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DIPLOMARBEIT

Comparison of analysis strategies for the determination of the ITRF

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Abstract

Reference systems are essential for navigation and surveying as well as for determining the shape of the Earth and its position and orientation in space. They are required for the precise determination of satellite orbits and serve as basis for Earth science applications such as measurements of the Earth's gravity field, atmosphere studies, detection of plate tectonic motion or global sea level change monitoring.
The International Terrestrial Reference Frame is the realization of the *International Terrestrial Reference System*. It is generated by combining observational data of the space geodetic techniques *Global Navigation and Satellite Systems*, *Very Long Baseline Interferometry*, *Satellite Laser Ranging* and *Doppler Orbitography and Radiopositioning Integrated by Satellite*. The purpose of combining the techniques is to exploit their individual strengths, since each technique is characterized by varying potential when it comes to determining certain geodetic parameters.
The combination centers of the *International Earth Rotation and Reference Systems Service* are responsible for the creation of the ITRF. They apply different strategies for the combination of the data. Comparisons of the independent solutions improve the reliability and provide an assessment of the accuracy of the ITRF. In this thesis the similarities and differences of the two approaches are discussed.
Furthermore, based on the VLBI campaign *CONT14*, both methods are compared by estimating coordinates of the participating stations. Thereby the results of the combination of solutions depend more on arbitrary operator decisions. However, when identical conditions are applied to define the geodetic datum, the resulting station coordinates only differ on the sub-millimeter level.

Kurzfassung

Referenzsysteme sind für Navigationszwecke und Vermessungsaufgaben unverzichtbar. Sie werden für die hochgenaue Bestimmung von Satellitenbahnen benötigt und dienen als Grundlage für die Referenzierung geophysikalischer oder geologischer Parameter. Dazu zählen unter anderem die Modellierung des Erdschwerefeldes, Untersuchungen der Erdatmosphäre, die Beschreibung von tektonischen Vorgängen und die Ermittlung des Meeresspiegelanstiegs.

Referenzsysteme werden durch Referenzrahmen realisiert, so auch das *Internationale Terrestrische Referenzsystem* (ITRS) durch den *Internationalen Terrestrischen Referenzrahmen* (ITRF). Dieser wird aus Beobachtungsdaten der geodätischen Weltraumverfahren *Global Navigation Satellite Systems*, *Very Long Baseline Interferometry*, *Satellite Laser Ranging* und *Doppler Orbitography and Radiopositioning Integrated by Satellite* errechnet. Jedes Verfahren besitzt hinsichtlich der Bestimmung geodätischer Parameter diverse Stärken und Schwächen. Werden die Beobachtungsdaten dieser Verfahren kombiniert, kommen die individuellen Vorteile zur Geltung und Nachteile werden durch die anderen Verfahren kompensiert.

Für die Berechnung des ITRF sind die Kombinationszentren des *International Earth Rotation and Reference Systems Service* zuständig. Dabei werden für die Kombination der Daten zwei unterschiedliche Strategien angewandt, was zu zwei voneinander unabhängigen Realisierungen führt. Ein Vergleich der Ergebnisse steigert die Verlässlichkeit und ermöglicht Rückschlüsse auf die Genauigkeit. In dieser Arbeit werden die beiden Ansätze verglichen und ihre Gemeinsamkeiten und Unterschiede behandelt.

Außerdem werden, basierend auf der VLBI-Messkampagne *CONT14*, mit beiden Methoden Koordinaten für die entsprechenden Stationen geschätzt. Dabei hängen die Ergebnisse der Kombination von Lösungen deutlich stärker von willkürlichen Entscheidungen ab. Erfolgt jedoch die Definition des geodätischen Datums für beide Ansätze ident, unterscheiden sich die resultierenden Stationskoordinaten lediglich im Submillimeterbereich.

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Contents

1 Introduction

1.1 Reference systems and reference frames

Reference systems are required for describing the position and motion of the Earth and other celestial bodies including artificial satellites. They are essential for referencing positions and movements of objects on the surface of the Earth and for determining the Earth's gravity field as well as its variation in time. Basically, reference systems in astronomy and geodesy are four-dimensional, comprising three geometric coordinates and a time coordinate. They are defined by general statements which ideally describe the position of the origin and the orientation of the coordinate axes. A reference system has to be realized by a reference frame which consists of a set of well-determined physical reference points that uniquely fix the coordinate system so that it can be used for the quantitative description of positions or movements. Depending on the location of the reference points, we can distinguish between space-fixed celestial reference systems and terrestrial reference systems which are fixed to the Earth [Torge and Müller, 2012, ch. 2].

1.2 International Celestial Reference System and Frame

"An inertial system is needed in order to model the ephemerides of celestial bodies in space, including those of artificial satellites. At the classical point of view, such a system is characterized by Newton's laws of motion. It is either at rest or in the state of uniform rectilinear motion without rotation. A space-fixed system represents an approximation to an inertial system" [Torge and Müller, 2012, ch. 2.3.1].

The International Celestial Reference System (ICRS) is defined as a barycentric celestial reference system (BCRS), which means that its origin is located in the barycenter of the solar system. The orientation of the ICRS is realized by the coordinates of extragalactic radio sources (quasars) which are determined by VLBI observations (see section 3.1). Due to the large distances between the Earth and the radio sources (>1.5 billion light years) it can be assumed that the radio sources have no rotational motion [Schuh and Behrend, 2012; Torge and Müller, 2012, ch. 2.4.1].

The ICRS is realized through the International Celestial Reference Frame (ICRF), established and maintained by the International Earth Rotation and Reference Systems Service (IERS). The first realization of the ICRF, ICRF1, was constructed upon 212 defining extragalactic radio sources in 1995. Two extensions of the frame were created: ICRF1-Ext.1 by using VLBI data available until April 1999 and ICRF1-Ext.2 by using VLBI data available until May 2002 [Petit and Luzum, 2010]. According to Torge and Müller [2012], the coordinates of the radio sources of ICRF1 are determined with a precision better than 1 mas on the average and 0.1 mas for the most precisely observed objects.

ICRF2 was constructed in 2009 and utilizes more than 3400 radio sources, including 295 defining sources which, according to Torge and Müller [2012] were determined with a precision of about 0.05 mas on average, and 0.02 mas at best. Besides the improved precision, the major enhancements compared to ICRF1 are an increased position stability and a more heterogeneous distribution of radio sources [Petit and Luzum, 2010].

1.3 International Terrestrial Reference System and Frame

The description of terrestrial reference systems in general and the definition of the International Terrestrial Reference System refer to the IERS Conventions 2010 [Petit and Luzum, 2010].

An Earth-fixed reference system is needed for describing positions and movements of objects on and close to the Earth's surface. Earth-fixed implies that the reference system rotates with the Earth and coordinates of points on the surface of the Earth undergo only small variations with time, which are caused by geophysical effects (tectonic or tidal deformations). It serves as basis for national surveys, geoinformation systems and navigation. It is also used as geometric frame for the determination of the Earth's gravity field and other applications which aim to retrieve geophysical or geological properties of the Earth, as well as for modeling deformations of the Earth's body and other terrestrial variations with time.

A TRS is defined by its origin and a basis of the associated vector space. The basis vectors have to be right-handed, orthogonal and of the same length. The triplet of unit vectors collinear to the basis vectors expresses the orientation of the TRS and the common length of these vectors its scale.

The appropriate definition of a terrestrial reference system (TRS) is fundamental for the applications mentioned above. Any defect on the origin, orientation or scale would have an impact on the results and interpretation of geodetic or geophysical applications.

The ITRS definition fulfills the following conditions[1]

1. It is geocentric, its origin being the center of mass for the whole Earth, including oceans and atmosphere;

2. The unit of length is the meter (SI). The scale is consistent with the TCG[2] time coordinate for a geocentric local frame, in agreement with IAU[3] and IUGG[4] (1991) resolutions. This is obtained by appropriate relativistic modeling;

3. Its orientation was initially given by the BIH[5] orientation at 1984.0;

4. The time evolution of the orientation is ensured by using a no-net-rotation condition with regards to horizontal tectonic motions over the whole Earth.

Primary realizations of the ITRS are produced by the IERS and are called International Terrestrial Reference Frame (ITRF). Thirteen ITRF versions have been produced so far, starting with the ITRF88 and ending with the ITRF2014. The latter has been published in January 2016. The basic idea for the creation of the ITRF is to combine station positions (and velocities), using observations of the following space geodetic techniques: Very Long Baseline Interferometry (VLBI), Satellite

[1] reference: Petit and Luzum, 2010
[2] Temps Coordonné Geocentrique - Geocentric Coordinate Time
[3] International Astronomical Union
[4] International Union of Geodesy and Geophysics
[5] Bureau International d l'Heure - International Time Bureau

Laser Ranging (SLR), Global Navigation Satellite Systems (GNSS), and Doppler Orbitography Radiopositioning Integrated by Satellite (DORIS). The purpose of combining different space geodetic techniques is to benefit from their individual strengths when it comes to determining certain geodetic parameters, as explained in section 4.1.

The services which are responsible for the analysis of the observations of each technique are the International GNSS Service (IGS), the International VLBI Service for Geodesy and Astrometry (IVS), the International Laser Ranging Service (ILRS) and the International DORIS Service (IDS). The observations are processed by the analysis centers of each service.

Up to the ITRF2000, long-term global solutions (comprising station positions and velocities) of the four techniques were used as input for the ITRF generation. Starting with the ITRF2005, time series of station positions and Earth Orientation Parameters (EOP) have been used as input data for the ITRF construction. Table 1.1 displays the time span of the input data for the ITRF2008 as well as the temporal resolution for each of the contributing techniques. The time series of weekly or session-wise data are provided in SINEX[6] format. The final TRF computations are carried out by the IERS combination centers.

Table 1.1: ITRF2008 input data.[7]

Technique	Service	Time span	Resolution	Type
VLBI	IVS	1980.0 - 2009.0	session-wise	normal equation
GNSS	IGS	1997.0 - 2009.5	weekly	solution
SLR	ILRS	1983.0 - 2009.0	fortnightly, weekly	solution
DORIS	IDS	1993.0 - 2009.0	weekly	solution

The latest version for which literature is available is the ITRF2008, which, referring to Altamimi et al. [2011], comprises 934 stations located at 580 sites. Figure 1.1 shows that the distribution of the sites is rather inhomogeneous as 463 are located in the northern and 117 in the southern hemisphere, respectively.

[6] Solution INdependent EXchange
[7] source: `http://itrf.ensg.ign.fr/ITRF_solutions/2008/input_data.php`
[8] source: Altamimi et al., 2011

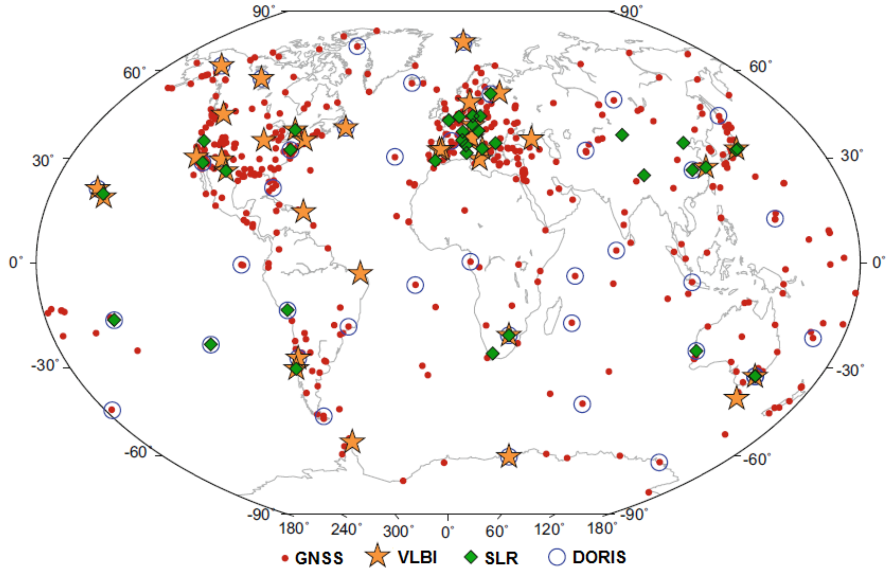


Figure 1.1: ITRF2008 station network.[8]

Currently three combination centers contribute to the determination of the upcoming versions: the Institut Géographique National (IGN, Paris), the Deutsches Geodätisches Forschungsinstitut (DGFI, Munich), and the Jet Propulsion Laboratory (JPL, Pasadena).

The combination centers at DGFI and IGN produce individual realizations (DTRF and ITRF, respectively) using different computation strategies which will be discussed in sections 4.3.2 and 4.3.3. Comparisons of the independent solutions improve the reliability and provide an assessment of the accuracy of the ITRF. Both combination methods make use of local ties (see section 4.2) in co-location sites where two or more geodetic techniques are operated. The local ties are used as additional observations with proper variances. They are usually derived from local surveys using either classical tachymeters or GNSS.

According to Altamimi et al. [2013], applications such as global sea level change detection require an accuracy and temporal stability of the origin and scale of the ITRF at the level of 1 mm and 0.1 mm/year. "This requirement is at least ten times higher than what is achievable today, mainly due to the degradation of the network of space geodetic techniques and their intrinsic systematic errors."

2 Mathematical fundamentals

This chapter summarizes the most important equations for the Gauss-Markov model and constitutive mathematical operations such as parameter reduction or stacking of normal equation systems. A compact and very comprehensible summary of least squares adjustment theory can be found in Thaller [2008, ch. 2]. Detailed explanations can be found in Niemeier [2008, ch. 4] or Navratil [2006, ch. 2].

Furthermore, this chapter contains the basics of geodetic datum definition as well as an introduction to similarity transformation along with the estimation of transformation parameters. The content of sections 2.4 to 2.6 has been adopted from the IERS Conventions 2010 Petit and Luzum [2010] and from Altamimi et al. [2002].

2.1 Least squares adjustment and Gauss-Markov model

The first fundamental step is to formulate a functional connection between u unknown parameters $\hat{X}$ and n independent observations $\hat{L}$. Each observation is expressed as a function of the unknown parameters:

$$\hat{L} = \varphi(\hat{X}) \ . \tag{2.1}$$

Since the observations are gained from measurements which succumb various error influences, equation (2.1) can only be fulfilled exactly if a vector of residuals v is introduced. It summarizes all random errors that influence the measurements. Thus, the vector of observations $\hat{L}$ can be separated into the original observations L and the residuals:

$$L + v = \hat{L} \ . \tag{2.2}$$

Inserting (2.2) into (2.1) yields

$$L + v = \varphi(\hat{X}) \ . \tag{2.3}$$

Usually the function φ that describes the relationship between observations and parameters is not linear. The Gauss-Markov model, however, requires a linear relationship. Equation (2.3) is linearized by a first-order Taylor series expansion. For this purpose, a priori values X_0 are used.

