\tilde{p}} \tilde{\pi}_M(p) = [\lambda\hat{T}(N-2) + (1-\mu)(1-\lambda)]\tilde{p}$$

The chain firm should be indifferent between any price in the support. Hence, $\tilde{\pi}_M(p) = \lim_{p\to\tilde{p}} \tilde{\pi}_M(p)$.

$$\lambda G(N-2;p)p + \frac{1-\mu}{2}(1-\lambda)p + \frac{1-\mu}{2}(1-\lambda)\tilde{p} = [\lambda\hat{T}(N-2)+(1-\mu)(1-\lambda)]\tilde{p} \quad (36)$$

Particularly, Equation 36 holds for $p = \underline{p}$. Then[24]

$$\underline{p} = \frac{\lambda\hat{T}(N-2) + \frac{1-\mu}{2}(1-\lambda)}{\lambda T(N-2) + \frac{1-\mu}{2}(1-\lambda)}\tilde{p} < \tilde{p}, \text{ since } T(N-2) > \hat{T}(N-2) \quad (37)$$

If the price is chosen from the higher interval, i.e. $p \in [\underline{\hat{p}}; \hat{p})$, then the profits of the chain firm are given by

$$\hat{\pi}_M(p) = \lambda\hat{G}(N-2;p)p$$

If the chosen price is arbitrarily close to the highest possible in the market, i.e. $p \to \hat{p}$, then the chain firm's profits are given by

$$\lim_{p\to\hat{p}} \hat{\pi}_M(p) = \lambda\nu_{NM}^{N-2}\hat{p}$$

By the indifference between prices in the support, $\hat{\pi}_M(p) = \lim_{p\to\hat{p}} \hat{\pi}_M(p)$.

$$\begin{aligned} \lambda\hat{G}(N-2;p)p &= \lambda\nu_{NM}^{N-2}\hat{p} \\ \Rightarrow \hat{G}(N-2;p)p &= \nu_{NM}^{N-2}\hat{p} \end{aligned} \quad (38)$$

Equation 38 holds true for $p = \underline{\hat{p}}$. Hence,

$$\hat{G}(N-2; p = \underline{\hat{p}})\underline{\hat{p}} = \nu_{NM}^{N-2}\hat{p}$$

[24] See appendix A.9 for a detailed derivation.

$$\Rightarrow \hat{T}(N-2)\underline{\hat{p}} = \nu_{NM}^{N-2}\hat{p}$$
$$\Rightarrow \underline{\hat{p}} = \frac{\nu_{NM}^{N-2}}{\hat{T}(N-2)}\hat{p} \tag{39}$$

Furthermore, the chain firm should be indifferent between prices in the lower interval and the upper interval. In particular, it should hold for $p \to \tilde{p}$ and $p \to \hat{p}$, i.e. $\lim_{p\to\tilde{p}} \tilde{\pi}_M(p) = \lim_{p\to\hat{p}} \hat{\pi}_M(p)$.

$$[\lambda\hat{T}(N-2) + (1-\mu)(1-\lambda)]\tilde{p} = \lambda\nu_{NM}^{N-2}\hat{p}$$
$$\Rightarrow \hat{p} = \frac{[\lambda\hat{T}(N-2) + (1-\mu)(1-\lambda)]}{\lambda\nu_{NM}^{N-2}}\tilde{p} \tag{40}$$

The profits of the regular firm, if it chooses a price $p \in [\underline{p}; \tilde{p})$, are given by

$$\tilde{\pi}_{NM}(p) = \alpha\left\{\lambda\left[(1-F(p))(1-\nu_M) + \nu_M\right]G(N-3;p) + \frac{1-\lambda}{N-2}\mu\right\}p + (1-\alpha)G(N-3;p)p$$

When it sets the price arbitrarily close to the upper bound of the lower interval ($p \to \tilde{p}$), the profits are

$$\lim_{p\to\tilde{p}} \tilde{\pi}_{NM}(p) = \left\{\alpha\left[\lambda\nu_M\hat{T}(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)\hat{T}(N-3)\right\}\tilde{p}$$

Again, the regular firm should be indifferent between any price in the support. Consequently, $\tilde{\pi}_{NM}(p) = \lim_{p\to\tilde{p}} \tilde{\pi}_{NM}(p)$.

$$\begin{aligned}\alpha\left\{\lambda\left[(1-F(p))(1-\nu_M) + \nu_M\right]G(N-3;p) + \frac{1-\lambda}{N-2}\mu\right\}p + (1-\alpha)G(N-3;p)p = \\ = \left\{\alpha\left[\lambda\nu_M\hat{T}(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)\hat{T}(N-3)\right\}\tilde{p}\end{aligned} \tag{41}$$

Again, Equation 41 should hold for any price $p \in [\underline{p}; \tilde{p})$. Specifically, for $p = \underline{p}$[25]:

$$\underline{p} = \frac{\alpha\left[\lambda\nu_M\hat{T}(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)\hat{T}(N-3)}{\alpha\left[\lambda T(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)T(N-3)}\tilde{p} < \tilde{p} \tag{42}$$

Pricing from the upper interval, i.e. $p \in [\underline{\hat{p}}; \hat{p})$, gives the following profits

$$\hat{\pi}_{NM}(p) = \left[\lambda(1-\alpha)(1-\hat{F}(p))\hat{G}(N-3;p) + \mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2}\right]p$$

[25] See appendix A.10 for a detailed derivation.

