



DISSERTATION

The Dynamics of Europe's Political Economy: a game theoretical analysis

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Abstract

Coalition formation models lie at the intersection of political science and economics. They deal with the creation of groups amongst agents as well as the determination of the outcome of many social, economic and political interactions. Although coalition formation is a well studied concept in the domain of economics, particularly in game theory, most models focus on a stable outcome and the *process* of coalition formation has received less attention. Indeed, the process of coalition formation is the point of departure of my research. The main goal of this thesis is to investigate the process of coalition formation with a focus on international agreements and the dynamics of Europe's political economy.

Two game-theoretical coalition formation models are built and tailored to the EU and international coalitions in general. One of the aims is to establish links between theoretical models and applications thereby getting a better understanding of dynamics of the EU. While the first model uses a sequential approach in which players iteratively form subcoalitions, the second model uses a simultaneous approach in which players form coalitions at once with all members. The sequential algorithm for coalition formation is inspired by the study of jets in high energy physics experiments and the simultaneous model is inspired by correlation clustering in computer science.

There are two main considerations. The first one is the computational complexity of coalition formation processes. The computational aspects of coalition formation games are of increased importance as these games are used to analyze situations with numerous players. However, in economics and political science there are limited number of analyses concerning with this issue. The second consideration is the determination of the distance between players. The majority of theoretical works in the literature assume a geometrical distance function in the Euclidean space. The examination of different distance functions and their role in shaping coalitions is the subject of this research.

The distance between two countries is measured by a distance function which has geometrical element, geographical distance, and non-geometrical elements, GDP, GDP per capita, population and regime type. Python implementations of coalition formation algorithms are presented. The results including data from 28 European countries illustrate the impact of the distance function in the process. Both models predict five founder member states (with the exception of Italy) of the EU.

The presented thesis is an interdisciplinary work. It is devoted to the questions of which coalitions will be formed, how one defines the distance between players, how resulting coalitions intertwine with the process and how to reduce the computational complexity of the process.

Zusammenfassung

Die theoretische Auseinandersetzung mit und Modellierung von Koalitionsmodellen erfolgt an der Schnittstelle von Politik- und Wirtschaftswissenschaften. Kern der Betrachtung sind dabei meist die Zusammenschlüsse einzelner Koalitionäre - wie hier z.B. Staaten - und die Ziele ihrer sozialen, wirtschaftlichen und politischen Interessen und Interaktionen. Wenngleich Koalitionen einen ausführlich untersuchten Bereich innerhalb der Wirtschaftswissenschaften, speziell der Spieltheorie, darstellen, so liegt der Fokus dabei meist auf dem Erreichen eines stabilen Gleichgewichts. Dem zugrundeliegenden Prozess der Koalitionsbildung wurde bisher nur wenig Bedeutung beigemessen. Dies bildet den Ausgangspunkt der vorliegenden Arbeit.

Ein solcher Prozess wird anhand internationaler Übereinkommen, unter besonderer Berücksichtigung der Dynamik der Politischen Ökonomie Europas, näher untersucht. Hierfür werden zwei spieltheoretische Modelle entworfen und die Bildung von Koalitionen anhand eines internationalen Staatenbündnisses auf europäischer Ebene angewendet. Zwei Anwendungsbeispiele der Modelle ermöglichen dabei besseres Verständnis für die Dynamik innerhalb der Europäischen Union. Das erste Modell mit sequentiellem Ansatz lässt die SpielerInnen schrittweise Subkoalitionen bilden, während das zweite Modell mit simultanem Ansatz die SpielerInnen zu einem bestimmten Zeitpunkt eine Koalition formen lässt. Der sequentielle Ansatz hat seinen Ursprung in Experimenten der Hochenergiephysik mit sogenannten "Jets"; der simultane Ansatz basiert auf korrelationsgesteuerte Clustering-Methoden der Computerwissenschaft.

Die Zielsetzung dieser Arbeit umfasst zwei wesentliche Aspekte: die Rechenkomplexität bei der Modellierung von Koalitionsbildungsprozessen sowie die Bedeutung der Distanzfunktion. Die Rechenkomplexität im Rahmen der Modellierung von Koalitionsbildungsprozessen. Diese ist von steigender Bedeutung, da anhand einzelner Spiele Koalitionen mit zahlreichen

SpielerInnen berechnet werden. In den Wirtschafts- und Politikwissenschaften haben sich bisher jedoch nur wenige Studien mit diesem Problem auseinandergesetzt. Der zweite wesentliche Teil dieser Arbeit ist die Beschäftigung mit der Abstandsfunktion zwischen den einzelnen SpielerInnen. In der Literatur wird derzeit mit überwiegender Mehrheit die Euklidische Abstand angenommen. In der vorliegenden Arbeit werden verschiedene Distanzfunktionen und ihre Rolle in der Koalitionsbildung betrachtet. Die Distanz zwischen zwei SpielerInnen - hier Staaten - wird anhand einer Distanzfunktion abgebildet, die geographische, geometrische und nicht-geometrische Größen, wie das Bruttoinlandsprodukt, das Pro-Kopf-Bruttoinlandsprodukt, die Bevölkerungsgröße wie auch der Art der Regime-Typen miteinbezieht.

Die entwickelten Algorithmen werden in Python implementiert und die Ergebnisse in der Arbeit präsentiert. Diese umfassen Daten aus 28 Staaten und zeigen den Einfluss der Distanzfunktion auf den Prozess der Koalitionsbildung. Beide der angewendeten Modelle berechnen – mit Ausnahme von Italien – fünf der sechs Gründerstaaten der Europäischen Union voraus.

Der in dieser Arbeit vorgestellte interdisziplinäre Zugang strebt die Beantwortung mehrerer grundlegender Fragen an: Welche Koalitionen werden gebildet? Wie wird die Distanz zwischen einzelnen SpielerInnen definiert? Welchen Zusammenhang gibt es zwischen den gebildeten Koalitionen und dem ihnen zugrundeliegenden Prozess? Wie kann die Berechnungskomplexität in der Modellierung eines solchen Prozesses reduziert werden?

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Chapter 1

Introduction

What are the reasons for forming coalitions with respect to political and economic cooperations among countries? How do countries form coalitions? What are the preferences concerning potential partners and how are they shaped? Who wins and who loses in a coalition? Why are some coalitions terminated while others are not? International coalitions include long-term and complex relationships. The main goal of this thesis is to investigate the formation of coalitions with a focus on international agreements and the dynamics of Europe's political economy. Within the presented research, several points will be considered such as the emergence and evolution of coalitions, heterogeneity among coalition members, and issues of potential enlargement.

Coalition formation is an extensively discussed topic in economic, political, and social analysis. The intention is not to repeat all of these discussions here. The objective of this thesis is to understand the process of coalition formation by applying game theory. There is a broad and growing literature in game theory investigating these topics. Recently, hedonic games, in which the payoff of a coalition only depends on its members ([Banerjee et al. \(2001\)](#)), has attracted much attention. Despite the simple nature of these models, a few open points warrant further study.

These are especially related to the European Union (EU) as an international/supranational organization. A game theoretical coalition formation model can be used and tailored to the EU in particular and international coalitions in general. One aim is to establish links between the theoretical model and the formation and dynamics of the EU, thereby gaining a better understanding of the potential entrance and exit of some countries.

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The issue of coalitions is not unique to the fields of social sciences. The aggregation of similar elements into groups is subject to many fields: mathematics, physics, computer science, etc. There have been several attempts to describe and analyze the formation of coalitions and networks in economics using concepts and tools from those fields¹. The models presented in this work particularly share similar characteristics with sequential recombination algorithms from particle physics and correlation clustering from computer science.

These models are introduced in order to be able to study fundamental aspects of coalition formation. To do so, I study the problem as a description of actual processes instead of focusing on formal stability conditions. That provides a strong motivation for studying properties of instability instead of trying to cure it. In both models, an algorithmic approach is used in which players form coalitions by following certain rules.

The models allow us to test their predictions to compare the different procedures. Even if the primary aim of this thesis is not to explore the formation of coalitions empirically, it is important that the models can be easily tested. The models can be applied to a wide range of problems, but I take the creation of the European Coal and Steel Community (ECSC) as an example and demonstrate its formation using the sequential and the simultaneous model.

Recently, sequential recombination algorithms have been successfully used in particle physics to analyze showers of particles produced in high energy collisions of elementary particles. In chapter 5, the process of coalition formation resembles this kind of clustering procedure which is a pairwise sequential recombination of particles.

Correlation clustering is a method of partitioning a set of elements into clusters. In correlation clustering, the formation of clusters takes place simultaneously. It can be used in several applications such as data mining. The number of clusters could be any value between one and the number of elements (Bansal et al. (2004)). In chapter 6, simultaneous coalition formation follows a similar structure.

