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The effect of rising longevity on optimal education investment and retirement age

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Abstract

This thesis investigates the effect of rising longevity on the individual's decision of optimal education, called Ben-Porath effect and the consequences on the optimal retirement age. Overlapping generations models with three periods: childhood, working age and old age, are suitable for modeling the human life cycle, since the transition of life cycles can be defined in line with individual decisions to optimize the lifetime utility. But also models with continuous and discrete flows of consumption and utility are investigated in this thesis. It will be shown that self fulfilling prophecies can occur in an economy that follows the Ben-Porath effect. The uncertainty of lifetime has a considerable impact on the dynamics of the Ben-Porath effect. Survival probabilities have strongly increased in the middle and old age during the last decades and lead to an increase in life expectancy. The Ben-Porath effect also depends on the change of age specific survival probabilities significantly. It can be shown as well, that the conditional probability to die at age t, given the survival until age t, called Hazard rate and conditions of living (like health and family circumstances) influence the Ben-Porath effect. Additionally, as the level of education affects the individual lifetime earnings and lifetime labor supply, it is generally necessary to investigate the effects of rising longevity on the retirement age at the same time.

Keywords: Longevity, optimal education, retirement age, uncertainty of lifetime, Hazard rate.

Contents

1 Motivation

Longevity, the duration of live, has seen a magnificent increase throughout the last decades. This circumstance can be causally linked with scientific advances in medical assistance. It has been a key point of demographic research already for many years. Increasing longevity affects societies in many different ways and can result in a significant change of the human life course. A longer life expectancy allows human beings to spend more time in different life cycle stages. In this thesis the effects of longevity on the optimal individuals' decisions to spend time or money for their education will be investigated and furthermore implications on the macroeconomic level will be studied. This is of great importance for academic research, policy and insurance industry, as longevity has been one of the biggest demographic developments during the last century and could lead to substantial changes in the way how lives are organized in the future.
Figure 1.1 illustrates the increasing life expectancy in Austria from 1970 until 2016 for men and women separately as well as the overall life expectancy throughout the population. In general, women have a higher life expectancy than men. It is often argued that women have a higher individual risk aversion factor and consequently live longer than men. It can be seen in Figure 1.1, that the overall life expectancy at birth has risen from 70 years in 1971 to nearly 82 years in 2016. Similar figures can be seen in other highly developed countries. Especially the survival probabilities at middle and old age have increased throughout the last decades, which will be crucial for the investigations in this thesis.

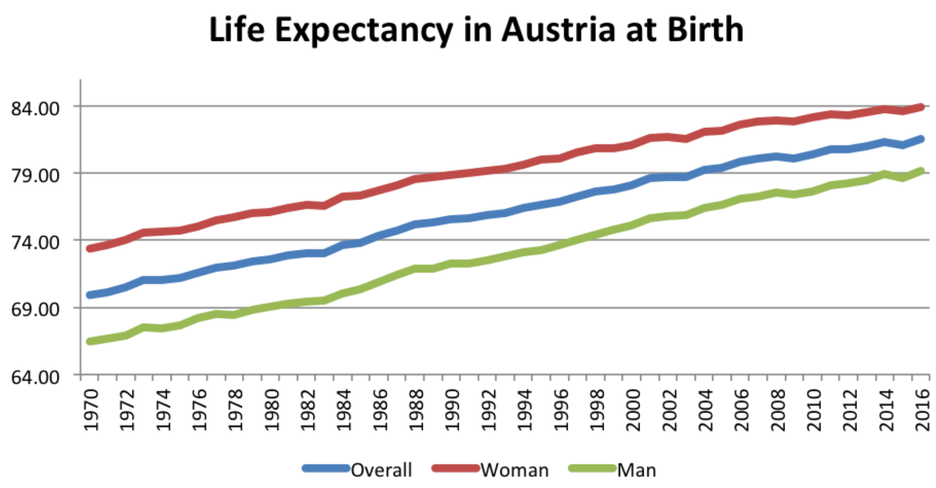


Figure 1.1: Own Visualization: Development of life expectancy in Austria. Source of data: *Statistik Austria: Lebenserwartung bei der Geburt 1970 bis 2016 nach Bundesländern und Geschlecht* (2017).

In order to study the effects of rising longevity on economic variables, the human life course is often divided into three periods: childhood, working age and old age. It is regularly assumed that the first period of life is used for education. Each individual thereby develops her or his own stock of human capital. In the second period of life this stock of human capital usually affects the wage rate positively. The old age is mostly assumed to be the retirement period of the life cycle. Overlapping Generation Models (see Appendix) are very suitable for modeling this structure.
From an economic point of view it is interesting to investigate, whether increasing life expectancy results in higher human capital accumulation. Ben-Porath (1967) first investigated the production of human capital considering life cycle earnings. The idea that rising longevity affects the optimal education choice of individuals positively, what consequently leads to increased levels of human capital, is nowadays called Ben-Porath effect. This argument can be formulated in the following way: If individuals are higher educated, they earn more money in their working period. Consequently they can afford more consumption throughout their whole life. Due to a higher life expectancy the rate of return on investments in education increases, since individuals have more time to use their stock of human capital to work.
Although a higher human capital intuitively could increase economic growth, this effect is not always proven empirically. Other effects of rising longevity, for example higher or lower savings, do have an impact on economic growth as well. Bloom, Canning, and Sevilla (2004) show that a 5-year increase in life expectancy generates a 21 % rise of the economic growth rate. Additionally one can argue that the positive Ben-Porath effect can only occur as long as individuals extend their working period as a result of increased longevity, since only in this case the rate of return on investments in education increases significantly as well.
Taking a closer look at longevity in our society, studies show, that the probabilities to survive another year at certain ages have increased more than in other stages of the live course. Throughout the last decades the probabilities to survive have increased mainly in the mid-life and old age periods, whereas they have stayed almost constant in the childhood (after the decrease of infant mortality). This process if often called rectangularization of the survival distribution function. Cervellati and Sunde (2013) show that the effects of increased longevity crucially depend on age specific survival probabilities and on the amount of life-time labor supply. The authors of this paper show, that age specific survival probabilities (in the working age) affect the individual decision on the optimal education period. On the one hand the rise in survival conditions affect the expected rate of return of investment in education, on the other hand the opportunity costs of staying in school an additional period rise as well.
Investigating the effect of longevity on human capital accumulation, it is important to consider its repercussion on life expectancy as well. One can argue, that better educated people have a more healthful way of living, therefore have an increased life expectancy and thus again are better educated. Figure 1.2 shows the effects, that will be investigated in this thesis. The effect of longevity on education (Ben-Porath effect) will be the key part of this thesis. Different impacts of the amount of education will be discussed. It is often assumed that the amount of education results in an individual level of human

capital, that can be aggregated in a closed economy to its total amount of human capital. As discussed above it seems plausible to assume that the total amount of human capital affects the realized longevity.

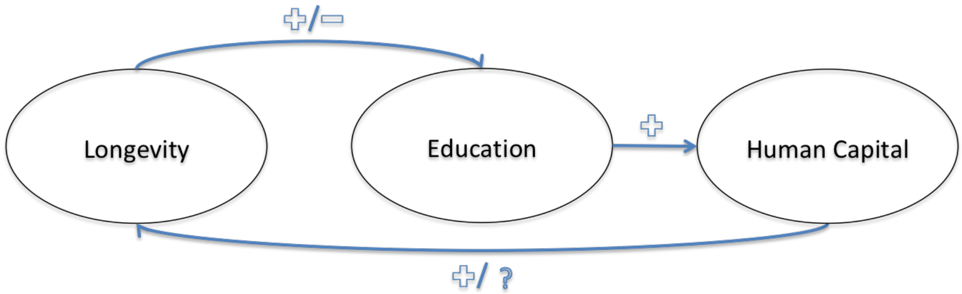
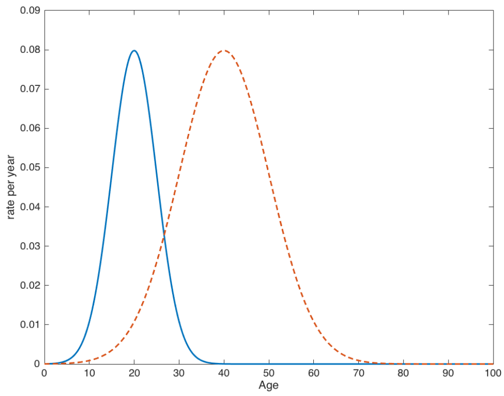


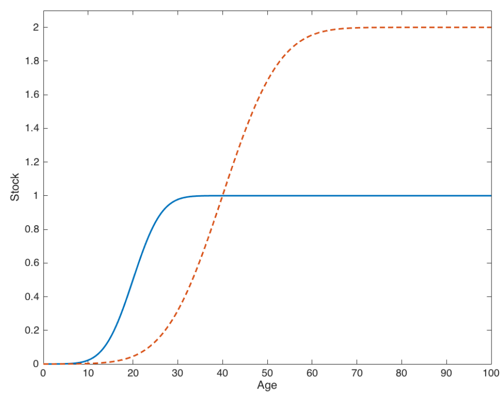
Figure 1.2: Longevity, Education and Human Capital.

Concerning the studies on an optimal education choice, the decay of human capital is of vital importance. In almost all overlapping generations models with human capital the first period of life of an agent is used to build up human capital. This stock of knowledge is crucial for the amount of wages in the second period of life. In the third period of life, the human capital, that was built up in the first period, is already partially decayed. This decay is considerable for the rate of return of investments in human capital, if the working period is not fixed (endogenous retirement age).
Additionally the total working period or the total labor supply of individuals will be considered. An increase of the length of life does not automatically lead to an increase of the working period, as studies like Hazan (2009) show. Therefore the optimal retirement age has to be taken into consideration as well in order to investigate the Ben-Porath effect. In fact the total human life course could change as a result of rising longevity.
In Lee and Goldstein (2003), the authors describe the effects of rising longevity on the human life course. They define the concept of proportional rescaling: As a result of rising life expectancy, every life cycle stage and transition of the life cycle expands in proportion to the increased life expectancy. They point out that proportional rescaling might be realized in some aspects of life, but it clearly does not hold for all aspects. However it seems a natural way of rescaling the life cycle as a result of increased life expectancy and can be seen as a benchmark option.
Lee and Goldstein distinguish between strong and weak proportional rescaling. In the strong form of proportional rescaling, rising longevity results in an adaption of the distribution of the timing of events. This means for example, that not only the mean age of giving birth increases, but also the spread around the mean age of giving birth increases proportionally to the increase of life expectancy. On the contrary, if rising longevity is only accompanied by an increase of the mean age of timings, but not by an increase of the spread of timings, the rescaling is called to be weak.
Concerning strong proportional rescaling of the life cycle, it has to be distinguished between stock and flow variables. The consumption per year, wage rate, interest rate, knowledge acquisition are flow variables all measured per unit of time. For example human capital and total lifetime utility are stock variables on the other side. Strong

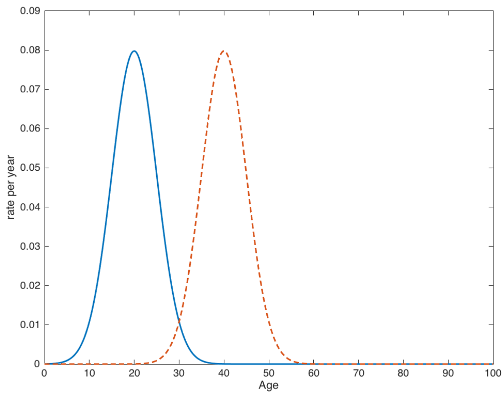
proportional rescaling can either be stock-constrained or flow-constrained. Figure 1.3 visualizes the effects of proportional rescaling on stock and flow variables, where Figures 1.3 (a) and (b) show the effects of strong proportional rescaling and Figures 1.3 (c) and (d) show the effects of weak proportional rescaling. These Figures visualize a hypothetical distribution of the age of women, when they give birth. Under strong proportional rescaling the mean age and the spread around the mean age of giving birth increase (double in this example) as a result of increased (doubled in this example) life expectancy. This leads to a doubled fertility in total. Under weak proportional rescaling only the mean age of giving birth increases (doubles in this example) as a result of increased (doubled in this example) life expectancy, but the spread stays at the same level. The total fertility stays at the same level under weak proportional rescaling.



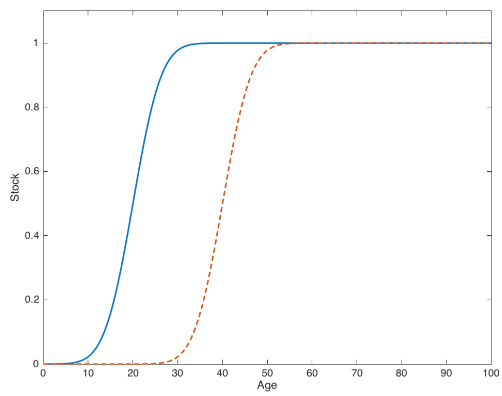
(a) Strong proportional rescaling: rate change



(b) Strong proportional rescaling: stock development change



(c) Wtrong proportional rescaling: rate change

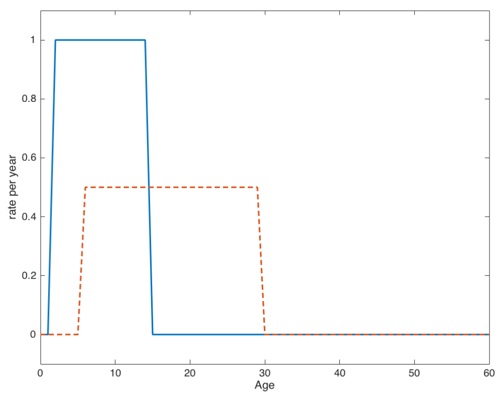


(d) Weak proportional rescaling: stock development change

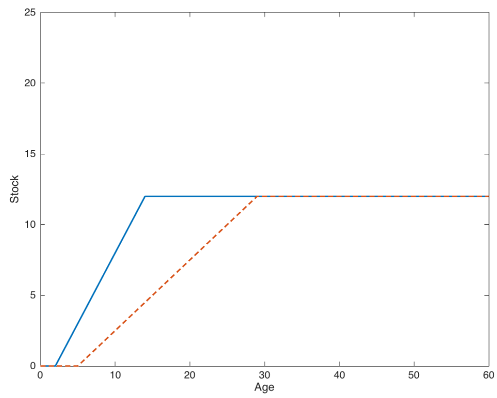
Figure 1.3: Own Visualization: Strong and weak proportional rescaling of the life cycle. Change in flow and stock variables. Source: Lee and Goldstein (2003)

Stock-constrained strong proportional rescaling means that a certain stock has the same level after the rescaling process. This means for example that the total lifetime earnings

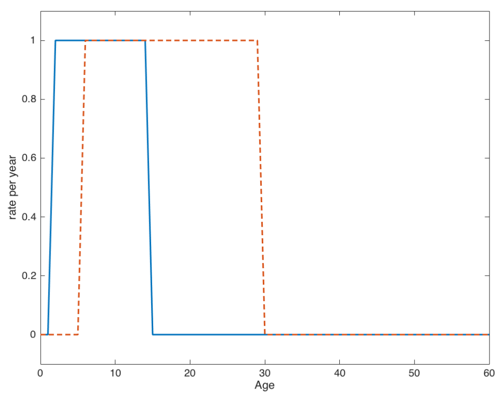
stay at the same level, although the life cycle stage of working has increased in proportion to the increased life expectancy. However, this implies that the wage rate measured in the same unit of time has decreased. In fact, stock-constrained strong proportional rescaling of the life cycle always goes in hand with a decrease of flow variables. Figures 1.4 (a) and (b) illustrate stock-constraint strong proportional rescaling.
Flow-constrained strong proportional rescaling on the other hand means that the flow variables have the same value after the rescaling process. For example the wage rate stays at the same value. But as the life cycle stage of working has increased in proportion to the increased life expectancy, the stock of total lifetime earnings increase as a result of flow-constrained strong proportional rescaling. Figures 1.4 (c) and (d) illustrate flow-constraint strong proportional rescaling.



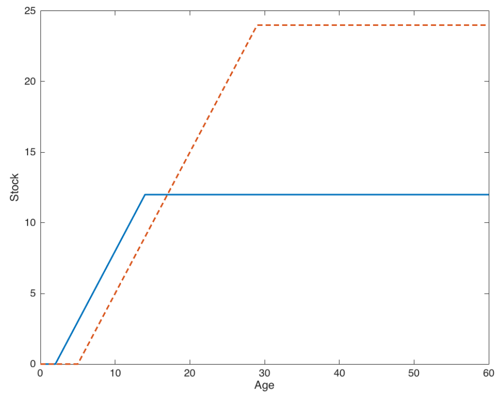
(a) rate change - proportional rescaling stock constrained



(b) stock change - proportional rescaling stock constrained



(c) rate change - proportional rescaling flow constrained



(d) stock change - proportional rescaling flow constrained

Figure 1.4: Own Visualization: Change in flow and stock variables under proportional rescaling. For example the rate of education and the level of the resulting stock of human capital (sum of yearly education rates) could follow this rescaling process. Source: Lee and Goldstein (2003)

In contrast to animals, where the life cycle can easily be divided into the pre maturity and maturity period, the transition of life cycles can not be defined in this simple way concerning the human life cycle as a result of a far more complex social behavior. Events and life cycles like schooling, moving out, marriage, child bearing can all be seen as a transition of the life cycle. Hence biological constraints like menarche and menopause as well as institutional constraints on schooling or retirement age have also be taken into consideration. The authors point out that while it can be observed that some social aspects of life are being delayed (such as childbearing and marriage), others seem to advance to younger ages (for example the beginning of education has shifted to younger ages from 1964 to 1998, following Lee and Goldstein (2003)). Additionally some aspects, that can be interpreted as transition of the life cycle are nowadays handled simultaneously, which makes it even more complicated to rescale the life cycle.

It might seem natural that, as life expectancy rises and survival probabilities, especially in the old age period, increase, the retirement age should rise as well. Historically seen, Lee and Goldstein (2003) point out that a long term trend of an earlier retirement age can be observed, although the decrease of the retirement age has fallen throughout the last decades. The authors mention that public and private pension systems have made it easier to retire early (or at the same age) even though the life expectancy has increased. Also the importance of leisure as a luxury good plays an important role.

Chapter 2 is based on Nishimura, Pestieau, and Ponthiere (2015) and will introduce a three period OLG model to examine the Ben-Porath effect under exogenous longevity and certainty about lifetime. This means that the lifetime is exogenously constant and that there is no risk for individuals to die earlier. Chapter 3 also considers a three period OLG model in order to show that the Ben-Porath effect can lead to self fulfilling prophecies. This theory is based on Cipriani and Makris (2006). In Chapters 4 and 5 the uncertainty of living will be the central topic of investigation. In Chapter 4, based on Cervellati and Sunde (2013), a discrete model with survival probabilities will be introduced and it will be investigated, how age-specific changes in survival probabilities have an impact on the Ben-Porath effect. In this thesis there is also a quantitative investigation of this model using Austrian specific survival condition in a MATLAB (2015) implementation. In Chapter 5, the theory of Sheshinski (2009) is mainly based on survival distribution functions. The three period OLG model of Nishimura, Pestieau, and Ponthiere (2015) in Chapter 6 uses endogenous longevity in order to study the Ben-Porath effect. Therefore it will be assumed that longevity depends on education. In this Chapter also the social optimum of the economy will be investigated. Chapter 7 summarizes the results of all Chapters and gives an overview on the most important issues concerning the effect of rising longevity on optimal education and retirement age.

2 Optimal Education Choice with Exogenous Longevity

In this Chapter the overlapping generations model of Nishimura, Pestieau, and Ponthiere (2015) with three periods will be presented. The purpose of this macroeconomic model is to reexamine the effect of longevity on the optimal education choice of individuals, called Ben Porath-effect, that was investigated in Ben-Porath (1967) for the first time. Therefore the model is microeconomic founded. The investigations will first be based on exogenous longevity. This means that lifetime is exogenously fixed and an increase in this exogenous parameter will be studied.

2.1 Assumptions

The lives of agents in this model are divided into three periods: childhood, working age and old age. The length of the first and second period is fixed. If the lifetime of individuals rises, only the length of the old age period increases in this model. In order to extend their income, though, individuals can decide to spend some time for old age labor in the retirement age. In this first period of life, young individuals borrow money, which they spend for education. There is no possibility to work in the first period, as well as there is no possibility for education except in this period. There is no uncertainty of life-length in this model, which means that survival functions are rectangular.
The population of every generation N_t is assumed to stay constant, meaning that there are exactly as many young individuals born every period, as old people are dying.

$$N_t = N_{t+1} = N \quad \forall t$$

The production of the economy Y_t depends on the capital stock K_t and on the stock of effective labor L_t. The production function $F(K_t, L_t)$ is assumed to have constant returns to scale, meaning that $aF(K_t, L_t) = F(aK_t, aL_t)$.

$$Y_t = F(K_t, L_t)$$

The factors of production, labor supply and capital, are rewarded by their marginal productivities, the wage rate and the interest rate.

$$w_t = F_L(K_t, L_t)$$
$$R_t = F_K(K_t, L_t)$$

Capital per effective working unit k_t is defined as $k_t = \frac{K_t}{L_t}$. As a result of constant returns to scale of the production function, $F(K_t, L_t) = L_t F(\frac{K_t}{L_t}., 1) =: L_t f(k_t)$ and therefore factor rewards can be written in the following way.

$$w_t = F_L(K_t, L_t) = \frac{\partial}{\partial L} L_t f(k_t) = f(k_t) - L_t f'(k_t) K_t \frac{1}{L_t^2} = f(k_t) - k_t f'(k_t)$$
$$R_t = F_K(K_t, L_t) = \frac{\partial}{\partial K} L_t f(k_t) = f'(k_t) \frac{1}{L_t} L_t = f'(k_t)$$

Some neoclassical assumptions for f are made: First, it is assumed that the marginal productivity of capital per effective working unit tends towards infinity, as capital per effective working unit tends towards zero. Secondly, it is assumed that the marginal productivity of capital per effective working unit tends towards zero, as capital per effective working unit tends towards infinity. These assumptions will be necessary for an interior optimal level of education and capital in the utility maximization problem.