If the regular shop sets a price $p \to \hat{p}$, then the profits are

$$\lim_{p\to\hat{p}} \hat{\pi}_{NM}(p) = \left(\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\nu_{NM}^{N-3}\right)\hat{p}$$

The last term in this equation is the probability that all other regular firms set the price $p = \hat{p}$. In a standard way, the indifference condition between prices tells us that $\hat{\pi}_{NM}(p) = \lim_{p\to\hat{p}} \hat{\pi}_{NM}(p)$.

$$\begin{aligned}\left[\lambda(1-\alpha)(1-\hat{F}(p))\hat{G}(N-3;p) + \mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2}\right]p = \\ = \left(\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\nu_{NM}^{N-3}\right)\hat{p} \quad (43)\end{aligned}$$

For the price $p = \underline{\hat{p}}$, Equation 43 transforms to[26]

$$\underline{\hat{p}} = \frac{\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\nu_{NM}^{N-3}}{\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\hat{T}(N-3)}\hat{p} < \hat{p} \quad (44)$$

Finally, the regular firm should be indifferent between pricing from the lower interval and the upper interval. This should be true particularly for the prices $p \to \tilde{p}$ and $p \to \hat{p}$, i.e. $\lim_{p\to\tilde{p}} \tilde{\pi}_{NM}(p) = \lim_{p\to\hat{p}} \hat{\pi}_{NM}(p)$.

$$\begin{aligned}&\left\{\alpha\left[\lambda\nu_M\hat{T}(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)\hat{T}(N-3)\right\}\tilde{p} = \\ &= \left(\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\nu_{NM}^{N-3}\right)\hat{p} \\ &\Rightarrow \hat{p} = \frac{\alpha\left[\lambda\nu_M\hat{T}(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)\hat{T}(N-3)}{\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\nu_{NM}^{N-3}}\tilde{p} \quad (45)\end{aligned}$$

These equation should be consistent. Consequently, following equations have to have the same coefficients: Equation 37 and Equation 42, Equation 39 and Equation 44, and Equation 40 and Equation 45.

$$\frac{\lambda\hat{T}(N-2) + \frac{1-\mu}{2}(1-\lambda)}{\lambda T(N-2) + \frac{1-\mu}{2}(1-\lambda)} = \frac{\alpha\left[\lambda\nu_M\hat{T}(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)\hat{T}(N-3)}{\alpha\left[\lambda T(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)T(N-3)} \quad (46)$$

$$\frac{\nu_{NM}^{N-2}}{\hat{T}(N-2)} = \frac{\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\nu_{NM}^{N-3}}{\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\hat{T}(N-3)} \quad (47)$$

[26]See appendix A.11 for a detailed derivation.

$$\frac{[\lambda \hat{T}(N-2)+(1-\mu)(1-\lambda)]}{\lambda \nu_{NM}^{N-2}}=\frac{\alpha\left[\lambda \nu_M \hat{T}(N-3)+\frac{1-\lambda}{N-2}\mu\right]+(1-\alpha)\hat{T}(N-3)}{\mu\frac{1-\lambda}{N-2}+(1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2}+(1-\alpha)\lambda\nu_{NM}^{N-3}} \tag{48}$$

We have to impose some further conditions to make the proposed behavior of the uninformed consumers optimal. First, prior to search they should be indifferent between going to the one of the shops of the chain firm and visiting one of the regular firms. Since the uninformed consumer continues to search if she observes a high price at the chain firm, we should set equal not expected prices at the firms but rather expected costs, i.e. we should not take into account the scenario when the chain firm chooses prices from the upper interval. Hence, the condition is the following

$$\frac{1}{2}(1+\nu_M)\tilde{p}+\frac{1}{2}(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)+\frac{1-\alpha}{\alpha}s=$$
$$\nu_{NM}\hat{p}+(1-\nu_{NM})\beta\int_{\underline{p}}^{\tilde{p}} p\,dF(p)+(1-\nu_{NM})(1-\beta)\int_{\underline{\hat{p}}}^{\hat{p}} p\,d\hat{F}(p) \tag{49}$$

The second constraint is when the non-shopper visits one of the regular firms first and observes a price $\hat{p}$, she should be indifferent between buying and continuing to search. Since *a priori* she is indifferent between either of firms, there are two conditions we can impose: one for the chain firm and one for the regular firm.

$$\hat{p}=\alpha\left[\frac{1}{2}(1+\nu_M)\tilde{p}+\frac{1}{2}(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)\right]+(1-\alpha)\left[\frac{1}{2}\hat{p}+\frac{1}{2}\int_{\underline{\hat{p}}}^{\hat{p}} p\,d\hat{F}(p)\right]+s \tag{50}$$

$$\hat{p}=\nu_{NM}\hat{p}+(1-\nu_{NM})\beta\int_{\underline{p}}^{\tilde{p}} p\,dF(p)+(1-\nu_{NM})(1-\beta)\int_{\underline{\hat{p}}}^{\hat{p}} p\,d\hat{F}(p)+s \tag{51}$$

The last constraint is the indifference between buying and continuing to search after the uninformed consumer observes a price $\tilde{p}$ at the chain firm. By the same reasons as in the first model (see footnote 1), this means here to continue searching at the second shop of the chain firm.

$$\tilde{p}=\frac{\nu_M}{1+\nu_M}\tilde{p}+\frac{1}{1+\nu_M}\left(\nu_M\tilde{p}+(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p\,dF(p)\right)+s$$

Rearranging gives us

$$\tilde{p} = \frac{1+\nu_M}{1-\nu_M} s + \int_{\underline{p}}^{\tilde{p}} p \, dF(p) \tag{52}$$