One advantage of these approaches is that there is no need to determine the number of clusters in advance. These two algorithmic approaches share a similar structure of hedonic

¹The Landscape theory and the Ising model, which originated in the study of physics are used in analyzing aggregations in economics and politics, see, for example, Axelrod and Bennett (1993) and Galam (2008). The study of graphs, the Network theory, is widely used not only in mathematics and computer science but also in economics and sociology. See Jackson (2008) for a comprehensive introduction to social and economic networks.

games in which any number of coalitions can be formed and there is no restriction on the number of coalitions.

Needless to say, considering only two options, either sequential or simultaneous, for the coalition formation process is debatable (a mixed approach could also be adopted, for example). However, this keeps the models more traceable and allows us to focus on other aspects. Using two complementary approaches to the problem provides a better understanding of the coalition formation process. This can also contribute to the knowledge about similarities and differences of formation processes in different fields, so that the theory can be improved.

The thesis is organized as follows. In chapter 2, a survey of coalition formations in political science and game theory will be presented. Basics elements of coalition formation games and a review of theoretical approaches are given. Furthermore, shortcomings of these theories are discussed. Chapter 3 introduces clustering theory which will be later used for sequential and simultaneous coalition formation models. Clustering algorithms and their advantages and disadvantages are discussed. In chapter 4 a brief history of formation of the EU will be discussed and the data of countries' characteristics will be presented. The core of this thesis are chapter 5 and 6. In chapter 5 a recombination algorithm from particle physics is presented and a sequential coalition formation model is built. In chapter 6 correlation clustering method is discussed and a simultaneous coalition formation model is built. Furthermore, implementations of algorithms are presented for both models. The thesis ends with conclusions.

Chapter 2

Coalition Formation

The theory of coalition formation was initially an interest of political science and cooperative game theory. Models of coalition formation in game theory date back to [Von Neumann and Morgenstern \(1944\)](#). There are different approaches and applications in political games: [Riker \(1960\)](#) minimum size winning coalitions, [Leiserson \(1968\)](#) fewest actor principle, [Axelrod \(1970\)](#) conflict of interest and connected coalitions, [de Swaan \(1973\)](#) minimizing policy distance, [Peleg \(1981\)](#) coalition formation in dominated simple games, [Deemen \(1997\)](#) the center player theory.

Coalitions are formed when agents prefer to act together in a group for the benefit of all the members of the group. These groups can consist of only two agents, as in the case of a married couple or a bilateral treaty, or several agents, as in a shared flat or an economic/political union. There are two fundamental questions in the formation of coalitions: which coalitions can be formed and how coalitions distribute payoffs among their members ([Hajdukov \(2004\)](#)). These questions are intertwined since the members' payoffs in a coalition depend on which coalitions form and which coalitions form depend on the distribution of payoffs among members.

An interesting subset of coalitional game theory is matching problems. Matching models on marriage problems, college admission problems, and roommate problems are introduced by [Gale and Shapley \(1962\)](#). In these models, players have preference lists over potential partners or roommates and the challenge is to find whether a stable matching exists. There is always a stable matching in the marriage problem while any matching might be unstable for the roommate problem. From the perspective of mechanism design, several interesting

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applications were inspired from these matching problems such as kidney exchange, school admission, and labor market problems. Stability plays a crucial role in these markets. In a general sense, stability means that no set of agents are interested in leaving their current cooperation and creating a new one.

Another subset of coalitional game theory is hedonic games. They were first introduced by [Drèze and Greenberg \(1980\)](#). More recently, hedonic games have been studied by, among others, [Banerjee et al. \(2001\)](#), [Barberà and Gerber \(2003\)](#), [Bogomolnaia and Jackson \(2002\)](#), and [Burani and Zwicker \(2003\)](#). Similar to matching games, in hedonic games, each player has preferences over potential coalitions. (In fact, hedonic games encompass matching games.). A partition of the set of players might be determined according to these preferences. These models exhibit two main features. The first is that the payoffs can only be determined by the identity of the coalition's members. The second is the prediction of more than one coalition, namely a partition.

Theoretical models suggest a number of restrictions in hedonic games in order to guarantee the stability of coalitions. Coalition formations emerge as a consequence of models' assumptions. However, these restrictions are not always applicable for international coalitions. Furthermore, becoming exclusively preoccupied with stability and its assumptions prevent us from studying other aspects of coalition formation like the formation process itself.

The models of coalition formation deal with the players' preferences and assumptions of these preferences. The formation of a coalition can be a prolonged process spanning multiple rounds of negotiations. An extreme example is the proposed EU membership of Turkey. Different to static models, studying the dynamic structure of how players can change their preferences over time is worth examining in order to shed light on the nature of coalition formation.

Another critical point is determining how players' preferences are shaped. In hedonic games and most coalition formation theories, the preferences are exogenously given. These preferences can either be in a one-dimensional or a multi-dimensional space. While in a one-dimensional case ordinal preferences may be used, in a multi-dimensional case preferences are cardinally measured. In both settings, the general assumption is that players will cooperate

with their closest neighbors (the definition of what constitutes the closest neighbors may differ among models). I will discuss different models in the following sections.

Many models in hedonic games focus on the stability of coalition partitions. However, if preferences are unrestricted, this criterion is not always fulfilled. These models seek to find a stable partition by restricting preferences. Despite the simplicity of hedonic games, finding a stable partition is not easily guaranteed.

Hedonic games find many applications in economics. They vary from the proportional sharing in production to the housing market. However, multilaterals and international organizations are paid less attention. (Maybe one exception is hedonic formation models in environmental issues.). These models are particularly plausible in these areas because they allow for a partition instead of a bipolar result.

The chapter is organized as follows. In the next section, I will survey coalition formations in political science and game theory. Before I present the framework of coalition formation, there are three important points to discuss. The first is preferences and their assumptions in both coalition theories in general and in hedonic models in particular. The second point is the process of coalition formation. This point requires special attention since the formation process and stability are closely related. The last point is the stability of coalitions. Multiple stability concepts exist in the literature; therefore, I will review these concepts and discuss their connections.

2.1 Forming Coalitions

In cooperative game theory, games are classified in two types: Transferable Utility (TU) games and Nontransferable Utility (NTU) games. In TU games, the utility is freely transferable from one player to another. In other words, every coalition can divide its worth in any possible way among its members. In NTU games, some feasible set of payoff vectors for every coalition is given and there is no possibility for transferring utility among the members. In pure hedonic games and matching games, there are no transfers between players. Therefore, these games fall into latter category.

Although hedonic games are a particular type of NTU games, it is worth noting here that there are models which use elements of non-cooperative game theory. For example,

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[Bogomolnaia and Jackson \(2002\)](#) use the Nash stability, which is a non-cooperative notion of stability. [Bloch and Diamantoudi \(2011\)](#) consider a non-cooperative perspective in which players form coalitions without a social planner.

Let me start with the basics of n -players cooperative game theory and further discuss the concepts relevant to international coalitions. Consider a finite set of players $N = \{1, 2, \dots, n\}$. A coalition is a subset of N . In a game, 2^N different coalitions can be formed. Note that the empty coalition \emptyset and the grand coalition $\{N\}$ are also considered to be valid. While for a specific problem (in which players have a common scale to measure the worth of a coalition) it is convenient to consider transferable utility, in a general situation, nontransferable utility is more appropriate. Every TU game can be represented as an NTU game but the converse is not true.

Definition 2.1.1 *A TU game is a pair (N, v) where $v : 2^N \rightarrow R$ is the characteristic function assigning to each coalition S in 2^N a value $v(S)$ and satisfying $v(\emptyset) = 0$.*

An important class of TU games are superadditive games. In this class, merging any pair of disjoint coalitions never diminishes the total payoff. Therefore, players can be expected to form a grand coalition. Formally, if a TU game is superadditive for each pair $S, T \subset N$ and $S \cap T = \emptyset$, it holds that

$$v(S \cup T) \geq v(S) + v(T).$$

Superadditivity is very demanding. The assumption of superadditivity may not be imposed because of several reasons (see [Aumann and Dreze \(1974\)](#) [p.233]) such as heterogeneity, moral hazard, or communication issues (see [Greenberg \(1994\)](#)[p. 1309]).

It is not possible to transfer utility among players when there is no common currency in a game. A classical example of this is an exchange economy. In NTU games, the possibilities for each coalition is represented by a set of payoff vectors.

Definition 2.1.2 *An NTU game is a pair (N, V) where V is a mapping of feasible payoffs to each coalition S in $2^N \setminus \{\emptyset\}$ a value $V(S)$ satisfying $V(\emptyset) = \emptyset$ and*

- $V(S)$ is nonempty, closed, and convex.
- $V(S)$ is comprehensive

- $V(S) \cap R_+^S$ is bounded

A coalitional game in which the payoff to a member of the coalition depends only on the coalition members is called a hedonic game. Formally,

Definition 2.1.3 A hedonic game G is a pair (N, \succeq_i) , where N is a finite set and $i \in N$. The binary relation \succeq is a complete, reflexive, and transitive preference relation for agent i . If $S \succeq_i T$, agent i prefers coalition T as much as coalition S .