- $\lim_{k \to 0} f'(k) = \infty$
- $\lim_{k \to \infty} f'(k) = 0$

Effective labor consists of the population size of the working generation N_t multiplied with the stock of human capital h_t and the fraction of the working population of the old generation N_{t-1} multiplied with the human capital of the previous period h_{t-1}. The variable $0 < z_t < 1$ represents the part of the old age period, which an individual uses to work. It also can be interpreted as retirement age, given a specific length of life. Additionally the decay of human capital throughout one period α has to be taken into consideration. $\alpha = 1$ means that there is no decay of human capital, whereas $\alpha = 0$ means that human capital completely decays throughout one period of life.

$$L_t = h_t N_t + z_t \alpha h_{t-1} N_{t-1}$$

As a result of CRS (constant returns to scale) of $F(K_t, L_t)$ and a constant population of N, the production function per capita $\tilde{y}_t = \frac{Y_t}{N}$ can be written using capital per capita $\tilde{k}_t = \frac{K_t}{N}$ as follows. (Note that the difference between $\tilde{k}_t$ and k_t is, that $\tilde{k}_t$ describes the relative stock of capital to a whole generation that is alive, whereas k_t describes the stock of capital relatively to the amount of labor force.)

$$\tilde{y}_t = \frac{Y_t}{N} = F(\tilde{k}_t, h_t + z_t \alpha h_{t-1})$$

The depreciation rate of physical capital δ is assumed to be 1, meaning that physical capital completely decays in one period. In this model, the working generation saves money s_t for consumption in their old age period Ns_t and the young individuals borrow money for their education Ne_t. Consequently the capital-market clearing condition is

$$K_{t+1} + Ne_t = Ns_t \ .$$

In order to secure positive physical capital $s_t > e_t$ is assumed. Using the definition of effective labor, k_{t+1} can be written as follows.

$$\begin{aligned} k_{t+1} =& \frac{K_{t+1}}{Nh(e_t) + Nz_{t+1}\alpha h(e_{t-1})} \\ =& \frac{N(s_t - e_t)}{N\left(h(e_t) + z_{t+1}\alpha h(e_{t-1})\right)} \\ =& \frac{s_t - e_t}{h(e_t) + z_{t+1}\alpha h(e_{t-1})} \end{aligned} \tag{2.1}$$

The stock of human capital h_t (of people in the working period) depends on the amount of education (in their childhood) $h_t = h(e_{t-1})$. Without any education, the minimal individual stock of human capital is assumed to be $h(0) = 1$. Education affects the stock of human capital strictly positively but the positive effect decreases with the amount of education. At a level of no education, its marginal increase affects the stock of education infinitely large whereas as the level of education tends towards infinity, its marginal increase does not affect the stock of human capital any longer.

- $h(0) = 1$
- $h'(e_{t-1}) > 0$
- $h''(e_{t-1}) < 0$
- $\lim_{e_{t-1}\to 0} h'(e_{t-1}) = \infty$
- $\lim_{e_{t-1}\to \infty} h'(e_{t-1}) = 0$

For example the isoelastic function $h(e_{t-1}) = \frac{e_{t-1}^{1-\phi}}{1-\phi} + 1$ fulfills these assumptions. See Chapter 8 Appendix for the definition and properties of the isoelastic or CRRA (constant relative risk aversion) function. Figure 2.1 illustrates the values of isoelastic functions $h(e_{t-1})$ with different values of ϕ. The function is strictly increasing for $e_{t-1} > 0$. The higher the value of e_{t-1}, the lower the marginal gain of human capital $h(e_{t-1})$ due to an additonal increase of education e_{t-1} (for $\phi > 0$).

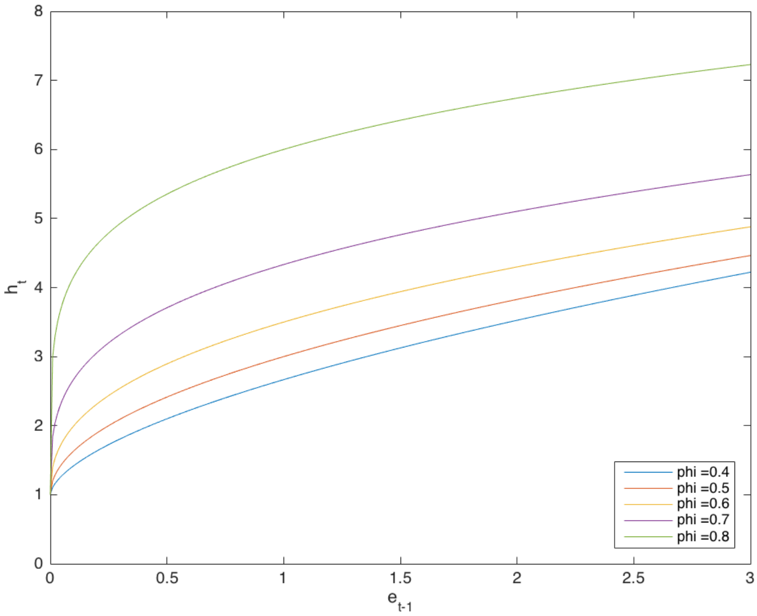


Figure 2.1: Accumulation of human capital for an isoelastic function.

In this model, utility functions are assumed in order to model the individual's preferences. Utility can be gained from consumption in the second and third period of life. The amount of education in the first period of life does neither provide direct utility nor disutility. As mentioned above, individuals can partly work in the third period of life. This old age labor is assumed to create disutility. Hence the lifetime utility of an individual U_t is modeled as

$$U_t = u(c_t) + l_{t+1}u(d_{t+1})$$

The consumption during the working period is given by c_t. Individuals in the old age consume d_{t+1}, that can be regarded as welfare during the third period of life. d_{t+1} is affected by material resources $\tilde{d}_{t+1}$, disutility of old age work $v(z_{t+1}, l_{t+1})$ and life-length l_{t+1}. Clearly material resources increase the welfare whereas old age labor has a negative effect on welfare, relatively to the length of the third period.

$$d_{t+1} = \frac{\tilde{d}_{t+1} - v(z_{t+1}, l_{t+1})}{l_{t+1}}$$

Some assumptions on the disutility function of old age labor are made: First, this disutility rises in the time spent working in the third period of life. Second, it rises stronger, the higher the level of the time spent working. And third, the longer the old age, the less disutility emerges from working in this period. Thus the assumptions on $v(z_{t+1}, l_{t+1})$ can be written as following.

- $v_z(z_{t+1}, l_{t+1}) > 0$
- $v_{zz}(z_{t+1}, l_{t+1}) > 0$
- $v_l(z_{t+1}, l_{t+1}) < 0$

In order to simplify the investigations Nishimura, Pestieau, and Ponthiere (2015) use a specific disutility function for old age labor with the above mentioned properties.

$$v(z_{t+1}, l_{t+1}) = \frac{(z_{t+1})^2}{2\gamma l_{t+1}} \qquad , \gamma > 0 \tag{2.2}$$

Hence the derivations of this function can be calculated explicitly.

- $v_z(z_{t+1}, l_{t+1}) = \frac{z_{t+1}}{\gamma l_{t+1}}$
- $v_l(z_{t+1}, l_{t+1}) = \frac{-(z_{t+1})^2 2\gamma}{(2\gamma l_{t+1})^2} = -\frac{(z_{t+1})^2}{2\gamma (l_{t+1})^2}$
- $v_{lz}(z_{t+1}, l_{t+1}) = -\frac{z_{t+1}}{\gamma (l_{t+1})^2}$
- $v_{ll}(z_{t+1}, l_{t+1}) = \frac{(z_{t+1})^2 16\gamma^3 l_{t+1}}{(2\gamma l_{t+1})^4} = \frac{(z_{t+1})^2}{\gamma l_{t+1}}$

It is assumed that each agent spends all its earnings. Hence, the budget constraint of an individual in the working period can be written as

$$c_t = w_t h(e_{t-1}) - e_{t-1} R_t - s_t \ . \tag{2.3}$$

The consumption in the working period c_t is equal to the income $w_t h(e_{t-1})$ minus the costs of education of the previous period $e_{t-1}R_t$ minus the savings for the old age s_t. In the third period, the cohorts can spend their savings as well as additional old age income $z_{t+1}\alpha w_{t+1} h(e_{t-1})$, which leads to the budget constraint in this period.

$$\tilde{d}_{t+1} = z_{t+1}\alpha w_{t+1} h(e_{t-1}) + R_{t+1} s_t \tag{2.4}$$

In the following section, the utility maximization of individuals will be studied. Under the above discussed assumption, the optimal values of consumption, old age welfare, education and old age labor at time t will be calculated. These values are called temporary equilibrium. An economy is called to be in a steady state, if the decision variables do not change in time. The Ben-Porath effect will be investigated based on economy in a steady state. First, all investigations are made with exogenous longevity. In Chapter 6 of this thesis, longevity will depend on the level of education.

2.2 Utility Maximization

Individuals maximize their lifetime utility at time t by choosing the amount of savings s_t, the amount of education e_{t-1} and the amount of old age labor z_{t+1} with the assumption of perfect foresight (they expect R^e_{t+1} and w^e_{t+1}). Let longevity in the third period of life be $l_{t+1} = l$. Regarding the budget constraints (2.3) and (2.4), the substituted and consequently unrestricted optimization problem of every individual can be written as

$$\max_{e_{t-1}, s_t, z_{t+1}} \quad u[w_t h(e_{t-1}) - e_{t-1} R_t - s_t] + lu \left[\frac{z_{t+1} \alpha w^e_{t+1} h(e_{t-1}) - v(z_{t+1}, l) + R^e_{t+1} s_t}{l} \right] \tag{2.5}$$

Differentiation with respect to s_t, z_{t+1} and e_{t-1} leads to the first-order conditions (FOCs) for optimality.

$$\begin{aligned} u'(c_t) &= R^e_{t+1} u'(d_{t+1}) \\ \alpha w^e_{t+1} h(e_{t-1}) &= v_z(z_{t+1}, l) \\ u'(d_{t+1}) \left[z_{t+1} \alpha w^e_{t+1} h'(e_{t-1}) \right] &= u'(c_t)[R_t - w_t h'(e_{t-1})] \end{aligned}$$

Using the special form of the disutility of old age labor function (2.2) and the FOC with respect to s_t, the FOCs can be rearranged to the following forms.

$$u'(c_t) = R^e_{t+1} u'(d_{t+1}) \tag{2.6}$$
$$z_{t+1} = \alpha w^e_{t+1} h(e_{t-1}) \gamma l \tag{2.7}$$
$$R^e_{t+1} R_t = R^e_{t+1} w_t h'(e_{t-1}) + \alpha^2 (w^e_{t+1})^2 h(e_{t-1}) \gamma l h'(e_{t-1}) \tag{2.8}$$

Equation (2.6) is the Euler equation for this problem. It states that the marginal utility of consumption in period t has to equal to the discounted marginal utility of consumption in period $t+1$, taking into consideration the expected interest rate R^e_{t+1}. The second FOC (2.7) states, that the level of income of old age labor has to equal the marginal disutility of old age labor in an optimum. The last FOC (2.8) points out, that the marginal welfare gains from education (RHS) have to equal the marginal costs (LHS) of education. The first term of the RHS in equation (2.8) is the marginal welfare gain from education from the second period of life, the second term the welfare gain arising in the third period of life. The temporary equilibrium of the utilization maximization with exogenous longevity can be summarized as following.

$$\begin{aligned} c_t &= w_t h(e_{t-1}) - e_{t-1} R_t - s_t \\ d_t &= \frac{z_{t+1} \alpha w^e_{t+1} h(e_{t-1}) - v(z_{t+1}, l) + R^e_{t+1} s_t}{l} \\ u'(c_t) &= R^e_{t+1} u'(d_{t+1}) \\ z_{t+1} &= \alpha w^e_{t+1} h(e_{t+1}) \gamma l \\ R^e_{t+1} R_t &= R^e_{t+1} w_t h'(e_{t-1}) + \alpha^2 (w^e_{t+1})^2 h(e_{t-1}) \gamma l h'(e_{t-1}) \\ L_t &= h_t(e_{t-1}) N + z_t \alpha h_{t-1}(e_{t-2}) N \\ K_t &= N(s_{t-1} - e_{t-1}) \end{aligned}$$

$$w_t = F_L(K_t, L_t)$$
$$R_t = F_K(K_t, L_t)$$

It could be the case, that $e_{t-1} = 0$ or $e_{t-1} = \infty$ are optimal decisions in sense of welfare maximization. However, the above made assumption guarantee an interior optimal education level. For $e_{t-1} \to 0$

$$\lim_{e_{t-1}\to 0} R^e_{t+1} w_t h'(e_{t-1}) + \alpha^2 (w^e_{t+1})^2 h(e_{t-1}) \gamma l h'(e_{t-1}) = \infty > R^e_{t+1} R_t$$

since $\lim_{e_{t-1}\to 0} h'(e_{t-1}) = \infty$. Hence, at the level of zero education, the marginal welfare gain of education is greater than the marginal costs of education. On the other hand $\lim_{e_{t-1}\to \infty} h'(e_{t-1}) = 0$, so at an infinite level of education, marginal cost of education are greater than marginal gains of additional education.

$$\lim_{e_{t-1}\to \infty} R^e_{t+1} w_t h'(e_{t-1}) + \alpha^2 (w^e_{t+1})^2 h(e_{t-1}) \gamma l h'(e_{t-1}) = 0 < R^e_{t+1} R_t$$

This means that $e_{t-1} = 0$ and $e_{t-1} = \infty$ cannot be optimal. Consequently there exists an interior optimal level of education in the individual's utility maximization problem. In order to specify conditions for uniqueness of the level of optimal education, the FOC with respect to e_{t-1} (2.8) will be differentiated again.

$$\begin{aligned}
&\frac{\partial}{\partial e_{t-1}} \left(R^e_{t+1} w_t h'(e_{t-1}) + \alpha^2 (w^e_{t+1})^2 h(e_{t-1}) \gamma l h'(e_{t-1}) \right) \\
=&\frac{\partial}{\partial e_{t-1}} \left(h'(e_{t-1}) [R^e_{t+1} w_t + \alpha^2 (w^e_{t+1})^2 h(e_{t-1}) \gamma l] \right) \\
=&h''(e_{t-1}) [R^e_{t+1} w_t + \alpha^2 (w^e_{t+1})^2 h(e_{t-1}) \gamma l] + [h'(e_{t-1})]^2 \alpha^2 (w^e_{t-1})^2 \gamma l \\
=&\left[\alpha^2 (w^e_{t+1})^2 \gamma l\right] \left[h''(e_{t-1}) h(e_{t-1}) + [h'(e_{t-1})]^2\right] + h''(e_{t-1}) R^e_{t+1} w_t
\end{aligned} \tag{2.9}$$

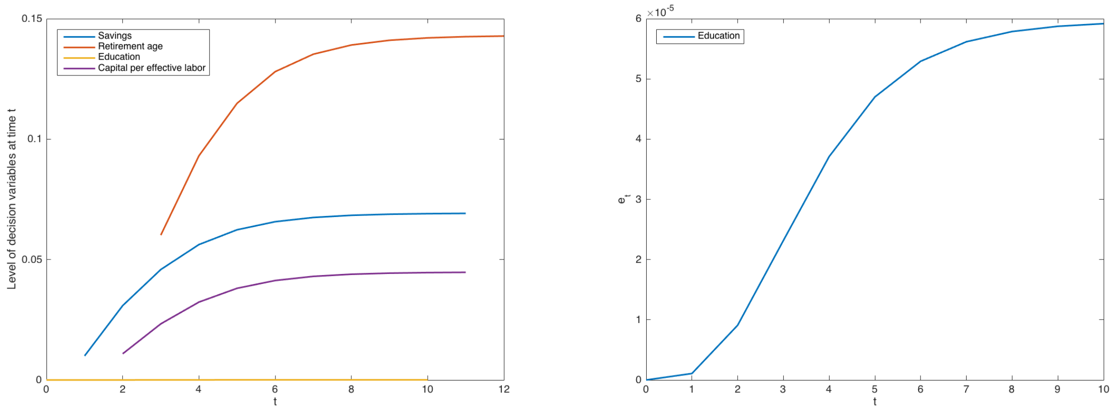
Clearly $\alpha^2 (w^e_{t+1})^2 \gamma l$ is positive, as $h''(e_{t-1}) R^e_{t-1} w_t$ is negative. For $[h''(e_{t-1}) h(e_{t-1}) + [h'(e_{t-1})]^2] < 0 \Leftrightarrow |h''(e_{t-1})| h(e_{t-1}) > [h'(e_{t-1})]^2]$, the RHS of (2.9) is negative. This means that the RHS of equation (2.8) is strictly monotonically decreasing in e_{t-1}. Hence, the optimal interior level of education has to be unique. Consequently the optimal values for old age labor and savings in the temporary equilibrium are unique as well.
For example the isoelastic function $h(e_{t-1}) = \frac{e^{1-\phi}_{t-1}}{1-\phi} + 1$ fulfills this criteria for $\phi \in (0.5, 1)$.

$$\begin{aligned}
&\frac{|-\phi e^{-\phi-1}_{t-1}|}{e^{-\phi}_{t-1}} > \frac{e^{-\phi}_{t-1}}{\frac{e^{1-\phi}_{t-1}}{1-\phi} + 1} \\
\Leftrightarrow \qquad &\phi e^{-\phi-1}_{t-1} \left(\frac{e^{1-\phi}_{t-1}}{1-\phi} + 1 \right) > e^{-2\phi}_{t-1}
\end{aligned}$$

$$\Leftrightarrow \qquad \phi e_{t-1}^{\phi-1} > 1 - \frac{\phi}{1-\phi} = \frac{1-2\phi}{1-\phi}$$

$$\Leftrightarrow \qquad e_{t-1}^{\phi-1} > \left[\frac{1-2\phi}{\phi(1-\phi)}\right]$$

For a numerical example, a logarithmic utility function and this isoelastic human capital function with $\phi = 0.75$ is used. The production side of the economy is modeled using a Cobb-Douglas production function $F(K, L) = K^a \cdot L^b$ with $a = b = 0.5$. Parameters $\gamma = 1, l = 1$ are set. Starting optimization at time $t = 0$, it is not possible to implement the assumption of perfect foresight, since future factor rewards do depend on the future capital stock (per effective working units), that itself is determined by decision variable of future optimization problems. It is therefore assumed that individuals expect the capital per effective labor to stay constant $k_{t+1}^e = k_t$ and following $R_{t+1}^e = R_t$ and $w_{t+1}^e = w_t$. With initial values for education $e_0 = 0$ and $s_1 = 0.01$, the time paths are visualized in Figure 2.2. This illustration shows that the economy develops in direction of a steady state.



(a) Time series of decision variables and capital per effective labor. Note that the levels of education increase as well, as visualized in Figure 2.2 (b).

(b) Time series of education. The absolute levels of education are far below the levels fo savings in every period. This is a result of the assuptions on the functional forms especially on $h(e_{t-1})$. However the goal of this implementation was to show the developpment towards a steady state.

Figure 2.2: Own Visualization: Time series of quantitative investigation of Nishimura, Pestieau, and Ponthiere (2015) under exogenous longevity.

2.3 Steady State

In this section, conditions for the existence of a steady state will be presented. In this model a steady state is characterized by

$$s_t = s_{t+1} \quad \forall t \qquad\qquad e_t = e_{t+1} \quad \forall t \qquad\qquad z_t = z_{t+1} \quad \forall t$$

This consequently means $c_t = c_{t+1}$ and $d_t = d_{t+1} \ \ \forall t$ as well. From equation (2.6) one can generally implicitly express the level of savings as function of current and expected factor rewards.

$$s_t \equiv s(w_t, R_t, w^e_{t+1}, R^e_{t+1}, e_{t-1})$$

In case of a unique interior optimal solution for the level of education, this level of e_{t-1} can be written as function of the factor rewards only (Regarding equation (2.8)).

$$e_{t-1} \equiv e(w_t, R_t, w^e_{t+1}, R^e_{t+1})$$

Consequently the savings can be written as function of the factor rewards only as well.

$$\begin{aligned} s_t &\equiv s(w_t, R_t, w^e_{t+1}, R^e_{t+1}, e(w_t, R_t, w^e_{t+1}, R^e_{t+1})) \\ &\equiv S(w_t, R_t, w^e_{t+1}, R^e_{t+1}) \end{aligned}$$

Taking into consideration the development of physical capital (2.1), a two dimensional state space in capital and education can be expressed by

$$k_{t+1} = \frac{S(w(k_t), R(k_t), w(k^e_{t+1}), R(k^e_{t+1})) - e_t}{h(e_{t-1}) + \alpha^2 w(k_t)[h(e_{t-2})]^2 \gamma l}$$

$$e_{t-1} = e(w(k_t), R(k_t), w(k^e_{t+1}), R(k^e_{t+1}))$$

Regarding the conditions for a steady state, the following conditions have to be fulfilled in this model for the existence of a steady state.

$$k = \frac{S(w(k), R(k), w(k), R(k)) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma l} \equiv \frac{\tilde{S}(k) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma l} \equiv \check{e}(k) \tag{2.10}$$

$$e = e(w(k), R(k), w(k), R(k)) \equiv \bar{e}(k) \tag{2.11}$$

Let the level of education, that satisfies equation (2.10), be named $\check{e}(k)$. For $\check{e}(k) = \bar{e}(k)$ the economy is in a steady state. The following assumptions have to be made, in order to guarantee the existence of a steady state:

- $\check{e}(k)$ is continuous
- There exists only one $k^a > 0$, so that $\check{e}(k^a) = 0$

- $\check{e}'(k^a) < 0$
- $\check{e}(0) \geq 0$
- $\check{e}'(0) > \bar{e}'(0)$

It is proven in the Appendix of Chapter 2 that

$$\bar{e}'(k) > 0 \tag{2.12}$$
$$\bar{e}(0) = 0 \tag{2.13}$$

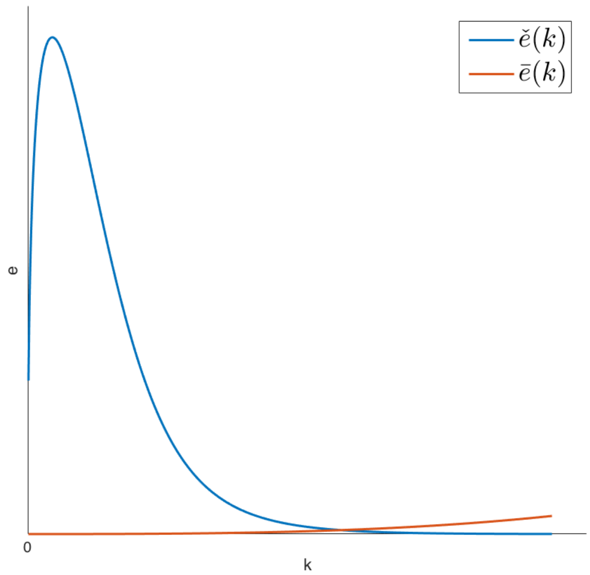


Figure 2.3: Own Visualization of conditions for steady state, Nishimura, Pestieau, and Ponthiere (2015) under exogenous longevity.