Again, to make things a little simpler we numerically analyze the case of $N = 3$. In this case, we have the following system of equations to solve

$$F(p) = 1 - \frac{\alpha(\lambda\nu_M + \mu(1-\lambda) + (1-\alpha)}{\lambda\alpha(1-\nu_M)} \frac{\tilde{p} - p}{p} \tag{53}$$

$$\hat{F}(p) = 1 - \frac{\nu_{NM}}{(1-\nu_{NM})(1-\beta)} \frac{\hat{p} - p}{p} \tag{54}$$

$$\frac{\lambda(\nu_M + (1-\nu_{NM})(1-\beta)) + \frac{1-\mu}{2}(1-\lambda)}{(1-\nu_{NM})\beta} = \frac{\alpha(\lambda\nu_M + \mu(1-\lambda) + (1-\alpha)}{\lambda\alpha(1-\nu_M)} \tag{55}$$

$$\frac{\nu_{NM}}{(1-\nu_{NM})(1-\beta)} = \frac{\mu(1-\lambda) + (1-\alpha)(1-\mu)(1-\lambda)}{\lambda(1-\alpha)} \tag{56}$$

$$\frac{\lambda(\nu_M + (1-\nu_{NM})(1-\beta)) + (1-\mu)(1-\lambda)}{\lambda\nu_{NM}} = \frac{\alpha(\lambda\nu_M + \mu(1-\lambda) + (1-\alpha)}{\mu(1-\lambda) + (1-\alpha)(1-\mu)(1-\lambda)} \tag{57}$$

$$\frac{1}{2}(1+\nu_M)\tilde{p} + \frac{1}{2}(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p \, dF(p) + \frac{1-\alpha}{\alpha} s =$$

$$\nu_{NM}\hat{p} + (1-\nu_{NM})\beta \int_{\underline{p}}^{\tilde{p}} p \, dF(p) + (1-\nu_{NM})(1-\beta)\int_{\underline{\hat{p}}}^{\hat{p}} p \, d\hat{F}(p) \tag{58}$$

$$\hat{p} = \alpha \left[\frac{1}{2}(1+\nu_M)\tilde{p} + \frac{1}{2}(1-\nu_M)\int_{\underline{p}}^{\tilde{p}} p \, dF(p) \right] + (1-\alpha) \left[\frac{1}{2}\hat{p} + \frac{1}{2}\int_{\underline{\hat{p}}}^{\hat{p}} p \, d\hat{F}(p) \right] + s \tag{59}$$

$$\tilde{p} = \frac{1+\nu_M}{1-\nu_M} s + \int_{\underline{p}}^{\tilde{p}} p \, dF(p) \tag{60}$$

$$\hat{p} = \frac{\alpha(\lambda\nu_M + \mu(1-\lambda) + (1-\alpha)}{\mu(1-\lambda) + (1-\alpha)(1-\mu)(1-\lambda)} \tilde{p} \tag{61}$$

We have seven equations to derive seven endogenous variables, namely α, β, μ, $\nu_M, \nu_{NM}, \tilde{p}, \hat{p}$. First five parameters are probabilities, so we know that they are bound between zero and one. We cannot *a priori* bound prices. Hence, we should reduce our system omitting them before we solve the system numerically. Last two equations are plugged to all others to derive a system of unknown endogenous probabilities.

This system of equations is highly nonlinear, so we have to be careful with the choice of initial guess[27].

Table 11: Correlated equilibrium values ($s = 0.01$).

λ	ν_M	ν_{NM}	μ	α	β	$\underline{\tilde{p}}$	$\tilde{p}$	$\underline{\hat{p}}$	$\hat{p}$	$\mathbb{E}p_M$	$\mathbb{E}p_{NM}$	π_M	π_{NM}
0.1	0.0357	0.8435	0.9999	0.3598	0.6166	0.2080	0.2154	0.2157	0.2311	0.2211	0.2281	0.0195	0.2080
0.13	0.0260	0.7992	0.9998	0.3800	0.6311	0.1515	0.1592	0.1594	0.1742	0.1642	0.1704	0.0181	0.1515
0.16	0.0168	0.7564	0.9998	0.4015	0.6461	0.1165	0.1244	0.1245	0.1387	0.1287	0.1342	0.0168	0.1165
0.19	0.0082	0.7150	0.9999	0.4245	0.6614	0.0928	0.1009	0.1010	0.1146	0.1046	0.1094	0.0156	0.0928
0.22	0.0004	0.6751	1.0000	0.4488	0.6769	0.0758	0.0841	0.0841	0.0971	0.0871	0.0913	0.0144	0.0758
0.54	0.0016	0.3728	0.9993	0.7997	0.8601	0.0169	0.0297	0.0297	0.0367	0.0267	0.0272	0.0074	0.0169
0.6	0.0370	0.3570	0.9725	0.9028	0.9170	0.0136	0.0287	0.0300	0.0345	0.0245	0.0246	0.0074	0.0134
0.65	0.0782	0.3602	0.8550	0.9729	0.9670	0.0108	0.0280	0.0324	0.0343	0.0229	0.0232	0.0080	0.0103
0.7	0.0780	0.3601	0.9415	0.9697	0.9578	0.0096	0.0265	0.0311	0.0334	0.0215	0.0220	0.0084	0.0095
0.75	0.0594	0.2651	0.8543	0.9574	0.9465	0.0065	0.0217	0.0255	0.0293	0.0169	0.0161	0.0058	0.0063
0.8	0.0433	0.1945	0.8563	0.9634	0.9590	0.0045	0.0187	0.0218	0.0255	0.0139	0.0118	0.0040	0.0044
0.85	0.0332	0.1355	0.8371	0.9615	0.9594	0.0031	0.0163	0.0191	0.0241	0.0116	0.0087	0.0028	0.0030
0.89	0.0127	0.1331	0.8919	0.9623	0.9477	0.0020	0.0141	0.0153	0.0205	0.0094	0.0067	0.0024	0.0020

Table 12: Correlated equilibrium values ($s = 0.05$).