The strict and indifference relation of a player i are as follows:

$$S \succ_i T \iff [S \succeq_i T \wedge T \not\succeq_i S],$$

$$S \sim_i T \iff [S \succeq_i T \wedge T \succeq_i S].$$

A feasible allocation in such a game is a partition of players. A partition is the distribution of all players in non-overlapping coalitions¹, i.e. it is supposed that each player belongs to one and only one coalition. Note that, if we consider that players are not anonymous, then the number of coalition partitions is more than simply the partition number of N .

Given a cooperative game, the question is how to determine the equilibrium. There are several solution concepts of which the core, the stable set and the Shapley value are widely used.

The set of all payoff profiles is called the core if no group of players deviate to form a new coalition in which they are better off. Determining the core is mathematically challenging in NTU games (Scarf (1967)) and might be empty.

A stable set is a set of payoff vectors which satisfies two properties: internal and external stability. An outcome is internally stable if it is immune to any deviation within the set and externally stable if any outcome outside of the set is not a possible solution. Additionally, the stable set may be empty or there can be many stable sets. Neither the core nor the stable set always predict a unique outcome.

These two concepts focus on the *stability* of an outcome. Other solution concepts of cooperative games may focus on *fairness*. The Shapley value is based on the idea that players

¹There are models that allow overlapping coalitions. However, an international organization like the EU does not allow the formation of overlapping coalitions. Therefore, this study ignores this possibility.

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should receive outcomes proportional to their contributions. The Shapley value is the unique payoff vector that satisfies the following assumptions:

- Symmetry: If two players' contributions are equal for every coalition, then the values assigned to them are equal.
- Efficiency: The total gain is distributed among the members.
- Null Player: A player is a null player if she or he does not influence the worth of any coalition.
- Additivity: The value of the sum of two games is the sum of the values of two games.

The following example can be formalized as a TU and an NTU game. Let three players A, B, and C divide a dollar. Consider we have an NTU game. If they divide a dollar in a way in which any two of them divide it equally, then the core contains the outcomes of $(\frac{1}{2}, \frac{1}{2}, 0)$, $(\frac{1}{2}, 0, \frac{1}{2})$, $(0, \frac{1}{2}, \frac{1}{2})$. All three outcomes are stable but the core cannot predict which one of them occurs.

If we consider a TU game and the players decide by simple majority voting on the division of a dollar then the core is empty. All singletons $\{1\}, \{2\}, \{3\}$ have a value of 0. All pairs and the grand coalition $\{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}$ have a value of 1. The grand coalition cannot be in the core because all pairs can deviate. However, none of the pairs can be in the core because we can always find an objection for all the pairs since the utility is transferable.

There is a similar problem, the roommate problem, in hedonic games and matching problems. Three students have preferences over which other to be roommates with and only two students can stay in one room. In the roommate problem, unlike in the marriage problem, there is no stable matching. Suppose that their preferences are $2 \succ_1 3$ for the first, $3 \succ_2 1$ for the second and $2 \succ_3 1$ for the third student. In all possible two roommate cases, one student always wishes to move in with the other student. The core is empty due to the cyclical relation. The roommate problem resembles the idea of the Condorcet Paradox. Roommate-like problems, resulting from a cycle, cause an empty core.

One of the main points in such games is to find the conditions for a nonempty core. For example, if the players do not care about the identity of their roommate but rather the size of

the coalition, then the core consists of $\{\{1,2\},\{3\}\}$, $\{\{1,3\},\{2\}\}$ and $\{\{2,3\},\{1\}\}$. This condition guarantees a nonempty core for this specific example. However, anonymity is not on hold in most cases of international coalitions. The population, culture, GDP, and other concerns will be relevant reasons for coalition formation. I will discuss several restrictions on preferences more in detail in [2.2](#).

In traditional coalition formation games, the winning coalition, the minimal size winning coalition, and the minimal size connected coalition can be found in numerous models. Coalition formation can be examined in a parliamentary system in which the theory can predict a winning coalition, namely the government, and a losing one. Similarly, in a non-democracy, players can be in a winning coalition, an oligarchy, or a losing coalition ([Acemoglu et al. \(2008\)](#)). Hedonic games allow for numerous coalitions to be formed at the same time and in arbitrary sizes. This feature allows us to examine the situations in which there are not necessarily a winning and a losing coalition.

Another well-known theory is the minimal size principle. Among the winning coalitions, the theory predicts that a minimal size coalition will be formed. The intuition behind this principle is that rational players try to maximize their outcomes and to do so they seek to keep their relative size as big as possible. This leads players to try to minimize the coalition size in terms of seats, resources, ...etc.

The last theory is the minimal size connected coalition. The underlying idea in this theory is that players try to minimize the conflict of interest among the coalition members. This means they prefer to form a coalition with players close to them in terms of ideology or other aspects.

There are a number of models that apply the theory of hedonic games with combinations of these principles of winning coalitions. For example, [Breton et al. \(2007\)](#) combine Gamson's rule² and hedonic games. They examine the situation in which heterogeneous (in terms of endowment) agents play a weighted majority game. They identify players as a combination of their weight and ideology in a multi-dimensional Euclidean space. Further, they consider single and double division methods and examine the stability of coalitions.

Closely related to this work, there are models that consider heterogeneous agents as an element in the model and examine the coalition formation process. The first group of studies

²Gamson's rule states that the outcome of the winning coalition is shared among the members in proportion to their weights.

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aim to show the role of heterogeneity and identity in hedonic coalition formation games. [Lazarova and Dimitrov \(2012\)](#) consider heterogeneous players in terms of nationality, ethnic background, or skill type who try to achieve a local (intra-group) or global (inter-group) status through being members of a group. They show the existence of the core stable partition when the players care only about the local status or only about the global status, both as substitutes and as complements.

[Desmet et al. \(2006\)](#) provide an interesting study of the trade-off between increasing returns in the provision of public goods and the cost of cultural heterogeneity. They empirically test the relationship between genetic distance, linguistic and cultural distance, and the stability of regional and national borders within Europe. Furthermore, they examine the likelihood of secession and unification between pairs of countries. As a result, they argue that larger nations benefit from increasing returns in the provision of public goods but suffer the costs of greater cultural heterogeneity. This affects agents preferences over different coalitions and the likelihood of unification or break-up.

The second group of studies aim to analyze the process of coalition formation. Understanding this process is particularly important because international coalitions exhibit a dynamic formation pattern. There are different procedures that players go through to form a coalition. On the one hand, large coalitions such as the EU often start off being small and expand over time. Therefore, one way to form a coalition is by gradually accepting new members. There is a crucial point to emphasize here. In section 2.3, I am going to discuss the consequences of both entry and exit options more in detail. This means that gradual formation can be interpreted as accepting new members or excluding current members. On the other hand, some coalitions such as environmental agreements try to start off being as big as possible. In this case, the players follow a greedy method in which they seek to maximize participation in the coalition and still gradually expand.

Additional theories including endogenous coalition formation are [Hart and Kurz \(1983\)](#), [Ray and Vohra \(1999\)](#), [Konishi and Ray \(2002\)](#). Others including dynamic elements are [Perry and Reny \(1994\)](#), [Bloch and Diamantoudi \(2011\)](#) and [Seidmann and Winter \(1998\)](#). Another strand of literature studies farsighted stability. [Chwe \(1994\)](#), and [Diamantoudi and Xue \(2003\)](#) capture a farsighted aspect of rational players in their models. [Barberà and Gerber \(2003\)](#) also analyze hedonic games with farsighted consideration.

The chapter proceeds as follows. I introduce three elements, namely preferences, process and stability. In section 2.2, I discuss these preferences in various models. First, I give possible restrictions on preferences and their definitions. Second, I discuss coalition formation in one-dimensional and multi-dimensional space. Furthermore, I look at the difference of using ordinal versus cardinal and fixed versus changing preferences. Finally, this section ends with the idea of forming coalitions with neighbors and the connectedness of a coalition. In Section 2.3, the process is presented as an element of coalition formation. The section provides two main approaches to the coalition formation process. In the last section, I discuss stability and instability and their relation to the preferences and the process.

2.2 Preferences

Assume that players know their preferences on coalition partitions and are able to rank them. Rational players seek to maximize their payoffs. Another assumption is complete information. In non-cooperative games, the role of beliefs and uncertainties are well-known concepts. There is relatively little work in cooperative games concerning these elements. Although beliefs and uncertainty are interesting topics, they are beyond the scope of this work.

The outcome of coalition formation games depends on the assumptions on preferences. Thus, in this chapter I will examine these assumptions. As it has been pointed out, the majority of models aim to find a stable partition. However, I will also discuss the adequacy of these assumptions on preferences in international coalition formation games.

2.2.1 Restrictions on Preferences

The Condorcet paradox shows us that in the simple three players society, the aggregate outcome may fall into a cycle. Black (1948) proved that if individual preferences are single-peaked, which violates the condition of unrestricted domain of preferences, the simple

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majority rule satisfies all other axioms of Arrow's impossibility theorem. In coalition formation games, *the roommate problem* resembles a similar cyclical relationship³.