Let $k^a > 0$ be the unique value of k, so that $\check{e}(k^a) = 0$. Obviously $0 = \check{e}(k^a) < \bar{e}(k^a)$, since $\bar{e}(k) > 0$. As $\check{e}(k)$ is continuous and since $\check{e}'(0) > \bar{e}(0)$, there has to exist a unique intersection level k^*, so that $\check{e}(k^*) = \bar{e}(k^*)$. This is equivalent to the existence of a steady state. Figure 2.3 illustrates the existence of the optimal value of the capital stock k^* at the intersection of $\bar{e}(k)$ and $\check{e}(k)$ under the above assumptions. Using the assumptions of the numerical example of section 2.2, the steady state is realized at the levels $s = 0.0763, z = 0.1785, e = 0.0083, k = 0.0261$.

2.4 The Ben-Porath Effect

In order to investigate the Ben-Porath effect in this model, the economy is assumed to be in a steady state.

Conditions (2.6) - (2.8) evaluated in a steady state can be used to see, that

$$wh'(e)\left[R+\alpha^2 wh(e)\gamma l\right]-R^2=F(l,\hat{e}(l))=\Delta \tag{2.14}$$

must hold in a steady state. Let $\hat{e}(l)$ be the level of education corresponding to the level of longevity as a result of (2.14). Consequently, using the Implicit Function Theorem, the effect of longevity on education can be calculated in the following way.

$$\begin{aligned}0=\frac{d}{dl}F(l,\hat{e}(l))&=\frac{\partial F(l,\hat{e}(l))}{\partial l}+\frac{\partial F(l,\hat{e}(l))}{\partial \hat{e}(l)}\frac{\partial \hat{e}(l)}{\partial l}=F_l(l,\hat{e}(l))+F_{\hat{e}(l)}(l,\hat{e}(l))\hat{e}'(l)\\ \Rightarrow \hat{e}'(l)&=-\frac{F_l(l,\hat{e}(l))}{F_{\hat{e}(l)}(l,\hat{e}(l))}=-\frac{F_l(l,\hat{e}(l))}{F_{\hat{e}(l)}(l,\hat{e}(l))}=-\frac{\Delta_l}{\Delta_e}\end{aligned}$$

Differentiation leads to

$$\begin{aligned}\Delta_l=&h'(e)\alpha^2 w^2 h(e)\gamma\\ \Delta_e=&wh''(e)R+w^2\alpha^2\gamma l[h''(e)h(e)+[h'(e)]^2]\end{aligned}$$

Thus the total derivation of education with respect to longevity can be written as

$$\frac{de}{dl}=-\frac{\Delta_l}{\Delta_e}=\frac{h'(e)\alpha^2 w^2 h(e)\gamma}{-[\alpha^2 w^2\gamma l]\left[h''(e)h(e)+[h'(e)]^2\right]-h''(e)Rw} \tag{2.15}$$

As shown in equation (2.9), $-\Delta_e>0$. Hence the effect on the optimal education level of longevity in this setting is

- increasing with the square of the wage rate w.
- decreasing with the square rate of the human capital decay $1/\alpha$.
- decreasing with the strength of the marginal disutility of labor in the old age period $1/\gamma$.

Additionally, some special cases are mentioned and interpreted, as they provide a better understanding of the Ben-Porath effect.

- No Ben-Porath effect if human capital decays totally!
  If $\alpha=0\Rightarrow\frac{de}{dl}=0$. This means that if there is complete decay of human capital during one period, longevity has no effect on the optimal level of education. This is a consequence of a very important assumption in this model: Longevity only increases the length of the third period of live. In contrast to models with age specific survival probabilities, (ceteris paribus) rising longevity does not increase

the rate of return of investments in human capital in this setting, if $\alpha = 0$. As soon as $\alpha > 0$ rising longevity affects the rate of return of investments in human capital and the Ben-Porath effect exists.

- No Ben-Porath effect if individuals refuse old age labor completely!
  For $\gamma \to 0 \quad \Rightarrow \quad v_z(z_{t+1}, l_{t+1}) = \frac{z_{t+1}}{\gamma l_{t+1}} \to \infty \quad \Rightarrow \quad \frac{de}{dl} \to 0$
  If agents strongly prefer not to work in the old age period, the effect of longevity on the optimal education level tends to be zero. This is a result of the structure of the model. Individuals cannot extend their working period. The transition of the life cycle is technically fixed and only old age labor can be used to gain additional income.

- A regime with a mandatory retirement age at the beginning of the third period of life $z = \alpha w h(e)\gamma l = 0$ also leads to $\frac{de}{dl} = 0$

2.5 Appendix Chapter 2

2.5.1 Prove for equation (2.12) and equation (2.13)

The FOC for the optimal level of education (2.8) in a steady state can be written as

$$w(k)h'(e)[R(k) + \alpha^2 w(k)[h(e)]\gamma l] - R(k)^2 = F(k, \bar{e}(k)) \equiv \Delta = 0 \tag{2.16}$$

As a result of the Implicit Function Theorem:

$$\begin{aligned} 0 &= \frac{d}{dk}F(k, \bar{e}(k)) = \frac{\partial F(k, \bar{e}(k))}{\partial k} + \frac{\partial F(k, \bar{e}(k))}{\partial \bar{e}(k)}\frac{\partial \bar{e}(k)}{\partial k} = F_k(k, \bar{e}(k)) + F_{\bar{e}(k)}(k, \bar{e}(k))\bar{e}'(k) \\ &\Rightarrow \bar{e}'(k) = -\frac{F_k(k, \bar{e}(k))}{F_{\bar{e}(k)}(k, \bar{e}(k))} = -\frac{F_k(k, \bar{e}(k))}{F_{\bar{e}(k)}(k, \bar{e}(k))} = -\frac{\Delta_k}{\Delta_e} \end{aligned}$$

Regarding the second-order condition with respect to e, equation (2.9), it is shown that $\Delta_e < 0$. Using $R'(k) = f''(k)$ and $w'(k) = f'(k) - f'(k) - kf''(k) = -kf''(k)$, the numerator can be transformed to

$$\begin{aligned} \Delta_k &= h'(e)\left[w'(k)[R(k) + \alpha^2 w(k)h(e)\gamma l] + w(k)[R'(k) + \alpha^2 w'(k)h(e)\gamma l]\right] - 2R(k)R'(k) \\ &= h'(e)\left[2\alpha^2 w(k)w'(k)h(e)\gamma l + w'(k)R(k) + w(k)f''(k)\right] - 2R(k)f''(k) \\ &= h'(e)\left[2\alpha^2 w(k)(-kf''(k))h(e)\gamma l - R(k)kf''(k) + w(k)f''(k)\right] - 2R(k)f''(k) \\ &= h'(e)w(k)f''(k)\underbrace{(-2\alpha^2 kh(e)\gamma l + 1)}_{<1}\underbrace{-h'(e)R(k)kf''(k)}_{>0} - 2R(k)f''(k) \\ &> h'(e)f''(k)w(k) - f''(k)R(k) = f''(k)(h'(e)w(k) - R(k)) \end{aligned}$$

Transforming equation (2.16), it is known that

$$R - w(k)h'(e) = \frac{\alpha^2 w(k)^2 h(e)h'(e)\gamma l}{R} > 0$$

so that $\bar{e}'(k) > 0$. To specify $\bar{e}(0)$, equation (2.16) can be transformed to

$$h'(e)\alpha^2 w(k)^2 h(e)\gamma l = f'(k)[f'(k) - h'(e)w(k)]$$

For $k \to \infty \Rightarrow w(k) = f(k) - f'(k)k \to 0$. Since $\lim_{k\to 0} f'(k) = \infty$, the equation above can only hold for $k = 0$ if $e = 0$. Hence $\bar{e}(0) = 0$, what was the last point to prove.

3 Self-Fulfilling Prophesies concerning Life Expectancy and Longevity

In this chapter, self-fulfilling prophecies concerning longevity will be investigated, based on Cipriani and Makris (2006). If a believe throughout a population causes itself to be realized as a result of positive interaction between the belief and the behavior of people, one speaks of a self fulfilling prophecy. It will be shown that a higher life expectancy throughout the population can result in a longer lifetime in an economy that follows the Ben-Porath effect. Therefore a simple OLG model with constant population will be introduced. The individual's probability to survive the first and second period is 100 % and the probability to survive to the third period is π. It is assumed that individuals use the first period of life for education only, spending $e_t \in (0,1)$ amount of time in this period for education. The amount of time spent on education creates disutility. Interpreted in another way one can state that time not spent for schooling in the first period of life (leisure) results in utility. Otherwise utility arises due to consumption in the second (adulthood) c_t^a and possibly in the third (old age) c_t^o period of life. All agents have the same preferences and maximize their lifetime utility U at time t according to

$$U_t = log(1 - e_t) + \delta log(c_t^a) + \phi \pi_t log(c_t^o)$$

In this context, $\delta \in (0,1)$ and $\phi \in (0,1)$ are individual discount factors for consumption. Taking uncertainty to survive to the third period of life into consideration, these parameters can be interpreted as individual risk aversion preferences as well. For reasons of simplicity, these parameters are constant in time.

By assumption, the survival probability π_t depends on the average human capital of the economy $\pi_t = \pi(\bar{h_t}) \in (\rho, 1), \rho > 0$. This function is assumed to be monotonically increasing in the average level of human capital. The amount of human capital is assumed to develop the following way with $\gamma \in (0,1]$.

$$h_t = h_{t-1}(1 + \gamma e_t) \tag{3.1}$$

As a result of the uncertainty to survive to the third period of life, the rate of return on capital is higher than the interest rate, since dying individuals (after the second period of life) do not consume their savings in the third period of life. Additionally marginal utilities of education are always assumed to be positive. This leads to the following budget constraint for individuals.

$$\begin{aligned} c_t^a =& (1 - s_t) w h_t \\ c_t^o =& s_t \frac{R}{\pi_t} w h_t \end{aligned}$$

Every individual earns wh_t. This amount can be spent for consumption in the second period of life c_t^a and for consumption in the third period of life c_t^o. $s_t \in [0, 1]$ is the part of the income, that is saved for consumption in the old age period. The sum of all savings in the economy is the amount of the capital stock of the current period.
So the individuals utility maximization problem at time t can be written as following. (Note that individuals use an expected survival probability π_t for this optimization.)

$$\max_{e_t, s_t} \quad \log(1 - e_t) + \delta \log((1 - s_t) w h_t) + \phi \pi_t \log\left(s_t \frac{R}{\pi_t} w h_t\right) \tag{3.2}$$

Thus, the first-order conditions can be calculated in the following way.

$$\begin{aligned} \frac{\partial U_t}{\partial s_t} &= \delta \frac{-wh_t}{(1 - s_t) w h_t} + \frac{\phi \pi_t \pi_t}{s_t R w h_t} \frac{R w h_t}{\pi_t} \\ &= -\frac{\delta}{(1 - s_t)} + \frac{\phi \pi_t}{s_t} = 0 \\ &\Leftrightarrow \delta s_t = (1 - s_t) \phi \pi_t \\ &\Leftrightarrow s_t(\delta + \phi \pi_t) = \phi \pi_t \\ &\Leftrightarrow s_t = \frac{\phi \pi_t}{\delta + \phi \pi_t} \end{aligned} \tag{3.3}$$

$$\begin{aligned} \frac{\partial U_t}{\partial e_t} &= -\frac{1}{(1 - e_t)} + \frac{\delta(1 - s_t) w h_{t-1} \gamma}{(1 - s_t) w h_{t-1}(1 + \gamma e_t)} + \frac{\phi \pi_t s_t \frac{R}{\pi_t} w h_{t-1} \gamma}{s_t \frac{R}{\pi_t} w h_{t-1}(1 + \gamma e_t)} \\ &= -\frac{1}{(1 - e_t)} + \frac{\delta \gamma}{(1 + \gamma e_t)} + \frac{\phi \pi_t \gamma}{1 + \gamma e_t} = 0 \\ &\Leftrightarrow -\frac{1 + \gamma e_t}{(1 - e_t)} = \delta \gamma + \phi \pi_t \gamma \\ &\Leftrightarrow 1 + \gamma e_t = \delta \gamma + \phi \pi_t \gamma - \delta \gamma e_t - \phi \pi_t \gamma e_t \\ &\Leftrightarrow (\gamma + \delta \gamma + \phi \pi_t \gamma) e_t = \gamma(\delta + \phi \pi_t) - 1 \\ &\Leftrightarrow e_t = \frac{\gamma(\delta + \phi \pi_t) - 1}{\gamma(1 + \delta + \phi \pi_t)} \end{aligned} \tag{3.4}$$

Equation (3.3) represents the Euler equation for this OLG model. A marginal increase of savings (individually discounted) has to be equal to the resulting additional old age consumption (individually discounted and regarding the survival probability) in an optimum. Equation (3.4) pins down the condition for the individual optimal level of education. For

$\gamma(\delta+\phi\pi_t) > 1$ the optimal level of education is positive. Hence, this parameter restriction is assumed. For given individual factors δ and ϕ, this restriction means that the parameter γ, measuring the influence of education on the development of human capital, must not be too small in order to not allow negative education. The influence of education on human capital has to be big enough to secure positive education levels. Following the optimal level of education, the optimal stock of human capital in an equilibrium ($\bar{h}_t = h_t$) evolves according to

$$h_t = h_{t-1}\left(1 + \frac{\gamma(\delta+\phi\pi(\bar{h}_t)) - 1}{1+\delta+\phi\pi(\bar{h}_t)}\right) = h_{t-1}\frac{(1+\gamma)(\delta+\phi\pi(\bar{h}_t))}{1+\delta+\phi\pi(\bar{h}_t)} \tag{3.5}$$

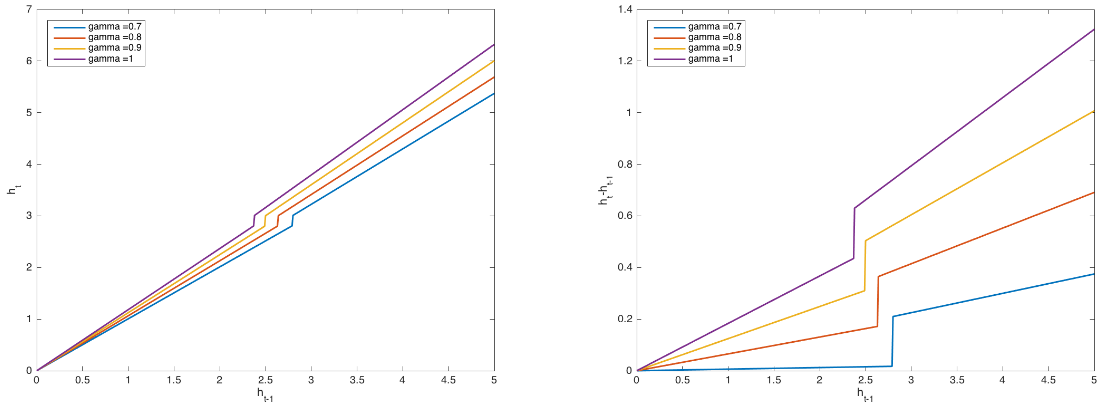
Maximizing their lifetime utility, individuals use their expected survival probability. The effect of this expectation on the individual's optimal level of education is positive:

$$\begin{aligned}\frac{\partial e_t}{\pi_t} &= \frac{\gamma\phi(\gamma(1+\delta+\phi\pi_t)) - (\gamma(\delta+\phi\pi_t)-1)\gamma\phi}{[\gamma(1+\delta+\phi\pi_t)]^2} \\ &= \frac{\phi\gamma(1+\delta+\phi\pi_t) - \phi\gamma(\delta+\phi\pi_t) + \phi}{\gamma(1+\delta+\phi\pi_t)^2} \\ &= \frac{\phi\gamma+\phi}{\gamma(1+\delta+\phi\pi_t)^2} = \frac{\phi(\gamma+1)}{\gamma(1+\delta+\phi\pi_t)^2} > 0\end{aligned}$$

In general, the development of human capital is not uniquely determined by equation (3.5), since the level of h_t cannot be written explicitly as a function $g(h_{t-1})$. However, considering a specific example, the human capital development can be visualized. Therefore consider the following setting:
Suppose that the economy is at time $t = 0$ and that the population can either be pessimistic, expecting a survival probability $\underline{\pi}$ or optimistic, expecting $\overline{\pi}$, with $\rho < \underline{\pi} < \overline{\pi} < 1$. It is assumed that all individuals expect the same probability rate. As shown above, the individual's optimal level of education increases in the expected survival probability. The higher the level of individual education, the higher the realized level of individual and average human capital. Assume that the function for the survival probability is

$$\pi(\bar{h}_t) = \begin{cases} \underline{\pi} & \text{, if } \bar{h}_t < h_t^* \\ \overline{\pi} & \text{, if } \bar{h}_t \geq h_t^* \end{cases}$$

In this function h_t^* is a barrier. If the average human capital $\bar{h}_t$ is smaller than h_t^*, the realized longevity becomes $\underline{\pi}$, otherwise $\overline{\pi}$. Depending on the level of human capital h_t one assumption for this barrier h_t^* has to be made: For given h_0 at $t = 0$, it is assumed that $\frac{(1+\delta+\phi\underline{\pi})h_0^*}{(1+\gamma)(\delta+\phi\underline{\pi})} > h_0 \geq \frac{(1+\delta+\phi\overline{\pi})h_0^*}{(1+\gamma)(\delta+\phi\overline{\pi})}$. Following equation (3.5) and the function for the survival probability self-fulfilling prophecies can be observed: If individuals expect a high survival probability $\overline{\pi}$, it follows that h_1 will be greater or equal to h_0^*, which leads to a high survival rate. On the contrary, if individuals expect a low survival probability $\underline{\pi}$, the average human capital $\bar{h}_1$ will be smaller than h_0^*, which leads to a low survival rate.



(a) Development of human captial.　　(b) Growth of human capital.

Figure 3.1: Own Visualization: Development of human capital at different influence levels of education γ. Source: Cipriani and Makris (2006).

For a numerical example, $\delta = 1$, $\phi = 0.9$, $\underline{\pi} = 0.5$, $\overline{\pi} = 0.5$ and $h_t^* = 3$ are chosen. For a positive level of education, δ must therefore be greater than 2/3. The development of human capital is visualized in Figure 3.1.
In a short summary, the following points can be observed in this model.

- Life expectancy π_t affects the amount of investment in education e_t positively.
- The higher the amount of investment in education, the higher the stock of human capital $h_t = h_{t-1}(1 + \gamma e_t)$.
- If the amount of human capital increases, the probability to survive could increase (but cannot decrease!) depending on the form of the function $\pi_t(\bar{h}_t)$.

This simple model shows that self-fulfilling prophecies concerning life expectancy and realizing lifetime can occur in an economy that follows the positive Ben-Porath effect.

4 Optimal Education Choice considering Survival Probabilities

In this Chapter, the model of Cervellati and Sunde (2013) will be presented as an example for the investigation of the effects of rising longevity on the optimal education period under uncertainty about lifetime. This discrete model is based on other assumptions than the models in the previous Chapters.