Furthermore, it is assumed that the residuals Δx are small compared to the a priori values. From the Taylor series expansion, the so-called design matrix A is obtained. It consists of all first order derivatives of the functions φ_1 to φ_n with respect to the unknown parameters $\hat{X}_1$ to $\hat{X}_u$:

$$A = \begin{pmatrix} \frac{\delta\varphi_1}{\delta\hat{X}_1} & \frac{\delta\varphi_1}{\delta\hat{X}_2} & \cdots & \frac{\delta\varphi_1}{\delta\hat{X}_u} \\ \frac{\delta\varphi_2}{\delta\hat{X}_1} & \frac{\delta\varphi_2}{\delta\hat{X}_2} & \cdots & \frac{\delta\varphi_2}{\delta\hat{X}_u} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\delta\varphi_n}{\delta\hat{X}_1} & \frac{\delta\varphi_n}{\delta\hat{X}_2} & \cdots & \frac{\delta\varphi_n}{\delta\hat{X}_u} \end{pmatrix} \tag{2.4}$$

Equation 2.3 now reads as follows:

$$L + v = \varphi(X_0) + A \cdot \Delta x \quad . \tag{2.5}$$

After introducing the reduced vector of observations l (observed-computed)

$$l = L - \varphi(X_0) \tag{2.6}$$

equation 2.5 now reads as

$$v = A \cdot \Delta x - l \tag{2.7}$$

which is also called ‘equation of residuals’.

Before (2.7) can be solved, stochastic information needs to be introduced. The main diagonal of the covariance matrix of observations Σ_{ll} contains the variances of the individual observations. The other elements correspond to the covariances. If the observations are uncorrelated, those elements are zero. The cofactor matrix of observations Q_{ll} is obtained by introducing the a priori variance factor of unit weight σ_0^2:

$$Q_{ll} = \frac{1}{\sigma_0^2}\Sigma_{ll} \quad . \tag{2.8}$$

For a reciprocal weighting of the observations according to their variances (observations with larger variance are assigned less weight), the weight matrix P is introduced by inversion:

$$P = Q_{ll}^{-1} \ . \tag{2.9}$$

In case of uncorrelated observations P is a diagonal matrix containing the variances of the observations σ_i^2 and the a priori variance factor of unit weight σ_0^2:

$$P = \begin{pmatrix} \frac{\sigma_0^2}{\sigma_1^2} & & & 0 \\ & \frac{\sigma_0^2}{\sigma_2^2} & & \\ & & \ddots & \\ 0 & & & \frac{\sigma_0^2}{\sigma_n^2} \end{pmatrix} \ . \tag{2.10}$$

Now the fundamental requirement of the least squares method, namely the minimization of the weighted square sum of the residuals can be introduced:

$$v^T P v \rightarrow min \ . \tag{2.11}$$

This requirement leads to the so-called normal equation system

$$A^T P A \cdot \Delta x = A^T P l \tag{2.12}$$

with the normal equation matrix

$$N = A^T P A \tag{2.13}$$

and the right hand side vector

$$b = A^T P l \ . \tag{2.14}$$

The unknown residuals Δx can be derived from (2.12) as follows:

$$\Delta x = N^{-1} b \ . \tag{2.15}$$

If observations and parameters are linked by a linear relationship, Δx is identical to $\hat{X}$ and the parameters can be obtained directly from equation (2.15). If a linearization had to be performed, the unknowns Δx represent an addition to the a priori values X_0 and the parameters are obtained by

$$\hat{X} = X_0 + \Delta x \ . \tag{2.16}$$

After computing the residuals according to (2.7), the minimized weighted square sum can be derived and divided into two parts:

$$v^T P v = (A \cdot \Delta x - l)^T P (A \cdot \Delta x - l) = \ldots = l^T P l - \Delta x^T b \ . \tag{2.17}$$

Apparently, $l^T P l$ only depends on the observations, whereas $\Delta x^T b$ also depends on the solution. Consequently, for computing the weighted square sum of residuals in a combination of datum-free normal equation systems, only $l^T P l$ has to be known from the single input normal equation systems, whereas the second part is derived from the combined solution itself [Thaller, 2008].

With the square sum of residuals the a posteriori variance of unit weight can be computed:

$$\hat{\sigma}_0^2 = \frac{v^T P v}{n - u} \ . \tag{2.18}$$

The covariance matrix of the estimated parameters can be obtained with the law of error propagation:

$$Q_{xx} = \hat{\sigma}_0^2 \cdot N^{-1} \ . \tag{2.19}$$

Based on the above equations, various mathematical operations can be applied to a normal equation system. Those relevant for this thesis are explained in the following sections.

2.2 Parameter reduction

As the size of a normal equation matrix increases, its inversion becomes more sophisticated. Therefore, due to limited computer capacities in the past, it was often necessary to keep the number of parameters to be estimated (and hence the size of the normal equation matrix) as small as possible.

The purpose of parameter reduction is to reduce the size of the normal equation system without affecting the solution for the remaining parameters. Nowadays, parameters are pre-eliminated when there are several parameter types (for example station coordinates, earth orientation parameters and source positions in a SINEX file from a VLBI session) but there is only one parameter type of interest for a certain application. The general approach is to split the normal equation system into two parts: the parameters to be estimated Δx_1 and the parameters that will be reduced (pre-eliminated) Δx_2. The normal equation system then looks as follows:

$$\begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix} \cdot \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{2.20}$$

or, decomposed:

$$N_{11} \cdot \Delta x_1 + N_{12} \cdot \Delta x_2 = b_1 \quad , \tag{2.21}$$

$$N_{21} \cdot \Delta x_1 + N_{22} \cdot \Delta x_2 = b_2 \quad . \tag{2.22}$$

Solving (2.22) for Δx_2 yields

$$\Delta x_2 = N_{22}^{-1} b_2 - N_{22}^{-1} N_{21} \cdot \Delta x_1 \quad . \tag{2.23}$$

Introducing (2.23) into (2.21) then yields:

$$N_{11} \cdot \Delta x_1 + N_{12}(N_{22}^{-1} b_2 - N_{22}^{-1} N_{21} \cdot \Delta x_1) = b_1$$

$$(N_{11} - N_{12} N_{22}^{-1} N_{21}) \cdot \Delta x_1 = b_1 - N_{12} N_{22}^{-1} b_2 \quad . \tag{2.24}$$

With

$$\bar{N}_{11} = N_{11} - N_{12} N_{22}^{-1} N_{21} \tag{2.25}$$

$$\bar{b}_1 = b_1 - N_{12} N_{22}^{-1} b_2 \tag{2.26}$$

equation (2.24) can be written as

$$\bar{N}_{11} \cdot \Delta x_1 = \bar{b}_1 \quad . \tag{2.27}$$

It is important to mention that the reduced parameters are estimated implicitly and can be retrieved with equation (2.23).

2.3 Combination of normal equation systems

If two or more normal equations contain a set of common parameters (for example station coordinates), the systems can be 'stacked'. This means that common parameters are merged into one parameter.

Before the systems can be merged, certain requirements have to be fulfilled: All systems have to be of the same size. If this is not the case, the systems that do not contain all parameters that shall be estimated have to be expanded by adding zero rows and columns to the normal equation matrix and zero elements to the right hand side vector. Furthermore, the elements in every system have to be sorted in the same order.

For two combined observation equations $v = A \cdot \Delta x - l$ the components would look as follows:

$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \ , \ l = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} \ , \ v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad . \tag{2.28}$$

Assuming that the observations of the two systems are uncorrelated, the weight matrix of the combined system can be constructed upon the a priori variance factor of unit weight σ_0^2 and the variance factors of each normal equation system σ_{0i}^2:

$$P = \begin{pmatrix} \frac{\sigma_0^2}{\sigma_{01}^2} P_1 & 0 \\ 0 & \frac{\sigma_0^2}{\sigma_{02}^2} P_2 \end{pmatrix} \quad , \tag{2.29}$$

The resulting combined normal equation system looks as follows:

$$(A_1^T P_1 A_1 + A_2^T P_2 A_2) \cdot \Delta x = A_1^T P_1 l_1 + A_2^T P_2 l_2 \quad . \tag{2.30}$$

It is important to keep in mind, that only the normal equation matrices and the right hand side vectors are summed up.

$$(N_1 + N_2) \cdot \Delta x = b_1 + b_2 \tag{2.31}$$

2.4 Transformation between terrestrial reference systems

Here, geocentric terrestrial reference systems for which the origin is close to the Earth's center of mass (geocenter), the orientation is equatorial (the Z axis is the direction of the pole) and the scale is close to an SI meter are considered.

Under these assumptions, a three-dimensional similarity transformation can be applied to transform the Cartesian coordinates of any point close to the Earth from

TRS (1) to TRS (2):

$$X^{(2)} = T_{1,2} + \lambda_{1,2} \cdot R_{1,2} \cdot X^{(1)} \ . \tag{2.32}$$

$T_{1,2}$ is a translation vector, $\lambda_{1,2}$ a scale factor and $R_{1,2}$ a rotation matrix. In the application of equation (2.32), the IERS uses the linearized formulas and notation. The standard transformation between two reference systems is a Euclidean similarity of seven parameters: three translation components, one scale factor, and three rotation angles, designated respectively, $T1$, $T2$, $T3$, D, $R1$, $R2$, $R3$, and their first time derivatives: $\dot{T}1$, $\dot{T}2$, $\dot{T}3$, $\dot{D}$, $\dot{R}1$, $\dot{R}2$, $\dot{R}3$.

The transformation of a coordinate vector X_1, expressed in reference system (1), into a coordinate vector X_2, expressed in reference system (2), is given by

$$X_2 = X_1 + T + DX_1 + RX_1 \ , \tag{2.33}$$

where $T = T_{1,2}$, $D = \lambda_{1,2} - 1, R = (R_{1,2} - I)$, and I is the identity matrix so that

$$T = \begin{pmatrix} T1 \\ T2 \\ T3 \end{pmatrix} , \quad R = \begin{pmatrix} 0 & -R3 & R2 \\ R3 & 0 & -R1 \\ -R2 & R1 & 0 \end{pmatrix} \ .$$

It is assumed that equation (2.33) is linear for sets of station coordinates provided by space geodetic techniques. Generally, $X1$, $X2$, T, D, and R are functions of time. Differentiating equation (2.33) with respect to time yields

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \dot{D}X_1 + D\dot{X}_1 + \dot{R}X_1 + R\dot{X}_1 \ . \tag{2.34}$$

D and R are at the 10^{-5} level and $\dot{X}$ is about 10 cm per year, so the terms $D\dot{X}1$ and $R\dot{X}1$ which represent about 0.1 mm over 100 years are negligible. Therefore, equation 2.34 can be written as

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \dot{D}X_1 + \dot{R}X_1 \ . \tag{2.35}$$

2.5 Definition of the geodetic datum

As mentioned in section 2.4, seven parameters are needed to fix a TRF at a given epoch, to which their time derivatives are added to define the TRF time evolution. The selection of the 14 parameters, called 'datum definition', establishes the TRF origin, scale, orientation and their time evolution.

Space geodetic techniques are not sensitive to all the parameters of the TRF datum definition (see section 4.1). Therefore, their observations do not contain all the necessary information to completely establish a TRF. Hence, some additional information is needed to complete the datum definition.

If this is the case, the normal equation matrix N is singular because it has a rank deficiency corresponding to the number of datum parameters that can not be defined by the observations of a technique. In practice, the IERS analysis centers add one of the following constraints, which can be applied either to all or only to a subset of stations, to remove the rank deficiency:

1. Removable constraints: solutions for which the estimated station positions and/or velocities are constrained to external values within an uncertainty $\sigma \approx 10^{-5}\,\mathrm{m}$ for positions and $\sigma \approx 10^{-5}\,\mathrm{m/year}$ for velocities. This type of constraint is easily removable, see for instance Altamimi et al. [2002].

2. Loose constraints: solutions where the uncertainty applied to the constraints is $\sigma \geq 1\,\mathrm{m}$ for positions and $\geq 10\,\mathrm{cm/y}$ for velocities.

3. Minimum constraints, used solely to define the TRF using a minimum amount of required information.

In case of removable or loose constraints, this amounts to adding the following observation equation:

$$\hat{X} - X_0 = 0 \ , \tag{2.36}$$

where $\hat{X}$ is the vector of estimated parameters (positions and/or velocities) and X_0 is that of the a priori parameters.

In case of minimum constraints, the added equation is of the form

$$B(\hat{X} - X_0) = 0 \ , \tag{2.37}$$

where $B = (A^T A)^{-1} A^T$ and A is the design matrix of partial derivatives, constructed upon a priori values, given either by

$$A = \begin{pmatrix} . & . & . & . & . & . & . \\ 1 & 0 & 0 & x_0^i & 0 & z_0^i & -y_0^i \\ 0 & 1 & 0 & y_0^i & -z_0^i & 0 & x_0^i \\ 0 & 0 & 1 & z_0^i & y_0^i & -x_0^i & 0 \\ . & . & . & . & . & . & . \end{pmatrix} \tag{2.38}$$

when solving for station positions only, or

$$A = \begin{pmatrix} . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 1 & 0 & 0 & x_0^i & 0 & z_0^i & -y_0^i & & & & & & & \\ 0 & 1 & 0 & y_0^i & -z_0^i & 0 & x_0^i & & & & 0 & & & \\ 0 & 0 & 1 & z_0^i & y_0^i & -x_0^i & 0 & & & & & & & \\ & & & & & & & 1 & 0 & 0 & x_0^i & 0 & z_0^i & -y_0^i \\ & & & \approx 0 & & & & 0 & 1 & 0 & y_0^i & -z_0^i & 0 & x_0^i \\ & & & & & & & 0 & 0 & 1 & z_0^i & y_0^i & -x_0^i & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . \end{pmatrix} \tag{2.39}$$

when solving for station positions and velocities.

The fundamental difference between the two approaches is that in equation (2.36), $\hat{X}$ is forced to be equal to X_0 (to a given σ), while in equation (2.37) $\hat{X}$ is expressed in the same TRF as X_0 using the projector B which contains all the necessary information for defining the underlying TRF. It is important to keep in mind that both approaches depend on the configuration and quality of the subset of stations X_0 upon which the constraints are applied.

In terms of normal equations, equation (2.37) can be written as

$$B^T \Sigma_\theta^{-1} B(\hat{X} - X_0) = 0 \ , \tag{2.40}$$

where Σ_θ is a diagonal matrix containing small variances for each of the transformation parameters. The general form of the singular normal equation constructed upon space geodetic observations, as obtained in equations (2.12) to (2.15), can be written as

$$N\Delta x = b \ , \tag{2.41}$$

where $\Delta x = \hat{X} - X_0$ stands for the linearized unknowns and b is the right-hand side of the normal equation.

Adding Equation 2.40 to the normal equation 2.41 allows it to be inverted and simultaneously to express the estimated solution $\hat{X}$ in the same TRF as the a priori solution X_0. Note that the first seven columns of the design matrix A correspond to the seven datum parameters (3 translations, 1 scale factor and 3 rotations). Therefore, this matrix should be reduced to those parameters which need to be defined (e.g. 3 rotations in almost all techniques and 3 translations in case of VLBI).