In the presence of a cyclical outcome, the core of the game is empty. Most of the studies on hedonic games and matching problems try to impose a set of restrictions on preferences in order to be able to guarantee a core partition. These restrictions are applied either on individual preferences or on the preference profiles. In the following, we will have a closer look at these assumptions.

The first property, separability, is often used in models of hedonic coalition formation (see [Banerjee et al. \(2001\)](#), [Burani and Zwicker \(2003\)](#), [Dimitrov et al. \(2006\)](#)). The idea is that the preference ordering in a subset S does not depend on the choice of alternatives outside of S . In other words, players can categorize the alternatives as good, bad, or neutral for them. Formally,

Definition 2.2.1 *A preference profile is separable if for any $i \in N$, for any $S \in N_i$ and for any $j \notin S$*

$$S \cup \{j\} \succeq_i S \Leftrightarrow \{i, j\} \succ_i \{i\} \text{ and } S \cup \{j\} \preceq_i S \Leftrightarrow \{i, j\} \preceq_i \{i\}.$$

This means that adding a good player leads to a better coalition, while adding a bad player leads to a worse coalition. When is this applicable to international coalitions? Separability is a rather mild assumption and can be observed in many situations. However, the well-known phenomenon “politics makes strange bedfellows” can occur in both party formations and international coalitions.

Preferences can be represented in additive form if one can determine the value of a coalition as simply the sum of the values of its members. If preferences are separable and can be represented by a utility function of an additive form, they are additively separable. Formally,

Definition 2.2.2 *A game is additively separable if $\forall i \in N$ there exists a function $v : N \rightarrow R$ such that*

$$S \succ_i T \Leftrightarrow \sum_{j \in S} u_i(j) > \sum_{j \in T} u_i(j).$$

³In contrast to the Condorcet Paradox, single-peaked preferences alone are not sufficient for solving the cycle in hedonic games and matching games. [Tan \(1991\)](#) identifies a necessary and sufficient restriction on preferences for solving the roommate problem.

In games with additively separable preferences, each player attaches a value to every other player. [Dimitrov et al. \(2006\)](#) consider the additively separable preferences by defining the preference profiles based on aversion to enemies and appreciation of friends. They show that an individually stable and a contractual individually stable coalition structure always exist under this consideration⁴.

Even though additive separability in matching games yields positive results ([Barbera, B., Bossert, W. and Pattanaik, 2004](#), p. 963), in hedonic games the core may still be empty.

Additive separability is a stronger requirement than separability. This means that additive separability implies separability but the converse is not always true. To illustrate this, imagine four countries⁵. They consider cooperating on an environmental issue in order to reduce the cost. Thus, they prefer more players to fewer. However, conflicts can exist between countries that affect their preferences. Assume that the countries are placed on a line according to their geographical positions.



Country 1's preference is $\{1, 3\} \succ \{1, 2\} \succ \{1, 4\}$ among the possible pairs according to the shortest distance. However, it prefers $\{1, 2, 4\} \succ \{1, 3, 4\}$ because it can be more costly to have a disconnected cooperation. Nevertheless, countries 2 and 4 can have a conflict. If they solve this conflict and therefore become part of a coalition, then the benefit increases. Thus, the preference order for country 1 is:

$$\{1, 2, 4\} \succ \{1, 3, 4\} \succ \{1, 3\} \succ \{1, 2\} \succ \{1, 4\} \succ \{1\}.$$

This preference order is separable. However, country 1 prefers on the one hand $\{1, 3\} \succ \{1, 2\}$ and on the other hand $\{1, 2, 4\} \succ \{1, 3, 4\}$. This violates the additive separability.

Separability and additive separability are assumed as natural restrictions in coalition formation models. However, as we have discussed, they do not always hold – especially in the context of the international coalitions.

Another restriction on preferences is symmetry. This argument is very easy: A preference profile is symmetric if players have the same value as each other.

⁴For definitions of stability see Section 2.4

⁵The statement is true for the number of countries $n > 3$.

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Definition 2.2.3 A preference profile is symmetric if

$$u_i(j) = u_j(i) \text{ for all } i, j.$$

[Bogomolnaia and Jackson \(2002\)](#) show that a Nash stable partition exists in hedonic games with preferences that satisfy additively separability and symmetry. A detailed discussion about symmetric additive separable preferences can be found in [Burani and Zwicker \(2003\)](#).

A similar restriction is mutuality. Mutuality does not require the degree of a player's preference but it requires that players find each other mutually acceptable.

Definition 2.2.4 A preference profile satisfies mutuality if

$$\{i, j\} \succ_i \{i\} \Leftrightarrow \{i, j\} \succ_j \{j\}.$$

Symmetry is a stronger assumption than mutuality, i.e. symmetry implies mutuality but the converse is not always true. While symmetry is cardinal, mutuality can be represented ordinal.

There can be situations in which players only care about the size of a coalition. Every player is indifferent among coalitions of the same size. Thus, in a game, players are anonymous and homogeneous.

Definition 2.2.5 A game satisfies anonymity if for any player $i \in N$ and for any two coalitions $S, T \in N$ it holds that if $|S| = |T|$ then $S \sim_i T$.

[Darmann et al. \(2012\)](#) study the *Group Activity Selection Problem*. This problem can be formalized as an anonymous hedonic game. In their model, each player participates in one activity at most, and her preferences regarding activities depend on the number of participants in the activity. Furthermore, they examine the case where players have preferences for activities in addition to the group size.

Again, in international coalitions, preferences are non-anonymous and players have constraints over the identities of members in addition to their preferences on the number of members. This can be due to several reasons: the decision mechanism in the coalition, cultural or historical relationships, and so on.

Anonymity is sufficient for obtaining a core stable partition with the combination of additive separability ([Banerjee et al. \(2001\)](#)) and an individually stable coalition partition with the combination of single-peakedness ([Bogomolnaia and Jackson \(2002\)](#)).

The next two restrictions are single-peaked and intermediate preferences which are commonly used in social choice theory to eliminate the Condorcet cycle. We say that players' preferences are single-peaked if their preference ordering for alternative coalitions is shaped by their relative distance from their peak point. An alternative closer to this peak point is preferred over more distant alternatives. Thus, the alternatives on a line are increasing (decreasing) then decreasing (increasing). Formally,

Definition 2.2.6 *A game satisfies single-peaked preferences if player i has a unique peak point p_i and for all alternatives a_1 and a_2 such that*

$$a_1 < a_2 \leq p \text{ or } a_1 > a_2 \geq p \Rightarrow a_2 \succ_i a_1.$$

There are two types of single-peakedness: ordinal and cardinal. [Brams et al. \(2002\)](#) consider a setting in which preferences are ordinally but not cardinally single-peaked. The following preferences demonstrate this with 4 players.

Player 1 : $2 \succ 3 \succ 4$

Player 2 : $3 \succ 4 \succ 1$

Player 3 : $2 \succ 1 \succ 4$

Player 4 : $3 \succ 2 \succ 1$

These preferences are ordinally single-peaked with respect to the ordering 1-2-3-4. Assume for contradiction that there is a cardinal single-peaked ordering. Since player 1 and 4 are always ranked last, they are the players on the most left and most right. According to player 2's preferences $d(3,4) < d(3,1)$, while according to player 3's preferences $d(3,1) < d(3,4)$. Hence, the above preference profile is not cardinally single-peaked.

Single-peakedness can be required only in one-dimensional space. It cannot restrict preferences in multi-dimensions. In section [2.2.2](#), we will discuss one-dimensional and

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multi-dimensional models. I will introduce a property which is similar to single-peakedness in multi-dimensional space: the intermediate preference property. It requires that if two players have the same preference ordering of two alternatives, then any other players who are in-between have the same preference ordering. The intermediate preference property relies on the ordering of outcomes rather than players. Formally,

Definition 2.2.7 *A preference profile satisfies the intermediate preference property if player k between players i and j for any two alternatives a_1 and a_2 such that*

$$[a_2 \succ_i a_1 \text{ and } a_2 \succ_j a_1] \Rightarrow a_2 \succ_k a_1.$$

These are the most commonly assumed restrictions on preferences. However, they are not sufficient for guaranteeing stability. A game that satisfies additive separability, symmetry, mutuality, the tree single-peaked property, and the tree intermediate preference property can have an empty core (for an example, see [Banerjee et al. \(2001\)](#)).

As I pointed out before, most works on hedonic games focus on the conditions necessary to achieve a stable coalition outcome. I do not intend to set new assumptions and find new restrictions. However, there will be a close link between the process of coalition formation and these restrictions. To understand the process, it is necessary to understand the basis of the restrictions. In [Section 2.3](#), I will discuss the process of coalition formation more in detail. Next, I give two restrictions which guarantee a nonempty core.

The first restriction is the *Top-Coalition Property* ([Banerjee et al. \(2001\)](#)). The authors give two conditions. The first restriction is the *weak top-coalition property* which guarantees the existence of a core. The second restriction is the *top-coalition property* which guarantees the uniqueness of core. The top-coalition property states that if there is a coalition in which all members prefer this coalition to any other coalition for any nonempty subset of players, then the core is unique and nonempty.