4.1 Assumptions and Utility Maximization

Age is a discrete variable measuring the years from 0 to T, which means that T is the maximum lifetime of an individual. The probability to survive until an age of t is $p_t \in (0,1), \quad t = 1, ..., T-1$ with $p_T = 0$. The life of an agent is divided into three periods. In the first period from 0 to S, individuals can spend time for education and build up human capital, in the second period, from $S+1$ to R, agents work. The third part is once again the retirement period. So in the first period of life $s_t \in [0,1]$ amount of time is used for education every year and $l_t = 1 - s_t$ is the corresponding amount of leisure. In the working period $L_t \in [0,1]$ is the amount of labor supply and $l_t = 1 - L_t$ the amount of leisure every year.
Agents maximize their expected lifetime utility at time $t = 0$. Utility arises from consumption c_t and leisure l_t. Additionally individuals are assumed to have a time preference of ρ. Hence, the expected discounted lifetime utility at time $t = 0$ can be written as

$$U = \sum_{t=1}^{T} \rho^{t-1} p_t [u(c_t) + v(l_t)] \tag{4.1}$$

Concerning the lifetime budget constraint of an agent, perfect capital and annuity markets are assumed, in order to simplify investigations. The interest rate r is exogenously given as well as the functional form of the wage rate, that depends on human capital $w(h(S))$ with $w'(h(S)) > 0$ meaning that a higher level of human capital leads to a higher wage rate. The amount of individual human capital is determined by the amount of years of education and the intensity s_t in every year: $h(S) = \sum_{t=1}^{S} g(s_t)$, with $g'(s_t) > 0$ and $g''(s_t) < 0$. This means that the intensity of schooling has a positive effect on the human capital, but the impact decreases with the level of the intensity. The expected income discounted to time $t = 0$ has to equal the expected consumption discounted to time $t = 0$, which leads to

$$\sum_{t=S+1}^{R} r^{t-1} p_t L_t w(h(S)) = \sum_{t=1}^{T} r^{t-1} p_t c_t \tag{4.2}$$

Individuals can choose their consumption path $c_t \quad t = 1, ..., T$, their intensity of education in the schooling period $s_t \quad t = 1, ..., S$, the amount of labor supply $L_t \quad t = S+1, ..., R$ in the second period of life and they can choose the length of their schooling years S at the beginning of their life. This problem is mathematically solved in two steps. In the first step individuals optimize their expected lifetime utility 4.1 using c_t and l_t for $t = 1...T$ as decision variables, subject to the budget constraint 4.2. The retirement age R and the length of education S is exogenous for the first optimization step. This first step is repeated for different values of $S = 1, 2, 3, 4, ...$ In the second step, total lifetime utilities with S years of schooling $U(S)$ are compared to find the optimal amount of years for education S. Consequently the Lagrangian function for the basic problem to be maximized with respect to c_t and l_t is

$$L = \sum_{t=1}^{T} \rho^{t-1} p_t [u(c_t) + v(l_t)] - \lambda \left[\sum_{t=1}^{T} r^{t-1} p_t c_t - \sum_{t=S+1}^{R} r^{t-1} p_t L_t w(h(S)) \right]$$

Differentiation leads to the first-order Conditions:

$$\frac{\partial L}{\partial c_t} = \rho^{t-1} p_t u'(c_t) - \lambda r^{t-1} p_t = 0 \tag{4.3}$$

$$\Rightarrow \rho^{t-1} u'(c_t) = \lambda r^{t-1}, \quad t = 1, ..., T \tag{4.4}$$

$$\frac{\partial L}{\partial l_t} = \rho^{t-1} p_t v'(l_t) - \lambda \left(- \sum_{i=S+1}^{R} r^{i-1} p_i L_i(S) \right) w'(h(S)) g'(s_t)(-1) = 0$$

$$\Rightarrow \rho^{t-1} p_t v'(l_t) = \lambda w'(h(S)) g'(s_t) \sum_{i=S+1}^{R} r^{i-1} p_i L_i(S), \quad t = 1, ..., S \tag{4.5}$$

$$\frac{\partial L}{\partial l_t} = \rho^{t-1} p_t v'(l_t) - \lambda(-1) r^{t-1} p_t (-1) w(h(S)) = 0$$

$$\Rightarrow \rho^{t-1} v'(l_t) = \lambda r^{t-1} w(h(S)), \quad t = S+1, ..., R \tag{4.6}$$

Equation (4.3) implies equation (4.4), which is the Euler equation of this problem, that shows that the amount of consumption decreases with time for $\rho < r$: If the individual discount rate ρ is smaller than the interest rate r, ρ^{t-1} tends faster to zeros than r^{t-1} for increasing t. As the Lagrangian multiplier λ is constant over time, this effect has to be compensated by $u'(c_t)$, which increases for a falling consumption path over time, as $u''(c_t) < 0$. Equation (4.5) states a condition for the optimal intensity of education for every year in the education period. The discounted and expected additional marginal gain of leisure during the education period has to equal the discounted and expected additional income by increased human capital and consequently increased wages multiplied with the Lagrangian multiplier. Equation (4.6) pins down a condition for the optimal labor supply. The discounted additional marginal gain of leisure during the working period has to equal the discounted income multiplied with the Lagrangian multiplier λ. Equation (4.4) shows that for $u'(.) > 0$ and $u''(.) < 0$ the optimal path of consumption is unique. If $v'(), w'() > 0$ and $v''(), w''() < 0$ is assumed, the path of leisure time l_t (and consequently schooling intensity s_t and labor supply L_t) is uniquely determined by equations (4.5) and (4.6). In the second mathematical optimization step, individuals look for the optimal length of their education period S, given the resulting optimal paths of consumption c_t and leisure l_t.

Intuitively, the second optimization step can be interpreted the following way: At an education period of length $S-1$, individuals compare the benefits and opportunity cost of staying one additional year at school, resulting in a schooling period S. The financial benefits of one additional year of education are given by the increase of the discounted (to time $t = 0$) total expected future lifetime earnings, that is given by

$$[w(h(S)) - w(h(S-1))] \cdot \underbrace{\sum_{t=S+1}^{R} r^{t-1} p_t L_t(S)}_{ETLS} . \tag{4.7}$$

The first factor in brackets of (4.7) is the difference of the wage rates that results of one additional year of education. Following the definition of Hazan (2009), the second term of the financial benefits is the Expected Total Labor Supply $ETLS$, a measure for time that is spent working throughout lifetime. On the other hand, the discounted (to time $t = 0$) opportunity cost of one additional year of schooling, resulting out of an income loss in this year is given by the wage rate with $S-1$ years of schooling multiplied with the amount of labor supply in this year and the survival probability

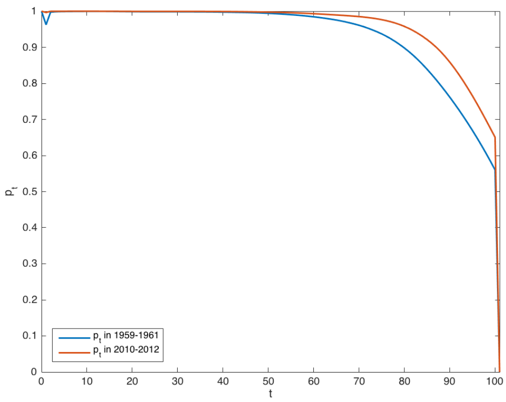
$$w(h(S-1)) \cdot r^{t-1} p_S L_S(S-1). \tag{4.8}$$

The term (4.8) points out that only the probability to survive until age S, namely p_S, is influential for the opportunity cost of one additional year of schooling. Time S is the time of entry into the labor market. On the other hand benefits of another year of education (4.7) are influenced by survival probabilities until age t $\quad S+1 \leq t \leq R$.

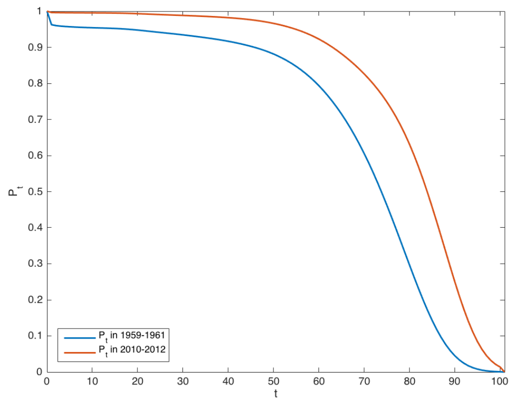
These theoretical results imply, that survival probabilities of different ages differently influence the decision of the optimal schooling period. This means that in order to specify the effect of longevity on the optimal education decision, it is important to analyze which survival probability changes lead to an increase life expectancy.

4.2 Quantitative Analysis

This section will present a numerical analysis of this model that is based on the implementation of Cervellati and Sunde (2013), Section 4. In contrast to Cervellati and Sunde (2013), the uncertainty of death is implemented using death probabilities of the Austrian population provided by *Statistik Austria: Sterbetafeln* (2017). Figure 4.1 illustrates the probabilities to survive until a specific age according to the statistics of Austria's population in 1959-1961 and in 2010-2012. Figure 4.1 (a) shows the probabilities at age t to survive another year p_t and Figure 4.1 (b) shows survival probabilities to survive until a specific age t, P_t. Consequently $P_T = \prod_{t=0}^{T} p_t$. Figure 4.1 clearly shows the rectangularization of the survival distribution function.



(a) Probabilities to survive another year at age t.



(b) Probability to survive until age t.

Figure 4.1: Survival Probabilities in Austria 1959-1961 and 2010-2012. Source *Statistik Austria: Sterbetafeln* (2017)

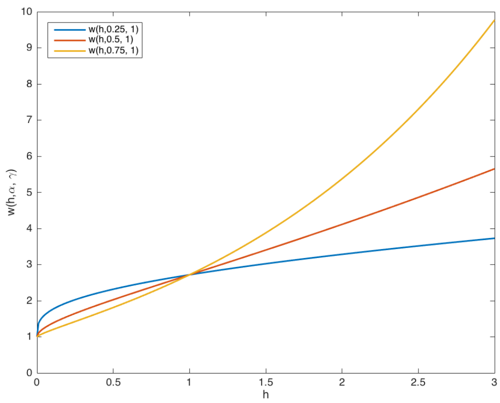
For the numerical implementation of this model, some functional forms have to be assumed. Following Cervellati and Sunde (2013), the utility function $u(c_t)$ is assumed to be a CRRA function (compare definition in Chapter 8 Appendix) and the earnings functions $w(h)$ is an exponential function.

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$
$$w(h) = \bar{w} \cdot e^{\gamma \cdot \theta(h(S))}$$

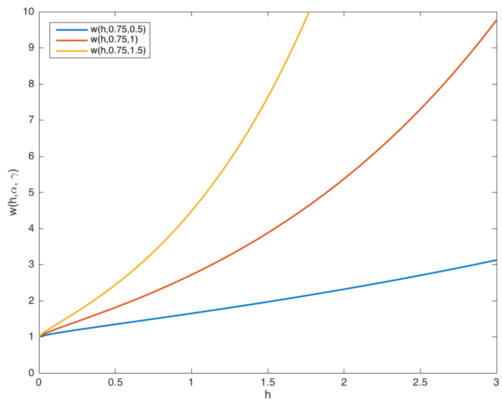
with

$$\theta(h(S)) = h(S)^{\alpha} \quad 0 < \alpha < 1$$
$$h(S) = \sum_{1}^{S} s_t \qquad with \quad g(s_t) = s_t = 1 - l_t$$

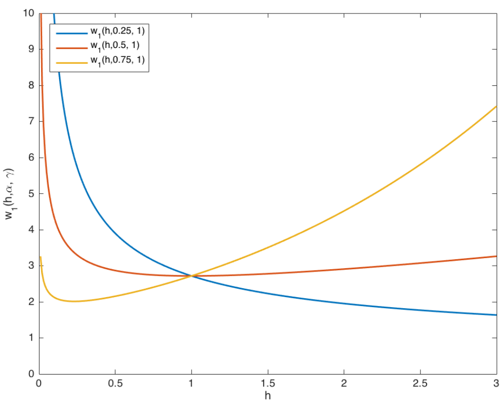
Figure 4.2 visualizes the earnings function and its partial derivatives $w_h(h) = \frac{\partial w(h)}{\partial h}$ and $w_{hh}(h) = \frac{\partial w(h)}{\partial^2 h}$ for different parameters settings. Figure 4.2 (a) shows that a higher value of α leads to a steeper form of the earnings function, if the level of human capital h is greater than 1, whereas Figure 4.2 (b) points out that a higher value of γ leads to a steeper form of the earnings function for all values of human capital h. Figures 4.2 (c) and (d) show the reaction of the earnings for a marginal increase of human capital. Figure 4.2 (c) illustrates that for a high value of α, the marginal earnings gain increases with the level of human capital h whereas the gain decreases for a low value of α. Figure 4.2 (d) shows that the higher value of γ, the stronger the marginal earnings gain for a marginal higher level of human capital h.



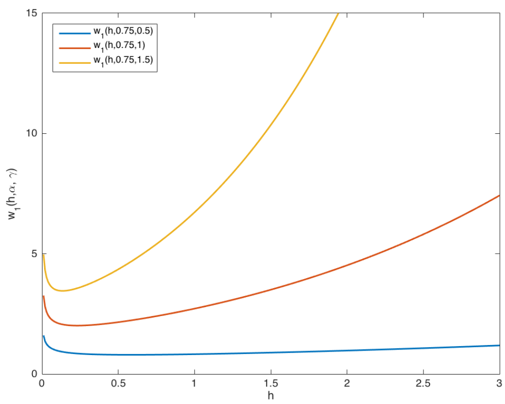
(a) Influence of parameter α on $w(h, \alpha, \gamma)$.



(b) Influence of parameter γ on $w(h, \alpha, \gamma)$.



(c) Influence of parameter α on $w_h(h, \alpha, \gamma)$.

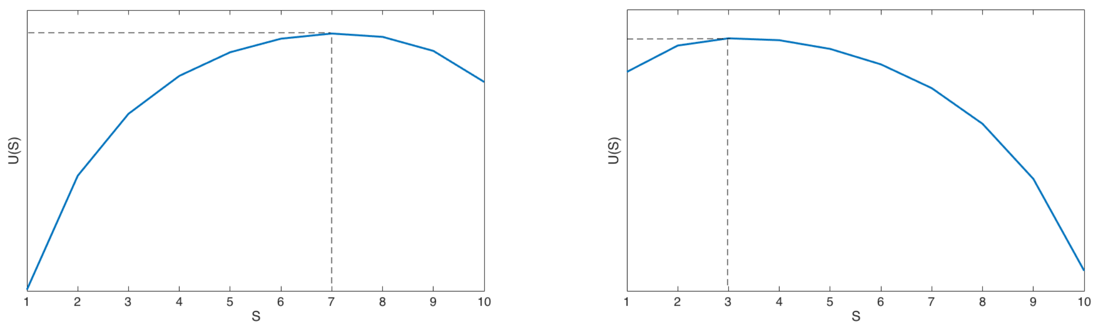


(d) Influence of parameter γ on $w_h(h, \alpha, \gamma)$.

Figure 4.2: Own Visualization of the earnings function. Source Cervellati and Sunde (2013).

Following Cervellati and Sunde (2013) the parameters $\bar{w}$ and γ are set to 1 and $\alpha = 2/3$ in the Benchmark. The measure of relative risk aversion σ is set to 2. In this numerical implementation, the interest rate r and the individual time preference ρ are set to 1, meaning that there is no decay of capital.

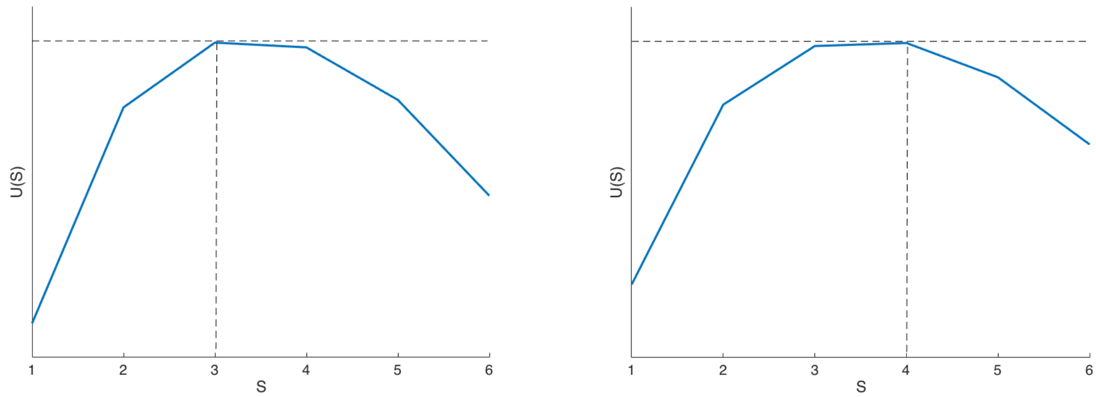
Equation (4.3) and the assumption $r = \rho = 1$ imply that the path of consumption c_t is constant over time and equals c. Equation (4.6) and the assumption $r = \rho = 1$ imply that the path of labor supply L_t is constant over time L. This means that the first step of the mathematical optimization for a given number S of years spent at school is to find the optimal value of consumption c, of labor supply L and S optimal values for the intensity of schooling s_t for the S years spent at school. Additionally the budget constraint, equation (4.2) has to be considered. Hence, S equations for the optimal intensity of schooling s_t, one equation for the optimal amount of consumption c, one for the optimal labor supply L and the budget constraint have to be solved for the equilibrium. In general (depending on the functional forms) these equations are nonlinear. Therefore MATLAB (2015) and its implemented function *lsqnonlin* is used to solve this system. For reasons of simplicity, the life of individuals is divided into parts of five years. Individuals can than decide to spend certain periods of life for education. The retirement age R is set to 65 in this implementation.
The results of the Benchmark scenario are the following. The population chooses to spend 7 periods (35 years) for schooling. This long period (35 years of schooling) can be explained as a result of the form of the earnings function with its parameters. Especially parameter $\alpha = 2/3$ has a huge positive effect on the rate of return of investment in education, as visualized in Figures 4.2. The intensity of education for every period of schooling is: $s_1 = 0,4639; s_2 = 0,5022; s_3 = 0,5023; s_4 = 0,5028; s_5 = 0,5034; s_6 = 0,5040; s_7 = 0,5047$. Following the intensity of education $s_t = 1 - l_t$, the optimal amount of consumption in every period is $c = 1,7055$ and the amount of labor supply in every period between $S+1$ and R is at $L = 0,4639$. Figure 4.3 illustrates the change in lifetime utility as a result of a change in parameter α from 2/3 to 1/3. A reduction of α affects the form of the earnings function as visualized in Figure 4.2. It leads to an optimal education period of 15 years, what seems more realistic regarding the developments of Austria's population's schooling behavior.



(a) Lifetime utility given S with $\alpha = 2/3$. (b) Lifetime utility given S with $\alpha = 1/3$.

Figure 4.3: Optimal lifetime utility with respect to the form of the Mincerian Earnings function.

In another numerical implementation of this model, the effect of a change in survival probabilities is investigated. On the one hand, survival probabilities in Austria in 1959-1961 are used, on the other hand survival probabilities in Austria in 2010-2012 are used provided by *Statistik Austria: Sterbetafeln* (2017). For this implementation α is set to $1/3$, γ and $\bar{w}$ are 1 and the parameter of relative risk aversion ϕ is set to $1,5$, meaning a slight decrease in the individual measure of relative risk aversion. The retirement age is assumed to be 75. Figure 4.4 shows that the optimal periods of schooling change from an optimal level of 3 periods in 1960 to an optimum of 4 periods in 2010 as a result of increasing survival probabilities between 1960 and 2010.



(a) Optimal Lifetime Utility with S periods of education in 1960.

(b) Optimal Lifetime Utility with S periods of education in 2011.

Figure 4.4: The effect of rising survival probabilities on the optimal education period.

These qualitative and quantitative results show that increasing survival probabilities, as they changed in Austria throughout the last decades, do affect the optimal period of education positively. The theoretical investigation have shown that the structure of the change of survival probabilities is crucial for the Ben-Porath effect. The Expected Total Labor Supply is only influenced by survival probabilities during the working period and massively affects the benefits of additional time of schooling.

5 Influence of Survival Conditions on Education and Retirement Age

In this chapter the influence of survival distribution functions and factors that influence longevity like health and family circumstances (to be called conditions of living from here on) on the Ben-Porath effect are studied based on the theory of Sheshinski (2009). Environment conditions like pollution of drinking water and air, the medical assistance or the social environment influence the life expectancy and consequently they influence the Ben-Porath effect. This will be the key point of investigation in this chapter. Uncertainty of lifetime and the influence of exogenous conditions of living affect the decisions on the optimal choices of education and retirement age to a great extent, as the investigations will show.

5.1 Survival Distribution Functions

In this model, age is assumed to be continuous from 0 to a maximum lifetime of T. For the survival distribution of individuals, the following assumptions are made:

- Conditions of living are summarized in α which is continuous in $(0, 1)$.
- The survival distribution function of any individual is given by $F(t, \alpha, T)$.
- $F(0, \alpha, T) = 1,\ F(T, \alpha, T) = 0 \quad \forall \alpha \in (0, 1)$
- Lifetime distribution function $G(t, \alpha, T) := 1 - F(t, \alpha, T)$
- $F(., ., .)$ is differentiable with respect to $t:\ f(t, \alpha, T) := F_t(t, \alpha, T) < 0 \quad \forall \alpha, 0 < t \leq T$
  $\Rightarrow g(t, \alpha, T) = G_t(t, \alpha, T) = -f(t, \alpha, T) > 0 \quad \forall \alpha, 0 < t \leq T$
- $F_\alpha(t, \alpha, T) = \frac{\partial F(t,\alpha,T)}{\partial \alpha} < 0.$

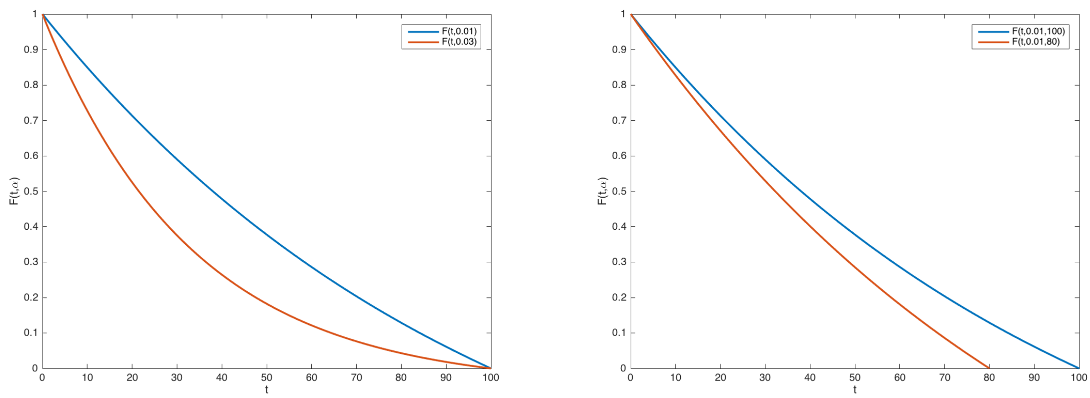
$F(t, \alpha, T)$ is the probability, that the event of death D is at time t or afterwards, given conditions of living $\alpha:\ F(t, \alpha, T) = Pr(D > t|\alpha)$. Consequently $f(t, \alpha, T)$ can be interpreted as probability to survive at age t for a marginal period. On the other hand $G(t, \alpha, T) = Pr(D \leq t)$ and $g(t, \alpha, T)$ can be interpreted as probability to die at age t, given the exogenous parameter α. Consequently the life expectancy, exogenously given conditions of living α is

$$E_\alpha = E(t|\alpha) = \int_0^T t \cdot g(t, \alpha, T)dt \tag{5.1}$$

A low value of $\alpha \to 0$ corresponds to the best possible conditions of living. In contrast, the higher the value of α, the worse the condition to live. Consequently $F_\alpha(t, \alpha, T) < 0$ is assumed. For several illustrations in this chapter, the survival distribution function

$$F(t, \alpha, T) = \frac{e^{-\alpha t} - e^{-\alpha T}}{1 - e^{-\alpha T}} \tag{5.2}$$

is used. This function $F(t, \alpha, T)$ is differentiable in t and α with both derivatives being negative $\forall t \in (0, T]$. For $T \to \infty$ the survival distribution function $F(t, \alpha, T) \to e^{-\alpha t}$, which is the exponential function. Figure 5.1 shows the effects of a change in α and in the maximal lifetime T on the survival distribution function. For further investigations in this chapter, the dependence of the survival distribution function on the maximum lifetime T will not be noted, in order to simplify notation $F(t, \alpha, T) = F(t, \alpha)$. The maximum lifetime T will be constant throughout this chapter. Figure 5.1 illustrates the effects of a change in the conditions of living α and a change in the maximum lifetime T on a survival distribution function.