λ	ν_M	ν_{NM}	μ	α	β	$\underline{\tilde{p}}$	$\tilde{p}$	$\underline{\hat{p}}$	$\hat{p}$	$\mathbb{E}p_M$	$\mathbb{E}p_{NM}$	π_M	π_{NM}
0.1	0.0357	0.8435	0.9999	0.3598	0.6166	1.0398	1.0772	1.0786	1.1553	1.1053	1.1403	0.0975	1.0398
0.13	0.0260	0.7992	0.9998	0.3800	0.6311	0.7577	0.7959	0.7970	0.8709	0.8209	0.8519	0.0905	0.7576
0.16	0.0168	0.7564	0.9998	0.4015	0.6461	0.5826	0.6219	0.6226	0.6936	0.6436	0.6710	0.0839	0.5826
0.19	0.0082	0.7150	0.9999	0.4245	0.6614	0.4641	0.5044	0.5048	0.5729	0.5229	0.5470	0.0778	0.4640
0.22	0.0004	0.6751	1.0000	0.4488	0.6769	0.3788	0.4203	0.4203	0.4857	0.4357	0.4567	0.0721	0.3788
0.54	0.0016	0.3728	0.9993	0.7997	0.8601	0.0844	0.1484	0.1486	0.1836	0.1336	0.1359	0.0370	0.0844
0.6	0.0370	0.3570	0.9725	0.9028	0.9170	0.0679	0.1434	0.1498	0.1723	0.1223	0.1229	0.0369	0.0672
0.65	0.0790	0.3599	0.8563	0.9735	0.9679	0.0545	0.1407	0.1628	0.1721	0.1152	0.1163	0.0403	0.0518
0.7	0.0761	0.3582	0.9412	0.9699	0.9585	0.0476	0.1316	0.1540	0.1654	0.1064	0.1086	0.0415	0.0468
0.75	0.0545	0.2706	0.8500	0.9585	0.9461	0.0314	0.1066	0.1237	0.1416	0.0826	0.0786	0.0287	0.0303
0.8	0.0429	0.1976	0.8619	0.9614	0.9561	0.0228	0.0936	0.1087	0.1281	0.0697	0.0595	0.0203	0.0222
0.85	0.0291	0.1499	0.8523	0.9594	0.9527	0.0154	0.0807	0.0924	0.1172	0.0572	0.0442	0.0149	0.0151
0.89	0.0057	0.1432	0.9325	0.9647	0.9488	0.0097	0.0693	0.0718	0.0938	0.0457	0.0322	0.0120	0.0096

Table 11, Table 12, Table 13 summarize equilibrium values for some search costs. We see that there are two separate intervals of λ for which there is an equilibrium. Besides, due to high nonlinearity of the system the parameters are not monotonic within the high interval. Furthermore, although expected price at the regular firm is higher, nevertheless uninformed consumers are indifferent between visiting either of them. This is due to the fact that there might be a search beyond the first quote in equilibrium. Profits of the regular firm are higher than profits of the chain firm for almost all values of λ. With the increase

[27] Here we present equilibrium values for the initial guess $(\nu_M, \nu_{NM}, \mu, \alpha, \beta) = (0.1, 0.4, 0.5, 0.5, 0.5)$. The choice of first three variables initial values is due to equilibrium values of the first type of equilibrium. Since we do not know anything about the latter two, we choose the average value.

in search costs the range of parameters where the chain firm have higher profits gets larger, but it is still small compare to the set of values where regular firm have higher profits.

To sum up, this type of equilibrium seems to close the gap between first two types

Table 13: Correlated equilibrium values ($s = 0.3$).

λ	ν_M	ν_{NM}	μ	α	β	$\underline{\tilde{p}}$	$\tilde{p}$	$\underline{\hat{p}}$	$\hat{p}$	$\mathbb{E}p_M$	$\mathbb{E}p_{NM}$	π_M	π_{NM}
0.1	0.0357	0.8435	0.9999	0.3598	0.6166	6.2388	6.4631	6.4717	6.9321	6.6321	6.8416	0.5847	6.2386
0.13	0.0260	0.7992	0.9998	0.3800	0.6311	4.5459	4.7757	4.7821	5.2252	4.9252	5.1113	0.5429	4.5456
0.16	0.0168	0.7564	0.9998	0.4015	0.6461	3.4958	3.7315	3.7359	4.1618	3.8618	4.0262	0.5036	3.4956
0.19	0.0082	0.7150	0.9999	0.4245	0.6614	2.7843	3.0264	3.0286	3.4375	3.1375	3.2819	0.4670	2.7842
0.22	0.0004	0.6751	1.0000	0.4488	0.6769	2.2730	2.5220	2.5221	2.9142	2.6142	2.7403	0.4328	2.2730
0.54	0.0016	0.3728	0.9993	0.7997	0.8601	0.5065	0.8905	0.8916	1.1014	0.8014	0.8154	0.2217	0.5064
0.6	0.0370	0.3570	0.9725	0.9028	0.9170	0.4072	0.8606	0.8991	1.0335	0.7335	0.7373	0.2214	0.4031
0.65	0.0754	0.3588	0.8774	0.9796	0.9760	0.3247	0.8425	0.9684	1.0101	0.6877	0.6865	0.2356	0.3110
0.7	0.0415	0.3394	0.9078	0.9809	0.9749	0.2226	0.6880	0.7565	0.7935	0.5366	0.5153	0.1885	0.2165
0.75	0.0509	0.2651	0.8807	0.9654	0.9577	0.1869	0.6391	0.7344	0.8205	0.4924	0.4576	0.1631	0.1815
0.8	0.0314	0.2068	0.9032	0.9684	0.9637	0.1284	0.5458	0.6102	0.6952	0.4000	0.3355	0.1150	0.1260
0.85	0.0167	0.1752	0.9253	0.9627	0.9516	0.0879	0.4712	0.5089	0.6249	0.3289	0.2583	0.0931	0.0870
0.9	0.0076	0.1419	0.9194	0.9606	0.9363	0.0554	0.4090	0.4299	0.5955	0.2693	0.1959	0.0761	0.0549