This is motivated by the result of [Farrell and Scotchmer \(1988\)](#) which uses the common ranking property. According to this property, there is a linear ordering over all coalitions which coincides with any player's preference ordering over coalitions. This is sufficient for core stability. The common ranking property implies the top-coalition property but the

converse relation does not hold. Therefore, the top-coalition property is a relaxed version of the common ranking property.

Definition 2.2.8 (1) Given a non-empty set of players $V \subseteq N$, a nonempty subset $S \subseteq V$ is a top-coalition of V iff for any $i \in S$ and any $T \subseteq V$ with $i \in T$, we have $S \succeq_i T$.

(2) Given a non-empty set of players $V \subseteq N$, a non-empty subset $S \subseteq V$ is a weak top-coalition of V iff S has an ordered partition $\{S^1, \dots, S^l\}$ such that

(i) for any $i \in S^1$ and any $T \subseteq V$ with $i \in T$, we have $S \succeq_i T$ and

(ii) for any $k > 1$, any $i \in S^k$, and any $T \subseteq V$ with $i \in T$, we have $T \succeq_i S \Rightarrow T \cap (\bigcup_{m < k} S^m) = \emptyset$.

A coalition formation game G satisfies the (weak) top-coalition property iff for any nonempty set of players $V \subseteq N$, there exists a (weak) top-coalition of V .

The second sufficient condition for a non-empty core in hedonic games is the *Ordinal Balancedness* (Bogomolnaia and Jackson (2002)). The authors adopt the Scarf-balancedness condition which is used to prove the existence of a non-empty core in NTU games.

To check whether a game is ordinally balanced, they check for each balanced family of coalitions, a partition of N , in which each player prefers her coalition in the partition to her worst coalition in the balanced family.

Definition 2.2.9 A collection of coalitions \mathcal{B} is balanced if there exists a vector of positive weights d_S , such that for each player $i \in N$, $\sum_{S \in \mathcal{B}, i \in S} d_S = 1$. A coalition formation game G is ordinally balanced if for each balanced collection of coalitions \mathcal{B} there exists a coalition partition such that for each i there exists $S \in \mathcal{B}$ with $i \in S$ such that $S_\Pi(i) \succeq_i S^6$.

These two conditions are sufficient to find a stable partition in hedonic games but none of them is necessary.

The restrictions on preferences that we examined up to now are imposed on preference profiles. Another direction in the literature is restricting individual preferences rather than preference profiles (Alcalde and Romero-Medina (2006), Pápai (2004)). These conditions

⁶ According to their notation Π is a coalition partition set and $S_\Pi(i)$ denotes the set $S_k \in \Pi$ such that $i \in S_k$.

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are called the *Union Responsiveness Condition*, the *Intersection Responsiveness Condition*, and the *Essentiality*. [Alcalde and Romero-Medina \(2006\)](#) show that these three conditions are independent and each alone guarantees the existence of a core stable partition.

Lastly, [Alcalde and Revilla \(2001\)](#) describe another restriction on each individual's preferences, the *Top-Responsiveness Condition*. This condition also ensures that the core is nonempty.

2.2.2 One-dimensional versus Multi-dimensional models

One way to categorize coalition formation games is to differentiate them in terms of dimensionality. The early models mostly consider one-dimensional space. Considering only a one-dimensional space of alternatives can simplify the problem because single-peaked preferences can be used. Furthermore, if the number of voters is odd, then there is no Condorcet Paradox. The outcome is also not manipulable.

Most coalition formation models are developed in the context of political elections and social choice theory. The theories I have introduced so far are one-dimensional theories. ([Riker \(1960\)](#) minimum size winning coalitions, [Leiserson \(1968\)](#) fewest actor principle, [Axelrod \(1970\)](#) conflict of interest and connected coalitions, [de Swaan \(1973\)](#) minimizing policy distance, [Peleg \(1981\)](#) coalition formation in dominated simple games, [Deemen \(1997\)](#) the center player theory).

[Axelrod \(1970\)](#) model of conflict of interest can be seen as the combination of minimum size winning coalitions and the minimal range principle in which parties in a parliament can be arranged in a unidimensional space and make proposals about which coalitions should be formed. For example, parties can compete on the left-right policy dimension. One-dimensional models do not necessarily depend on policies, but they might depend on power like [Peleg \(1981\)](#) dominated simple game⁷, for example. Although there may be certain policy issues for which one dimension is sufficient, this is a tight limitation. One dimension is not accurate in party politics when parties have different perspectives on multiple policies. Consider a conservative-liberal scale: a party might be conservative on government spending but liberal on social issues ([Grofman and Straffin \(1984\)](#)). This is also related to the single-peaked preferences as I have discussed in section 2.2.1.

⁷A game is simple if a coalition either wins or not, with no outcomes in between.

Some problems naturally have more than one dimension. One may consider settings where alternatives take positions in two-dimensional rather than one-dimensional space. This is especially relevant in economics as opposed to political science. [Hotelling \(1929\)](#) model considers that consumers have preferences concerning the prices of products and the locations of shops. In multi-dimensional space, Condorcet cycles can once again occur. Furthermore, the Condorcet cycle is not the only source of instability.

Later theories, i.e. multi-dimensional theories, are based on spatial modeling. In the spatial models, there is a relationship between the preferences and the locations or positions of the alternatives. Among others, [Grofman \(1982\)](#) generalizes the Axelrod's connected coalitions of multi-dimensional space. [Downs \(1957\)](#) modifies the idea of Hotelling's spatial model to political competition. One of the main differences between one-dimensional and multi-dimensional models is how the distance between players is measured. While the spatial theories use the Euclidean distance, one-dimensional theories may apply ordinal orderings of parties on the ideological dimension only. I will discuss ordinal and cardinal preferences in the next section.

When the policy space is assumed to be multi-dimensional, many political issues can be considered at the same time. The distance between players is calculated as follows: each player i may choose a policy position x_i from an n -dimensional Euclidean policy space \mathbb{R}^n , $n \geq 1$. A distance between two positions $x_i = (x_{i1}, \dots, x_{in})$ and $x_j = (x_{j1}, \dots, x_{jn})$ is given by

$$d(x_i, x_j) = \sqrt{\sum_{k=1}^n (x_{ik} - x_{jk})^2}. \quad (2.1)$$

There is a relationship between dimensionality and stability. [Milchtaich and Winter \(2002\)](#) study stability and segregation in group formation. In their model, players seek to join a group that consists of people similar to them. In addition, they assume that there is a limit to the possible number of groups. When they assume one dimension where players can be represented by points on the line, at least one stable partition exists. However, a stable partition might fail into two or more dimensions. The model is closely related to [Schelling \(1971\)](#) dynamic model of segregation. [Milchtaich and Winter \(2002\)](#) show that a stable partition is also segregating. I will discuss Schelling's model and segregation in coalition formation in section [2.2.4](#).

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We will now turn our attention to hedonic games. [Breton et al. \(2007\)](#) examine one-dimensional and multi-dimensional variants of Gamson's game and apply a pure hedonic coalition formation, where each player is distinguished by multiple arrays of characteristics. In a single dimension, they determine that a winning coalition is stable if it has the least endowment among all winning coalitions. Then, they study the case in multi-dimensional space. First, they consider a single division method where the payoffs of players are a share of their weighted endowment in the coalition. Under the single division method, they show that a winning coalition is stable if it has the least total endowment among all winning coalitions. This case is identical to the one-dimensional case. However, they then consider the double division method where for each characteristic, the weighted share of each player is calculated and then the total share is calculated as a weighted average according to the weight for each characteristic. Under the double division method, a stable coalition may fail to exist. Finally, they give the *Congruence* condition in order to guarantee a non-empty core. Their condition generalizes the *Top-Coalition Property* which I introduced in the previous section.

2.2.3 Cardinal and Ordinal Preferences

In game theory, utilities and preferences can be ordinally or cardinally represented. We call the preferences ordinal if players order or rank their preferences and cardinal if they are also commensurable. Hence, if players' preferences can be described on a cardinal scale, it implies that they can also be represented ordinally, but the converse is not true.

In the *Theory of Political Coalitions*, [Riker \(1960\)](#) compares the cardinality of the winning coalitions. He shows that a winning coalition that has the smallest number of seats among all the winning coalitions in the parliament can be formed. As we will see in this section and in section 2.3, [Brams et al. \(2002\)](#) support Riker's idea of a minimal size winning coalition. However, they use both ordinal and cardinal preferences to examine coalition formation using two processes.

Before I discuss ordinal and cardinal preferences, let me review definitions that I will use in this section.