(a) A lower value of α increases the probability to survive until age t.

(b) A higher maximum lifetime also increases the probability to survive until age t and beyond.

Figure 5.1: Visualization of a survival distribution function.

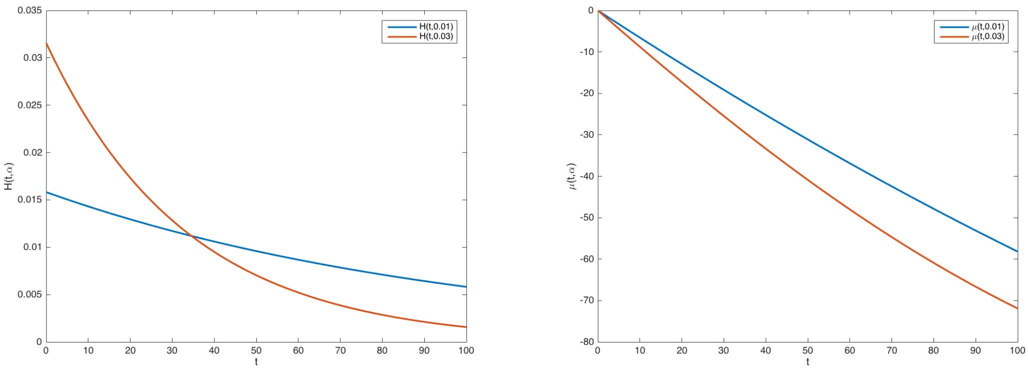
The conditional probability to die at age t, given the survival until age t is called hazard rate: $H(t, \alpha) = \frac{g(t,\alpha)}{F(t,\alpha)}$. Its reaction to a marginal change of the exogenously given parameter α for the conditions of living will be crucial for the following investigations. In order to simplify notation, the function $\mu(t, \alpha)$ is introduced and related to the hazard rate in the following way.

$$\mu(t,\alpha) := \frac{1}{F(t,\alpha)}\frac{\partial F(t,\alpha)}{\partial \alpha} = \frac{F_\alpha(t,\alpha)}{F(t,\alpha)} \tag{5.3}$$

$$\Rightarrow \mu_t(t,\alpha) = \frac{\partial}{\partial t}\frac{F_\alpha(t,\alpha)}{F(t,\alpha)} = \frac{f_\alpha(t,\alpha)F(t,\alpha) - f(t,\alpha)F_\alpha(t,\alpha)}{F(t,\alpha)^2}$$

$$\begin{aligned} H_\alpha(t,\alpha) &= \frac{g_\alpha(t,\alpha)F(t,\alpha) - g(t,\alpha)F_\alpha(t,\alpha)}{F(t,\alpha)^2} = \\ &= \frac{-f_\alpha(t,\alpha)F(t,\alpha) + f(t,\alpha)F_\alpha(t,\alpha)}{F(t,\alpha)^2} = -\mu_t(t,\alpha) \end{aligned} \tag{5.4}$$

Hence, the behavior of $\mu(t,\alpha)$ in time is the negative of the reaction of the hazard rate to a change in α. Figure 5.2 visualizes the Hazard rate and the function $\mu_t(t,\alpha)$ for the survival distribution function of equation (5.2).



(a) Hazard rate for different values of α. (b) $\mu(t,\alpha)$ for different values of α.

Figure 5.2: Visualization of the Hazard rate and corresponding function $\mu(t,\alpha)$.

Figure 5.3 visualizes $H_\alpha(t,\alpha) = -\mu_t(t,\alpha)$ for different values of α. It shows that $H_\alpha(t,\alpha) > 0$, strictly increasing and tends to infinity for $t \to \infty$.

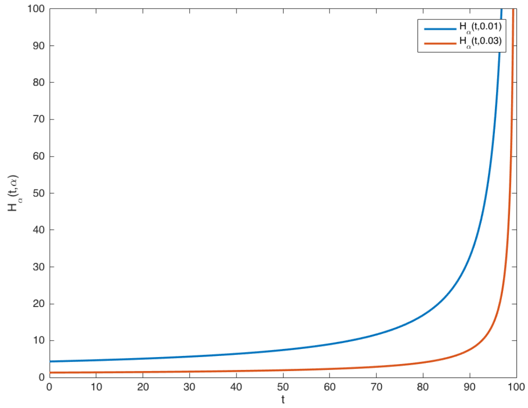


Figure 5.3: Visualization of $H_\alpha(t,\alpha) = -\mu_t(t,\alpha)$.

5.2 Utility Maximization

The life of individuals is divided into three periods. The first period $(0, e)$ is completely used for schooling. The variable e can be seen as amount of education. In this model education does not cost any money. The longer the first period, the higher the amount of education of an agent. As there is no intensity of education in this model, the amount of time spent for education e equals the individual stock of human capital $h = e$. The second period (e, R) can be interpreted as working period, where individuals earn money $w(t, e)$ (Again in this model leisure time is not implemented). Hence, the variable R can be seen as retirement age. The last period is the old age (R, T), where individuals do not work any more. Individuals maximize their lifetime utility at time 0 under perfect foresight. On the one hand utility arises out of consumption $u(c(t))$. The function $u(c(t))$ is assumed to have positive and diminishing marginal returns $u'(.) > 0$ and $u''(.) < 0$. On the other hand disutility arises due to labor supply $a(t)$, with $a'(.) > 0$, so that the lifetime utility U of an agent can be written as

$$U = \int_0^T u(c(t))F(t,\alpha)dt - \int_e^R a(t)F(t,\alpha)dt \tag{5.5}$$

In order to simplify the investigations, the interest rate r is assumed to be $r = 0$. Additionally perfectly competitive longevity insurance markets are assumed in Sheshinski (2009). This allows agents to maximize their lifetime utility based on expected cash flows of earnings and expected cash flows of consumption. The wage rate $w(t, e)$ is assumed to increase in e and to have diminishing marginal productivity: $w_2(t,e) = \frac{\partial w(t,e)}{\partial e} > 0$ and $w_{22}(t,e) = \frac{\partial^2 w(t,e)}{\partial e^2} < 0$. Hence the lifetime budget constraint of an agent can be written as

$$\int_0^T c(t)F(t,\alpha)dt - \int_e^R w(t,e)F(t,\alpha)dt = 0 \quad . \tag{5.6}$$

Individuals optimize their lifetime utility U (5.5) with respect to the decision variables consumption $c(t)$, education e, retirement age R subject to the budget constraint equation (5.6). Consequently the individuals maximization problem can be solved using the Lagrangian function L.

$$\begin{aligned}\max_{c(t),e,R} L = &\int_0^T u(c(t))F(t,\alpha)dt - \int_e^R a(t)F(t,\alpha)dt \\ &- \lambda\left(\int_0^T c(t)F(t,\alpha)dt - \int_e^R w(t,e)F(t,\alpha)dt\right)\end{aligned}$$

Differentiation leads to the first-order conditions:

$$\begin{aligned}\frac{\partial L}{\partial c(t)} &= \int_0^T u'(c(t))F(t,\alpha)dt - \lambda\int_0^T F(t,\alpha)dt = 0 \\ &\Leftrightarrow u'(c(t)) = \lambda \\ \frac{\partial L}{\partial R} &= -a(R)F(R,\alpha) + \lambda w(R,e)F(R,\alpha) = 0 \\ &\Leftrightarrow a(R) = u'(c)w(R,e) \\ \frac{\partial L}{\partial e} &= a(e)F(e,\alpha) + \lambda\left[-w(e,e)F(e,\alpha) + \int_e^R \frac{\partial}{\partial e}w(t,e)F(t,\alpha)dt\right] \\ &= a(e)F(e,\alpha) + \lambda\left[-w(e,e)F(e,\alpha) + \int_e^R w_2(t,e)F(t,\alpha)dt\right] = 0 \\ &\Leftrightarrow a(e) - \lambda w(e,e) + \frac{\lambda}{F(e,\alpha)}\int_e^R w_2(t,e)F(t,e)dt = 0\end{aligned}$$

If consumption has positive and diminishing marginal utilities ($u'(c(t)) > 0$ and $u''(c(t)) < 0$), the condition $u'(c(t)) = \lambda$ implies that the path of consumption over time is constant in an optimum. Using the budget constraint, the optimal level of consumption can be explicitly calculated for optimal values of e and R.

$$c = \frac{\int_e^R w(t,e)F(t,\alpha)dt}{\int_0^T F(t,\alpha)dt}$$

For further investigations, the wage rate is assumed to be independent of age $w(t,e) = w(e)$ and for reasons of simplicity the disutility of labor at the beginning of the working period is assumed to be zero $a(e) = 0$. So the first-order conditions can be summarized as following:

$$\lambda = u'(c(t)) \qquad \Rightarrow \qquad c = \frac{\int_e^R w(e)F(t,\alpha)dt}{\int_0^T F(t,\alpha)dt} \tag{5.7}$$

$$0 = u'(c)w(e) - a(R) \qquad \equiv \varphi(R,e,\alpha) \tag{5.8}$$

$$0 = \frac{1}{F(e,\alpha)} \int_e^R F(t,\alpha)dt - \frac{w(e)}{w'(e)} \qquad \equiv \psi(R,e,\alpha) \tag{5.9}$$

In this case, a pair (R^*, e^*), that satisfies conditions (5.8) and (5.9) for a specific level of conditions of living α is called equilibrium.

5.3 Equilibrium

In order to guarantee uniqueness of the optimal level of education and retirement age (R^*, e^*) some additional assumptions have to be made. For

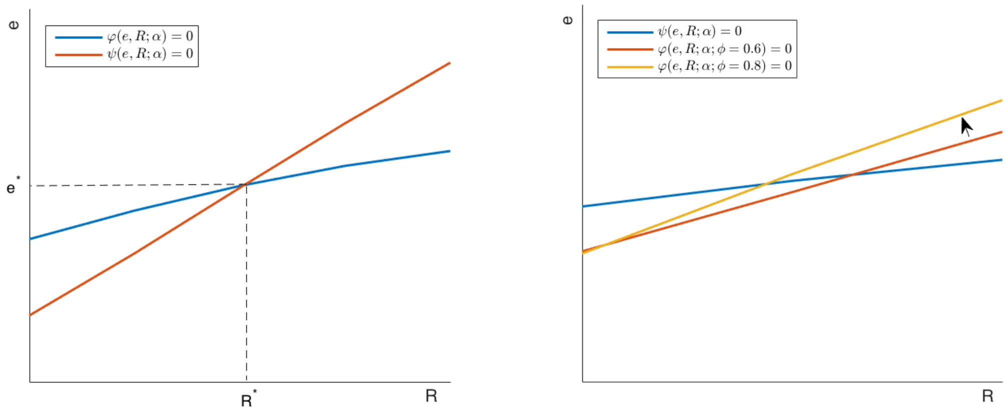
$$\frac{w''(e)}{w'(e)} - \frac{f(e,\alpha)}{F(e,\alpha)} < 0 \tag{5.10}$$

and (5.11)

$$\sigma = -\frac{u''(c)c}{u'(c)} \in (0.5, 1) \tag{5.12}$$

let $e(R;\varphi = 0)$ be the value that satisfies $\varphi(R,e;\alpha) = 0$ for given R and constant α and let $e(R;\psi = 0)$ be the value that satisfies $\psi(R,e;\alpha) = 0$ for given R and constant α. It is proven in the Appendix Chapter 5, that $e(R;\varphi = 0)$ and $e(R;\psi = 0)$ have a unique intersection level R^* and consequently the optimal utility maximizing values (c^*, R^*, e^*) are unique. Condition (5.10) connects the behavior of the wage rate function with the hazard rate that is related to the survival distribution function $F(t,\alpha)$. In the Appendix at the end of this chapter it is shown that $\psi_e(R,e,\alpha) < 0$, $\varphi_e(R,e,\alpha) > 0$, $\psi_R(R,e,\alpha) > 0$ and $\varphi_R(R,e,\alpha) < 0$. Figure 5.4 visualizes the unique equilibrium. Therefore $a(t) = (t-e)/2$ and CRRA functions are used for $u(c) = \frac{c^{1-\phi}-1}{1-\phi}$ and $w(e) = \frac{e^{1-\delta}-1}{1-\delta}$. These functions satisfy the necessary conditions and guarantee a unique intersection. It is visualized in Figure 5.4 b, that a higher value of ϕ, the measure of relative risk aversion of individuals, leads to a steeper form of φ and consequently to lower optimal values for e and R.

In the following, the effect of an increasing or decreasing $\mu(t,\alpha)$ in time on the equilibrium (e^*, R^*) will be discussed. Therefore the effect of conditions of living α on the curves $\varphi(R,e,\alpha)$ and $\psi(R,e,\alpha)$ has to be calculated.



(a) Unique intersection of $\varphi(.) = 0$ and $\psi(.) = 0$.

(b) The measure of relative risk aversion of individuals ϕ influences only the curve $\varphi(.) = 0$.

Figure 5.4: Own Visualization: Unique equilibrium (R^*, e^*). Source Sheshinski (2009).

$$
\begin{aligned}
\varphi_\alpha(R,r,\alpha) &= u''(c)\frac{\partial c}{\partial \alpha}w(e)\\
\frac{\partial c}{\partial \alpha} &= \frac{\int_e^R w(e)F_2(t,\alpha)dt \int_0^T F(t,\alpha)dt - \int_e^R w(e)F(t,\alpha)dt \int_0^T F_2(t,\alpha)dt}{(\int_0^T F(t,\alpha)dt)^2}\\
&= \frac{w(e)\int_e^R w(e)F(t,\alpha)dt}{\int_0^T F(t,\alpha)dt}\underbrace{\left[\frac{\int_e^R F_2(t,\alpha)dt}{\int_e^R F(t,\alpha)dt} - \frac{\int_0^T F_2(t,\alpha)dt}{\int_0^T F(t,\alpha)dt}\right]}_{\nu(e,R,\alpha)}
\end{aligned}
$$

As $F_2(t,\alpha) < 0$ and $\int_0^T F(t,\alpha)dt > \int_e^T F(t,\alpha)dt$ it follows that $\nu(e,T,\alpha) > 0$.

$$
\begin{aligned}
\nu_R(e,R,\alpha) &= \frac{F_2(R,\alpha)\int_e^R F(t,\alpha)dt - \int_e^R F_2(t,\alpha)dt F(R,\alpha)}{(\int_e^R F(t,\alpha)dt)^2}\\
&= \frac{\int_e^R F_2(R,\alpha)F(t,\alpha) - F_2(t,\alpha)F(R,\alpha)dt}{(\int_e^R F(t,\alpha)dt)^2}\\
&= \frac{F(R,\alpha)\int_e^R \left[\frac{F_2(R,\alpha)}{F(R,\alpha)} - \frac{F_2(t,\alpha)}{F(t,\alpha)}\right]F(t,\alpha)dt}{(\int_e^R F(t,\alpha)dt)^2}\\
&= \frac{F(R,\alpha)}{\int_e^R F(t,\alpha)dt}\int_e^R \underbrace{\left[\frac{F_2(R,\alpha)}{F(R,\alpha)} - \frac{F_2(t,\alpha)}{F(t,\alpha)}\right]}_{*}\frac{F(t,\alpha)}{(\int_e^R F(t,\alpha)dt)}
\end{aligned}
$$

For $\mu_t(t,\alpha) < 0$ the term in square brackets, $*$, is less than 0. Consequently $\nu_R(e,R,\alpha) < 0$. As shown above, $\nu(e,T,\alpha) > 0$. Both results together show that $\nu(e,R,\alpha) > 0$ for $0 \leq R \leq T$. This leads to the conclusion, that $\varphi_\alpha(R,r,\alpha) < 0$. Regarding the two

dimensional state space (R, e), the effect of alpha on the curve $\varphi(R, e, \alpha) = 0$ can be explained in the following way. For a constant R, an increased value of α leads to an increased value of e, since $\varphi_e(R, e, \alpha) > 0$. This means that the curve $\varphi(R, e, \alpha) = 0$ shifts upwards and to the left as a result of an increased value of α (worse condition of living).
In a similar way, the effect of α to $\psi(R, e, \alpha)$ can be calculated.

$$\begin{aligned}
\psi_\alpha(R, e, \alpha) &= -F(e, \alpha)^{-2} \int_e^R F(t, \alpha) dt + \frac{\int_e^R F_2(t, \alpha) dt}{F(e, \alpha)} \\
&= \frac{1}{F(e, \alpha)} \left[\int_e^R F_2(t, \alpha) dt - \frac{F_2(e, \alpha) \int_e^R F(t, \alpha) dt}{F(e, \alpha)} \right] \\
&= \frac{1}{F(e, \alpha)} \int_e^R \underbrace{\left[\frac{F_2(t, \alpha)}{F(t, \alpha)} - \frac{F_2(e, \alpha)}{F(e, \alpha)} \right]}_{**} F(t, \alpha) dt
\end{aligned}$$

For $\mu_t(t, \alpha) < 0$ the term in square brackets, $**$, is less than 0. Consequently $\psi_\alpha(R, r, \alpha) < 0$. Regarding the two dimensional state space (R, e), the effect of alpha on the curve $\psi(R, e, \alpha) = 0$ can be explained in the following way. For a constant R, an increased value of α leads to a decrease of e, since $\psi_e(R, e, \alpha) < 0$. This means that the curve $\psi(R, e, \alpha) = 0$ shifts downwards an to the right as a result of an increased value of α (worse condition of living).

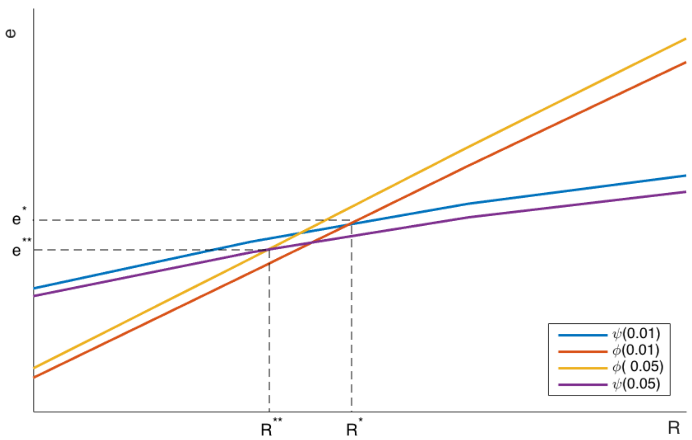


Figure 5.5: Own Visualization: The effect of increasing conditions of living α on the optimal values (R^*, e^*). Source Sheshinski (2009).

Figure 5.5 visualizes the effect of α on the equilibrium in this model. A decrease of the conditions of living leads to a lower optimal level of education e^* and retirement age R^*. It is worth mentioning once again that this statement does only hold, if $\mu_t(t, \alpha) < 0$. To put it in other words: The optimal values of education e^* and retirement age R^* decrease as a result of a fall in conditions of living α, if the the effect of α on the hazard rate $H_\alpha(t,\alpha)$ is positive.

5.4 Appendix Chapter 5

5.4.1 Uniqueness of equilibrium

A positive determinant of the Jacobi Matrix $J(\Psi(R,e;\alpha))$ with $\Psi(R,e;\alpha) = (\varphi(R,e;\alpha), \psi(R,e;\alpha))^t$ is a sufficient condition for a global maximum (R^*, e^*) of this function. To proof this, several partial derivatives have to be calculated.

$$
\begin{aligned}
\varphi_R(R,e,\alpha) &= u''(c)w(e)\frac{\partial c}{\partial R} - a'(R) \\
&\rightarrow \frac{\partial c}{\partial R} = \frac{w(e)F(R,\alpha)}{\int_0^T F(t,\alpha)dt} \\
\varphi_R(R,e,\alpha) &= \frac{u''(c)w(e)^2F(R,\alpha)}{\int_0^T F(t,\alpha)dt}\frac{u'(c)}{u'(c)} - a'(R) \\
&= u'(c)w(e)\left(\underbrace{\frac{u''(c)}{u'(c)}\frac{w(e)\int_e^R F(t,\alpha)dt}{\int_0^T F(t,\alpha)dt}}_{=-\sigma<0}\frac{F(R,\alpha)}{\int_e^R F(t,\alpha)dt}\right) - a'(R) \\
&= u'(c)w(e)\left(-\sigma\frac{F(R,\alpha)}{\int_e^R F(t,\alpha)dt}\right) - a'(R) < 0
\end{aligned}
$$

$$
\begin{aligned}
\psi_e(R,e,\alpha) &= \frac{-f(e,\alpha)}{F(e,\alpha)^2}\int_e^R F(t,\alpha)dt + \frac{1}{F(e,\alpha)}(-F(e,\alpha)) - \frac{w'(e)^2 - w(e)w''(e)}{w'(e)^2} \\
&= -\frac{f(e,\alpha)\int_e^R F(t,\alpha)dt}{F(e,\alpha)^2} - 1 - 1 + \frac{w(e)w''(e)}{w'(e)^2} \\
&= \frac{w(e)}{w'(e)}\left(\frac{w''(e)}{w'(e)}\right) - \frac{f(e,\alpha)\int_e^R F(t,\alpha)dt}{F(e,\alpha)^2} - 2 \\
&= \frac{w(e)}{w'(e)}\left(\frac{w''(e)}{w'(e)} - \frac{f(e,\alpha)}{F(e,\alpha)}\right) - 2
\end{aligned}
$$

For $\frac{w''(e)}{w'(e)} - \frac{f(e,\alpha)}{F(e,\alpha)} < 0$, $\psi_e(R,e,\alpha) < 0$.

$$
\begin{aligned}
\varphi_e(R,e,\alpha) &= u''(c)\frac{\partial c}{\partial e}w(e) + u'(c)w'(e) \\
&\rightarrow \frac{\partial c}{\partial R} = \frac{w'(e)\int_e^R F(t,\alpha)dt - w(e)F(t,\alpha)}{\int_0^T F(t,\alpha)dt}
\end{aligned}
$$

$$\begin{aligned}
\varphi_e(R,e,\alpha) &= u''(c)w(e)\frac{w'(e)\int_e^R F(t,\alpha)dt - w(e)F(t,\alpha)}{\int_0^T F(t,\alpha)dt} + u'(c)w'(e) \\
&= u'(c)w'(e) + \frac{u'(c)c}{u'(c)}u''(c)w'(e) - \frac{u'(c)c}{u'(c)}u''(c)\frac{w(e)}{\int_e^R F(t,\alpha)dt} \\
&= u'(c)\left(w'(e)(1-\sigma) + \sigma\frac{w(e)F(t,\alpha)}{\int_e^R F(t,\alpha)dt}\right)
\end{aligned}$$

For $\sigma < 1$, it follows that $\varphi_e(R,e,\alpha) > 0$. As a result of equation (5.9), $\varphi_e(R,e,\alpha)$ can further be simplified around the optimal values of e^* and R^*.

$$\varphi_e(R,e,\alpha) = u'(c)w'(e) > 0$$

$$\psi_R(R,e,\alpha) = \frac{F(R,\alpha)}{F(e,\alpha)} > 0$$

For the determinant of the Jacobian Matrix $J(\Psi(R,e;\alpha))$ to be positive, one more assumption is needed.

$$\begin{aligned}
det(J(\Psi)) &= \varphi_R\psi_e - \varphi_e\psi_R \\
&= u'(c)w(e)\left(\sigma\frac{F(R,\alpha)}{\int_e^R F(t,\alpha)dt} + \frac{a'(R)}{a(R)}\right)\left(2 + \frac{w(e)}{w'(e)}\left(\frac{f(e,\alpha)}{F(e,\alpha)} - \frac{w''(e)}{w'(e)}\right)\right) \\
&\quad - u'(c)w(e)\frac{F(R,\alpha)}{F(e,\alpha)} \\
&< u'(c)w(e)\left(2\sigma\frac{F(R,\alpha)}{F(e,\alpha)} - \frac{F(R,\alpha)}{F(e,\alpha)}\right)
\end{aligned}$$

Hence, for $2\sigma - 1 > 0$, the determinant of the Jacobi Matrix is positive and the optimal (maximizing) pair (R^*, e^*) is unique.