2.6 Estimation of transformation parameters

Least squares adjustment is commonly used to estimate the seven transformation parameters and their rates between two TRFs. For this purpose, equations (2.33) and (2.35) are rewritten as

$$X_2 = X_1 + A\theta \tag{2.42}$$

$$\dot{X}_2 = \dot{X}_1 + A\dot{\theta} \tag{2.43}$$

where θ and $\dot{\theta}$ are the vectors of the seven transformation parameters and their rates, respectively. A is the design matrix of partial derivatives constructed upon approximate station positions, as in (2.38) and (2.39).
Least squares adjustment yields solutions for θ and $\dot{\theta}$ of equations (2.42) and (2.43) as follows:

$$\theta = (A^T P_x A)^{-1} A^T P_x (X_2 - X_1) \tag{2.44}$$

$$\dot{\theta} = (A^T P_v A)^{-1} A^T P_v (\dot{X}_2 - \dot{X}_1) \tag{2.45}$$

The estimated transformation parameters (and their rates) depend on the choice of the weight matrix P_x for station positions, and P_v for station velocities.

3 Space geodetic techniques

Space geodetic techniques use extraterrestrial objects, such as artificial satellites or quasars to perform measurements and determine processes in the Earth system. The major geodetic applications are the determination of station positions and their variations in time, the description of the Earth's rotation and its orientation in space as well as the determination of the Earth's gravity field and its variation in time.

The basics of the space geodetic techniques VLBI, GNSS, SLR and DORIS and their observation equations are outlined in this chapter. Observation data from these techniques serve as basis for the ITRF combination.

3.1 Very Long Baseline Interferometry

The bigger part of the content of this section has been adopted from Schuh and Böhm [2013].

Very Long Baseline Interferometry is a purely geometrical space geodetic technique which uses radio telescopes to measure the difference in arrival times of signals from extragalactic radio sources (quasars). The positions of the quasars are regarded to be fixed which makes them suitable for realizing an inertial system - the ICRF, which is described in section 1.2. Since VLBI is the only space geodetic technique that is capable of establishing a connection to the celestial reference system, it is also the only technique which can provide direct measurements of nutation parameters and of the Earth rotation angle (UT1-UTC) and therefore deliver an entire set of Earth orientation parameters, which are essential for the determination of positions and for navigational purposes on Earth and in space.

3.1.1 VLBI observation equation

The geometric principle of VLBI is based on the assumption that due to the large distances between the quasars and the Earth the signals arrive on Earth as plane wave fronts. The triangle for the determination of the baseline vector becomes rectangular and provides a direct relation between the baseline vector b and the direction to the radio source s_0 (see figure 3.1). The scalar product τ represents the difference of arrival times of the signal at stations 1 and 2 with the sign convention $\tau = t_2 - t_1$. With the velocity of light c, τ can be determined as follows:

$$\tau = -\frac{b \cdot s_0}{c} = t_2 - t_1 \quad . \tag{3.1}$$

The delay τ is time-dependent, and the largest impact to its variation is due to the fact that the interferometer is fixed to the Earth and therefore rotates with respect to the celestial reference system.

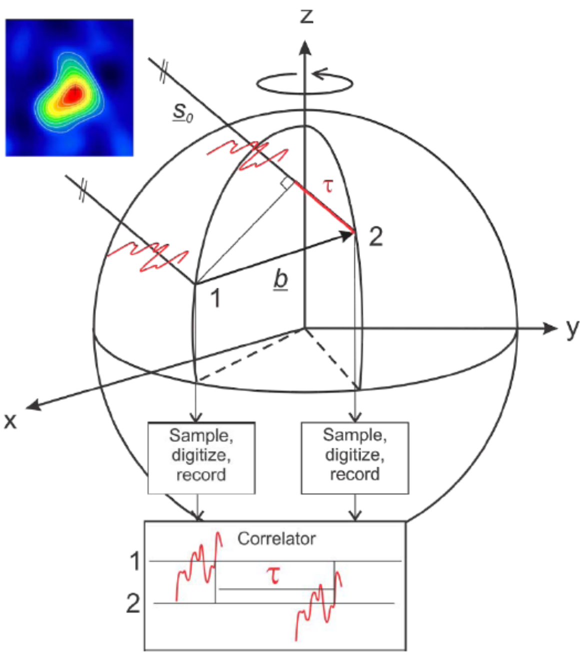


Figure 3.1: Geometric VLBI model.[1]

For geodetic applications two frequencies are observed: S-band (2.3 GHz, corresponding to a wavelength of approximately 13 cm) and X-band (8.4 GHz, corresponding to a wavelength of about 3.6 cm). The recorded signals are time-tagged with very stable and precise time stamps from hydrogen masers. The data is sent to correlation centers where so-called fringes are generated in order to obtain the group delay observable τ which is relevant for geodetic and astrometric applications. With those delays, the baseline b and further geodetic parameters can be derived with sub-centimeter accuracy.

[1]source: Schuh and Böhm, 2013

Furthermore VLBI provides very accurate measurements of the angle between the Earth-fixed baseline vector b and the space-fixed radio sources s_0. In order to evaluate equation (3.1) with parameter estimation techniques, b and s_0 have to be transformed into a common system. That way, even smallest changes in the baseline lengths or in the angles between the reference systems can be detected, and Earth orientation parameters can be monitored with unprecedented accuracy.

On their way from the quasars to the telescopes and especially in the Earth's atmosphere, the signals are distorted by various effects. Expanding equation (3.1) by taking into account all major error influences, the following equation can be obtained:

$$c\,\tau = \; b\,s_0 + c \cdot \delta t + \delta Ion + \delta Tr + \delta Rel + \delta A + \epsilon \tag{3.2}$$

with

c	speed of light [ms^{-1}]
δt	clock error [s].
δIon	ionospheric delay [m]
δTr	tropospheric delay [m]
δRel	relativistic effects [m]
δA	axis offset [m]
ϵ	measurement error [m].

3.2 Global Navigation Satellite Systems

Currently there are two fully operational global navigation satellite systems: Russian GLONASS[2] and NAVSTAR-GPS[3], whose basics will be outlined in this section.

The NAVSTAR-GPS was the first spaceborne radio navigation system based on timing and ranging, which became operational worldwide. The development of the system was started in 1973 by the U.S. Department of Defense.

The basic idea of GPS is that at least four satellites are visible from any location on the Earth at any time. This is achieved by having up to more than 30 satellites

[2]GLObalnaya NAvigationaya Sputnikovaya Sistema
[3]NAVigation System with Time And Ranging - Global Positioning System

orbiting the Earth in six nearly circular orbital planes. Each orbital plane is characterized by an inclination of 55°, an altitude of about 20200 km and an orbital period of exactly half a sidereal day (11 hours, 58 minutes).

3.2.1 GNSS observation equation

Theoretically, the three unknown coordinates of the receiver could be computed from the observed distances to three satellites (intersection of spherical shells, figure 3.2). However, satellite and receiver clocks are not synchronized. Therefore, a fourth distance measurement for the determination of the clock synchronization error is required.

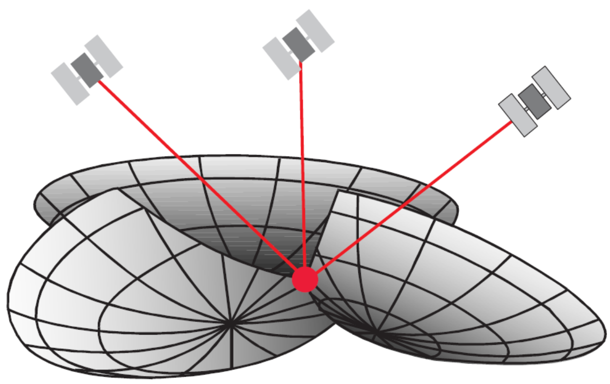


Figure 3.2: Intersection of spherical shells.[4]

All GPS signals are based on a basic frequency of 10.23 MHz which is generated by atomic clocks. The so-called carrier waves are obtained by multiplication with this basic frequency:

L1 = $154 \cdot 10.23$ MHz = 1575.42 MHz (corresponding to ≈19 cm wavelength)
L2 = $120 \cdot 10.23$ MHz = 1227.60 MHz (corresponding to ≈24 cm wavelength)
L5 = $115 \cdot 10.23$ MHz = 1176.45 MHz (corresponding to ≈25.5 cm wavelength)

The frequencies are modulated with codes (the C/A-code[5] and the P-code[6] among others) and the so-called navigation message, which contains information about the satellite itself (satellite ephemeris, satellite clock information, health status, etc.).

Basically, there are two different GPS measurement principles: The so-called pseudo-range measurements which use the C/A-code or the P-code and the carrier phase

[4] source: Leica Geosystems, *Einführung in die GPS-Vermessung*, v 1.0
[5] Coarse/Acquisition-code
[6] Precision-code

measurements which are based on the carrier frequencies L1, L2 and L5. According to Torge and Müller [2012] the accuracy limits for C/A-code and P-code measurements are approximately $\pm 3\,\mathrm{m}$ and $\pm 0.3\,\mathrm{m}$ respectively.

Geodetic applications like global reference frame realization or Earth rotation studies, however, require high-precision observations which can only be achieved with carrier phase measurements. Carrier phase measurements are based on the comparison of the phase of the emitted and the received signal as well as the determination of the integer number of complete carrier cycles between the satellite and the receiver (the so-called ambiguity).

On its way from the satellite to the receiver the signal is biased by various error influences. All those influences have to be taken into account when forming the observation equation for an observation between a receiver R and a satellite S with a frequency f at a certain time t:

$$\begin{aligned} \Phi_R^S(t) = & \rho_R^S + c \cdot \delta t_R - c \cdot \delta t^S + \delta Tr_R^S - \delta Ion_{R,f}^S + \delta M_{R,f}^S + \delta Orb_R^S \\ & + \delta Rel_R^S + \delta A_{R,f}^S + \lambda_f \cdot [\Phi_R - \Phi^S + \cdot N_R^S] + \epsilon_{R,f}^S \end{aligned} \tag{3.3}$$

with

c	speed of light [ms^{-1}]
ρ_R^S	geometrical distance between satellite and receiver [m]
$c \cdot \delta t_R$	receiver clock error [m]
$c \cdot \delta t^S$	satellite clock error [m]
$\delta Ion_{R,f}^S$	ionospheric path delay [m]
δTr_R^S	tropospheric path delay [m]
$\delta M_{R,f}^S$	multipath effects [m]
δOrb_R^S	ephemeris errors [m]
δRel_R^S	relativistic effects [m]
$\delta A_{R,f}^S$	phase center eccentricities of receiver and satellite antennas [m]
Φ_R	initial receiver phase [cycles]
Φ^S	initial satellite phase [cycles]
N_R^S	initial phase ambiguity [cycles]
λ_f	wavelength [m]
$\epsilon_{R,f}^S$	measurement error [m].

It is important to keep in mind, that every term with a subscript f depends on the frequency. A complete derivation of the observation equation and details of the theoretical background can be found in Hofmann-Wellenhof et al. [2008].

3.3 Satellite Laser Ranging

The principle of SLR, which is displayed in figure 3.3, is rather simple. Ultra-short visible or infrared pulses are emitted by a laser telescope, reflected at a satellite and received at the telescope again. The travel time of the signal is measured very precisely by an interval counter which is controlled by an atomic clock. The distance ρ between the telescope and the satellite can be derived from the elapsed time τ:

$$\rho = \frac{1}{2} c \tau \ . \tag{3.4}$$

The observed satellites are equipped with retro-reflectors. There are satellites that have been developed for SLR only - for example STARLETTE[7] or LAGEOS[8]. On the other hand SLR can be used for the precise orbit determination of satellites with arbitrary purposes such as the gravity missions CHAMP[9] and GRACE[10] or altimetry satellites like TOPEX[11] or ENVISAT[12].

SLR is a very precise ranging technique, where distances are measured directly and not differentially as in GNSS or VLBI. Furthermore, SLR operates in the optical frequency band (according to the ILRS website[13] the common wavelength is 532 nm). Therefore, another advantage is the favorable propagation of laser light in the atmosphere. On the other hand, laser measurements depend on the weather conditions, since optical signals can not propagate through clouds.

3.3.1 SLR observation equation

The basic observable is the time difference τ between the transmission of a laser pulse and its reception at the station. Multiplying τ with the speed of light results in twice the distance between the station and the satellite. However, the obtained

[7] Satellite de Taille Adaptée avec Réflecteurs Laser pour les ETudes de la TErre
[8] LAser GEOdynamics Satellite
[9] CHAllenging Minisatellite Payload
[10] Gravity Recovery And Climate Experiment
[11] TOPography EXperiment
[12] ENVIronmental SATellite
[13] `http://ilrs.gsfc.nasa.gov/technology/groundSegment`
[14] source: `http://gpsworld.com/wp-content/uploads/2013/09/EA-fig1.jpg`

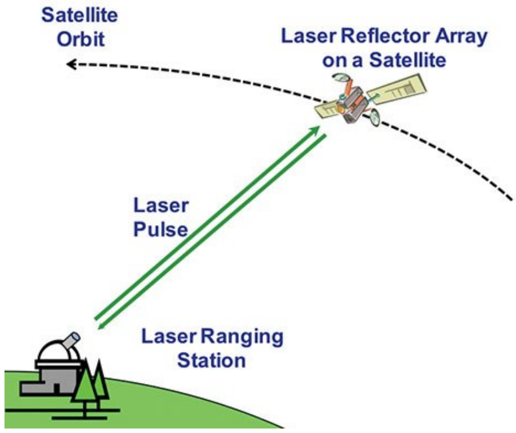


Figure 3.3: SLR principle.[14]

distance does not represent the geometrical distance which could be derived from the positions of the station and the satellite. Therefore, various correction terms have to be applied to obtain the observation equation:

$$\frac{1}{2}\,c\,\tau \;=\; \rho + \delta Tr + \delta Rel + \delta Bias + \delta CoM + \epsilon \tag{3.5}$$

with

c	speed of light [ms^{-1}].
τ	elapsed time [s]
ρ	distance between satellite and telescope [m]
δTr	tropospheric correction [m]
δRel	relativistic effects [m]
$\delta Bias$	range bias [m]
δCoM	center of mass correction [m]
ϵ	measurement error [m].

3.4 Doppler Orbitography and Radiopositioning Integrated by Satellite

DORIS is a French satellite system which was developed by CNES[15] in partnership with GRGS[16] and IGN. It uses the Doppler shift for the precise determination of satellite orbits and locating ground stations.

There are approximately 60 beacons distributed over the whole globe which emit radio signals on two frequencies: 401.25 MHz and 2036.25 MHz (corresponding to wavelengths of approximately 75 cm and 15 cm respectively). The signals are received and processed by DORIS receivers which are mounted on satellites. Examples for such satellites are JASON[17]-1, JASON-2 or ENVISAT.

3.4.1 DORIS observation equation

The basic principle of DORIS is displayed in figure 3.4. The received frequency f_r is shifted against the emitted frequency f_e due to the relative velocity $\dot{s} = ds/dt$ between the emitting beacon and the receiver (Doppler effect).