Definition 2.2.10 *A minimal winning coalition is a coalition in which each member is necessary for the coalition in order to win. The set of winning coalitions is denoted by W and if a coalition is not winning then it is an element of set L . Formally, S is a minimal winning coalition if $S \in W$ and $\forall T : (T \subset S \Rightarrow T \in L)$. The set of all winning coalitions is W^{MIN} .*

Definition 2.2.11 *A minimal size winning coalition is a winning coalition which has the smallest size among all winning coalitions. This is relevant especially in weighted voting games. Formally, let $w_i \geq 0$ be weights, q be threshold and $(q; w_1, \dots, w_n)$ be a weighted voting game. A winning coalition S is of minimum size if $\forall T \in W, w(S) \leq w(T)$.*

Connected coalitions are introduced in [Axelrod \(1970\)](#) *Conflict of Interest Theory*. The model assumes ordinal preferences in one-dimensional space and predicts that only connected coalitions are formed. The idea is that if the conflict between parties is smaller, it is more likely they will form a coalition together. Additionally, winning connected coalitions have minimal size. In contrast to this model, [Brams et al. \(2002\)](#) show that minimal winning coalitions may be disconnected in their model. More interestingly, connected individual preferences of players can result in a disconnected majority coalition.

Definition 2.2.12 *A connected coalition is a coalition in which there is no other player between linearly ordered players. Formally, $\forall i, k \in S$ and $x_i > x_j > x_k$ then $j \in S$.*

In one-dimensional theories, the policy positions of parties and coalitions are assigned to one ideological dimension ordinally but in multi-dimensional space cardinal distances are used. How does one determine whether a coalition is connected? While in one dimension a connected coalition contains every player that is in between its two members, in a multi-dimensional space a connected coalition forms a convex hull. [Grofman \(1982\)](#), in the model of *Proto-coalition Formation*, generalizes connectedness to multi-dimensional space. However, connected coalitions in multi-dimensional space become more numerous, and therefore, less helpful – maybe all minimal winning coalitions are connected ([Grofman and Straffin \(1984\)](#)). Similarly, [Schofield \(1993\)](#) assumes that a party holds a position in a policy space and that preferences for coalitions are based on Euclidean distances. Each winning coalition is associated with a compromise set which is a convex hull of the preferred positions of the parties.

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Ordinal preferences can be seen as a source of cycles in the Condorcet paradox or the Roommate problem. A natural question is whether cardinal preferences solve the problem of cycles. First, consider the Condorcet example. Now in addition to the ranking of preferences, the individuals have intensity over alternatives. It prevents cycles in the case of majority voting with three individuals and three alternatives.

However, Samuelson's conjecture states that Arrow's impossibility theorem holds for cardinal preferences where individuals express their preferences by a von-Neumann Morgenstern utility function. [Kalai and Schmeidler \(1977\)](#) prove that if the number of players $n \geq 4$ and cardinal preferences are continuous, the aggregation rule satisfies cardinal independence of irrelevant alternatives and unanimity requirements if and only if it is cardinally dictatorial.

We have discussed the *cardinal* and *ordinal* preferences in coalition formation theories. Now, I review a particular result in [Brams et al. \(2002\)](#). In that model, players' preferences are divided into two groups: ordinal and cardinal single-peaked preferences. Even if the cardinality does not solve Arrow's problem, single-peakedness solves it with both ordinal and cardinal preferences. However, in coalition formation, the core is even more demanding. [Brams et al. \(2002\)](#) state the following result:

Theorem 2.2.1 ([Brams et al. \(2002\)](#)) *There may be no stable majority coalition even if preferences are cardinally or ordinally single-peaked.*

Their counterexample includes 4 players trying to form a majority coalition. The preferences are cardinally single-peaked (therefore also ordinally single-peaked) with respect to ordering 1-2-3-4. The preferences are as follows.

Player 1 : $2 \succ 3 \succ 4$

Player 2 : $1 \succ 3 \succ 4$

Player 3 : $4 \succ 2 \succ 1$

Player 4 : $3 \succ 2 \succ 1$

In this example, there is no core stable majority coalition. To illustrate, consider the possible majority coalitions. 123 is unstable because player 3 prefers 234; however, 234 is

also unstable because player 2 prefers 123. Two disconnected majority coalitions, 124 and 134, are also unstable because player 1 prefers 123 and player 4 prefers 234.

While in most hedonic games players have ordinal preferences on coalitions they want to join, models exist which consider purely cardinal preferences. [Burani and Zwicker \(2003\)](#) study hedonic games with additively separable and symmetric preferences where players' preferences are purely cardinal. They introduce a decomposition of the utility profiles representing symmetric additively separable preferences into two components, the cardinal component and the alternating component. When they consider purely cardinal preferences, they show that a coalition structure always exists that is both core and Nash stable, and when the preferences are restricted to be purely alternating the core may be empty.

2.2.4 Forming a coalition with neighbors

In the literature, coalition formation is mostly studied as party formation. However, international affairs is a rich area in which players need to form coalitions for mutual benefit or dependence, but it also involves conflict or opposition. A good example is the Organization of Petroleum Exporting Countries (OPEC). On the one hand, OPEC members gain benefits as being part of the cooperation and restricting the oil supply. On the other hand, each member wants to take as large as possible a share in aggregate production. These can appear as centrifugal and centripetal forces. In addition, members have economic and political objectives. Dynamic models examining strategic behaviors in OPEC are already to be found in the literature (for example see [Moran \(1981\)](#)).

In international relationships or any other kind of coalition formation such as marriage, clubs, and etc., the assumption that individuals prefer to associate with people similar to them is considered a uniting force. Under this assumption, one of the most interesting results is Schelling's model of segregation.

Schelling's dynamic model is the first attempt to understand the reasons and mechanisms of segregation in a society. In the model there are two types of agents, black and white, trying to live in a neighborhood where they are happy with their neighbors consisting of these two types of agents. They have individual preferences for a certain neighborhood composition, namely a black-white ratio in a neighborhood. These preferences are results of a tolerance

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level for the other types of agents. The agents have a certain tolerance level to live with the other type of agents, but they always prefer to be a majority in their neighborhood.

Initially, the agents are randomly distributed on a line or plane. According to an exogenously given order, they move in turns to the closest position where they will be happy with its black-white ratio. Even though they are not against living in a mixed neighborhood, aggregation of individual preferences surprisingly results in a segregated outcome.

This simple model has a great power to explain segregation in society or coalition formation into two groups, but it cannot offer the reason as to why there are sometimes more than two groups. Hedonic games may help to explain such a phenomenon.

Preferences play a central role in coalition formation in hedonic models. In fact, the determination of feasible coalitions depends on nothing but the players' preferences concerning their potential coalition members. In light of Schelling's dynamic model of segregation, a number of models introduce segregating and integrating properties. [Milchtaich and Winter \(2002\)](#) and [Lazarova and Dimitrov \(2012\)](#) discuss segregation within a setting with status-based preferences and with two types of agents (low and high). Both models state conditions for stable partitions to be segregating. Differently, in [Barberà et al. \(2013\)](#) non-segregated groups may arise within core stable structures. They consider three type of agents (low, medium, and high) and two principles for distributing the benefits (agents choose Egalitarianism or meritocracy by majority voting).

2.2.5 Fixed and changing preferences

Countries can enter and leave coalitions and characteristics of governments may change over time. This poses questions regarding the reaction of all countries in coalitions when other countries enter or exit. Such questions are of particular relevance to the EU. Since its initial conception, the EU has been continuously expanding, with surprising speed during some periods. More recently, the unprecedented question of the possibility of countries leaving the EU has arisen. This leads us to a further consideration of internal subgroup dynamics which could surface with groups of countries trying to push some members to exit.

The assumption of fixed preferences is often made in the literature on coalition formation models. Both during formation and afterwards, the preferences and nature of coalitions may

change. As a result, models that treat the preferences as given cannot examine and predict situations due to changes in preferences.

Changes in players' preferences and values take time. While some changes may immediately take place, others may have an impact only over a long time horizon. These changes may be relevant to economic power (some currently rich EU members were relatively poorer at an earlier time) or, regime type or identity (memberships of former communist countries).

Particularly, if the coalition has no end term, new entrants and other issues become more relevant. The next chapter will discuss the role of process in coalition formation.

2.3 Process

The importance of the process of coalition formation is usually underestimated. The main concern in cooperative game theory, as well in hedonic games, is the stability of coalitions. However, the coalition formation process is pivotal for achieving a stable outcome. Two main approaches are used in coalition formation: simultaneous formation in which decisions are made in one stage and sequential formation in which decisions are made over time. Different processes, simultaneous and sequential, may lead to different outcomes. Furthermore, the outcomes of the two different sequential procedures may drastically differ.

A further categorization can be made in terms of choosing coalition partners. This can be done using different algorithms. On the one hand, a choice can be “greedy” – meaning players always make the choice that looks best at the moment. For example, players can prefer to form a coalition with the closest neighbors until they reach the threshold for the majority. They do not try to solve all the possible, related subproblems or future problems. On the other hand, players can follow a dynamic approach to coalition formation. In this approach, they consider subproblems and future possibilities.

Surprisingly, in some cases these two strategies predict the same solution. However, here I will outline two models in which these strategies result in different outcomes. These models are related to international coalitions in general and the EU in particular. Namely, they are *Managing the Evolution of Multilateralism* (Downs et al. (1998)) and *Single-Peakedness and Disconnected Coalitions* (Brams et al. (2002)). In both models, the process of coalition formation plays a critical role.