7 Conclusion

We consider a model with heterogeneous firms which compete by setting prices for a homogeneous good. One type of firm is not standard. It is assumed that it can set two prices in the market, i.e. it is a chain firm (it has two shops). There are two types of consumers present in the economy. First, informed consumers who know all prices in the market and, hence, buy at the firm which set the lowest price. Second, uninformed consumers who get their first price for free, but each other price comes at costs. The latter are engaged in a sequential search. Unlike Stahl (1989), where identical firms have a trade-off between setting a high price and attracting only uninformed consumers and setting a low price and competing for informed consumers, the possibility of choosing two prices allows chain firm to compete for both types of consumers simultaneously. Hence, in all equilibria we observe price dispersion.

Three types of equilibria are considered. All types exist only for certain parameter values. These equilibria fully cover the parameter space. First, we derive an equilibrium where uninformed consumers are *a priori* indifferent which firm they go to first. This type of equilibrium exists when the share of informed consumers is approximately below 63%. Interestingly that is this share goes the gap between profits of different types of firms gets smaller and eventually a regular firm has higher profits. Second, we consider an equilibrium where uninformed consumers prefer to visit first a regular firm. Although it seems counterintuitive, since a chain firm is supposed to have more market power, this type of equilibrium naturally follows from the results of the previous type. Similar conclusion are drawn from it.

Since first two types of equilibria do not cover the entire parameter space, we consider a third type — a positive correlation device between first two types. This equilibrium exhibits non-linear dynamics and it exists for two separate sets of parameters.

Profits of both firms in all types of equilibria are uniformly higher in comparison to the benchmark Stahl (1989) model. It can be explained by the fact that a merger (the chain firm can be considered as two firms with the same management) leads to a lower competition and, hence, higher prices.

As a future agenda, it is necessary to perform comparative statics, asymptotic analysis, and welfare analysis. Profits are not the only criteria to judge how good the model is. Simulations for general N should also be performed. Graphs of equilibrium distributions are interesting to compare given different parameter values.

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A Detailed derivations

A.1 Distribution function of prices in (2)

$$
\begin{aligned}
F(p) &= 1 - \frac{\left[\nu_{NM}^{N-2}\frac{\tilde{p}}{p} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\frac{\tilde{p}-p}{p}\right]^{\frac{1}{N-2}} - \nu_{NM}}{1-\nu_{NM}} \\
&= \frac{1 - \left[\nu_{NM}^{N-2}\frac{\tilde{p}}{p} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\frac{\tilde{p}-p}{p}\right]^{\frac{1}{N-2}}}{1-\nu_{NM}} \\
&= \frac{1}{1-\nu_{NM}} - \frac{\left[\left(\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)\frac{\tilde{p}}{p} - \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right]^{\frac{1}{N-2}}}{1-\nu_{NM}} \\
&= \frac{1}{1-\nu_{NM}} - \left[\frac{a}{p} + b\right]^{\frac{1}{N-2}},
\end{aligned}
$$

where

$$
a = \frac{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}}\tilde{p}
$$

and

$$
b = -\frac{\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}}
$$

A.2 Lower bound of prices in (6)

$$
\begin{aligned}
&F(\underline{p}) = 0 = 1 - \frac{\left[\nu_{NM}^{N-2}\frac{\tilde{p}}{\underline{p}} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\frac{\tilde{p}-\underline{p}}{\underline{p}}\right]^{\frac{1}{N-2}} - \nu_{NM}}{1-\nu_{NM}} \\
&\Rightarrow 1 = \nu_{NM}^{N-2}\frac{\tilde{p}}{\underline{p}} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\frac{\tilde{p}-\underline{p}}{\underline{p}} \\
&\Rightarrow \left(\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)\frac{\tilde{p}}{\underline{p}} = 1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda} \\
&\Rightarrow \underline{p} = \frac{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}\tilde{p} < \tilde{p}, \text{ since } \nu_{NM}^{N-2} < 1
\end{aligned}
$$

A.3 Equilibrium condition in (7)

$$
\begin{aligned}
&\lambda\nu_{NM}^{N-2}\tilde{p} + \frac{(1-\mu)(1-\lambda)}{2}(\tilde{p}-\underline{p}) = \lambda\nu_M\nu_{NM}^{N-3}\tilde{p} + \mu(1-\lambda)(\tilde{p}-\underline{p}) \\
\Rightarrow&\lambda\nu_{NM}^{N-2}\tilde{p} + \frac{(1-\mu)(1-\lambda)}{2}\left(1 - \frac{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}\right)\tilde{p} =
\end{aligned}
$$