Neglecting higher order terms, the Doppler frequency shift for satellites with velocities much smaller than the speed of light is given by

$$f_r - f_e = -\frac{f_e}{c}\dot{s} \quad . \tag{3.6}$$

The Doppler shift is proportional to $\dot{s}$, a reverse in sign occurs at the time of closest approach of the satellite to the beacon (i.e. when the beacon is on the perpendicular of the satellite ground track, $\dot{s} = 0$). In principle, a range difference (range rate) can be determined from (3.6) by integration over time. In practice, f_r is compared with a stable reference frequency f_0 which is generated in the receiver, with $f_0 \approx f_e$. Integration over a time interval yields the Doppler count:

$$N_{ij} = \int\limits_{t_i+\Delta t_i}^{t_j+\Delta t_j} (f_0 - f_r)dt \quad . \tag{3.7}$$

[15]Centre National d'Etudes Spatiales
[16]Groupe de Recherche de Géodésie Spatial
[17]Joint Altimetry Satellite Oceanography Network

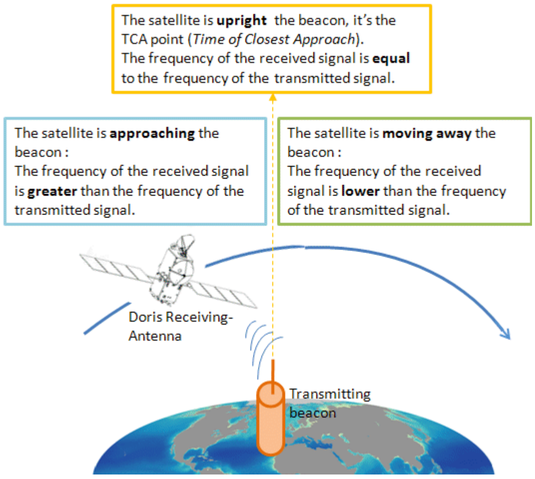


Figure 3.4: DORIS principle.[18]

With (3.6) the observation equation can be obtained:

$$N_{ij} = (f_0 - f_e)(t_j - t_i) + \frac{f_0}{c}(s_j - s_i) \quad . \tag{3.8}$$

with

c	speed of light [ms^{-1}].
N	integrated Doppler count
f_0	reference frequency [s^{-1}]
f_e	emitted frequency [s^{-1}]
$t_j - t_i$	integration time [s]
$s_j - s_i$	range rate [m]

Just like GNSS and VLBI, DORIS measurements do not depend on weather conditions. The capability of gathering large amounts of data in comparably short time spans leads to an increase of accuracy of the orbit determination with time.

[18]source: `http://www.aviso.altimetry.fr/en/techniques/doris/principle.html`

4 Combination of space geodetic techniques

This chapter aims to explain advantages and disadvantages of the individual techniques and why it is beneficial to combine them. Furthermore, requirements for the combination such as local tie vectors and the different combination approaches of the IERS combination centers are introduced.

4.1 Common and technique-specific parameters

As already mentioned in section 1.3, each technique has its strengths and weaknesses when it comes to determining certain geodetic parameters such as the origin and the scale for the ITRF. Table 4.1 indicates that there are common parameters such as station positions or terrestrial pole coordinates which can be retrieved from each of the techniques introduced in chapter 3, but also technique-specific parameters.

Table 4.1: Parameters from space geodetic techniques. (x) indicates that the parameter can be retrieved but does not contribute to the ITRF.

	Station Coordinates	Earth Orientation Parameters (EOP)			Datum Parameters	
		Pole Coord.	ΔUT1	Nutation	Origin	Scale
VLBI	x	x	x	x		x
GNSS	x	x			(x)	(x)
SLR	x	x			x	x
DORIS	x	x			(x)	(x)

VLBI for example, is the only technique which provides access to extragalactic radio sources and hence to celestial reference frames. Furthermore, it is the only technique that is able to measure ΔUT1 and nutation parameters in an absolute sense. The satellite techniques only contribute to ΔUT1 and the nutation parameters by their first derivatives in time, i.e. LOD[1] and nutation rates.

[1]Length Of Day

The geocenter, however, can only be determined by satellite techniques. Theoretically, any of the satellite techniques that have been introduced in chapter 3 could be used. In practice, the origin of the ITRF2008 is determined solely from SLR measurements. The reason for this is that according to Seitz et al. [2012] origin realizations from GNSS and DORIS are affected by insufficient modeling of solar radiation pressure. Furthermore, SLR satellites, which travel on low Earth orbits, are more sensitive to the Earth's gravity field and therefore have a better capability of determining the geocenter.

Information about the scale is provided by all the space geodetic techniques from chapter 3. For the determination of the ITRF2008 scale GNSS and DORIS are not considered, because the GNSS scale is correlated with satellite antenna phase center offsets and the DORIS scale time series shows a significant drift of about -0.28 ± 0.01 ppb/year (corresponding to -1.8 mm/year at the Earth's surface) starting in 2001. According to Seitz et al. [2012], the reason for this drift is unknown. Therefore, the scale of the ITRF2008 "is defined by nullifying the scale factor and its rate with respect to the mean of VLBI and SLR long-term solutions as obtained by stacking their respective time series" [Altamimi et al., 2011].

Whenever a new ITRF is created, its orientation is aligned to the previous version: The ITRF2008 orientation is defined in such a way that there are zero rotation parameters at epoch 2005.0 and zero rotation rates between ITRF2008 and ITRF2005. 179 reference stations located at 131 sites, including 107 GNSS, 27 VLBI, 15 SLR and 12 DORIS sites serve as so-called datum stations upon which these two conditions are applied [Petit and Luzum, 2010].

SLR and VLBI can be considered as the major contributors to the ITRF defining datum parameters. However, retrospection of figure 1.1 illustrates, that both techniques only have a small number of observing stations available. In addition, the distribution of the stations is inhomogeneous. Those disadvantages lead to a rather poor accuracy for the determination of temporal highly resolved parameters such as EOP.

In contrast, DORIS and especially GNSS benefit from a large number of favorably distributed stations as well as numerous observations. Figure 1.1 also indicates, that GNSS contributes to most co-location sites (see section 4.2) and is therefore essential for the combination of the station networks of the different techniques.

Since ITRF2005, EOP are included in the ITRF computations - see Altamimi et al. [2007]. This provides clear advantages in the development of the ITRF. The coordinates of the terrestrial pole are very reliable for verifying the consistency of the combined frame. Furthermore, the combination of terrestrial pole coordinates can be considered as a 'global tie'.

"If the complete time series of pole coordinates are combined, this global tie is introduced at all epochs for which pole coordinates are available. The combination of LOD can be described as the combination of a global daily rotation with respect to the z-axis of the ITRF. Finally, the common adjustment of the reference frame and a full set of EOP guarantees the consistency between all parameters" [Seitz et al., 2012].

4.2 Local tie vectors

For the creation of the ITRF the individual technique networks have to be connected. The crucial link for this process are so-called local tie vectors. Local ties are determined at co-location sites. Co-location site means that two or more space geodetic instruments operate or have operated at close locations (usually less than 1 km). The reference points of the instruments (e.g. the intersection of axis of an SLR telescope or a VLBI antenna) are determined in a local survey with a tachymeter or with GNSS. Local tie vectors represent the coordinate differences of these reference points.

GNSS plays a key role for the determination of local ties: 84 co-location sites were used for the ITRF2008 combination. 82 of them contain permanent GNSS stations [Altamimi et al., 2011]. Table 4.2 displays the poor numbers of co-locations for the other techniques (eight VLBI-SLR, ten VLBI-DORIS and ten SLR-DORIS co-locations) which amplify the importance of GNSS even further. However, "the drawback of GNSS being the connecting technique is that any intrinsic GNSS error would be transferred to the ITRF2008 estimated parameters" [Altamimi et al., 2011].

Table 4.2: ITRF2008 local ties.[2]

GNSS-DORIS	GNSS-SLR	GNSS-VLBI	DORIS-SLR	DORIS-VLBI	SLR-VLBI
45	48	44	10	10	8

Even though local ties are essential for the combination process, their utilization is also a big challenge: Most local ties are measured by national agencies which operate co-location sites. This means that the measurements are carried out by different people, using different equipment and different strategies. For the ITRF2008 combination only 63 % of the local ties were available with complete variance-covariance information of the local network adjustment [Altamimi et al., 2011]. Furthermore, the inhomogeneous distribution of observation sites which has been adressed in section 1.3 naturally leads to an inhomogeneous distribution of local ties.

Another challenge are the discrepancies between local ties and estimates from the space geodetic techniques. Referring to Altamimi et al. [2012], slightly more than 50 % of the available SLR and VLBI tie vectors to GNSS exhibit residuals larger than 6 mm, and about 30 % have residuals larger than 10 mm. Table 4.3 summarizes the discrepancies of local ties from GNSS to the other techniques. It is very difficult to distinguish the reason for those discrepancies. According to Altamimi et al. [2012], they can be caused by erroneous local ties, erroneous space geodetic estimates or both.

Table 4.3: Local tie discrepancies.[2]

Discrepancy mm	GNSS-DORIS %	GNSS-SLR %	GNSS-VLBI %
<6	34	43	47
6-10	12	29	24
>10	54	28	29

Some large tie discrepancies may come from the effect of uncalibrated radomes covering the GNSS antennas in some IGS sites. The fact that the reference points of the instruments, such as the axis intersection of SLR telescopes or VLBI antennas, are not physically accessible is another limiting factor in terms of local tie accuracy. Reference points of VLBI telescopes for example are represented by markers on the ground. Therefore, it is important to distinguish the eccentricity from the ground marker to the reference point very precisely.

[2]numbers from Altamimi et al., 2011

Seitz et al. [2012] show that there is a relation between the standard deviations of local ties and the deformation of the individual technique-specific networks. The mean network deformation can be obtained by the root mean square of station position residuals after a seven-parameter similarity transformation between the combined and the single-technique solutions. Their investigations show, that the network deformation increases with the number of local ties.

However, Altamimi et al. [2013] found out that the precision of the transfer of the SLR origin and the scale from SLR and VLBI to a GNSS frame increases with the number of local ties. Therefore, for the combination processes described in sections 4.3.2 and 4.3.3 it is important to introduce a set of well-distributed local ties with proper variance information in order to minimize the deformation of the technique-specific networks but also to retain the consistency of the datum defining parameters.

4.3 Combination methods

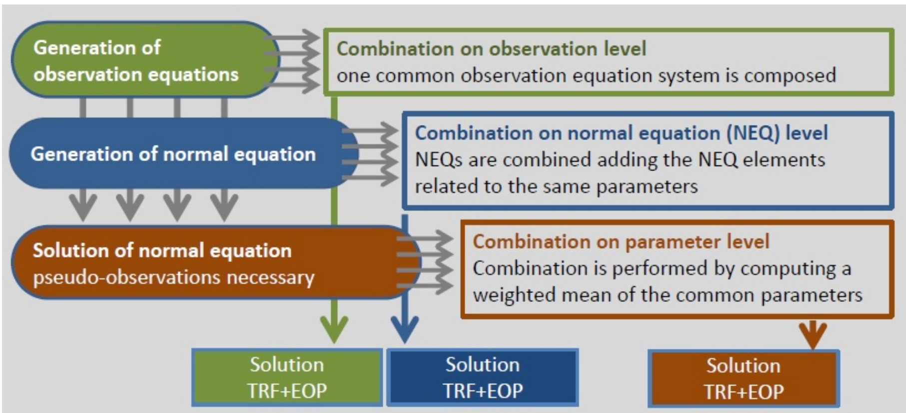


Figure 4.1: Combination methods on different levels of least squares adjustment.[3]

For the combination of the different techniques the observations are evaluated with a least squares adjustment in a linearized Gauss-Markov model as explained in section 2.1. As displayed in figure 4.1, the combination can be carried out on three different levels of the Gauss-Markov model: the observation level, the normal equation level or the parameter (solution) level.

[3]source: Seitz, 2012

4.3.1 Combination of observations

Theoretically the combination on observation level would provide the most satisfactory results, because the observations from all techniques are evaluated in one adjustment, i.e. all observations are processed with the same software. In order to avoid systematic differences due to heterogeneous computational frameworks it is essential to use identical fundamental constants, parameterizations and physical models for all observation types [Coulot et al., 2007].

This leads to the convenience that outliers can be detected with the information from all techniques. Furthermore, individual observations of any technique can be weighted accordingly, in order to determine the combined parameters in an optimal way [Seitz, 2009]. Moreover, the combination of observations should allow the determination and reduction of differences between techniques so that EOP and station position time series can be provided in a unified global combined TRF. It could also improve the validation of local ties and facilitate the creation of an additional link between the satellite techniques GNSS, SLR and DORIS by estimating a combined geocenter motion [Coulot et al., 2007].

However, for the realization of this approach, a very sophisticated software package is required. Such a software package should be able to handle the different observation types from all techniques and compute technique specific parameters such as range biases for SLR, clock parameters for VLBI and many more. Furthermore, it should be able to generate geodetic products such as CRFs, TRFs or EOP. According to Coulot et al. [2007] "it is obvious that such an ambitious computation is still utopic as the problems involved are numerous and quite arduous. However, such a combination is clearly the goal to reach in the future." Therefore, the IERS established the Working Group on the Combination at the Observation Level (WG COL), which aims to develop methods and software packages for the combination on observation level.

4.3.2 Combination of normal equations - DTRF

The ITRS realization computed at DGFI is labeled DTRF. It is one example for the combination on normal equation level. It is based on the combination of constraint-free normal equation systems which are built upon the observations of the space geodetic techniques introduced in chapter 3.

Constraint-free in this context means that the parameters in the normal equation systems are not affected by any constraints and that no conditions are applied to solve technique-specific datum defects. "However, some of the a priori reduced technique-specific parameters, like for example empirical parameters of the orbit modeling or tropospheric parameters in case of VLBI, are - and of course must be - constrained individually" [Seitz et al., 2012].

The input normal equations are combined data computed by the corresponding technique centers from contributions of the analysis centers. As already mentioned in section 1.3, the data is stored in SINEX files. SINEX enables the storage of normal equations as well as solutions. For the latest ITRS realization performed by the DGFI (DTRF2008), GNSS, SLR and DORIS input data are provided as solutions and the normal equations have to be reconstructed using the information about the applied constraints given in the SINEX files. The mathematical foundation for the reconstruction of the normal equations can be found in Angermann et al. [2004] or Seitz et al. [2012]. The VLBI data are available in form of normal equations and can be used directly from the SINEX files.

The combination procedure consists of two parts. In the first step, the time series of normal equation systems provided by the technique centers are combined to one normal equation system for each technique, as in equations (2.12) to (2.14). The normal equations are combined (stacked) by adding the elements of the individual normal equation systems which correspond to the same parameters (see section 2.3).

If the normal equations contain different parameters, and hence differ in size, the individual normal equations have to be expanded to the amount of all parameters that shall be solved. This is done by adding zero rows and columns to the normal equation matrix and zero elements to the right hand side vector. Finally, all normal equation systems have to be sorted in the same order of parameters before they can be combined. Station velocities are set up as new parameters and discontinuities are considered by setting up new positions and velocities for the corresponding stations after the events.

In the second step the normal equation systems of the different techniques are combined along with local tie vectors and restrictions concerning station velocities at co-location sites. The velocities for all stations at the same site are set equal if they do not differ significantly with respect to a $3\,\sigma$ - criterion [Seitz et al., 2012].