6 Optimal Education Choice with Endogenous Longevity

In this chapter the optimal education choice of individuals will be considered in a model with endogenous longevity. The theory is based on Nishimura, Pestieau, and Ponthiere (2015).

6.1 Model assumptions

This model uses the same structure as the model in Chapter 2. All variables are defined analogously. The only difference is, that longevity is not taken exogenously, but instead it will be assumed that, longevity positively depends on the level of individual education. It is assumed that the lifetime in the third period of life l_{t+1} depends on the level of education at time $t-1$, e_{t-1}.

$$l_{t+1} = l(e_{t-1})$$

It is assumed that there exists a minimum length of lifetime in the third period l. This length increases with the amount of education in the first period. The higher the amount of education, the lower the marginal increase in $l(e_{t-1})$. If the level of education $l(e_{t-1})$ tends towards 0, the longevity gain from a marginal increase of education tends towards infinity and if the level of education $l(e_{t-1})$ tends towards infinity, the longevity gain from a marginal increase of education tends towards zero. This means that at a very low level of education, a marginal increase of education affects longevity massively, whereas at a very high level of education, a marginal increase of education does not affect longevity strongly. Additionally a maximum length of lifetime $\bar{l}$ will be assumed.

- $l(0) = l > 0$
- $l'(e_{t-1}) > 0$
- $l''(e_{t-1}) < 0$
- $\lim_{e_{t-1}\to 0} l'(e_{t-1}) = \infty$
- $\lim_{e_{t-1}\to \infty} l'(e_{t-1}) = 0$
- $\lim_{e_{t-1}\to \infty} l(e_{t-1}) = \bar{l} < 1$

6.2 Utility Maximization

The total individual's lifetime utility to be maximized at time t can be written as

$$\begin{aligned}\max_{e_{t-1},s_t,z_{t+1}} \; & u[w_t h(e_{t-1}) - e_{t-1}R_t - s_t] \\ & + l(e_{t-1})u\left(\frac{z_{t+1}\alpha w^e_{t+1}h(e_{t-1}) - v(z_{t+1}, l(e_{t-1})) + R^e_{t+1}s_t}{l(e_{t-1})}\right)\end{aligned}$$

Differentiation with respect to s_t, z_{t+1} and e_{t-1} leads to the first-order conditions for optimality:

$$u'(c_t) = R^e_{t+1}u'(d_{t+1}) \tag{6.1}$$

$$\alpha w^e_{t+1}h(e_{t-1}) = v_z(z_{t+1}, l(e_{t-1})) \tag{6.2}$$

$$\begin{aligned}u'(c_t)[R_t - w_t h'(e_{t-1})] &= l'(e_{t-1})u(d_{t+1}) \\ &+ l(e_{t-1})u'(d_{t+1})\Bigg[\frac{z_{t+1}\alpha w^e_{t+1}h'(e_{t-1}) - v_l(.,.))l'(e_{t-1})}{l(e_{t-1})} \\ &- \frac{\left(z_{t+1}\alpha w^e_{t+1}h(e_{t-1}) - v(z_{t+1}, l(e_{t-1})) + R^e_{t+1}s_t\right)l'(e_{t-1})}{l(e_{t-1})^2}\Bigg]\end{aligned} \tag{6.3}$$

Using $v(z_{t+1}, l(e_{t-1})) = \frac{z^2_{t+1}}{2\gamma l(e_{t-1})}$, equation (6.2) and using $d_{t+1} = \frac{z_{t+1}\alpha w^e_{t+1}h'(e_{t-1}) + R^e_{t+1}s_t - v(z_{t+1}, l(e_{t-1}))}{l(e_{t-1})}$, equation (6.3) can be transformed, so that the necessary conditions for optimality are

$$u'(c_t) = R^e_{t+1}u'(d_{t+1}) \tag{6.4}$$

$$z_{t+1} = \alpha w^e_{t+1}h(e_{t-1})\gamma l(e_{t-1}) \tag{6.5}$$

$$\begin{aligned}u'(c_t)[R_t - w_t h'(e_{t-1})] &= u'(d_{t+1})\left[z_{t+1}\alpha w^e_{t+1}h'(e_{t-1})\right] \\ &+ l'(e_{t-1})\left[u(d_{t+1}) - u'(d_{t+1})v_l(z_{t+1}, l(e_{t-1})) - u'(d_{t+1})d_{t+1}\right].\end{aligned} \tag{6.6}$$

Comparing the FOCs in this setting and in the setting of exogenously given longevity in Chapter 2, only the FOC with respect to education, equation (6.6), differs from equation (2.8). The additional term $l'(e_{t-1})\left[u(d_{t+1}) - u'(d_{t+1})v_l(z_{t+1}, l(e_{t-1})) - u'(d_{t+1})d_{t+1}\right]$ appears, since the level of education now has an effect on the duration of life in the third period. Let $\epsilon = \frac{u'(d)d}{u(d)}$ be the elasticity of the utility function $u(.)$. This elasticity is a measure for the relative change of the utility (output) as a result of a change of consumption (input). Equation (6.3) implies that $\frac{u'(c_t)}{u'(d_{t+1})} = R^e_{t+1}$. Consequently equation (6.6) becomes

$$\begin{aligned}R^e_{t+1}[R_t - w_t h'(e_{t-1})] &= z_{t+1}\alpha w^e_{t+1}h'(e_{t-1}) \\ &+ \frac{l'(e_{t-1})u(d_{t+1})}{u'(d_{t+1})} - \left[v_l(z_{t+1}, l(e_{t-1})) + d_{t+1}\right]l'(e_{t-1})\end{aligned}$$

$$
\begin{aligned}
&\Leftrightarrow \\
R^e_{t+1}R_t &= h'(e_{t-1})\left[z_{t+1}\alpha w^e_{t+1} + w_t R^e_{t+1}\right] \\
&\quad + l'(e_{t-1})\left[\frac{1}{\epsilon}d_{t+1} - v_l(z_{t+1}, l(e_{t-1})) - d_{t+1}\right] \\
&\Leftrightarrow \\
R^e_{t+1}R_t &= h'(e_{t-1})\left[\alpha^2(w^e_{t+1})^2 h(e_{t-1})\gamma l(e_{t-1}) + w_t R^e_{t+1}\right] \\
&\quad + l'(e_{t-1})\left[\frac{1}{\epsilon}d_{t+1} - v_l(z_{t+1}, l(e_{t-1})) - d_{t+1}\right].
\end{aligned} \tag{6.7}
$$

If $\frac{1}{\epsilon}d_{t+1} - v_l(z_{t+1}, l(e_{t-1})) - d_{t+1}$ is positive, a marginal increase in the amount of education creates utility, as $l'(e_{t-1})$ is assumed to be greater than 0. This means that in this setting of endogenous longevity, the marginal welfare gains of additional education is greater than the effect in the model with exogenous longevity, as long as $\frac{1}{\epsilon}d_{t+1} - v_l(z_{t+1}, l(e_{t-1})) - d_{t+1} > 0$. So the necessary temporary equilibrium conditions in the case of endogenous longevity can be summarized as following.

$$
\begin{aligned}
c_t &= w_t h(e_{t-1}) - e_{t-1}R_t - s_t \\
d_t &= \frac{z_{t+1}\alpha w^e_{t+1}h(e_{t-1}) - v(z_{t+1}, l(e_{t-1})) + R^e_{t+1}s_t}{l(e_{t-1})} \\
u'(c_t) &= R^e_{t+1}u'(d_{t+1}) \\
z_{t+1} &= \alpha w^e_{t+1}h(e_{t+1})\gamma l(e_{t-1}) \\
R^e_{t+1}R_t &= h'(e_{t-1})\left[\alpha^2(w^e_{t+1})^2 h(e_{t-1})\gamma l(e_{t-1}) + w_t R^e_{t+1}\right] \\
&\quad + l'(e_{t-1})\left[\frac{1}{\epsilon}d_{t+1} - v_l(z_{t+1}, l(e_{t-1})) - d_{t+1}\right] \\
L_t &= h_t(e_{t-1})N_t + z_t\alpha h_{t-1}(e_{t-2})N_{t-1} \\
K_t &= N(s_{t-1} - e_{t-1}) \\
w_t &= F_L(K_t, L_t) \\
R_t &= F_K(K_t, L_t)
\end{aligned}
$$

As a result of the assumptions on the derivatives of the longevity and human capital function (see Section 2.1), there exists an interior optimal level of education. For $0 < \epsilon < 1$ and $|h''(e_{t-1})|h(e_{t-1})l(e_{t-1}) > (h'(e_{t-1}))^2 l(e_{t-1}) + 2h'(e_{t-1})h(e_{t-1})l'(e_{t-1})$, the optimal level of education is unique (Proof in the Appendix of Chapter 6). Consequently there exists a unique temporary equilibrium under the above assumptions.

6.3 Steady State

As in Section 2.3, a steady state is characterized by

$$s_t = s_{t+1} \quad \forall t \qquad\qquad e_t = e_{t+1} \quad \forall t \qquad\qquad z_t = z_{t+1} \quad \forall t$$

This consequently means $c_t = c_{t+1}$ and $d_t = d_{t+1} \ \forall t$ as well. In order to investigate the existence of a steady state, the first-order conditions (6.4), (6.5) and (6.7) can be used to find conditions for a two dimensional state space in e and k. Using equation (6.4), the savings per young worker s_t can be written as a function of present and future (expected) factor rewards and of the education level, since c_t and d_{t+1} are only functional forms of present and future factor rewards and the education level. Assuming existence and uniqueness of the temporary equilibrium, the education level itself can be written as function of present and future factor rewards and of the savings per young worker (regarding equation (6.7)).

$$\begin{aligned} s_t &\equiv \hat{s}(w_t, R_t, w^e_{t+1}, R^e_{t+1}, e_{t-1}) \\ e_{t-1} &\equiv \hat{e}(w_t, R_t, w^e_{t+1}, R^e_{t+1}, s_t) \\ &= \hat{e}(w_t, R_t, w^e_{t+1}, R^e_{t+1}, \hat{s}(w_t, R_t, w^e_{t+1}, R^e_{t+1}, e_{t-1})) \end{aligned}$$

Using equation (6.5), the capital accumulation (see definition in equation (2.1)) can be written as following.

$$k_{t+1} = \frac{\hat{s}(w(k_t), R(k_t), w(k^e_{t+1}), R(k^e_{t+1}), e_{t-1}) - e_t}{h(e_{t-1}) + \alpha^2 w(k_t)[h(e_{t-2})]^2 \gamma l(e_{t-2})} \quad .$$

Since the optimal level of education only depends on current and future factor rewards, it also can be written as a function of the per capita capital stock.

$$e_{t-1} \equiv e(w(k_t), R(k_t), w(k^e_{t+1}), R(k^e_{t+1}), \hat{s}(w(k_t), R(k_t), w(k^e_{t+1}), R(k^e_{t+1}, e_{t-1}))$$

Regarding the conditions for a steady state ($k_{t+1} = k_t$ and $e_{t+1} = e_t$), the steady state in this model, where longevity depends on the level of education, has to satisfy the following conditions:

$$k = \frac{\hat{s}(w(k), R(k), w(k), R(k), e) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma l(e)} \equiv \frac{\tilde{s}(k,e) - e}{h(e) + \alpha^2 w(k)[h(e)]^2 \gamma l(e)} \tag{6.8}$$

$$e = \hat{e}(w(k), R(k), w(k), R(k), \tilde{s}(k,e)) \equiv \tilde{e}(k, \tilde{s}(k,e)) \tag{6.9}$$

It will be shown that there exists a steady state - a pair (k, e) that satisfies equations (6.8) and (6.9). Equation (6.8) can be rearranged to

$$h(e)\left[1+\alpha^2 w(k)h(e)\gamma l(e)\right] = \frac{\tilde{s}(k,e)-e}{k} \quad . \tag{6.10}$$

For a given k, let $\check{e}(k)$ be the solution of equation (6.10) and $\bar{e}(k)$ be the solution value of e for $e = \tilde{e}(k, \tilde{s}(k,e))$. Considering equation (6.10) it makes sense to assume that an increase of k also increases $\check{e}(k)$, $\check{e}'(k) > 0$: A higher per capita capital stock k goes hand in hand with a lower interest rate because $R(k) = f'(k)$ and $f''(k) < 0$. A higher per capita capital stock k also goes hand in hand with a higher wage rate, since $w'(k) = -kf''(k) > 0$. On the one side a fall of the interest rate results in an increase in the rate of return of investment into education, since intuitively the price for borrowing money for education falls. On the other side an increase of the wage rate w can be interpreted as a fall of the costs for education. If additionally $\bar{e}(0) = 0$ is assumed, the function $\bar{e}(k)$ is increasing passing through the pair $(e,k) = (0,0)$.
Let $k^a > 0$ be the value of k, such that $\check{e}(k^a) = 0$. As $\bar{e}(0) = 0$ and $\bar{e}'(k) > 0$, $\bar{e}(k^a)$ has to be greater than $\check{e}(k^a)$. Assuming $\check{e}'(0) > \bar{e}'(0)$ and $\check{e}(0) \geq 0$, there has to be a $\bar{k}$ with $\check{e}(\bar{k}) < \bar{e}(\bar{k})$. Hence, assuming continuity, there exists an intersection $e^* = \bar{e}(k^*)$ and $e^* = \check{e}(k^*)$ fulfilling both equations (6.8) and (6.9). This intersection characterizes the existence of a steady state in the case of endogenous longevity with perfect foresight.
The conditions for the existence of a steady state can be summarized as follows:

- Let $\bar{e}(k)$ be the level of e satisfying $e = \tilde{e}(k, \tilde{s}(k,e))$ for a given k, than it has to be assumed that
  - $\bar{e}(0) = 0$
  - $\bar{e}'(k) > 0$
- Let $\check{e}(k)$ be the level of e satisfying $h(e)[1+\alpha^2 w(k)h(e)\gamma l(e)] + \frac{e}{k} = \frac{\tilde{s}(k,e)}{k}$ for a given level of capital k
  - The level of k, so that $\check{e}(k) = 0$ is unique
  - $\check{e}(k)$ is continuous
  - $\check{e}'(0) > \bar{e}'(0)$
  - $\check{e}(0) \geq 0$

6.4 Ben-Porath Effect

In order to investigate the effect of longevity on the optimal education choice in this model, $\frac{\partial e}{\partial l}$ will be calculated and studied. Rearrangement of equation (6.7), evaluated in steady state leads to the following condition, that holds if the economy is in a steady state

$$R = wh'(e)\left[1+\frac{\alpha z}{R}\right] + \frac{l'(e)d}{R}\left[\frac{1}{\epsilon}-1\right] - \frac{l'(e)v_l(z,l(e))}{R} \tag{6.11}$$

This can be interpreted in the following way: The left hand side, R, pins down the marginal cost of additional education, which is of course the interest rate. The right hand side is the gain from marginal additional education, consisting of three terms. The first term of the RHS of equation 6.11 $wh'(e)\left[1+\frac{\alpha z}{R}\right]$ corresponds to the earnings in the second, and the discounted earnings in the third period of life. The second term $\frac{l'(e)d}{R}\left[\frac{1}{\epsilon}-1\right]$ is a result of the positive effect of education on longevity $l'(e) > 0$ and increases the gain of additional marginal education. The third term $\frac{l'(e)v_l(z,l(e))}{R}$ arises as a result of the positive effect of education on the disutility of old age labor. This means that disutility of old age labor decreases, if the level of education rises ceteris paribus: $e\uparrow \Rightarrow \ l(e)\uparrow \Rightarrow \ v(z,l(e))\downarrow$.
In order to investigate the Ben-Porath effect, $\tilde{l}(e) = \lambda l(e)$ is used to describe the FOC with respect to e and $z = \alpha w h(e)\gamma\lambda l(e)$.

$$h'(e)w\left[R+\alpha z\right] + \lambda l'(e)\left[d\left(\frac{1}{\epsilon}-1\right) - v_l(z,\lambda l(e))\right] - R^2 = \Delta$$

Analogously to section 2.4, using the Implicit Function Theorem $\frac{de}{d\lambda}$ can be calculated.

$$\begin{aligned}\Delta_\lambda &= h'(e)w^2\alpha^2 h(e)\gamma l(e) + l'(e)\left[d\left(\frac{1}{\epsilon}-1\right) - v_l(z,\lambda l(e))\right] \\ &+\lambda l'(e)\left[\frac{\partial d}{\partial\lambda}\left(\frac{1}{\epsilon}-1\right) - v_{lz}(z,\lambda l(e))\alpha w h(e)\gamma l(e) - v_{ll}(z,\lambda l(e))l(e)\right]\end{aligned} \quad (6.12)$$

Using the special form of the disutility function and its derivatives, this equation can be rearranged. Therefore, note, that

$$\begin{aligned}&\lambda l(e)v_{ll}(z,l(e)) + z v_{lz}(z,\lambda l(e)) = \lambda l(e)\frac{z^2}{\gamma l(e)} + z(-\frac{z}{\gamma l(e)^2}) \\ &= \frac{z^2}{\gamma l(e)^2} - \frac{z^2}{\gamma l(e)^2} = 0\end{aligned}$$

and consequently equation (6.12) can be written as

$$\Delta_\lambda = h'(e)w^2\alpha^2 h(e)\gamma l(e) - l'(e)v_l(z,\lambda l(e)) + l'(e)\left(\frac{1}{\epsilon}-1\right)\left(d+\lambda\frac{\partial d}{\partial\lambda}\right)$$

Hence

$$\frac{de}{d\lambda} = \frac{\Delta_\lambda}{-\Delta_e} = \frac{h'(e)w^2\alpha^2 h(e)\gamma l(e) - l'(e)v_l(z,\lambda l(e)) + l'(e)\left(\frac{1}{\epsilon}-1\right)\left(d+\lambda\frac{\partial d}{\partial\lambda}\right)}{-\Delta_e} \quad (6.13)$$

For an interpretation of the Ben-Porath effect (6.13) in this setting it will be shown that

$\left(d + \lambda \frac{\partial d}{\partial \lambda}\right) > 0$. Therefore note that

$$\begin{aligned}
d =& \frac{z\alpha wh(e) - v(z, l(e)) + Rs}{\lambda l(e)} = \\
=& \frac{\alpha^2 w^2 h(e)^2 \gamma \lambda l(e) - \frac{\alpha^2 w^2 h(e)^2 \gamma^2 \lambda^2 l(e)^2}{2\gamma\lambda l(e)} + Rs}{\lambda l(e)} \\
=& \frac{\alpha^2 w^2 h(e)^2 \gamma \lambda l(e) - \frac{1}{2}\alpha^2 w^2 h(e)^2 \gamma \lambda l(e)}{\lambda l(e)} + \frac{Rs}{\lambda l(e)} \\
=& \frac{\alpha^2 w^2 h(e)^2 \gamma \lambda l(e)}{2} + \frac{Rs}{\lambda l(e)}
\end{aligned}$$

So, the FOC with respect to s, (6.4) can be written as

$$F(\lambda, s(\lambda)) = -u'(wh(e) - eR - s) + u'(\frac{\alpha^2 w^2 h(e)^2 \gamma \lambda l(e)}{2} + \frac{Rs}{\lambda l(e)}) = 0 \tag{6.14}$$

Consequently $\frac{\partial s}{\partial \lambda}$ can be calculated.

$$\frac{\partial s}{\partial \lambda} = -\frac{F_\lambda}{F_{s(\lambda)}} = -\frac{-\frac{R^2 u''(d)s}{\lambda l(e)\lambda}}{u''(c) + \frac{R^2}{\lambda l(e)} u''(d)} = \left(\frac{\frac{R^2}{\lambda l(e)} u''(d)}{u''(c) + \frac{R^2}{\lambda l(e)} u''(d)}\right) \frac{s}{\lambda}$$

Thus $\frac{\partial d}{\partial \lambda}$ can be calculated.