$$
\begin{aligned}
&= \lambda\nu_M\nu_{NM}^{N-3}\tilde{p} + \mu(1-\lambda)\left(1 - \frac{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}\right)\tilde{p} \\
\Rightarrow &\lambda\nu_{NM}^{N-2} + \frac{(1-\mu)(1-\lambda)(1-\nu_{NM}^{N-2})}{2\left(1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)} = \lambda\nu_M\nu_{NM}^{N-3} + \frac{\mu(1-\lambda)(1-\nu_{NM}^{N-2})}{1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}} \\
\Rightarrow &2\lambda\nu_{NM}^{N-2}\left(1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right) + (1-\mu)(1-\lambda)(1-\nu_{NM}^{N-2}) = \\
&= 2\lambda\nu_M\nu_{NM}^{N-3}\left(1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right) + 2\mu(1-\lambda)(1-\nu_{NM}^{N-2}) \\
\Rightarrow &2\lambda(\nu_{NM}^{N-2} - \nu_M\nu_{NM}^{N-3}) + (1-\mu)(1-\lambda)\nu_{NM}^{N-2} + (1-\mu)(1-\lambda)(1-\nu_{NM}^{N-2}) = \\
&= \nu_M\nu_{NM}^{N-3}(1-\mu)(1-\lambda) + 2\mu(1-\lambda)(1-\nu_{NM}^{N-2}) \\
\Rightarrow &2\lambda(\nu_{NM}^{N-2} - \nu_M\nu_{NM}^{N-3}) + (1-\mu)(1-\lambda)(1-\nu_M\nu_{NM}^{N-3}) = 2\mu(1-\lambda)(1-\nu_{NM}^{N-2}) \\
\Rightarrow &2\lambda(\nu_{NM}^{N-2} - \nu_M\nu_{NM}^{N-3}) + (1-\lambda)(1-\nu_M\nu_{NM}^{N-3}) = \mu(1-\lambda)\left[2(1-\nu_{NM}^{N-2}) + (1-\nu_M\nu_{NM}^{N-3})\right] \\
\Rightarrow &\mu(1-\lambda) = \frac{2\lambda(\nu_{NM}^{N-2} - \nu_M\nu_{NM}^{N-3}) + (1-\lambda)(1-\nu_M\nu_{NM}^{N-3})}{2(1-\nu_{NM}^{N-2}) + (1-\nu_M\nu_{NM}^{N-3})}
\end{aligned}
$$

A.4 Calculation of the integral in (12)

We will calculate the integral using integration by parts

$$
\begin{aligned}
\int_{\underline{p}}^{\tilde{p}} p\,dF(p) = \left|\begin{array}{cc} p = u & dF(p) = dv \\ dp = du & F(p) = v \end{array}\right| &= \left|uv - \int v\,du\right| \\
&= pF(p)\Big|_{p=\underline{p}}^{p=\tilde{p}} - \int_{\underline{p}}^{\tilde{p}} F(p)\,dp \\
&= \tilde{p} - \int_{\underline{p}}^{\tilde{p}} F(p)\,dp
\end{aligned}
$$

To evaluate $\int_{\underline{p}}^{\tilde{p}} F(p)\,dp$, we use Equation 2

$$
\begin{aligned}
\int_{\underline{p}}^{\tilde{p}} F(p)\,dp &= \int_{\underline{p}}^{\tilde{p}} \left(\frac{1}{1-\nu_{NM}} - \left[\frac{a}{p} + b\right]^{\frac{1}{N-2}}\right) dp \\
&= \int_{\underline{p}}^{\tilde{p}} \frac{dp}{1-\nu_{NM}} - \int_{\underline{p}}^{\tilde{p}} \left[\frac{a}{p} + b\right]^{\frac{1}{N-2}} dp
\end{aligned}
$$

$$= \frac{\tilde{p} - \underline{p}}{1 - \nu_{NM}} - \int_{\underline{p}}^{\tilde{p}} \left[\frac{a}{p} + b\right]^{\frac{1}{N-2}} dp$$

Since the integral $\int_{\underline{p}}^{\tilde{p}} \left[\frac{a}{p} + b\right]^{\frac{1}{N-2}} dp$ cannot be calculated analytically for general N, we use hypergeometric function ${}_2F_1(a, b, c, x)$ to express a closed-form solution

$$\int_{\underline{p}}^{\tilde{p}} \left[\frac{a}{p} + b\right]^{\frac{1}{N-2}} dp = \frac{N-2}{N-3} p \left(\frac{a}{p} + b\right)^{\frac{1}{N-2}} \left(\frac{bp}{a} + 1\right)^{-\frac{1}{N-2}} \times$$
$$\times {}_2F_1\left(1 - \frac{1}{N-2}, -\frac{1}{N-2}, 1 - \frac{1}{N-2}, -\frac{bp}{a}\right)\Bigg|_{p=\underline{p}}^{p=\tilde{p}} \qquad (62)$$

Before calculating values of the integral at the boundaries $p = \tilde{p}$ and $p = \underline{p}$, let us calculate some parts of it at these boundaries

$$\frac{a}{\tilde{p}} + b = \frac{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}} \frac{\tilde{p}}{\tilde{p}} - \frac{\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}} = \frac{\nu_{NM}^{N-2}}{(1-\nu_{NM})^{N-2}}$$
$$\frac{a}{\underline{p}} + b = \frac{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}} \frac{\tilde{p}}{\underline{p}} - \frac{\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}}$$
$$= \frac{1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}} - \frac{\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})^{N-2}} = \frac{1}{(1-\nu_{NM})^{N-2}}$$
$$\frac{b\tilde{p}}{a} = -\frac{\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}$$
$$\frac{b\underline{p}}{a} = -\frac{\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}$$
$$\frac{b\tilde{p}}{a} + 1 = \frac{\nu_{NM}^{N-2}}{\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}$$
$$\frac{b\underline{p}}{a} + 1 = \frac{1}{1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}$$
$$\left(\frac{a}{\tilde{p}} + b\right)^{\frac{1}{N-2}} \left(\frac{b\tilde{p}}{a} + 1\right)^{-\frac{1}{N-2}} = \frac{\nu_{NM}}{1-\nu_{NM}} \frac{\nu_{NM}^{-1}}{\left(\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)^{-\frac{1}{N-2}}} = \frac{\left(\nu_{NM}^{N-2} + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)^{\frac{1}{N-2}}}{1-\nu_{NM}}$$
$$\left(\frac{a}{\underline{p}} + b\right)^{\frac{1}{N-2}} \left(\frac{b\underline{p}}{a} + 1\right)^{-\frac{1}{N-2}} = \frac{\left(\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)^{\frac{1}{N-2}}}{1-\nu_{NM}} \left(1 + \frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)^{\frac{1}{N-2}}$$