The geodetic datum is realized by adding minimum constraints. For the datum definition of the DTRF2008 the contributing techniques are the same as for ITRF2008. This means, that the origin is realized from SLR data only and the scale from SLR and VLBI. After estimating variance factors for the local ties and the technique-wise normal equation systems, the combined normal equation system is solved. Figure 4.2 shows a simplified flowchart of the combination process.

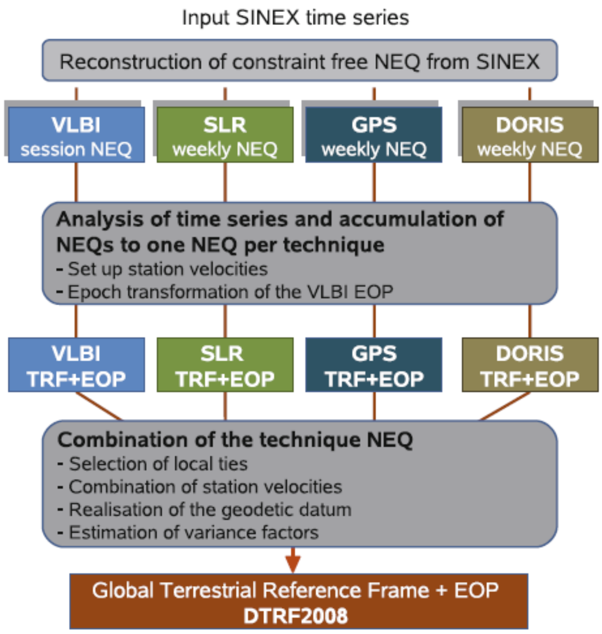


Figure 4.2: DGFI computation strategy.[4]

"The DTRF2008 solution comprises station positions, station velocities, coordinates of the terrestrial and the celestial pole (nutation parameters), UT1-UTC and LOD. The reference epoch of the station positions is 2005.0" [Seitz et al., 2012].

[4]source: Seitz et al., 2012

The combination of normal equations can - under certain conditions - be regarded as a good approximation of the combination of observations. If the observational data are processed with the same fundamental constants, the same physical models, and the same a priori values for parameters and no constraints are added before combining the normal equation systems, the combination at normal equation level should lead to similar results as the combination at observation level. However, in contrast to the combination on observation level, outliers are detected technique-wise and not with the entirety of the data from all techniques. “The effect on the solution is, particularly in the case of ITRF computation, assumed to be unverifiable” [Seitz et al., 2012].

One further aspect concerning ITRS realization has to be discussed. As stated before, no constraints are supposed to be applied on the data before the combination. However, according to Seitz et al. [2012], parameters which can not be estimated very stable, as e.g. clock parameters, are slightly constrained. In order to be able to handle the large normal equation matrices, some of those constrained parameters are reduced (see section 2.2) because they are not of direct interest. After the reduction, the constraints can not be removed anymore. “Thus, in order to avoid deformations of the solution, the a priori constraints on the reduced parameters must be introduced very carefully. A more rigorous way is not to constrain and reduce parameters before combination” [Seitz, 2012].

4.3.3 Combination of solutions - ITRF

The construction strategy applied by the IGN combination center is based on the combination of solutions (estimated parameters) and can be considered as a two-step approach as well. In the first step the individual technique time series of station positions and EOP are accumulated and in the second step the resulting long-term solutions of the four techniques are combined with local tie vectors.

The following characterization of the combination process has been adopted from the extensive descriptions in Altamimi et al. [2002, 2007, 2011].
The main two equations of the combination model involve a 14-parameter similarity transformation (see section 2.4), station positions and velocities as well as EOP and are written as:

$$\begin{aligned} X_s^i &= X_c^i + (t_s^i - t_0)\dot{X}_c^i + T_k + D_k X_c^i + R_k X_c^i + (t_s^i - t_k)[\dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i] \\ \dot{X}_s^i &= \dot{X}_c^i + \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \end{aligned} \tag{4.1}$$

$$\begin{aligned} x_s^p &= x_c^p + R2_k \\ y_s^p &= y_c^p + R1_k \\ UT_s &= UT_c - \frac{1}{f} R3_k \\ \dot{x}_s^p &= \dot{x}_c^p \\ \dot{y}_s^p &= \dot{y}_c^p \\ LOD_s &= LOD_c \ \ , \end{aligned} \tag{4.2}$$

where for each point i, X_s^i (at epoch t_s^i) and $\dot{X}_s^i$ are positions and velocities of technique solution s and X_c^i (at epoch t_0) and $\dot{X}_c^i$ are those of the combined solution c. For each individual frame k, as implicitly defined by solution s, D_k is the scale factor, T_k the translation vector and R_k the rotation matrix. The dotted parameters are their derivatives with respect to time.

As introduced in section 2.4 the translation vector T_k is composed of three origin components (T_x, T_y, T_z) and the rotation matrix of three small rotation (R_x, R_y, R_z). t_k is a conventionally selected epoch of the seven transformation parameters. In addition to equation (4.1) involving station positions and velocities, the EOP are added by equation (4.2), making use of pole coordinates x_s^p, y_s^p and universal time

UT_s as well as their daily rates $\dot{x}_s^p$, $\dot{y}_s^p$ LOD_s.

The link between the combined frame and the EOP is ensured via the three rotation parameters appearing in the first three lines of equation (4.2). The combination model given in equations (4.1) and (4.2) provides the following normal equation for each individual solution s:

$$\begin{pmatrix} A1_s^T \\ A2_s^T \end{pmatrix} P_s \begin{pmatrix} A1_s & A2_s \end{pmatrix} \begin{pmatrix} X \\ \theta \end{pmatrix} = \begin{pmatrix} A1_s^T P_s l_s \\ A2_s^T P_s l_s \end{pmatrix} \quad , \tag{4.3}$$

where $A1_s$ and $A2_s$ are design matrices, defined for each point i by

$$A1_s^i = \begin{pmatrix} I & dt_s^i I \\ 0 & I \end{pmatrix} \quad , \quad A2_s^i = \begin{pmatrix} A_s^i & dt_k^i A_s^i \\ 0 & A_s^i \end{pmatrix} \quad , \tag{4.4}$$

with $dt_s^i = t_s^i - t_0$, $dt_k^i = t_s^i - t_k$, and A_s^i is the design matrix relative to point i as defined by equation (2.38). P_s is the weight matrix (the inverse of the variance matrix of solution s) and, referring to equation (2.6), l_s is the vector of (observed-computed) parameters, in terms of least squares adjustment. The unknown parameters in equation (4.3) are X: station positions and velocities and θ: transformation parameters from the combined frame to frame k.

The first line of equation (4.1) and the entire equation (4.2) are used to estimate long-term solutions for each technique, by accumulating (rigorously stacking) the individual technique time series of station positions and EOP. In this process, the second line of equation (4.1), which includes the rates of the translation, scale and rotation parameters, is not included because station velocities are not available/estimable at a weekly or even daily basis. Moreover, a precise definition of the reference frame associated with the resulting long-term solution (comprising station positions at a reference epoch, station velocities and EOP) has to be clearly specified.

Transformation parameters are set up between each weekly (or session-wise) frame and the long-term frame to enable an independent datum definition of the latter. As a result, a rank deficiency is created in the normal equation matrix. This rank deficiency needs to be completed by defining the long-term frame origin, scale, orientation and their time evolution. Thereby it is essential that the long-term solutions are representative of the mean origin and mean scale information of the space geodetic techniques.

Therefore, minimum constraints as introduced in section 2.5, (see also Altamimi et al. [2007]), which have been designed for such purpose, are consequently used. In case of GNSS, SLR and DORIS no-net-rotation conditions are applied. In case of VLBI, which is no satellite technique and thus not sensitive to the Earth's center of mass, additionally no-net-translation conditions are needed to realize the origin of the frame.

In the second step, the long-term solutions of the four techniques are combined with local ties. Thereby it is essential to find suitable relative weighting factors for the local ties and for each input solution. This is a challenging process because of the imbalance between the space geodetic technique solutions, which are global by nature, and the small, local-by-nature networks of co-location sites. Since it is tricky to apply a given mathematical or statistical method of variance component estimation, the weighting procedure used for the ITRF2008 computation is a combination of the degree of freedom method as described in Altamimi et al. [2002, Appendix A, Sect. A5, Eq. A16] and an empirical weighting.

Concerning local ties, variance factors are estimated empirically in order to fulfill the following conditions: The normalized residuals should be smaller than 3 and the uncertainty per tie vector component should be at least 3 mm. According to Altamimi et al. [2011] "the reasons for these two conditions are that the precision of a local tie between physically inaccessible instrumental measurement reference points is unlikely to be better than 3 mm, and the agreement between local ties and space geodetic estimates is by far larger than 3 mm for most of the co-location sites. Consequently the local ties should be properly weighted in order to avoid contaminating the combined frame defining parameters by local tie and space geodetic discrepancies and at the same time to preserve consistency between individual technique solutions and ITRF2008."

Referring to Seitz [2012], "the combination at solution level is not a straight-forward approach due to the multiple application of pseudo-observations and the subsequent removal of datum information by estimating transformation parameters." Moreover, steps like the selection of stations which are used for the set-up of transformation parameters heavily depend on operator decisions.

4.4 Challenges

As already mentioned in section 1.3, the ITRF does not yet meet the level of requirement in terms of accuracy and stability over time for applications such as global sea level change detection.

Local ties are one limiting factor. As displayed in table 4.3, about 30 % of the discrepancies between space geodetic estimates and local ties from GNSS to VLBI as well as SLR, and more than 50 % from GNSS to DORIS are larger than 1 cm. It is difficult to distinguish the causes for those discrepancies, since they can be caused by errors in the local ties, errors in space geodetic estimates, or both. Altamimi et al. [2011] found out, that some large discrepancies in the height component might be related to uncalibrated radomes which cover the GNSS antennas. Also, the fact that the reference points of the geodetic instruments are physically inaccessible impedes the precise determination of local ties.

Another aspect that has to be improved is the statistical information about local ties. As stated in section 4.2, for the ITRF2008 combination only 63 % of the local ties were available with complete variance-covariance information of the local network adjustment.

In order to improve the accuracy of positions (and implicitly the agreement of local ties with space geodetic estimates), other technique-specific error influences have to be taken into account. For example, GNSS station position time series are imposed by seasonal signals which are caused by atmosphere loading effects. Figure 4.3 a displays that especially the height component is affected by such effects. Other influences are phase center variations and phase center offsets of the GNSS antennas [Altamimi, 2014].

Other technique-specific weaknesses are axis offset errors and elevation-dependent antenna deformations in case of VLBI or range biases and station timing/counter biases in case of SLR. The determination of the geocenter (especially the Z-component) with DORIS is distorted by the influence of solar radiation pressure [Altamimi, 2014]. Furthermore, for any technique, discontinuities in the station position times series as shown in figure 4.3 b, can be caused by equipment changes or seismic activities.

[5] figures from Altamimi, 2013

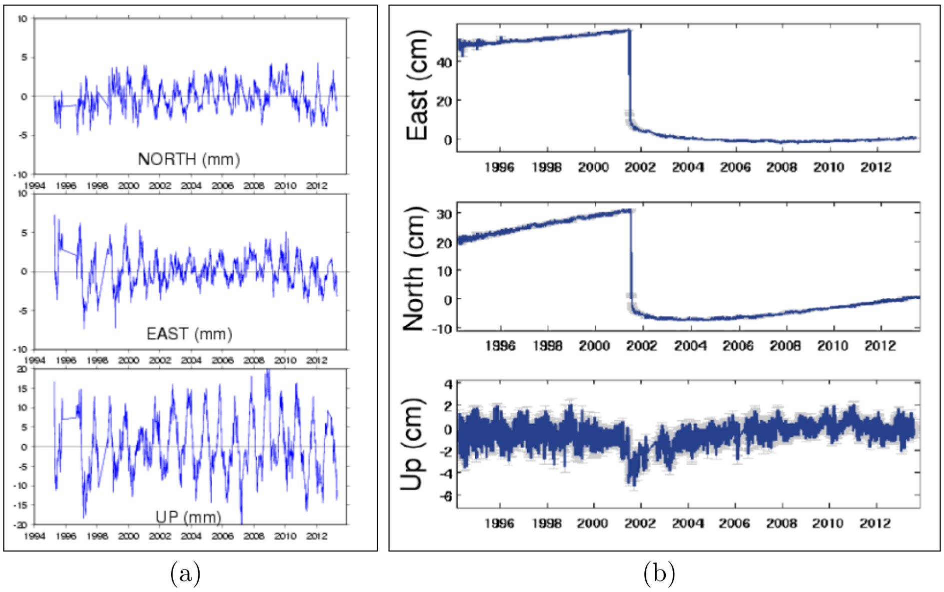


Figure 4.3: Error influences on station position residuals.[5]
(a) Seasonal signals at GNSS station in Brazilia (Brazil),
(b) post-seismic deformation at GNSS station Arequipa (Peru).

Therefore, it is important to develop or to enhance the existing models for all the effects mentioned above, in order to improve the accuracy and stability over time of upcoming ITRF versions. For some of the effects, new models have been developed already: The effect of seasonal variations can be reduced with sinusoidal functions and new parametric models which use logarithmic and/or exponential functions will be used for the correction of postseismic displacements [Altamimi, 2014].

5 Comparison of combination strategies

In this chapter the objectives of the International VLBI Service for Geodesy and Astrometry are summarized. Furthermore, the CONT14 campaign, which is the basis for the data used in the practical part, is outlined.

The aim of the practical part of this thesis was to derive station coordinates for all stations which participated in the CONT14 campaign. This is realized with the strategies introduced in sections 4.3.2 and 4.3.3. The results are compared in order to draw conclusions about the similarities and differences as well as possible advantages or disadvantages.

5.1 The International VLBI Service for Geodesy and Astrometry

As already explained in sections 3.1 and 4.1, VLBI can be used to determine parameters of celestial and terrestrial reference frames and the Earth orientation parameters as connection between the frames. One advantage of VLBI is the time span of observations for all parameters, as the geodetic applications of radio interferometry started already in the 1970s.

In the meantime, the potential of VLBI has increased rapidly, as - for example - documented in Schuh and Behrend [2012]. Thus, VLBI can deliver long time series for all parameters of interest in geodetic and geodynamic applications, e.g., changes in baseline lengths as an indicator for plate tectonics, or the fluctuations of EOP for a better understanding of the dynamic behavior of hydrosphere, atmosphere and the Earth's interior.

Concerning the EOP, it is worth mentioning once again that VLBI is the only technique which is able to determine UT1 and nutation angles in an absolute sense and to provide a stable inertial reference frame for long-term studies. Furthermore, estimated tropospheric and ionospheric delays can contribute to atmosphere studies.

According to Schuh and Behrend [2012] the organizational structure of geodetic techniques started to change in the mid-1990s. Since then, a shift from activities that were organized in local, national or regional frameworks to more global structures could be noticed.

On the initiative of the International Association of Geodesy (IAG), a number of technique-specific services were established, followed by an integrated observing system. The International VLBI Service for Geodesy and Astrometry (IVS) is one of these services. It operates under the auspices of the Global Geodetic Observing System (GGOS), which, according to Schuh and Behrend [2012], "coordinates and integrates the geodetic activities on a global scale."