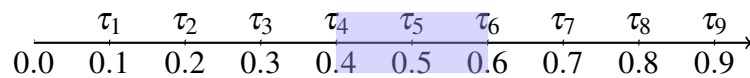
Coalition Formation

Downs et al. (1998) suggest sequential and inclusive constructions in forming coalitions among countries – a coalition can be in terms of trade, military defense, or environmental issues. The main assumption is that the initial group of members is more demanding than others. For example, they demand no or minimum trade barriers in a trade agreement. The countries have ideal points that reflect their demand for cooperation.

The inclusive formation is a one-step process in which all potential members are included in the coalition where they decide on an initial treaty level by majority voting. Due to changes in the ideal points of the countries over time, the treaty level may also change. The sequential formation is a multi-step process in which a coalition expands gradually with new members. Through the admission of new members, the coalition decides on a new treaty level. The admission of new members depends on their progress. It is shown that coalitions formed by a sequential construction are more likely to have a more cooperative treaty level than those formed by inclusive construction.⁸

To illustrate two formations, consider that nine countries are deciding on a common tax level by 2/3 majority voting.⁹ They are symmetrically ordered according to the preferred tax level $\tau \in [0, 1]$. Assume that the rich countries tend to prefer a lower tax level than the poorer countries and we follow an inclusive coalition formation process. In Figure 2.1, countries are ordered according to their preferred tax level. Given single-peaked preferences, the Median Voter Theorem gives the tax level of the median player, i.e. τ_5 .

Fig. 2.1 Inclusive Formation

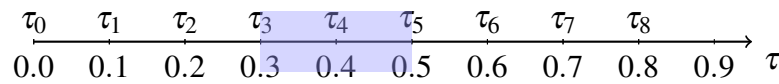


Now assume that the preferred tax levels are shifted due to changes in economies as in Figure 2.2. After the change of the countries' positions, the tax level stays at the same level, at τ_5 , because the core lies in between τ_4 and τ_6 where any tax level in this interval cannot be defeated. However, if the coalition would have formed after the change in tax level, then the tax level would be τ_4 .

⁸The authors assume that if a player is a member of coalition by sequential or inclusive process, it stays in that coalition. This means they only consider the entry option and not the exit option.

⁹The example is from Downs et al. (1998) and has been slightly modified.

Fig. 2.2 Inclusive Formation After Change



That means that, everything else equal, cooperation is deeper if the states integrate first and then join the coalition rather than integrate in the coalition.

Brams et al. (2002) define two processes in coalition formation. Rather than predicting a stable outcome the authors model the process of coalition formation. They show that both procedures may not result in a minimal connected majority coalition. Furthermore, they reason that the oversized coalition and strange bedfellows phenomena result from these two processes.

The first process is the Fallback (FB) process: Players look for coalition partners according to their rankings. If players mutually desire each other and comprise the majority of players, they form a majority coalition. Otherwise, the second most desirable players are considered and so on. A coalition is formed by this process when players consider possible coalition partners in their preference rankings until some majority coalition forms where all members consider each other mutually acceptable. The second process is the Build Up (BU) process. The difference to the FB process is that when two players find each other acceptable but cannot form a majority coalition, they fuse into a single player. The process continues until a majority coalition is formed.

In these two processes of coalition formation, a further division is made in accordance with the particular characteristics of preferences. Players' preferences are divided into two groups: ordinal and cardinal single-peaked preferences. The authors compare the connectedness of coalitions resulting from the two procedures. Disconnected coalitions can be formed under the FB procedure. However, when they consider cardinally single-peaked preferences, the FB procedure always produces connected coalitions. Furthermore, FB coalitions are not necessarily minimal under either ordinal or cardinally single-peaked preferences. Similarly, the BU procedure results in a majority coalition which is always connected under cardinally single-peaked preferences. However, when they consider ordinal single-peakedness, a BU coalition can be the unique disconnected FB coalition. Indeed, these

are the reasons why different procedures may produce strange bedfellows and oversized coalitions.

2.4 Stability - Why so much Stability

The core is the most used stability concept in cooperative game theory even though it is not very powerful for predicting which coalition will be formed – as we discussed in Section 2.1. A coalition is stable if and only if a coalition of players that could strictly gain by forming another coalition does not exist. Early models of hedonic games (Banerjee et al. (2001) and Bogomolnaia and Jackson (2002)) state sufficient conditions for the core stability, i.e. a non-empty core. Iehlé (2007) gives a condition necessary and sufficient for the existence of core stable coalition structures in such hedonic games.

The simplest form of the coalition formation problem does not assume any restrictions on preferences. If we look again at a coalition formation with three players, there are three possible coalition structures. Namely: the grand coalition, coalitions with two players together excluding the third player, and coalitions in which each player is alone. The question then pertains to the number of ways in which one can write the total number of players n as a sum of coalitions. The answer is the partition number denoted by $p(n)$. A partition of a positive integer n , also called an integer partition, is a way of writing n as a sum of positive integers.¹⁰ In our three players example, the partitions are $1 + 1 + 1$, $2 + 1$ and 3 , i.e. $p(3) = 3$.

The partition number increases enormously with the increasing magnitude of the number. If we have ten players, the partition number is 42. For fifty players, it is over 200.000 and it is over a million for sixty one players. Note that only the size of coalitions is considered (two sums that differ only in the order of their summands are considered to be the same partition). If the identity of members is also considered, the number of combinations increases even further.

¹⁰There is no closed formula for partition numbers. Euler developed a method for computing the partition number and Ramanujan found an equation for the asymptotic behavior of partition numbers.

As we have discussed, there are two main questions. Namely, whether the possible partitions are core stable, i.e. the verification problem and whether a core stable partition exists, i.e. the existence problem.

Definition 2.4.1 *A coalition partition Π is core stable if there is no $T \subset N$ such that $T \succ_i S_{\Pi}(i)$ for all $i \in T$ (otherwise T blocks Π).*

In addition to the core stability, there are other stability notions both in cooperative and non-cooperative perspectives. [Bogomolnaia and Jackson \(2002\)](#) examine stability in terms of group and individual deviations. In addition to core stability, they introduce three other stability concepts: *Nash*, *Individual*, and *Contractual Individual stability*. The most stringent stability concept is Nash stability: no player would rather join a different coalition in the given partition. If no player would rather join a different coalition and also be tolerated there, the given partition is individually stable. Note that Individual stable partitions are a subset of Nash stable partitions. Last, Contractual Individual stable partitions are a subset of Individual stable partitions in which no player would rather join a different coalition of the given partition, be tolerated there, and be allowed to leave.

Now, we look at two theorems that give sufficient conditions for a non-empty core.

Theorem 2.4.1 ([Banerjee et al. \(2001\)](#)) *Suppose game G satisfies the weak top-coalition property. Then, it has a non-empty core.*¹¹

I do not give a formal proof here but discuss the idea. The theorem states a sufficient condition for the existence of a core. If a coalition S satisfies the weak top-coalition property it has a partition of $\{S_1, S_2, \dots, S_l\}$ and set $l = 4$. Let me briefly illustrate the mechanism. In partition S_4 , any player needs at least one player from $S_1 \cup S_2 \cup S_3$ to form a more preferable coalition than S . In S_3 , each player prefers S unless they cooperate with at least one player from $S_1 \cup S_2$. Therefore, no player in S_3 is an option for players in S_4 . In S_2 each player prefers S unless they cooperate with at least one player from S_1 . Therefore, no player in S_2 is an option for players in S_4 and S_3 . Finally, in S_1 each player prefers S to any other coalition. In other words, each partition of S , given that there is no better coalition without an agent from earlier groups, is the best coalition for its members. Hence, there is no profitable coalitional deviation.¹²

¹¹If the preferences are strict then the core is unique. For the proof see [Banerjee et al. \(2001\)](#).

¹²See [Bogomolnaia and Jackson \(2002\)](#) for an example in which the condition is not necessary.

Coalition Formation

The second sufficient condition is from [Bogomolnaia and Jackson \(2002\)](#). The theorem is adapted from [Scarf \(1967\)](#) and [Greenberg \(1994\)](#). It states that every balanced game has a non-empty core.

Theorem 2.4.2 ([Bogomolnaia and Jackson \(2002\)](#)) *If a game is ordinally balanced, then there exists a core stable coalition partition.*

Again, I do not give the formal proof.¹³ A hedonic game is ordinally balanced if there is a partition for each balanced family of coalitions in which each player is better off than her worst situation in the balanced family. The weight function in a balanced collection of coalitions can be seen as players' contribution of time, effort, cost, etc. They give an algorithm for finding an individually stable coalition partition. An individually stable partition might exist when there is no core partition.