$$\begin{aligned}
\frac{\partial d}{\partial \lambda} =& \frac{\partial}{\partial \lambda} \frac{Rs}{\lambda l(e)} = \frac{R}{l(e)} \frac{\partial}{\partial \lambda} \left(\frac{s(\lambda)}{\lambda}\right) = \frac{R}{l(e)} \left(\frac{\lambda \frac{\partial s(\lambda)}{\partial \lambda} - s(\lambda)}{\lambda^2}\right) \\
=& \frac{R}{\lambda l(e)} \left(\frac{\partial s}{\partial \lambda} - \frac{s}{\lambda}\right) = \frac{R}{\lambda l(e)} \left(\frac{\frac{R^2 u''(d)}{\lambda l(e)}}{\frac{u''(c)\lambda l(e) + R^2 u''(d)}{\lambda l(e)}} \frac{s}{\lambda} - \frac{s}{\lambda}\right) \\
=& \frac{R}{\lambda l(e)} \left(\frac{R^2 u''(d)}{u''(c)\lambda l(e) + R^2 u''(d)} \frac{s}{\lambda} - \frac{s}{\lambda}\right) \\
=& \frac{Rs}{\lambda^2 l(e)} \left(\frac{R^2 u''(d)}{R^2 u''(d) \left(\frac{u''(c)\lambda l(e)}{R^2 u''(d)} + 1\right)} - 1\right) \\
=& \frac{Rs}{\lambda^2 l(e)} \left(\frac{1 - \left(1 + \frac{u''(c)\lambda l(e))}{R^2 u''(d)}\right)}{\frac{u''(c)\lambda l(e)}{R^2 u''(d)} + 1}\right) \\
=& \frac{Rs}{\lambda^2 l(e)} \left(\frac{-u''(c)}{u''(c) + \frac{R^2}{\lambda l(e)} u''(d)}\right) < 0
\end{aligned}$$

However

$$d+\lambda\frac{\partial d}{\partial \lambda} \geq \frac{Rs}{\lambda l(e)}+\lambda\frac{\partial s}{\partial \lambda}=\frac{Rs}{\lambda l(e)}\left(1-\frac{u''(c)}{u''(c)+\frac{R^2}{\lambda l(e)}u''(d)}\right)=\frac{Rs}{\lambda l(e)}\left(\frac{\frac{R^2}{\lambda l(e)}u''(d)}{u''(c)+\frac{R^2}{\lambda l(e)}u''(d)}\right)>0$$

Concerning the interpretation of the effect of longevity on education in this 3 period OLG model with endogenous longevity, calculated in equation (6.13), the following statements can be made:

- As shown in the Appendix at the end of this chapter $-\Delta_e > 0$. It can be seen easily that for $l(e) \to 0$ the effect of longevity on education is the same as in the setting with exogenous longevity, as only the first term of the numerator remains.
- For the second term: $-l'(e)v_l(z, \lambda l(e))$ it is important to remember that $l'(e) > 0$ and $v_l(z, \lambda l(e)) < 0$ is assumed. Thus, $-l'(e)v_l(z, \lambda l(e))$ points out, that there is a channel from education to longevity and consequently to disutility of old age labor. This channel increases the utility of additional education and hence increases the effect of longevity on education.
- The third term $l'(e)\left(\frac{1}{\epsilon}-1\right)\left(d+\frac{\partial d}{\partial \lambda}\right)$ affects the Ben-Porath effect positively as well, since $0 < \epsilon < 1$ and $d+\frac{\partial d}{\partial \lambda} > 0$. Clearly, the smaller the elasticity of utility ϵ, the greater the effect of longevity on the optimal education level. This means that the preferences of individuals are important for the size of the Ben-Porath effect in the case of endogenous longevity. It is interesting, that $\frac{\partial d}{\partial \lambda} < 0$, what means that increased life-length leads to less consumption in the third period of life. However it is shown above that $d+\frac{\partial d}{\partial \lambda} > 0$, that secures a positive effect of longevity on education as well for the last term $l'(e)\left(\frac{1}{\epsilon}-1\right)\left(d+\frac{\partial d}{\partial \lambda}\right)$.

6.5 Social Optimality

In this section, the social planners problem of this model will be investigated. The social planer intends to optimize the total utility of individuals (for all generations). The optimal values of consumption c_t, old age welfare d_t, education e_t, old age labor z_t and the per capita capital stock $\tilde{k}_t = K/N$ are chosen with respect to the resource constraint of the total economy, equation (6.15). The production of the closed economy in one period in this model consists of the amount of consumption of the working generation and of the retired generation and of the investment of the young generation in education and of the savings of the working generation. So the total production of the economy $F(K, L)$ must satisfy

$$F(K,L) = Nc_t + N\tilde{d}_t + Ne_t + K_t \quad .$$

The production function $F(K, L)$ has the property of constant returns to scale. Using the definition of effective labor (see section 2.1) $L_t = h_tN_t + z_t\alpha h_{t-1}N_{t-1}$ the production function can be written as

$$F(K,L) = NF(K/N, L/N) = NF(\tilde{k}_t, h(e_{t-1} + z_t\alpha h(e_{t-2}))).$$

Additionally it can be seen that the definition of old age welfare is equivalent to

$$d_t = \frac{\tilde{d}_t - v(z_t, l(e_{t-2}))}{l(e_{t-2})}$$
$$\Leftrightarrow$$
$$d_t l(e_{t-2}) = \tilde{d}_t - v(z_t, l(e_{t-2})) \quad .$$

Consequently the resource constraint of the total economy is

$$F(\tilde{k}_t, h(e_{t-1} + z_t\alpha h(e_{t-2}))) = c_t + d_t l(e_{t-2}) + v(z_t, l(e_{t-2})) + e_t + \tilde{k}_{t+1} \quad . \tag{6.15}$$

Let β be the time preference. The social planer's optimization problem can be solved using a Lagrangian function. The utility of all of all generations $\sum_{t=t_{start}} \beta^t[u(c_t) + l(e_{t-1})u(d_{t+1})]$ should be maximized subject to the resource constraint of the economy equation 6.15. Consequently the maximization problem for the social planner can be written:

$$\begin{aligned} \max_{c_t,d_t,e_t,z_t,k_t} L = \sum_{t=t_{start}} \beta^t[u(c_t) + l(e_{t-1})u(d_{t+1})] - \lambda_t\Big[c_t + d_t l(e_{t-2}) + v(z_t, l(e_{t-2})) \\ + e_t + \tilde{k}_{t+1} - F(\tilde{k}_t, h(e_{t-1} + z_t\alpha h(e_{t-2})))\Big] \end{aligned} \tag{6.16}$$

Differentiation leads to the first-order conditions:

$$\begin{aligned}
&\frac{\partial L}{\partial c_t} = 0 \Rightarrow \beta^t u'(c_t) = \lambda_t \qquad (6.17)\\
&\frac{\partial L}{\partial d_t} = 0 \Rightarrow \beta^{t-1} u'(d_t) = \lambda_t l(e_{t-2})\\
&\frac{\partial L}{\partial e_t} = 0 \Rightarrow \beta^{t+1} l'(e_t)u(d_{t+2}) - \lambda_{t+2}d_{t+2}l'(e_t) - \lambda_t + \lambda_{t+1}F_2(\tilde{k}_{t+1}, h(e_t) + z_{t+1}\alpha h(e_{t-1}))h'(e_t)\\
&\qquad + \lambda_{t+2}F_2(\tilde{k}_{t+2}, h(e_{t+1}) + z_{t+2}\alpha h(e_t))\alpha z_{t+2}h'(e_t) - \lambda_{t+2}v_l(z_{t+2}, l(e_t))l'(e_t) = 0\\
&\frac{\partial L}{\partial z_t} = 0 \Rightarrow \lambda_t F_2(\tilde{k}_t, h(e_{t-1} + z_t\alpha h(e_{t-2})))\alpha h(e_{t-2}) = \lambda_t v_z(z_t, l(e_{t-2}))\\
&\frac{\partial L}{\partial \tilde{k}_t} = 0 \Rightarrow \lambda_{t-1} = \lambda_t F_1(\tilde{k}_t, h(e_{t-1} + z_t\alpha h(e_{t-2})))
\end{aligned}$$

In a steady state $c_t = c_{t+1}$ holds. Hence equation (6.17) implies that $\lambda_t = \beta\lambda_{t-1}$ must hold. Additionally $F_2(.) = w$ and $F_1(.) = R$. Hence, the FOC with respect to e in a steady state can be summarized as follows

$$
\begin{aligned}
1 &= F_2(\tilde{k}, h(e) + z\alpha h(e))h'(e)(\beta + \beta^2 z\alpha) - l'(e)(-\beta u(d) + \beta^2 d + \beta^2 v_l(z, l(e))) \\
\beta^{-1} &= wh'(e)(1 + \beta z\alpha) + l'(e)\beta \left[\frac{u(d)}{u'(d)} - d - v_l(z, l(e))\right] \\
\beta^{-1} &= wh'(e)(1 + \beta z\alpha) + l'(e)\beta \left[d\left(\frac{1}{\epsilon} - 1\right) - v_l(z, l(e))\right] \qquad (6.18)
\end{aligned}
$$

For $\beta^{-1} = R$ this condition equals equation (6.7), which is the condition for the optimal education level in the individuals utility maximization problem evaluated in a steady state. This means that the social optimal values for consumption, education and retirement age are realized in the laissez faire economy as well, if the per capita capital stock of the laissez faire economy fulfills the modified golden rule :

$$
\frac{\partial F(\tilde{k}^*, h(e)(1 + z\alpha))}{\partial \tilde{k}} = \beta^{-1} \qquad (6.19)
$$

This result shows the importance of the time preference β that is decisive for the optimal values of savings, education and retirement age. If the capital stock in the decentralized model is lower than the social optimal capital stock, the interest rate R is higher than its social optimal value. Consequently the cost of education increases and the optimal value of education in the laissez faire economy is lower than socially optimal. In terms of governmental regulations of the laissez faire economy, this means that the social optimal values of the decision variables can be realized in the decentralized model as well by imposing a mandatory lump sum transfer from the working generation to the young generation, so that the optimal levels of education are funded.

6.6 Appendix Chapter 6

6.6.1 Proof of unique optimal level of education under endogenous longevity

Note that the FOC with respect to e_{t-1} fulfills

$$
\begin{aligned}
\lim_{e_{t-1}\to 0} & h'(e_{t-1})\left[\alpha^2 (w^e_{t+1})^2 h(e_{t-1})\gamma l(e_{t-1}) + w_t R^e_{t+1}\right] \\
&+ \frac{l'(e_{t-1})}{u'(d_{t+1})}\left[u(d_{t+1}) - u'(d_{t+1})d_{t+1} - u'(d_{t+1})v_l(z_{t+1}, l(e_{t-1}))\right] \\
&= \infty > R^e_{t+1} R_t
\end{aligned}
$$

and

$$\begin{aligned}
&\lim_{e_{t-1}\to\infty} h'(e_{t-1})\left[\alpha^2(w^e_{t+1})^2 h(e_{t-1})\gamma l(e_{t-1}) + w_t R^e_{t+1}\right]\\
&\quad + \frac{l'(e_{t-1})}{u'(d_{t+1})}\left[u(d_{t+1}) - u'(d_{t+1})d_{t+1} - u'(d_{t+1})v_l(z_{t+1}, l(e_{t-1}))\right]\\
&\quad = 0 < R^e_{t+1}R_t
\end{aligned}$$

This means, that the if the level of education tends towards zero, the marginal welfare gain from additional education is greater than the marginal costs of additional education. On the other hand, if the level of education tends towards infinity, the marginal welfare gain from additional education tends towards zero and hence is smaller than the marginal costs of additional education. So the levels 0 and ∞ of education cannot be optimal. As a result of differentiability assumptions (and continuity), there has to exist an interior optimal level of education in this setting of endogenous longevity as well. In order to specify conditions for uniqueness of the level of optimal education the FOC with respect to e_{t-1} will be differentiated again.

$$\begin{aligned}
&\frac{\partial}{\partial e_{t-1}}\Bigg(h'(e_{t-1})\left[\alpha^2(w^e_{t+1})^2 h(e_{t-1})\gamma l(e_{t-1}) + w_t R^e_{t+1}\right]\\
&\quad + \frac{l'(e_{t-1})}{u'(d_{t+1})}\left[u(d_{t+1}) - u'(d_{t+1})d_{t+1} - u'(d_{t+1})v_l(z_{t+1}, l(e_{t-1}))\right]\Bigg)\\
=&h''(e_{t-1})\left[\alpha^2(w^e_{t+1})^2 h(e_{t-1})\gamma l(e_{t-1}) + w_t R^e_{t+1}\right]\\
&\quad + h'(e_{t-1})\left[\alpha^2(w^e_{t+1})^2 h'(e_{t-1})\gamma l(e_{t-1}) + \alpha^2(w^e_{t+1})^2 h(e_{t-1})\gamma l'(e_{t-1})\right]\\
&\quad + \frac{l''(e_{t-1})u'(d_{t+1}) - l'(e_{t-1})u''(d_{t+1})\frac{\partial d_{t+1}}{\partial e_{t-1}}}{(u'(d_{t+1}))^2}\left[u(d_{t+1}) - u'(d_{t+1})d_{t+1} - u'(d_{t+1})v_l(z_{t+1}, l(e_{t-1}))\right]\\
&\quad + \frac{l'(e_{t-1})}{u'(d_{t+1})}\Bigg[u'(d_{t+1})\frac{\partial d_{t+1}}{\partial e_{t-1}} - u''(d_{t+1})\frac{\partial d_{t+1}}{\partial e_{t-1}}d_{t+1} - u'(d_{t+1})\frac{\partial d_{t+1}}{\partial e_{t-1}}\\
&\quad - u''(d_{t+1})\frac{\partial d_{t+1}}{\partial e_{t-1}}v_l(z_{t+1}, l(e_{t-1})) - u'(d_{t+1})v_{lz}(z_{t+1}, l(e_{t-1}))\frac{\partial z_{t+1}}{\partial e_{t-1}}\\
&\quad - u'(d_{t+1})v_{ll}(z_{t+1}, l(e_{t-1}))l'(e_{t-1})\Bigg]
\end{aligned}$$

$$
\begin{aligned}
=&h''(e_{t-1}) \left[\alpha^2 (w^e_{t+1})^2 h(e_{t-1})\gamma l(e_{t-1}) + w_t R^e_{t+1}\right] \\
&+ h'(e_{t-1}) \left[\alpha^2 (w^e_{t+1})^2 h'(e_{t-1})\gamma l(e_{t-1}) + \alpha^2 (w^e_{t+1})^2 h(e_{t-1})\gamma l'(e_{t-1})\right] \\
&+ l''(e_{t-1}) \left[d_{t+1}\left(\frac{1}{\epsilon}-1\right) - v_l(z_{t+1}, l(e_{t-1}))\right] \\
&+ l'(e_{t-1}) \Bigg[- \frac{u''(d_{t+1})u(d_{t+1})}{u'(d_{t+1})^2}\frac{\partial d_{t+1}}{\partial e_{t-1}} + \frac{u''(d_{t+1})d_{t+1}}{u'(d_{t+1})}\frac{\partial d_{t+1}}{\partial e_{t-1}} \\
&+ \frac{u''(d_{t+1})v_l(z_{t+1}, l(e_{t-1}))}{u'(d_{t+1})}\frac{\partial d_{t+1}}{\partial e_{t-1}} + \frac{\partial d_{t+1}}{\partial e_{t-1}} - \frac{u''(d_{t+1})d_{t+1}}{u'(d_{t+1})}\frac{\partial d_{t+1}}{\partial e_{t-1}} - \frac{\partial d_{t+1}}{\partial e_{t-1}} \\
&- \frac{u''(d_{t+1})v_l(z_{t+1}, l(e_{t-1}))}{u'(d_{t+1})}\frac{\partial d_{t+1}}{\partial e_{t-1}} - v_l z(z_{t+1}, l(e_{t-1}))\frac{\partial z_{t+1}}{\partial e_{t-1}} - v_l l(z_{t+1}, l(e_{t-1}))l'(e_{t-1}) \Bigg] \\
=&h''(e_{t-1}) \left[\alpha^2 (w^e_{t+1})^2 h(e_{t-1})\gamma l(e_{t-1}) + w_t R^e_{t+1}\right] \\
&+ h'(e_{t-1}) \left[\alpha^2 (w^e_{t+1})^2 h'(e_{t-1})\gamma l(e_{t-1}) + \alpha^2 (w^e_{t+1})^2 h(e_{t-1})\gamma l'(e_{t-1})\right] \\
&+ l''(e_{t-1}) \left[d_{t+1}\left(\frac{1}{\epsilon}-1\right) - v_l(z_{t+1}, l(e_{t-1}))\right] \\
&+ l'(e_{t-1}) \Bigg[- v_{ll}(z_{t+1}, l(e_{t-1}))l'(e_{t-1}) - \frac{u''(d_{t+1})u(d_{t+1})}{u'(d_{t+1})^2}\frac{\partial d_{t+1}}{\partial e_{t-1}} \\
&- v_{lz}(z_{t+1}, l(e_{t-1})) \left[\alpha w^e_{t+1}\gamma \left(h'(e_{t-1})l(e_{t-1}) + h(e_{t-1})l'(e_{t-1})\right)\right] \Bigg] \\
=&h''(e_{t-1}) \left[\alpha^2 (w^e_{t+1})^2 h(e_{t-1})\gamma l(e_{t-1}) + w_t R^e_{t+1}\right] \\
&+ h'(e_{t-1}) \left[\alpha^2 (w^e_{t+1})^2 h'(e_{t-1})\gamma l(e_{t-1}) + \alpha^2 (w^e_{t+1})^2 h(e_{t-1})\gamma l'(e_{t-1})\right] \\
&+ l''(e_{t-1}) \left[d_{t+1}\left(\frac{1}{\epsilon}-1\right) - v_l(z_{t+1}, l(e_{t-1}))\right] \\
&+ l'(e_{t-1}) \left[-v_{ll}(z_{t+1}, l(e_{t-1}))l'(e_{t-1}) - v_{lz}(z_{t+1}, l(e_{t-1}))z_{t+1}\left(\frac{h'(e_{t-1})}{h(e_{t-1})}\frac{l'(e_{t-1})}{l(e_{t-1})}\right)\right] \\
&+ l'(e_{t-1})(-\frac{u''(d_{t+1})u(d_{t+1})}{u'(d_{t+1})^2}\frac{\partial d_{t+1}}{\partial e_{t-1}})
\end{aligned}
\tag{6.20}
$$

Using $-v_{ll}(.,.) - v_{lz}(.,.)\frac{z_{t+1}}{l(e_{t-1})} = 0$ the forth term becomes $l'(e_{t-1})v_{lz}(z_{t+1}, l(e_{t-1}))z_{t+1}\frac{h'(e_{t-1})}{h(e_{t-1})}$.

$$
\begin{aligned}
&= \alpha^2 (w^e_{t+1})^2 \gamma \left[h''(e_{t-1}) h(e_{t-1}) l(e_{t-1}) + (h'(e_{t-1}))^2 l'(e_{t-1}) + 2h'(e_{t-1}) h(e_{t-1}) l'(e_{t-1}) \right] \\
&+ h''(e_{t-1}) w_t R^e_{t+1} \\
&+ l''(e_{t-1}) \left[d_{t+1} \left(\frac{1}{\epsilon} - 1 \right) - v_l(z_{t+1}, l(e_{t-1})) \right] \\
&+ l'(e_{t-1}) \left(- \frac{u''(d_{t+1}) u(d_{t+1})}{u'(d_{t+1})^2} \frac{\partial d_{t+1}}{\partial e_{t-1}} \right)
\end{aligned} \tag{6.21}
$$

Clearly, the second term is negative. As well is the third term, for $\epsilon < 1$. It can be shown that the forth term is negative by taking into consideration that a change of e_{t-1} results in a change of s_t:

$$
\begin{aligned}
d_{t+1} =& \frac{z_{t+1} \alpha w^e_{t+1} h(e_{t-1}) + R^e_{t+1} s_t - v(z_{t+1}, l(e_{t-1}))}{l(e_{t-1})} = \frac{\alpha^2 (w^e_{t+1})^2 (h(e_{t-1}))^2 \gamma}{2} \frac{R^e_{t+1} s_t}{l(e_{t-1})} \\
\frac{\partial d_{t+1}}{\partial e_{t-1}} =& \frac{2\alpha^2 (w^e_{t+1})^2 h(e_{t-1}) h'(e_{t-1}) \gamma}{2} + R^e_{t+1} \left(\frac{\frac{\partial s_t}{\partial e_{t-1}} l(e_{t-1}) - s_t l'(e_{t-1})}{(l(e_{t-1}))^2} \right) \\
=& \alpha^2 (w^e_{t+1})^2 h(e_{t-1}) h'(e_{t-1}) \gamma + \frac{R^e_{t+1}}{l(e_{t-1})} \left(\frac{\partial s_t}{\partial e_{t-1}} - \frac{l'(e_{t-1})}{l(e_{t-1})} s_t \right)
\end{aligned}
$$

Using equation (6.7) in form of

$$
\begin{aligned}
R_t - w_t h'(e_{t-1}) = \frac{1}{R^e_{t+1}} \Bigg[& \alpha^2 (w^e_{t+1})^2 h'(e_{t-1}) h(e_{t-1}) \gamma l(e_{t-1}) \\
& + l'(e_{t-1}) d_{t+1} (\frac{1}{\epsilon} - 1) - l'(e_{t-1}) v_l(z_{t+1}, l(e_{t-1})) \Bigg]
\end{aligned}
$$