The IVS, which was established in 1999, is an international collaboration of organizations which operate or support VLBI components. The IVS supports geodetic and astrometric work on reference systems and Earth science research in general, and provides the basis to all operational activities. According to the IVS terms of reference[1] its mission objectives are:

1. to foster and carry out VLBI programs. This is accomplished through close coordination of the participating organizations to provide high-quality VLBI data and products.

2. to promote research and development activities in all aspects of the geodetic and astrometric VLBI technique.

3. to advance the education and training of VLBI participants through workshops, reports, and other means.

4. to support the integration of new components into IVS.

5. to interact with the community of users of VLBI products. IVS represents VLBI in the Global Geodetic Observing System (GGOS) of the IAG and interacts closely with the International Earth Rotation and Reference Systems Service (IERS).

[1] `http://ivscc.gsfc.nasa.gov/about/org/documents/ivsTOR.html`

"In support of these objectives, the IVS coordinates VLBI observing programs, sets performance standards for VLBI stations, establishes conventions for VLBI data formats and data products, issues recommendations for VLBI data analysis software, sets standards for VLBI analysis documentation, and institutes appropriate VLBI product delivery methods to ensure suitable product quality and timeliness. The IVS coordinates its activities with the astronomical community because of the dual use of many VLBI facilities and technologies for both astronomy and astrometry/geodesy."

5.2 Data description - the CONT14 campaign

VLBI observations are not continuous (sessions usually last 24 hours but are not carried out every day). Therefore, to meet the requirement of continuous observations for many geodetic and geodynamic applications, the IVS is organizing special campaigns which provide continuous observations for a time span significantly longer than 24 hours, usually lasting two weeks. Those sessions are called CONT.

CONT14 is a campaign of continuous VLBI sessions, which was observed in May 2014 (6th May 2014 00:00 UT until 20th May 2014 24:00 UT). It is a continuation of the series of very successful continuous VLBI campaigns that were observed at irregular intervals since 1994. The most recent CONT campaigns were observed in roughly three-year intervals as CONT05 (September 2005), CONT08 (August 2008) and CONT11 (September 2011).

Figure 5.1: Geographical distribution of the CONT14 network.[2]

The CONT14 network comprises 17 stations at 16 sites. As can be seen in figure 5.1, ten stations are located in the northern and seven in the southern hemisphere (note that there are two stations at Hobart observation site). The stations are itemized in table 5.1.

Table 5.1: CONT14 station list.[2]

ID	Name	Code	Observatory Name and Location
7382	BADARY	Bd	Badary Radio Astronomical Observatory, Russia
7297	FORTLEZA	Ft	Space Radio Observatory of the Northeast (ROEN), Fortaleza, Brazil
7378	HART15M	Ht	Hartebeesthoek Radio Astronomy Observatory, South Africa
7374	HOBART12	Hb	Mt. Pleasant Radio Astronomy Observatory, Hobart, TAS, Australia
7242	HOBART26	Ho	Mt. Pleasant Radio Astronomy Observatory, Hobart, TAS, Australia
7375	KATH12M	Ke	Katherine Observatory, Katherine, NT, Australia
7298	KOKEE	Kk	Kokee Park Geophysical Observatory, Kauai, HI, USA
7243	MATERA	Ma	Centro di Geodesia Spaziale G. Colombo, Matera, Italy
7331	NYALES20	Ny	Ny Ålesund Geodetic Observatory, Spitsbergen, Norway
7213	ONSALA60	On	Onsala Space Observatory, Sweden
7345	TSUKUB32	Ts	Tsukuba VLBI Station, Japan
7377	WARK12M	Ww	Warkworth VLBI Station, New Zealand
7209	WESTFORD	Wf	Westford Antenna, Haystack Observatory, MA, USA
7224	WETTZELL	Wz	Fundamentalstation Wettzell, Germany
7376	YARRA12M	Ya	Yarragadee Observatory, Yarragadee, WA, Australia
7386	YEBES40M	Ys	Astronomical Center at Yebes, Spain
7381	ZELENCHK	Zc	Radioastronomical Observatory Zelenchukskaya, Russia

[2]source: `http://ivscc.gsfc.nasa.gov/program/cont14/`

5.3 Estimating coordinates for the CONT14 stations

5.3.1 Preprocessing steps

The data from the CONT14 campaign are provided in SINEX files. The files were generated with VieVS[3] by the IVS analysis center at the Department of Geodesy and Geoinformation (research group advanced geodesy) at the TU Wien.

The dataset consists of daily SINEX files, resulting in a total number of 15 files. Each file contains among other things, the normal equation matrix, right-hand-side vector, a priori parameter vector, and estimated parameter vector for one 24h session respectively. Each data type is located at a certain position in the file, called block.

The calculations, which are carried out with MATLAB[4], require some preprocessing steps. Figure 5.2 a shows an example of a normal equation matrix block. It consists of three columns containing the elements of the lower triangular form of the normal equation matrix along with two colums containing the corresponding row and column indices for the individual elements. Since the normal equation matrix is always symmetrical, only the upper or lower triangular portion needs to be stored. Therefore, the matrices have to be re-arranged into quadratic format and mirrored along the main diagonal. An example of a restructured normal equation matrix is displayed in figure 5.2 b.

```
+SOLUTION/NORMAL_EQUATION_MATRIX L
*Row__ Col__ Norm_Equ_Matrix_Value Norm_Equ_Matrix_Valu2 Norm_Equ_Matrix_Valu3
     1     1   1.7247745241003e+06
     2     1   1.4933665011202e+05   1.0243439460186e+06
     3     1  -4.3953618336500e+05   5.7602147916320e+05   7.5086732215252e+05
     4     1  -5.1072390106478e+03  -1.6003787280461e+00   1.6812873291857e+03
     4     4   3.1621377740036e+05
     5     1  -3.5780404174722e+03  -9.6483569834116e+03  -1.6168242058567e+03
     5     4   1.4705682692065e+05   7.7778864013564e+05
```

(a)

	1	2	3	4	5
1	1.7248e+06	1.4934e+05	-4.3954e+05	-5.1072e+03	-3.5780e+03
2	1.4934e+05	1.0243e+06	5.7602e+05	-1.6004	-9.6484e+03
3	-4.3954e+05	5.7602e+05	7.5087e+05	1.6813e+03	-1.6168e+03
4	-5.1072e+03	-1.6004	1.6813e+03	3.1621e+05	1.4706e+05
5	-3.5780e+03	-9.6484e+03	-1.6168e+03	1.4706e+05	7.7779e+05

(b)

Figure 5.2: Cutout of a normal equation matrix. (a) In the SINEX file, (b) after restructuring in MATLAB.

[3]Vienna VLBI Software, see `http://vievs.geo.tuwien.ac.at`
[4]MATrix LABoratory

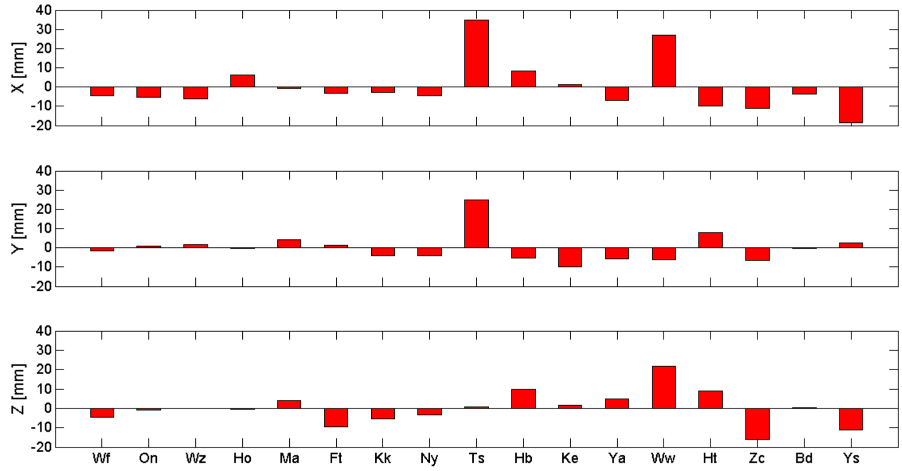
Furthermore, the order of parameters differs in each session. Thus, the normal equation matrices, right hand side vectors and estimated parameter vectors need to be sorted in a consistent way for all sessions.

5.3.2 Application of the combination of normal equations

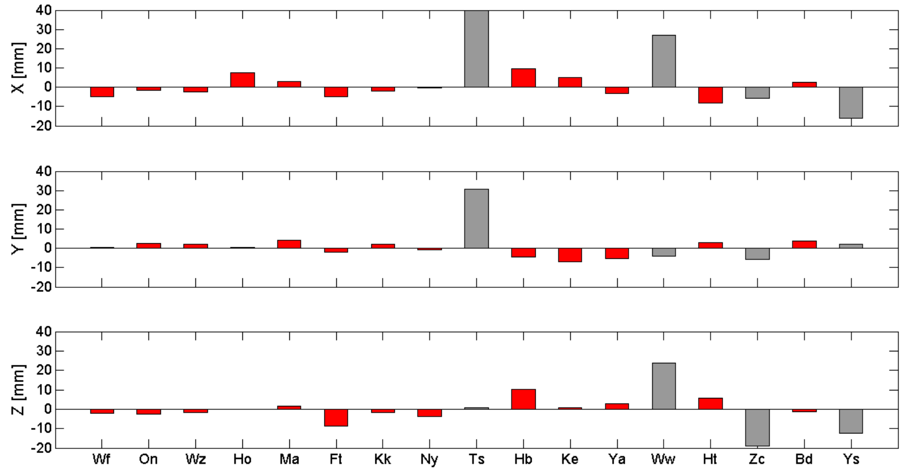
The normal equation systems in the SINEX files contain station positions, Earth orientation parameters and source coordinates, but only station positions are to be estimated. Therefore, the rows and columns of the normal equation matrices and the elements of the right hand side vectors which correspond to the source coordinates are deleted, which means that the source coordinates are fixed to their a priori values. The rows and columns of the normal equation matrices and the elements of the right hand side vectors which correspond to the EOP are reduced using equations (2.25) and (2.26). After the parameter reduction, the normal equation systems can be stacked as in equation (2.31).

The stacked normal equation matrix can not be inverted yet because it is singular due to the missing datum definition. To remove the rank deficiency of six (three translations, three rotations), minimum constraints as introduced in section 2.5 are applied. For this purpose the a priori station coordinates of the first session are used to create a constraint matrix as in equation (2.38). Solving the normal equation system yields the coordinate residuals. As can be seen in figure 5.3 a, the vast majority of the residuals is less than one centimeter with only a few exceptions, the residual for the X-coordinate of station Tsukuba being the largest with 34.7 mm.

In order to investigate possible reasons for the few large residuals, the stations with the largest residuals are excluded from the datum definition, which means that the elements of the corresponding rows in the constraint matrix are set to zero. Figure 5.3 b shows that excluding those stations from the datum definition hardly changes the overall outcome. Moreover, the largest residuals are not diminished. This leads to the assumption, that the large residuals are not caused by the selection of stations for constraining. A glance into the SINEX files reveals that the differences between the a priori values and the estimated values of each session agree with the residuals from the combined normal equation system. This suggests that the large residuals result from inaccurate a priori values.



(a)



(b)

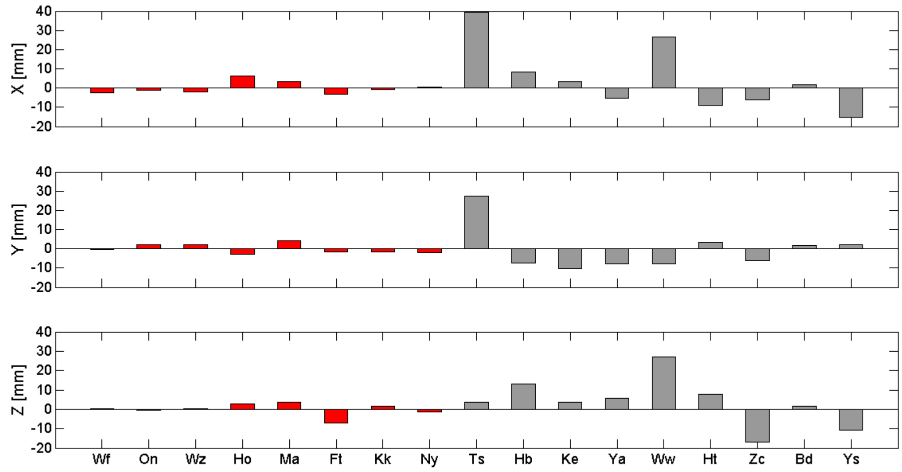
Figure 5.3: Combination of normal equations - coordinate residuals;
red: datum stations, grey: non-datum stations.
(a) constraining with all stations,
(b) constraining without Ts, Ww, Zc and Ys.

According to the USGS website[5] a magnitude 6.0 earthquake struck Japan on May 4, 2014. The epicenter was located approximately 150 km south-southwest of the city of Tsukuba. This event can be considered a possible cause for the discrepancies of the a priori and estimated coordinates of station Tsukuba.

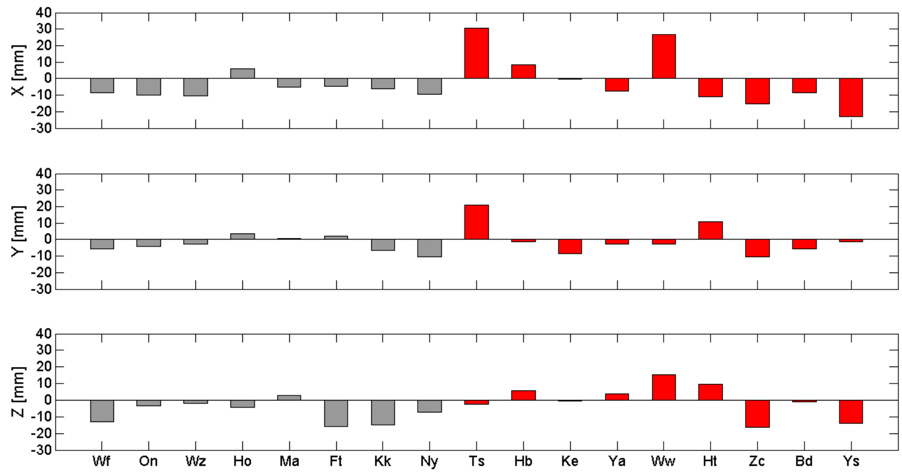
Concerning the impact of the selection of datum stations on the overall outcome, several attempts with varying sets of datum stations are carried out. Comparing figure 5.4 a (constraining with the first eight stations) with figure 5.4 b (constraining with the remaining nine stations) reveals that changing the datum stations can

[5]United States Geological Survey: `http://www.usgs.gov/`

cause differences larger than 1 cm for some residuals, for example the Z-coordinate of Westford (0.3 mm vs. -12.9 mm) or the X-coordinate of Badary (1.6 mm vs. -8.4 mm).



(a)



(b)

Figure 5.4: Combination of normal equations - impact of the selection of datum stations; red: datum stations, grey: non-datum stations. (a) constraining with stations Wf-Ny, (b) constraining with stations Ts-Ys.