Plugging in these values into Equation 62 as well as the expression for $\underline{p}$ from Equation 6, we get

$$\int_{\underline{p}}^{\tilde{p}} \left[\frac{a}{p}+b\right]^{\frac{1}{N-2}} dp = \frac{N-2}{N-3}\frac{\left(\nu_{NM}^{N-2}+\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)^{\frac{1}{N-2}}}{1-\nu_{NM}}\tilde{p}\times$$
$$\times\, {}_2F_1\left(1-\frac{1}{N-2},-\frac{1}{N-2},1-\frac{1}{N-2},\frac{\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{\nu_{NM}^{N-2}+\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}\right)-$$
$$-\frac{N-2}{N-3}\frac{\left(\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)^{\frac{1}{N-2}}}{1-\nu_{NM}}\left(1+\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)^{\frac{1}{N-2}}\frac{\nu_{NM}^{N-2}+\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{1+\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}\tilde{p}\times$$
$$\times\, {}_2F_1\left(1-\frac{1}{N-2},-\frac{1}{N-2},1-\frac{1}{N-2},\frac{\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{1+\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}\right)$$
$$= f(\nu_{NM},\mu;\lambda,N)\tilde{p}$$

Finally,

$$\begin{aligned}\int_{\underline{p}}^{\tilde{p}} p\, dF(p) &= \tilde{p} - \int_{\underline{p}}^{\tilde{p}} F(p)\, dp \\ &= \tilde{p} - \frac{\tilde{p}-\underline{p}}{1-\nu_{NM}} + \int_{\underline{p}}^{\tilde{p}} \left[\frac{a}{p}+b\right]^{\frac{1}{N-2}} dp \\ &= \frac{\underline{p}-\nu_{NM}\tilde{p}}{1-\nu_{NM}} + f(\nu_{NM},\mu;\lambda,N)\tilde{p} \\ &= \frac{\nu_{NM}^{N-2}-\nu_{NM}+\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}}{(1-\nu_{NM})\left(1+\frac{1-\mu}{2}\frac{1-\lambda}{\lambda}\right)}\tilde{p} + f(\nu_{NM},\mu;\lambda,N)\tilde{p} \\ &= g(\nu_{NM},\mu;\lambda,N)\tilde{p} \equiv g\tilde{p}\end{aligned}$$

A.5 Derivation of expression in (21)

$$[\frac{1}{2}(1+\nu_M)-\nu_{NM}]\tilde{p} = \nu_{NM}\frac{\nu_{NM}(1+\nu_M)-2\nu_M}{(1-\nu_M)(1-\nu_{NM})}s + [\frac{1}{2}(1+\nu_M)-\nu_{NM}]\left(\tilde{p}-\frac{1+\nu_M}{1-\nu_M}s\right)$$
$$\Rightarrow \nu_{NM}\frac{\nu_{NM}(1+\nu_M)-2\nu_M}{(1-\nu_M)(1-\nu_{NM})} - [\frac{1}{2}(1+\nu_M)-\nu_{NM}]\frac{1+\nu_M}{1-\nu_M} = 0$$
$$\Rightarrow \nu_{NM}^2(1+\nu_M) - 2\nu_M\nu_{NM} - [\frac{1}{2}(1+\nu_M)-\nu_{NM}](1+\nu_M)(1-\nu_{NM}) = 0$$
$$\Rightarrow -\frac{1}{2}(1+\nu_M)^2(1-\nu_{NM}) + \nu_{NM}(1+\nu_M) - 2\nu_M\nu_{NM} = 0$$
$$\Rightarrow -\frac{1}{2}(1+\nu_M)^2(1-\nu_{NM}) + \nu_{NM}(1-\nu_M) = 0$$

$$\Rightarrow \nu_{NM} = \frac{\frac{1}{2}(1+\nu_M)^2}{\frac{1}{2}(1+\nu_M)^2 + (1-\nu_M)}$$
$$\Rightarrow \nu_{NM} = \frac{(1+\nu_M)^2}{3+\nu_M^2}$$

A.6 Distribution function in (26)

$$(1-F_M(p))\left[(1-F_{NM}(p))(1-\nu_{NM})+\nu_{NM}\right]^{N-3} = \frac{1-\lambda}{\lambda(N-2)}\frac{\tilde{p}-p}{p}$$
$$\Rightarrow F_M(p) = 1 - \frac{\frac{1-\lambda}{\lambda(N-2)}\frac{\tilde{p}-p}{p}}{\left[(1-F_{NM}(p))(1-\nu_{NM})+\nu_{NM}\right]^{N-3}}$$
$$\Rightarrow F_M(p) = 1 - \frac{\frac{1-\lambda}{\lambda(N-2)}\frac{\tilde{p}-p}{p}}{\nu_{NM}^{N-3}\left(\frac{\tilde{p}}{p}\right)^{\frac{N-3}{N-2}}} = 1 - \frac{1-\lambda}{\lambda(N-2)\nu_{NM}^{N-3}}\left(\frac{\tilde{p}}{p}\right)^{\frac{N-2}{N-3}}\left(\frac{\tilde{p}}{p}-1\right)$$