$\frac{\partial s_t}{\partial e_{t-1}}$ can be calculated:

$$
\begin{aligned}
\frac{\partial s_t}{\partial e_{t-1}} &= - \frac{F_e}{F_s} \\
= &- \frac{-u''(c_t) \left(w_t h'(e_{t-1}) - R_t \right) + R^e_{t-1} u''(d_{t+1}) \left(\alpha^2 (w^e_{t+1})^2 h(e_{t-1}) h'(e_{t-1}) \gamma - \frac{R^e_{t+1} s_t}{(l(_e t-1))^2} l'(e_{t-1} \right)}{u''(c_t) + (R^e_{t+1})^2 \frac{1}{l(e_{t-1})} u''(d_{t+1})} \\
= &- \frac{\frac{u''(c_t)}{R^e_{t+1}} \left(\alpha^2 (w^e_{t+1})^2 h'(e_{t-1}) h(e_{t-1}) \gamma l(e_{t-1}) + l'(e_{t-1}) d_{t+1} (\frac{1}{\epsilon} - 1) - l'(e_{t-1}) v_l(z_{t+1}, l(e_{t-1})) \right)}{-u''(c_t) + (R^e_{t+1})^2 \frac{1}{l(e_{t-1})} u''(d_{t+1})} \\
&+ \frac{R^e_{t-1} u''(d_{t+1}) \left(\alpha^2 (w^e_{t+1})^2 h(e_{t-1}) h'(e_{t-1}) \gamma - \frac{R^e_{t+1} s_t}{(l(_e t-1))^2} l'(e_{t-1} \right)}{u''(c_t) + (R^e_{t+1})^2 \frac{1}{l(e_{t-1})} u''(d_{t+1})}
\end{aligned}
$$

$$= -\frac{\alpha^2(w^e_{t+1})^2 h(e_{t-1})h'(e_{t-1})\gamma\left(\frac{u''(c_t)}{R^e_{t+1}}l(e_{t-1}) + R^e_{t+1}u''(d_{t+1})\right)}{u''(c_t) + (R^e_{t+1})^2\frac{1}{l(e_{t-1})}u''(d_{t+1})}$$
$$-\frac{l'(e_{t-1})}{u''(c_t) + (R^e_{t+1})^2\frac{1}{l(e_{t-1})}u''(d_{t+1})}\cdot$$
$$\left(\frac{u''(c_t)}{R^e_{t+1}}\left(d_{t+1}(\frac{1}{\epsilon}-1) - v_l(z_{t+1}, l(e_{t-1}))\right) - \frac{(R^e_{t+1})^2 s_t u''(d_{t+1})}{(l(e_{t-1}))^2}\right)$$
$$= -\frac{l(e_{t-1})}{R^e_{t+1}}\alpha^2(w^e_{t+1})^2 h(e_{t-1})h'(e_{t-1})\gamma$$
$$-\frac{l'(e_{t-1})\cdot\left[\frac{u''(c_t)}{R^e_{t+1}}\left(d_{t+1}(\frac{1}{\epsilon}-1) - v_l(z_{t+1}, l(e_{t-1})) - \frac{(R^e_{t+1})^2 s_t u''(d_{t+1})}{(l(e_{t-1}))^2}\right)\right]}{u''(c_t) + (R^e_{t+1})^2\frac{1}{l(e_{t-1})}u''(d_{t+1})}$$

Hence

$$\frac{\partial d_{t+1}}{\partial e_{t-1}} = \alpha^2(w^e_{t+1})^2 h(e_{t-1})h'(e_{t-1})\gamma + \frac{R^e_{t+1}}{l(e_{t-1})}\left[\frac{l(e_{t-1})}{R^e_{t+1}}\alpha^2(w^e_{t+1})^2 h(e_{t-1})h'(e_{t-1})\gamma\right.$$
$$\left.-\frac{l'(e_{t-1})\cdot\left[\frac{u''(c_t)}{R^e_{t+1}}\left(d_{t+1}(\frac{1}{\epsilon}-1) - v_l(z_{t+1}, l(e_{t-1})) - \frac{(R^e_{t+1})^2 s_t u''(d_{t+1})}{(l(e_{t-1}))^2}\right)\right]}{u''(c_t) + (R^e_{t+1})^2\frac{1}{l(e_{t-1})}u''(d_{t+1})} - \frac{l'(e_{t-1})}{l(e_{t-1})}s_t\right]$$
$$= -\frac{l(e_{t-1})}{u''(c_t) + \frac{(R^e_{t+1})^2}{l(e_{t-1})}u''(d_{t+1})}\frac{R^e_{t+1}}{l(e_{t-1})}\left[\frac{u''(c_t)}{R^e_{t+1}}\left(d_{t+1}(\frac{1}{\epsilon}-1) - v_l(z_{t+1}, l(e_{t-1}))\right.\right.$$
$$\left.\left.-\frac{(R^e_{t+1})^2 s_t u''(d_{t+1})}{(l(e_{t-1}))^2}\right) + \frac{s_t}{l(e_{t-1})}\left(u''(c_t) + \frac{(R^e_{t+1})^2}{l(e_{t-1})}u''(d_{t+1})\right)\right]$$
$$= -\frac{l(e_{t-1})}{u''(c_t) + \frac{(R^e_{t+1})^2}{l(e_{t-1})}u''(d_{t+1})}\cdot$$
$$\left[\underbrace{\frac{u''(c_t)}{l(e_{t-1})}\left(d_{t+1}(\frac{1}{\epsilon}-1) - v_l(z_{t+1}, l(e_{t-1}))\right)}_{<0}\right.$$
$$\left.+\underbrace{\frac{(R^e_{t+1})^2 s_t u''(d_{t+1})(1 + r^e_{t+1}) + R^e_{t+1}s_t u''(d_{t+1})}{(l(e_{t-1}))^2}}_{<0}\right]$$

As $\frac{\partial d_{t+1}}{\partial e_{t-1}} < 0$, the FOC with respect to e_{t-1}, calculated in equation (6.7), is strictly monotonically decreasing in e_{t-1}, which means that the interior optimal level of education in this setting with endogenous longevity is unique as well.

7 Conclusion

This thesis investigates aspects, that influence the mechanism of the Ben-Porath effect, which states in general that the optimal amount of schooling increases as a result of increasing lifetime working hours (see Acemoglu and Johnson (2007)). For example lifetime utility, consumption, the wage rate, the interest rate, leisure time, survival probabilities and the conditions of living affect the dynamics of the Ben-Porath effect. As already mentioned in Chapter 1, biological and institutional constraints could either restrict the Ben-Porath effect or could even support it.
The three period OLG model of Nishimura, Pestieau, and Ponthiere (2015), that is presented in Chapters 2 and 6 shows, that the wage rate positively affects the impact of rising longevity on the optimal education decision, whereas human capital decay and the individual's rejection of old age labor decrease the consequences of rising longevity. In the case of endogenous longevity (Chapter 6) it is shown that for a higher influence of the level of education on longevity, the Ben-Porath effect becomes stronger. It is also mentioned in this Chapter, that the level of education affects the disutility of old age labor, which again supports the Ben-Porath effect.
The theory in Chapter 3, based on Cipriani and Makris (2006), points out that expectations throughout a population can lead to a self-fulfilling prophecy, if the economy follows the Ben-Porath effect. This means that a higher life expectancy (and an adapted, healthier lifestyle) can lead to an increased lifetime.
Chapter 4, based on Cervellati and Sunde (2013), discusses the influence of survival probabilities on the Ben-Porath effect. Thereby rising longevity is specified by the origins of the increased life expectancy. In fact, especially survival probabilities in the last part of life have increased throughout the last decades. It is shown that survival probabilities in high ages mainly increase the rate of return of investments in education, but do not heavily affect the cost of a longer education period. This can be seen as an indication that rising longevity throughout the last decades has increased the optimal level of education. This theoretical result can also numerically be observed in Chapter 4 on the basis of a quantitative analysis of this theory using Austrian survival probabilities *Statistik Austria: Sterbetafeln* (2017). In this numerical example, the optimal education period for individuals in 1960 and 2010 is calculated and an increase from 15 years in 1960 to 20 years in 2010 is observed.
Chapter 5, based on Sheshinski (2009), shows that the Hazard Rate can be used to specify the impact of the Ben-Porath effect. This Chapter additionally points out that the conditions of living positively affect the optimal education period and the retirement age.
In this thesis, four different models have been introduced in order to study the effect of longevity on the optimal education choice and retirement age. To model the Ben-Porath effect, overlapping generations models are commonly used. Their structure suites this

problem very well, since periods of life, where individuals are retired or in a schooling process can be modeled as own life cycle stages.
The second commonly used type of model in order to investigate the Ben-Porath effect has a structure, that is similar to a Ramsey model. Flows of consumption, education, earnings and other variables are thereby investigated. Age can either be modeled as a discrete variable or as a continuous variable. In discrete models flows of variables per unit of time (consumption per year) are studied.
In both commonly used types of models, different aspects can be implemented. Chapter 4 and Chapter 5 show that the implementation of survival probabilities or survival distribution function increase the complexity of a model massively. Also the implementation of leisure and an endogenously determined retirement age increase the complexity of models.
In a conclusion I want to point out that the adaption of the human life cycle as a result of increasing longevity is a very complex process. Consequently many different aspects influence the Ben-Porath effect. Hence it is not straight forward to state, that rising longevity positively affects the optimal education period and the retirement age. However most results also of this thesis show that in general, longevity has a positive effect on the optimal education period and the retirement age.

8 Appendix

8.1 OLG Models

The basic idea of overlapping generations models, that started with Samuelson (1958) and Diamond (1965) is to use the human life cycle and to create a heterogeneity in the population concerning the age of individuals. This is technically done by dividing the population into generations. Every generation lives for a certain amount of periods. Agents are assumed to have a finite life time, whereas the economy has an infinite life time. Figure 8.1 illustrates an overlapping generations model with three periods.

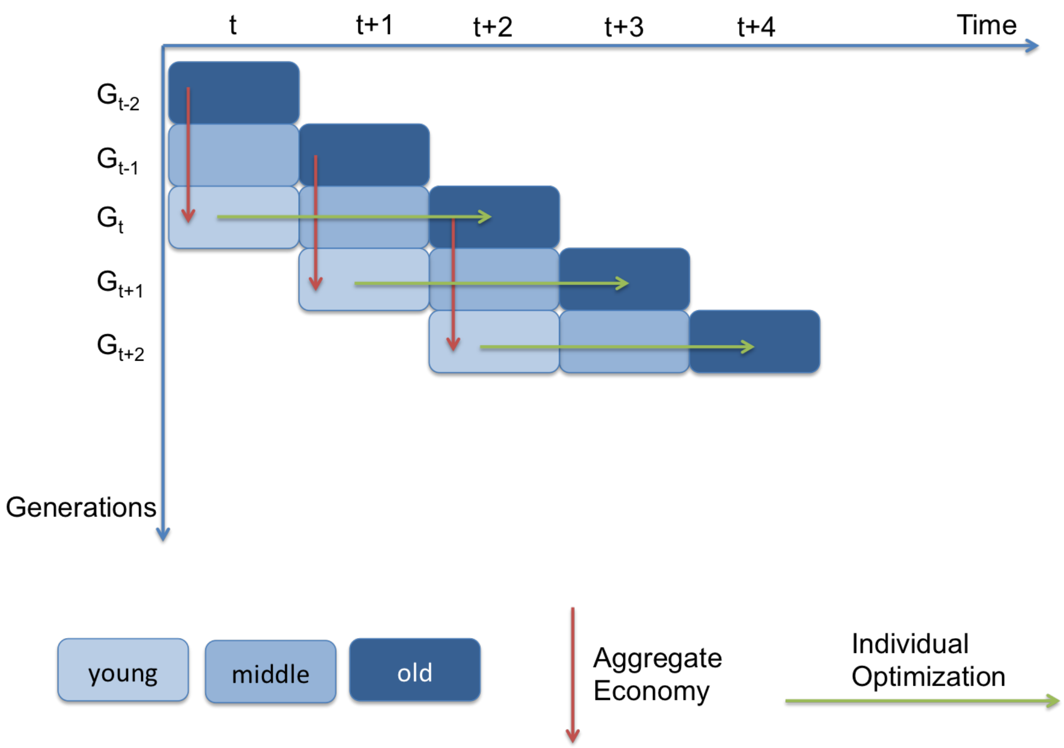


Figure 8.1: Overlapping Generations and Aggregate Economy

8.2 Elasticity

For a function $f(x) = y$, the point elasticity $\epsilon_{x,y}$ is a measure of a relative change of y as a result of a relative change of x.

$$\epsilon_{x,y} = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} \overset{\Delta \to 0}{=} \frac{\frac{\mathrm{d}y}{y}}{\frac{\mathrm{d}x}{x}} = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{x}{y} = y'\frac{x}{y} = \frac{\mathrm{d}\ \ln(y)}{\mathrm{d}\ \ln(x)}$$

For $0 < |\epsilon_{x,y}| < 1$ the variable y is called to be inelastic, since it changes relatively less as result of a change in x than the variable x relatively changes itself. The opposite holds for $|\epsilon_{x,y}| > 1$, where y is called to be elastic. For $|\epsilon_{x,y}| \to 0$, y is called to be absolutely inelastic and for $|\epsilon_{x,y}| \to \infty$, y is called to be completely elastic. For the function of relative risk aversion CRRA (introduced also in the Appendix), Figure 8.2 visualizes the elasticity of utility for different relative risk aversion parameters σ. This Figure shows, that the Elasticity of the CRRA function is negative for $0 < x < 1$. This means that y decreases for a marginal increase of x. On the other hand for $1 < x$, the elasticity is positive, implying that a marginal increase of x also increases the output y. Figure 8.2 additionally points out that the output of a CRRA function is very sensitive for values x near 1.

$$\epsilon_{c,u} = c^{-\phi} \cdot c \cdot \left(\frac{c^{1-\phi}-1}{1-\phi}\right)^{-1} = c^{1-\phi}\left(\frac{c^{1-\phi}-1}{1-\phi}\right)^{-1} \tag{8.1}$$

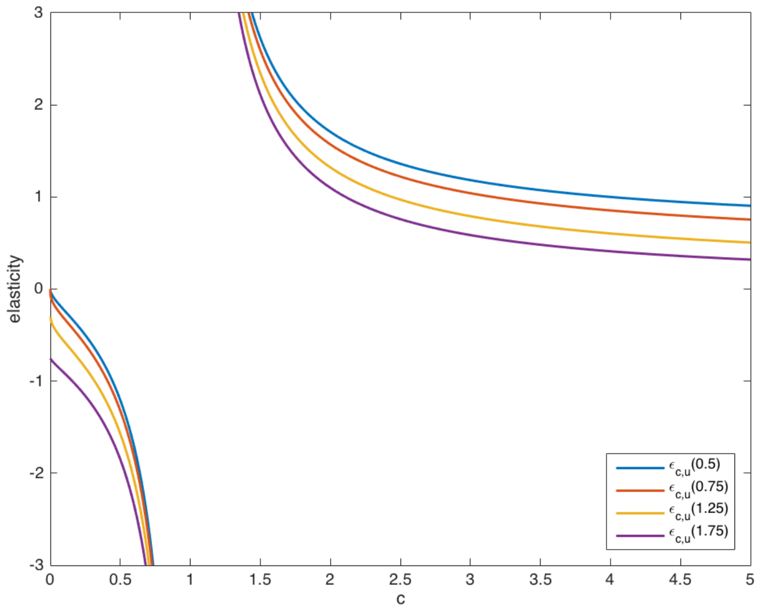


Figure 8.2: Elasticity of a CRRA function.

8.3 Inter temporal elasticity of substitution and relative risk aversion

The inter temporal elasticity of substitution is defined as marginal change in the growth of consumption due to a marginal increase in the growth of utility.

$$IES(c) = -\frac{\mathrm{d}\ln(c_{t+1}/c_t)}{\mathrm{d}(\ln(u'(c_{t+1})/u'(c_t)))}$$

For continuous consumption, the inter temporal elasticity of substitution can be written as

$$\begin{aligned} IES(c) = &- \frac{\partial(\dot{c}_t/c_t)}{\partial(\dot{u}'(c_t)/u'(c_t))} = -\frac{\partial(\dot{c}_t/c_t)}{\partial(u''(c_t)/dotc_t/u'(c_t))} = \\ &- \frac{\partial(\dot{c}_t/c_t)}{\partial(u''(c_t)/u'(c_t)\cdot\dot{c}_t/c_t)} = -\frac{u'(c_t)}{u''(c_t)\cdot c_t} \end{aligned}$$

The Arrow-Pratt measure of relative risk aversion is defined as

$$RRA(c) = -\frac{d(u'(c_t))}{d(c_t)}\frac{c_t}{u'(c_t)} = -\frac{u''(c_t)\cdot c_t}{u'(c_t)}$$

Per assumption utility rises in consumption $u'(c) > 0$ but the increase of utility decreases with the amount of consumption $u''(c) < 0$. An agent with a concave utility function is called to be risk averse, as its utility of the expected value of a gamble is greater than the expected value of the gamble $E(u(c)) < u(E(c))$. The following function $u(c)$ is called constant relative risk aversion (CRRA) function, since the Arrow-Pratt measure for relative risk aversion of an agent with this utility function is constant in consumption.

$$\begin{aligned} u(c) &= \frac{c^{1-\phi}-1}{1-\phi} \\ u'(c) &= c^{-\phi} \\ u'(c) &= -\phi c^{-\phi-1} \\ RRA(c) &= -\frac{-\phi c^{-phi-1}c}{c^{-\phi}} = \phi \\ IES(c) &= \frac{1}{\phi} \end{aligned} \tag{8.2}$$

Figure 8.3 illustrates the CRRA function for different values of ϕ. It shows that the utility gain of consumption for $0 < c < 1$ is very high, whereas it flattens for $c > 1$. For higher values of ϕ, the curvature increases.

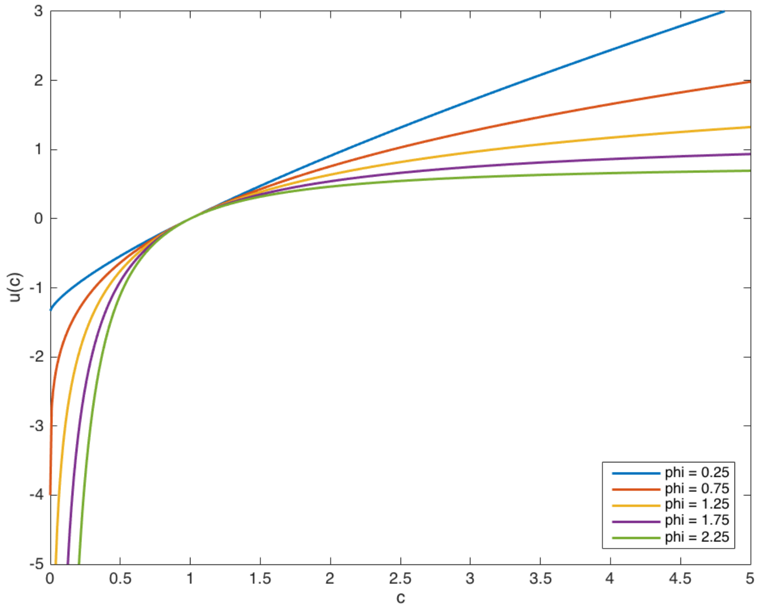


Figure 8.3: CRRA function visualized for different parameters.

8.4 Matlab Code Chapter 4

```
p=xlsread('surv_probs.xlsx',1,'A2:A102');
    %Read survival probabilities for one year
p5=ones(1,Tmax5);
p5(1) = p(1)*p(2)*p(3)*p(4)*p(5);
 for j = 2:Tmax5
     p5(j)=p5(j-1);
         for i = 1:5
             p5(j) = p5(j)*p((j-1)*5+i);
         end
 end
    %calculate survival probabilities for 5 year periods

solutions = zeros(10,13);
solutions(1,1:4) = lsqnonlin(@find_opt1, [1,2,0.9,0.5*ones(1,1)],
                            [-1000,zeros(1,3)],[1000,1000,ones(1,2)]);
solutions(2,1:5) = lsqnonlin(@find_opt2, [1,2,0.9,0.5*ones(1,2)],
                            [-1000,zeros(1,4)],[1000,1000,ones(1,3)]);
solutions(3,1:6) = lsqnonlin(@find_opt3, [1,2,0.9,0.5*ones(1,3)],
                            [-1000,zeros(1,5)],[1000,1000,ones(1,4)]);
solutions(4,1:7) = lsqnonlin(@find_opt4, [1,2,0.9,0.5*ones(1,4)],
                            [-1000,zeros(1,6)],[1000,1000,ones(1,5)]);
solutions(5,1:8) = lsqnonlin(@find_opt5, [1,2,0.9,0.5*ones(1,5)],
                            [-1000,zeros(1,7)],[1000,1000,ones(1,6)]);
solutions(6,1:9) = lsqnonlin(@find_opt6, [1,2,0.9,0.5*ones(1,6)],
                            [-1000,zeros(1,8)],[1000,1000,ones(1,7)]);

utility = zeros(10,1);
for j = 1:10
    for i = 1:j
        utility(j) = utility(j,2) + p5(i) * (u(solutions(j,2))
                     + u(1-solutions(j,i+3)));
    end
    for k = (j+1):Tmax5
        utility(j) = utility(j,2) + p5(k) * (u(solutions(j,2))
                     + u(1-solutions(j,3)));
    end
end

%%%%END%%%%

function [opt] = find_opt5(input)

Tmax5=20;
p=xlsread('surv_probs.xlsx',1,'A2:A102');
p5=ones(1,Tmax5);
p5(1) = p(1)*p(2)*p(3)*p(4)*p(5);
 for j = 2:Tmax5
     p5(j)=p5(j-1);
         for i = 1:5
             p5(j) = p5(j)*p((j-1)*5+i);
         end
 end
```

```
S = 5; %entspricht 10 Jahren
R = 15; %entspricht 13*5=65 Jahren

h = @(leisure) sum(leisure);

    BC1 = 0;
    for i = (S+1):R
        BC1 = BC1 +p5(i);
    end

    BC2 = 0;
    for i = 1:Tmax5
        BC2 = BC2 + p5(i);
    end

        lambda = input(1);
        c = input(2);
        Labor = input(3);
            edu_intense = zeros(1,S);
            for i = 1:S
                edu_intense(i) = input(i+3);
            end

    opt(1) = uabl(c)-lambda;
    opt(2) = uabl(1-Labor) - lambda * wage(h(edu_intense));
    opt(3) = wage(h(edu_intense))*BC1*Labor-c*BC2;
    opt(4) = p5(1) * uabl(1-edu_intense(1))
             - lambda * wage_abl(h(edu_intense))*Labor*BC1;
    opt(5) = p5(2) * uabl(1-edu_intense(2))
             - lambda * wage_abl(h(edu_intense))*Labor*BC1;
    opt(6) = p5(3) * uabl(1-edu_intense(3))
             - lambda * wage_abl(h(edu_intense))*Labor*BC1;
    opt(7) = p5(4) * uabl(1-edu_intense(4))
             - lambda * wage_abl(h(edu_intense))*Labor*BC1;
    opt(8) = p5(5) * uabl(1-edu_intense(5))
             - lambda * wage_abl(h(edu_intense))*Labor*BC1;

end

%%%%%END%%%%%
```

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