Therefore, since a priori coordinates are used for constraining, it seems reasonable not to use stations with inaccurate a priori values (i.e. those with the largest residuals) for the datum definition. Hence, the selection of stations in figure 5.3 b is used to constrain the normal equation matrix for the calculation of the coordinate residuals as in equation (2.15).

5.3.3 Application of the combination of solutions

For the combination of solutions a combination model as in equation (4.3) is created. The vector l_s represents the difference between the estimated coordinates of each session (input solutions) and the a priori coordinates which are used to generate the matrix A_s^i. Since no velocities are estimated, equation (4.4) can be simplified:

$$A1_s^i = I \ , \ A2_s^i = A_s^i \ . \tag{5.1}$$

As already stated in section 4.3.3, the combination model is, in fact, a normal equation system from which the unknown coordinate residuals and session-wise transformation parameters can be obtained when $A1$ and $A2$ are merged accordingly:

$$\begin{pmatrix} A1 & A2 \end{pmatrix} = \left(\begin{array}{cccccccccccccccccccccc}
1 & & & & & & & 1 & 0 & 0 & x_0^1 & 0 & z_0^1 & -y_0^1 & & & & & & & & \cdot \\
 & 1 & & & & & & 0 & 1 & 0 & y_0^1 & -z_0^1 & 0 & x_0^1 & & & & & & & & \cdot \\
 & & 1 & & & & & 0 & 0 & 1 & z_0^1 & y_0^1 & -x_0^1 & 0 & & & & & & & & \cdot \\
 & & & \ddots & & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & & & & & & \cdot \\
 & & & & 1 & & & 1 & 0 & 0 & x_0^n & 0 & z_0^n & -y_0^n & & & & & & & & \cdot \\
 & & & & & 1 & & 0 & 1 & 0 & y_0^n & -z_0^n & 0 & x_0^n & & & & & & & & \cdot \\
 & & & & & & 1 & 0 & 0 & 1 & z_0^n & y_0^n & -x_0^n & 0 & & & & & & & & \cdot \\
1 & & & & & & & & & & & & & & 1 & 0 & 0 & x_0^1 & 0 & z_0^1 & -y_0^1 & \cdot \\
 & 1 & & & & & & & & & & & & & 0 & 1 & 0 & y_0^1 & -z_0^1 & 0 & x_0^1 & \cdot \\
 & & 1 & & & & & & & & & & & & 0 & 0 & 1 & z_0^1 & y_0^1 & -x_0^1 & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}\right) ,$$

where n is the number of stations. The dimensions of $A1$ and $A2$ are $3n \,\mathrm{x}\, 3n$ and $3n \,\mathrm{x}\, m$ (m being the number of estimated transformation parameters), respectively. The transformation parameters are set up to enable an independent realization of the datum of the combined solution. This means that a rank deficiency in the normal equation matrix is created. In order to solve the normal equation system, the rank deficiency is removed by adding minimum constraints.

Regarding the weight matrix P, two cases are investigated. In the first case P is the identity matrix and in the second case P contains diagonal weights according to the variances (the lower the variance, the larger the weight) of the input solutions which are also enclosed in the SINEX files. Figure 5.5 displays that the resulting coordinate residuals are practically identical (the differences are smaller than 10^{-15} m).

Further investigations with varying weight matrices confirm those results. Therefore, it can be assumed, that the weight matrix has no impact on the estimated coordinates. As will be shown later, the coordinate residuals are similar to those obtained from the combination of normal equations.

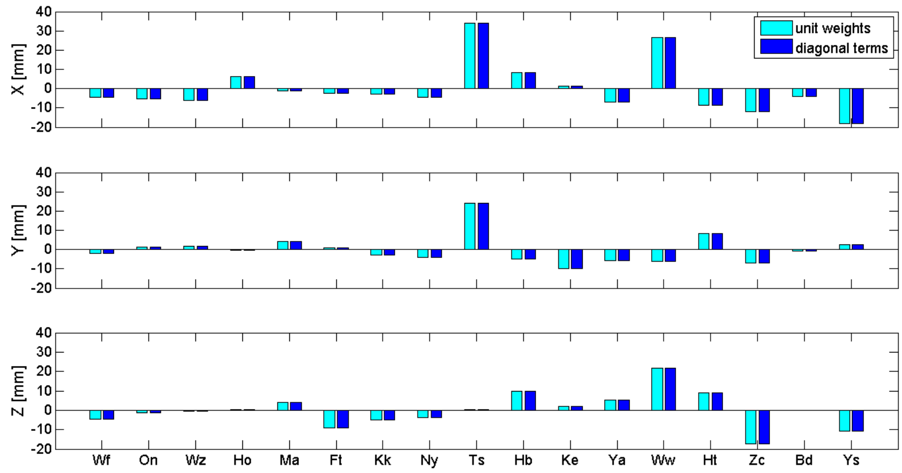
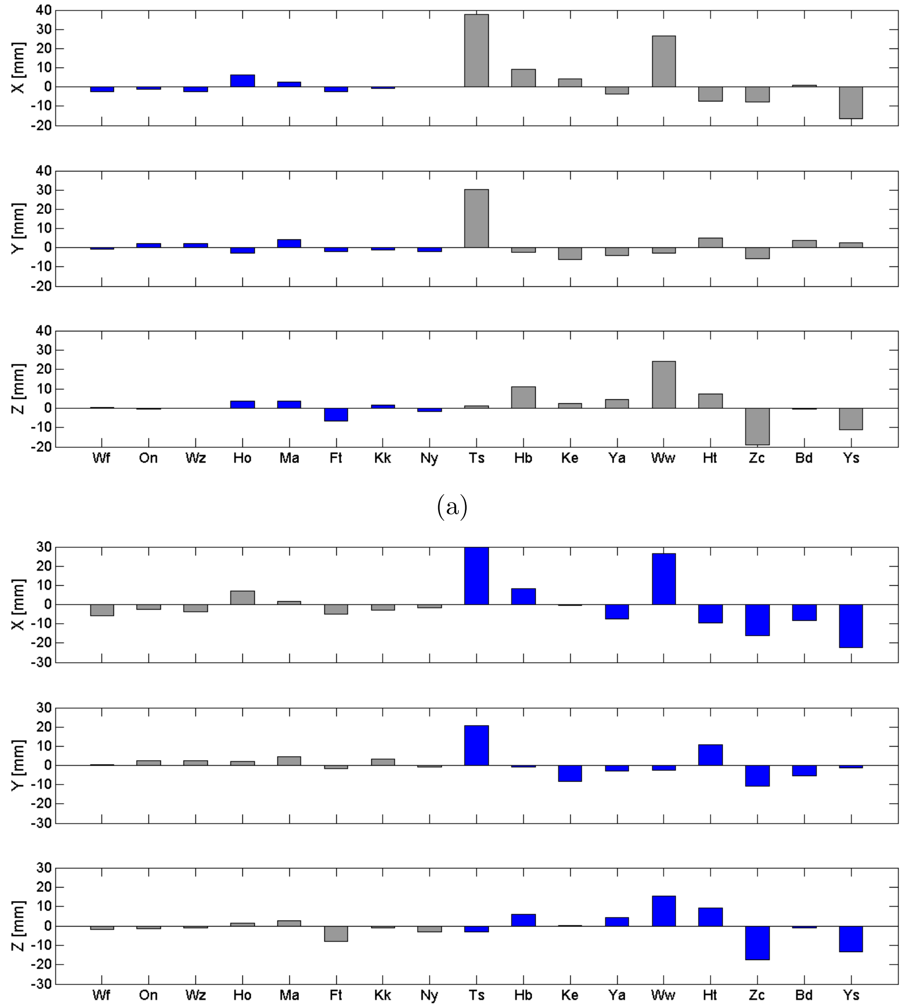


Figure 5.5: Combination of solutions - coordinate residuals;
cyan: P=I, blue: diagonal weights according
to the variances of the input solutions.

The influence of the selection of datum stations is analyzed in the same way as for the combination of normal equations. The results in figure 5.6 show, that the impact of the selection of datum stations can amount to almost 1 cm for some residuals in this approach as well. The residuals for the Y-coordinate of Tsukuba (30.2 mm vs. 20.8 mm) and for the Z-coordinate of Warkworth (24.3 mm vs. 15.4 mm) can be mentioned as examples.

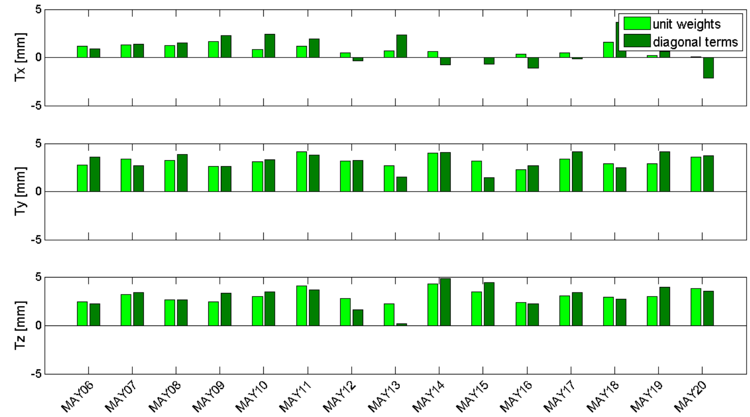


(a)

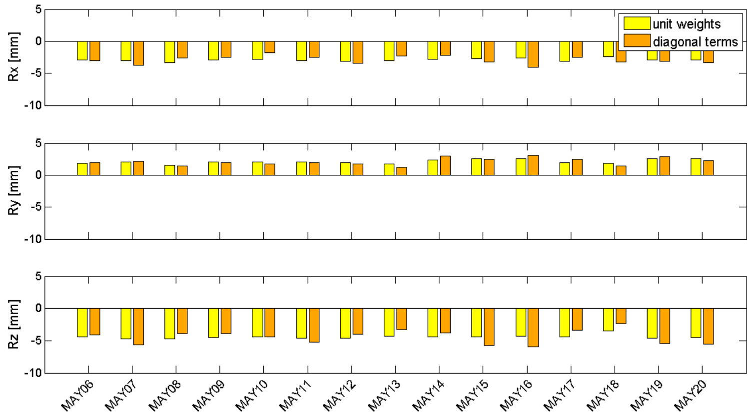
(b)

Figure 5.6: Combination of solutions - impact of the selection of datum stations; blue: datum stations, grey: non-datum stations. (a) constraining with stations Wf-Ny, (b) constraining with stations Ts-Ys.

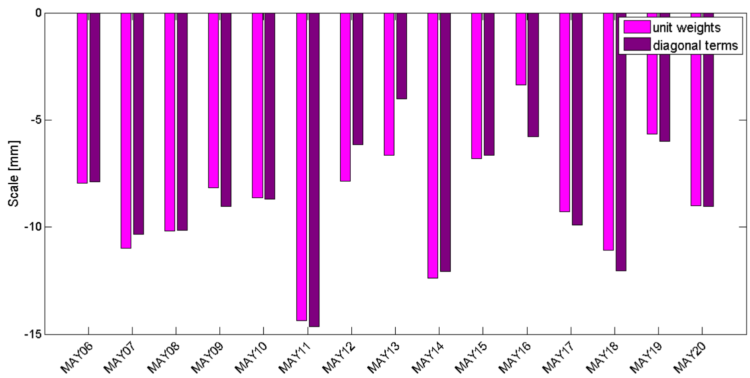
Unlike the coordinate residuals, the estimated transformation parameters slightly depend on the weight matrix. Figure 5.7 shows the estimated translation, rotation and scale time series. The small rotation angles and the scale are multiplied with the Earth radius (6371 km) in order to represent their impact on the Earth's surface.



(a) Translation.



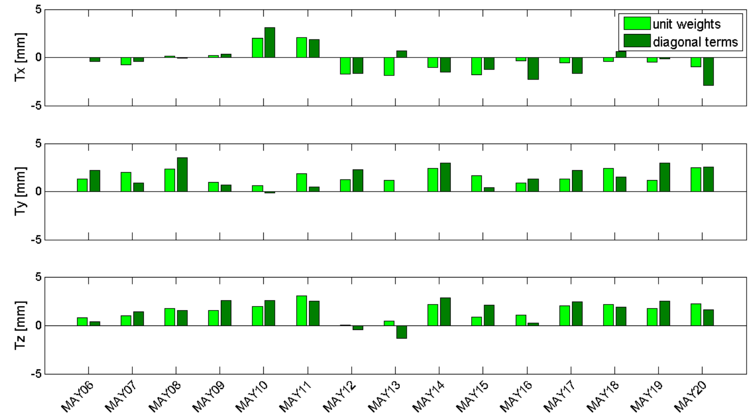
(b) Rotation.



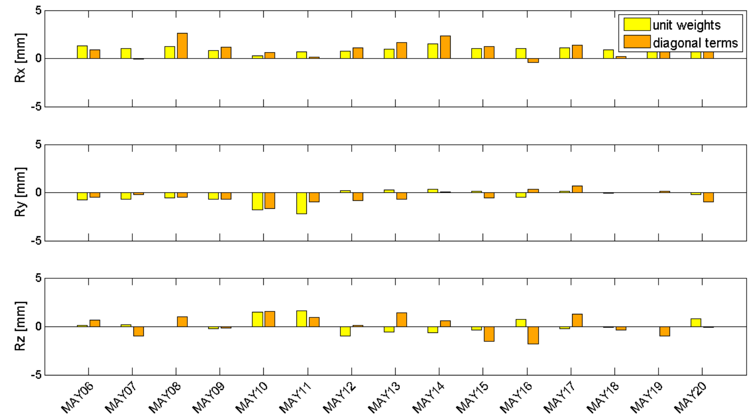
(c) Scale.

Figure 5.7: Estimated transformation parameters.

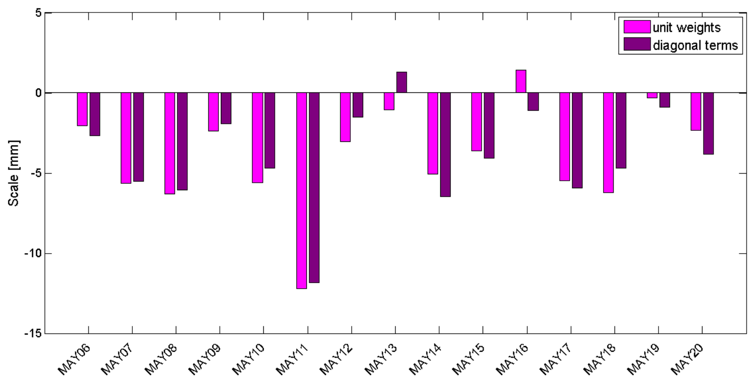
While the maximal impact of the weight matrix is only a few millimeters, the selection of datum stations has a greater influence on the transformation parameters. Figure 5.8 shows, that the agreement between the frames increases significantly when the stations with the largest residuals (Tsukuba, Warkworth, Zelenchukskaya and Yebes) are excluded from the datum definition.



(a) Translation.



(b) Rotation.



(c) Scale.