A.7 Expression in (27)

$$F_M(\underline{p} = \nu_{NM}^{N-2}\tilde{p}) = 0 = 1 - \frac{1-\lambda}{\lambda(N-2)\nu_{NM}^{N-3}}\frac{\frac{1}{\nu_{NM}^{N-2}}-1}{\left(\frac{1}{\nu_{NM}^{N-2}}\right)^{\frac{N-3}{N-2}}}$$
$$\Rightarrow 1 = \frac{1-\lambda}{\lambda(N-2)}\frac{1-\nu_{NM}^{N-2}}{\nu_{NM}^{N-2}}$$
$$\Rightarrow \frac{\lambda(N-2)}{1-\lambda} = \frac{1-\nu_{NM}^{N-2}}{\nu_{NM}^{N-2}}$$
$$\Rightarrow \nu_{NM}^{N-2}[\lambda(N-2)+(1-\lambda)] = 1-\lambda$$
$$\Rightarrow \nu_{NM} = \sqrt[N-2]{\frac{1-\lambda}{\lambda(N-2)+(1-\lambda)}}$$

A.8 Expected price at the regular firm in (29)

$$\mathbb{E}_{NM}(p) = \nu_{NM}\tilde{p} + (1-\nu_{NM})\int_{\underline{p}}^{\tilde{p}} p\, dF_{NM}(p)$$
$$= \nu_{NM}\tilde{p} + (1-\nu_{NM})\int_{\underline{p}}^{\tilde{p}} pF'_{NM}(p)\, dp$$
$$= \nu_{NM}\tilde{p} + (1-\nu_{NM})\int_{\underline{p}}^{\tilde{p}} p\left[-\frac{\nu_{NM}}{1-\nu_{NM}}\frac{1}{N-2}\left(\frac{\tilde{p}}{p}\right)^{\frac{1}{N-2}-1}\tilde{p}\left(-\frac{1}{p^2}\right)\right] dp$$

$$
\begin{aligned}
&= \nu_{NM}\tilde{p} + \frac{\nu_{NM}}{N-2}\int_{\underline{p}}^{\tilde{p}} \left(\frac{\tilde{p}}{p}\right)^{\frac{1}{N-2}} dp \\
&= \nu_{NM}\tilde{p} + \frac{\nu_{NM}}{N-2}\tilde{p}^{\frac{1}{N-2}} \frac{p^{-\frac{1}{N-2}+1}}{-\frac{1}{N-2}+1}\Bigg|_{p=\underline{p}}^{p=\tilde{p}} \\
&= \nu_{NM}\tilde{p} + \frac{\nu_{NM}}{N-3}\left(\tilde{p} - \nu_{NM}^{N-3}\tilde{p}\right) \\
&= \frac{\nu_{NM}(N-2) - \nu_{NM}^{N-3}}{N-3}\tilde{p}
\end{aligned}
$$

A.9 Lower bound price in (37)

$$
\begin{aligned}
&\lambda G(N-2; p = \underline{p})\underline{p} + \frac{1-\mu}{2}(1-\lambda)\underline{p} + \frac{1-\mu}{2}(1-\lambda)\tilde{p} = [\lambda\hat{T}(N-2) + (1-\mu)(1-\lambda)]\tilde{p} \\
&\quad \Rightarrow \left(\lambda T(N-2) + \frac{1-\mu}{2}(1-\lambda)\right)\underline{p} = [\lambda\hat{T}(N-2) + \frac{1-\mu}{2}(1-\lambda)]\tilde{p} \\
&\quad \Rightarrow \underline{p} = \frac{\lambda\hat{T}(N-2) + \frac{1-\mu}{2}(1-\lambda)}{\lambda T(N-2) + \frac{1-\mu}{2}(1-\lambda)}\tilde{p} < \tilde{p}, \text{ since } T(N-2) > \hat{T}(N-2)
\end{aligned}
$$

A.10 Lower bound price in (42)

$$
\begin{aligned}
&\alpha\left\{\lambda G(N-3; p = \underline{p}) + \frac{1-\lambda}{N-2}\mu\right\}\underline{p} + (1-\alpha)G(N-3; p = \underline{p})\underline{p} = \\
&= \left\{\alpha\left[\lambda\nu_M\hat{T}(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)\hat{T}(N-3)\right\}\tilde{p} \\
&\Rightarrow \left\{\alpha\left[\lambda T(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)T(N-3)\right\}\underline{p} = \\
&= \left\{\alpha\left[\lambda\nu_M\hat{T}(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)\hat{T}(N-3)\right\}\tilde{p} \\
&\Rightarrow \underline{p} = \frac{\alpha\left[\lambda\nu_M\hat{T}(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)\hat{T}(N-3)}{\alpha\left[\lambda T(N-3) + \frac{1-\lambda}{N-2}\mu\right] + (1-\alpha)T(N-3)}\tilde{p} < \tilde{p}
\end{aligned}
$$

A.11 Lower bound price in (44)

$$
\begin{aligned}
&\left[\lambda(1-\alpha)\hat{G}(N-3; p = \underline{\hat{p}}) + \mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2}\right]\underline{\hat{p}} = \\
&= \left(\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\nu_{NM}^{N-3}\right)\hat{p} \\
&\Rightarrow \left[\mu\frac{1-\lambda}{N-2} + (1-\mu)\frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda\hat{T}(N-3)\right]\underline{\hat{p}} =
\end{aligned}
$$

$$= \left(\mu \frac{1-\lambda}{N-2} + (1-\mu) \frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda \nu_{NM}^{N-3} \right) \hat{p}$$

$$\Rightarrow \underline{\hat{p}} = \frac{\mu \frac{1-\lambda}{N-2} + (1-\mu) \frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda \nu_{NM}^{N-3}}{\mu \frac{1-\lambda}{N-2} + (1-\mu) \frac{(1-\alpha)(1-\lambda)}{N-2} + (1-\alpha)\lambda \hat{T}(N-3)} \hat{p} < \hat{p}$$