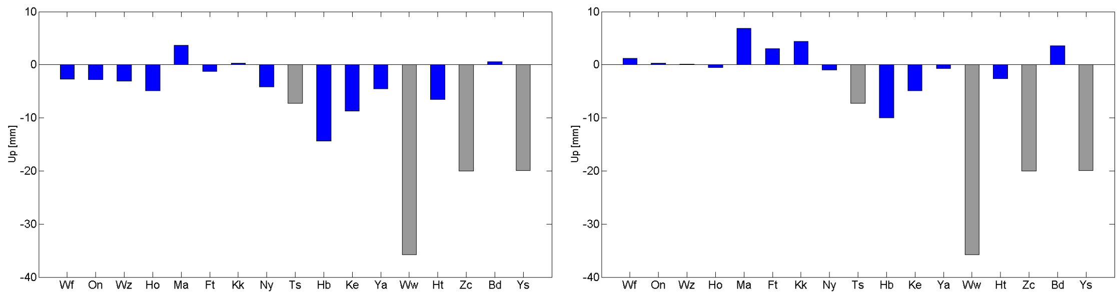
Figure 5.8: Impact of the selection of datum stations on the transformation parameters. Constraining without Ts, Ww, Zc, Ys.

There is one aspect which has to be discussed concerning the estimation of scale time series. As it is commonly known, any scale bias is absorbed by the local height-component of a station. Therefore the geocentric coordinate residuals are transformed into a local frame (East, North and Up components) with the transformation matrix R:

$$R = \begin{pmatrix} -sin\lambda & cos\lambda & 0 \\ -sin\phi cos\lambda & -sin\phi sin\lambda & cos\phi \\ cos\phi cos\lambda & cos\phi sin\lambda & sin\phi \end{pmatrix} . \tag{5.2}$$

ϕ and λ represent the geographical latitude and longitude of each station respectively. Figure 5.9 a shows the residuals of the height component, which are almost all negative when no scale is estimated. Estimating scale time series implies that the combination model contains a column for the scale for each session which means that the matrix of constraints also has to contain a column for the scale so that there is no rank deficiency in the normal equation matrix.

This leads to the residuals displayed in figure 5.9 b, where the sum of the residuals of the datum stations (blue bars) converges to zero.



(a) (b)

Figure 5.9: Correlation between scale and local height component;
blue: datum stations, grey: non-datum stations.
(a) no scale estimated, (b) scale estimated.

Comparing figures 5.7 and 5.8 shows that estimating session-wise transformation parameters delivers valuable information for identifying stations with inaccurate a priori coordinates. This information strengthens the assumption, that the stations Tsukuba, Warkworth, Zelenchukskaya and Yebes should not serve as datum stations. Figure 5.10 displays the resulting coordinate residuals.

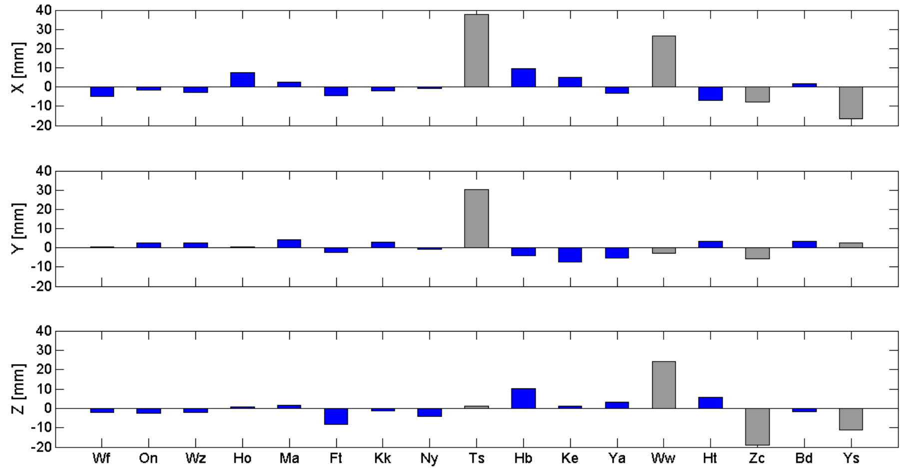


Figure 5.10: Coordinate residuals from combining solutions; blue: datum stations, grey: non-datum stations. Constraining without Ts, Ww, Zc and Ys.

5.3.4 Comparison and results

Figure 5.11 shows a comparison of the coordinate residuals from the combination of normal equations (figure 5.3 b) and from the combination of solutions (figure 5.10). Both approaches are realized with identical constraining conditions (minimum constraints applied upon all stations except Tsukuba, Warkworth, Zelenchukskaya and Yebes). The results agree at the sub-millimeter level with a few exceptions (the largest discrepancy being 2 mm for the X-coordinate of station Zelenchukskaya). The resulting station coordinates are summarized in table 5.2.

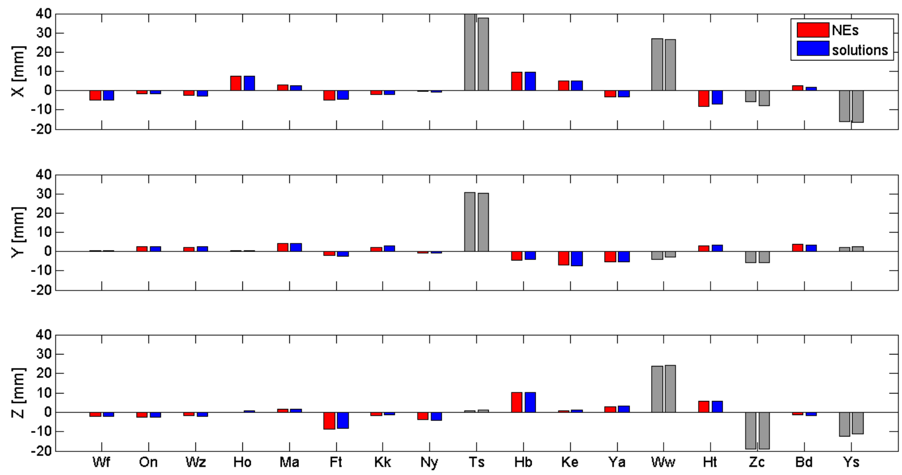


Figure 5.11: Comparison of coordinate residuals. red: normal equations, blue: solutions, grey: non-datum stations. Constraining without Ts, Ww, Zc and Ys.

Table 5.2: Estimated station coordinates.

Station	X [m]	Δ X [mm]	Y [m]	Δ Y [mm]	Z [m]	Δ Z [mm]
Combination of Normal Equations						
Combination of Solutions						
BADARY	-838201.0965		3865751.5646		4987670.9063	
	-838201.0970	0.5	3865751.5644	0.2	4987670.9058	0.5
FORTLEZA	4985370.0037		-3955020.4043		-428472.0722	
	4985370.0044	-0.7	-3955020.4049	0.6	-428472.0718	0.4
HART15M	5085490.8072		2668161.5114		-2768692.5895	
	5085490.8083	-1.1	2668161.5117	-0.3	-2768692.5896	0.1
HOBART12	-3949990.8240		2522421.2185		-4311707.9673	
	-3949990.8242	0.2	2522421.2187	-0.2	-4311707.9673	0
HOBART26	-3950237.3980		2522347.6938		-4311561.8204	
	-3950237.3979	-0.1	2522347.6938	0	-4311561.8199	-0.5
KATH12M	-4147354.6881		4581542.3763		-1573303.1354	
	-4147354.6884	0.3	4581542.3760	0.3	-1573303.1350	-0.4
KOKEE	-5543837.7853		-2054566.7724		2387852.5084	
	-5543837.7852	-0.1	-2054566.7717	-0.7	2387852.5088	-0.4
MATERA	4641938.4575		1393003.3383		4133325.8005	
	4641938.4569	0.6	1393003.3384	-0.1	4133325.8003	0.2
NYALES20	1202462.5144		252734.5185		6237766.2273	
	1202462.5142	0.2	252734.5185	0	6237766.2269	0.4
ONSALA60	3370605.7895		711917.7283		5349830.9242	
	3370605.7896	-0.1	711917.7284	-0.1	5349830.9240	0.2
TSUKUB32	-3957409.4358		3310228.7517		3737494.6678	
	-3957409.4377	1.9	3310228.7512	0.5	3737494.6681	-0.3
WARK12M	-5115324.4457		477843.2936		-3767192.7851	
	-5115324.4461	0.4	477843.2949	-1.3	-3767192.7848	-0.3
WESTFORD	1492206.3214		-4458130.5447		4296015.6086	
	1492206.3214	0	-4458130.5451	0.4	4296015.6086	0
WETTZELL	4075539.6151		931735.5482		4801629.5470	
	4075539.6149	0.2	931735.5483	-0.1	4801629.5465	0.5
YARRA12M	-2388896.1934		5043350.0108		-3078590.7892	
	-2388896.1935	0.1	5043350.0107	0.1	-3078590.7889	-0.3
YEBES40M	4848761.8189		-261484.1737		4123085.0668	
	4848761.8186	0.3	-261484.1732	-0.5	4123085.0680	-1.2
ZELENCHK	3451207.5082		3060375.4149		4391915.0413	
	3451207.5062	2.0	3060375.4147	0.2	4391915.0414	-0.1

6 Summary and outlook

For the determination of the ITRF data from the space geodetic techniques GNSS, VLBI, SLR and DORIS are combined in order to benefit from the technique's individual strengths. The combination can be carried out on the observation level, on the normal equation level or on the solution level. The combination on observation level is not applied today because software packages which can handle all the different data types are still in the process of development. The creation of such a software package is a goal to be pursued, because the combination on the observation level is the most rigorous approach.

The combination of normal equations is applied by the IERS combination center at the DGFI for calculating the DTRF. It can - under certain conditions - be considered a good approximation of the combination of observations. Theoretically, one advantage of the combination of normal equations is that constraints only have to be applied before solving the combined normal equation system. However, the input data of GNSS, SLR and DORIS are provided as solutions (and therefore are affected by constraints) and the normal equations have to be reconstructed. Hence, it would be desirable to provide all input data as normal equations so that no normal equations have to be reconstructed from constrained solutions.

The strategy used at the IGN for the determination of the ITRF is the combination of solutions. Every time a technique-wise solution is calculated, constraints have to be applied to define the datum. Transformation parameters have to be set up to enable an independent realization of the datum of the combined solution.

Both approaches are applied with data from the CONT14 campaign. The combination of normal equation requires the reduction of parameters which are not to be estimated and a consistent order of parameters before the individual normal equations can be stacked. For the combination of solutions an appropriate combination model has to be set up. Session-wise transformation parameters are estimated to enable an independent datum definition of the combined solution. In this process, arbitrary decisions such as the selection of stations which are used for estimating the transformation parameters can have a significant impact on the outcome. The results of both approaches depend on the applied constraints. When identical constraining conditions are used, the two approaches mostly agree on the sub-millimeter level.

For the inter-technique combination both strategies require local tie vectors. The local ties often disagree with the positions derived from the space techniques to an extent that exceeds the accuracy of the station positions. Therefore the set of local ties which is used for the combination has to be chosen very carefully. Other limiting factors in terms of accuracy are seasonal signals due to atmosphere loading and station position discontinuities which can be caused by seismic activity or equipment changes.

Despite those challenges the ITRF is more accurate than any technique-specific reference frame and can be considered the most accurate global terrestrial reference frame today. However, it does not yet meet the requirements of certain applications such as the monitoring of global sea level change. Therefore, it is essential not only to maintain but to improve the ITRF over time.

Bibliography

Altamimi, Z. (2013). Update on the International Terrestrial Reference Frame (ITRF). ICG-8, Dubai, UAE, Nov. 10-14, 2013.

Altamimi, Z. (2014). The International Terrestrial Reference Frame: Current status and future challenges. GSFC Seminar, Greenbelt, Feb. 10, 2014.

Altamimi, Z., Collilieux, X., Legrand, J., Garayt, B., and Boucher, C. (2007). ITRF2005: A new release of the International Terrestrial Reference Frame based on time series of station positions and Earth Orientation Parameters. *Journal of Geophysical Research*, Vol. 112(B09401):1–19.

Altamimi, Z., Collilieux, X., and Métivier, L. (2011). ITRF2008: An improved solution of the international terrestrial reference frame. *Journal of Geodesy*, Vol. 85:457–473.

Altamimi, Z., Collilieux, X., and Métivier, L. (2012). Analysis and results of ITRF2008, (IERS Technical Note ; No. 37).

Altamimi, Z., Collilieux, X., and Métivier, L. (2013). ITRF Combination: Theoretical and Practical Considerations and Lessons from ITRF2008. In Altamimi, Z. and Collilieux, X., editors, *Reference Frames for Applications in Geosciences*. Springer Verlag.

Altamimi, Z., Sillard, P., and Boucher, C. (2002). ITRF2000: A new release of the International Terrestrial Reference Frame for earth science applications. *Journal of Geophysical Research*, Vol. 107(B10, 2214).

Angermann, D., Drewes, H., Krügel, M., Meisel, B., Gerstl, M., Kelm, R., Müller, H., Seemüller, W., and Tesmer, V. (2004). ITRS Combination Center at the DGFI: A Terrestrial Reference Frame Realization 2003.

Coulot, D., Berio, P., Biancale, R., Loyer, S., Soudarin, L., and a. M. Gontier (2007). Toward a direct combination of space-geodetic techniques at the measurement level: Methodology and main issues. *Journal of Geophysical Research*, Vol. 112(B05410):1–21.

Hofmann-Wellenhof, B., Lichtenegger, H., and Wasle, E. (2008). *GNSS - Global Navigation Satellite Systems GPS, GLONASS, Galileo, and more*. SpringerWienNewYork.

Navratil, G. (2006). *Ausgleichungsrechnung II*. Lecture notes, TU Vienna.

Niemeier, W. (2008). *Ausgleichungsrechnung*. De Gruyter, 2nd edition.

Petit, G. and Luzum, B. (2010). *IERS Conventions (2010) - IERS Technical Note No. 36*.

Schuh, H. and Behrend, D. (2012). VLBI: A Fascinating Technique for Geodesy and Astrometry. *Journal of Geodynamics*, Vol. 61:68–80.

Schuh, H. and Böhm, J. (2013). Very Long Baseline Interferometry for Geodesy and Astrometry. In Xu, G., editor, *Sciences of Geodesy II, Innovations and Future Developments*, pages 339–376. Springer Verlag.

Seitz, M. (2009). *Kombination geodaetischer Raumbeobachtungsverfahren zur Realisierung eines terrestrischen Referenzsystems*. PhD thesis, TU Dresden.

Seitz, M. (2012). Comparison of different combination strategies applied for the computation of terrestrial reference frames and geodetic parameter series. In Kutterer, H., Seitz, F., Alkhatib, H., and Schmidt, M., editors, *The 1st International Workshop on the Quality of Geodetic Observation and Monitoring Systems (QuGOMS'11)*. Springer Verlag.

Seitz, M., Angermann, D., Bloßfeld, M., Drewes, H., and Gerstl, M. (2012). The 2008 DGFI realization of the ITRS: DTRF2008. *Journal of Geodesy*, Vol. 86(12):1097–1123.

Thaller, D. (2008). *Inter-technique combination based on homogeneous normal equation systems including station coordinates, Earth orientation and troposphere parameters*. PhD thesis, TU Munich.

Torge, W. and Müller, J. (2012). *Geodesy*. De Gruyter, 4th edition.

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