The idea of finding an individually stable coalition partition is as follows. Players only care about the size of the coalition of which they will be a part. Furthermore, their preferences are single-peaked. First, a coalition S_1 is formed in which players have the highest peaks about the size. Then, a coalition S_2 is formed from the remaining players depending on the same idea. If S_1 is open, then check the players according to highest peak size in S_2 . In the case that any player prefers S_1 to S_2 and all players in S_1 are better off by adding this player, then move her or him to S_1 until it is closed. This process is applied iteratively until all players are assigned to a coalition.

¹³See [Banerjee et al. \(2001\)](#) for an example in which the condition is not necessary.

Chapter 3

Clustering

Clustering is a technique for grouping elements (data objects) into meaningful structures. The results are called clusters. To do so, a distance measure is used. The aim is to maximize the similarity between elements of the same group and minimize the similarity between elements of different groups. In other words, elements belonging to the same cluster have high similarity and elements belonging to different clusters have low similarity. Therefore, clustering can be seen as a tool for discovering “natural” groups among elements.

Unfortunately, this general definition remains unclear because there are many different kinds of clustering problems and similarity is a subjective concept. Clustering is a vast subject, and obtaining a full overview is not possible within the scope of this work. This chapter aims to introduce the clustering techniques used in Chapter 5 and 6 and discuss their theoretical perspectives.

Clustering techniques provide for theoretical models in diverse fields. Even though every clustering method can be the object of criticism, these methods have to ascertain two general points. First, they have to answer the question of what similarity means. Generally, similarity is measured by a distance function. However, similarity can be an ambiguous term. [Bronstein and Mumford \(2009\)](#) give the following example to manifest possible consequences of working without technical restrictions. A centaur is a creature in Greek mythology with the head, arms, and torso of a man and the body and legs of a horse. A centaur and a man are similar, a centaur and a horse are similar, but we cannot conclude that a man and a horse are similar. To clarify such issues and avoid complications, the formal definition of similarity is given in Section 3.1.

The second point that needs to be addressed is how to group elements together. This is equivalent to choosing an algorithm for a specific clustering task. Only two clustering methods are considered in this work to study the process of coalition formation. The first one is partitional clustering, which constructs partitions of the set of elements according to a distance measure. The second one is hierarchical clustering which creates a hierarchical decomposition of a set of elements according to a distance measure.

In the following, the general distance measure is discussed. Any distance measure which satisfies certain properties can be used to evaluate coalitions. However, the results are not necessarily meaningful. At this point, the models should be guided by game theory.

3.1 Similarity and distance function

The concept of similarity can be confusing because it can be defined in different ways. In this work, the similarity between two elements is defined by a quantitative measure that is called a distance measure or function. Choosing a distance function is crucial to both models in Chapter 5 and 6.

Even though there are infinitely many choices for defining a distance measure, the choices should obey certain rules. The most used distance measure in the literature is the Euclidean distance. However, we are not restricted to using only Euclidean distance. First, we can generalize the distance function. To do so, we should give an abstract definition of distance between two elements of an arbitrary nonempty set.

Formally, a distance is a function defined on the Cartesian product of a set X , $d : X \times X \rightarrow \mathbb{R}$. It is called a metric on X if for every $x, y, z \in X$ the following conditions hold,

(M1) $d(x, y) \geq 0$ (*the Non-negativity*)

(M2) $d(x, y) = 0$ if and only if $x = y$ (*the identity axiom*)

(M3) $d(x, y) + d(y, z) \geq d(x, z)$ (*the triangle inequality*)

(M4) $d(x, y) = d(y, x)$ (*the symmetry axiom*)

A set X provided with a metric is called a *metric space*.

The low dimensional Euclidean metric is often used in classical clustering problems. Applications in economics, political science and game theory often involve a two-dimensional Euclidean metric. Two immediate problems appear when the Euclidean space is in a high dimension and the space is not Euclidean at all.

One should choose a metric depending on the application. In this work, I will use geometric and non-geometric elements in the distance measure even though using a mixture of these is not a usual approach.

3.2 Clustering algorithms

This section presents two clustering algorithms. In a coalition formation process, two strategies are considered. Players can either follow the bottom-up strategy or top-down strategy. In the bottom-up strategy, players move sequentially. In this sequential process, each element is assumed as a cluster by its own. Then, two elements are merged based on their distance. The new cluster inherits properties of its predecessors. The process stops when the merging is not feasible - depending on a rule. The idea is to form a coalition step-by-step. In the top-down strategy, players move simultaneously. In the simultaneous process, there are no intermediate steps before final clusters are reached. Instead, all players decide simultaneously.

In clustering analysis, the way in which to cluster elements is specified by a clustering algorithm. Clustering algorithms can be put into two groups: partitional and hierarchical clustering algorithms.

K-means is one of the best known partitional algorithms. The idea is to allocate N objects in a fixed number of clusters. For this one chooses K centroids, in other words center points. These points are random but it is preferable that they are not very close to each other. Then every object is attained to the closest centroid. Different configuration of initial centroid allocation results in different outcomes. To overcome this instability resulting from the sensitivity of k-means to initial centroid location, an algorithm can be run several times in which more configurations are explored. Then, the most desirable one can be confidently chosen. A typical algorithm is as following.

1. Choose K centroids at random

Clustering

2. Assign elements to closest centroid, forming K clusters
3. Calculate centroid (mean of distances) of each cluster, update centroids
4. Check if an element in a cluster is closer to another centroid. Reallocate if necessary
5. Repeat from step 3 until no object changes the cluster anymore.

The K-means algorithm has been met the criticism for the predetermined number of clusters. In chapter 6, correlation clustering which overcomes this criticism will be discussed in detail.

The second algorithm I want to review is for Hierarchical Clustering. It can be done two ways: either agglomerative, in which every object is considered as a cluster and the closet pairs gradually merge, or divisive, in which all objects are considered as one giant cluster and are gradually split. I am particularly interested in Agglomerative Hierarchical Clustering.

A basic Agglomerative Hierarchical Clustering works as follows.

1. Define a similarity matrix
2. Merge the closest two clusters
3. Update the similarity matrix
4. Repeat steps 2-4 until only one cluster remains

These two procedures result in different outcomes. Both procedures have a number of benefits and drawbacks. The one main advantage of sequential clustering is being able to study subcoalitions. The general problem of the two clustering algorithms is the number of clusters. The cluster number, k , must be determined beforehand. I will discuss this issue and how to overcome this difficulty in sequential coalition formation in Chapter 5 and in simultaneous coalition formation in Chapter 6.

3.2.1 Computational Complexity

The space and time requirements can be crucial to an algorithm. With a small number of objects, neither is important. Given that current computers are reasonably fast, we can choose

an algorithm that is easy to implement and not necessarily sophisticated, but, as the number of objects get larger, there is a trade-off between exactness and running time.

In k-means clustering, the computational complexity is linear in the number of objects. Namely, the running time is $O(IKMN)$, where I is the number of iterations, K is the number of clusters, M is the dimension of data and N is the number of objects. Typically, the result converges in a few iterations. That means k-means is linear in its inputs and, therefore, fast. The two main shortcomings are: i) the result can be highly sensitive to the choice of initial centroids and ii) a prior knowledge of k is not always possible. However, when there is no restriction on the number of clusters, we face a much “harder” problem. Chapter 6 studies an algorithm with such a problem and its complexity.

The brute force algorithm for hierarchical clustering is not very efficient. At each step, the distances between each pair is calculated to find the closest pair. The initial step takes n^2 time, and subsequent steps take $(n-1)^2$, $(n-2)^2$, and so on. Additionally, the time for finding the minimum, $n, (n-1), (n-2), \dots$, makes the overall running time of this algorithm $O(N^3)$. This running time is a significant limitation on the number of elements. Chapter 5 studies a more efficient algorithm and presents its complexity.

Chapter 4

The EU and International Coalitions

4.1 A Brief History of the EU

The EU is a unique economic and political cooperation example. It is over sixty years since six European countries formed the union. A proper discussion of its history and dynamics would be beyond the scope of this work. For a comprehensive discussion of its historical development and dynamics see [Tilly et al. \(2007\)](#). Yet the EU are experiencing the most trouble time in its history. These problems and Europe's future are discussed in [Hanappi \(2013a\)](#) and [Hanappi \(2016\)](#).

A customs agreement was signed between Belgium, Luxembourg and the Netherlands in 1948. This was the first concrete step towards a European entity ([Carchedi, 2001](#), p.11). This was followed by Belgium, France, West Germany, Italy, Luxembourg, and the Netherlands signing the so-called Treaty of Paris and thereby creating the ECSC. The actual integration of the EU started with the ECSC in 1951 when this group of nations, known as the 'inner Six', built a supranational authority ([Baldwin and Wyplosz \(2012\)](#)). This supranational authority controlled the single market of coal and steel industries. On the one hand, the aim of cooperation was to end wars, particularly between France and Germany. On the other hand, European countries were not economically strong enough in the world market on their own. Therefore, the formation of the ECSC had economic and political motivations and later gradually evolved to the EU.

The EU and International Coalitions

Since the creation of ECSC, several enlargements and structural changes have taken place¹. The EU has grown from six to twenty-eight member states as a result of these enlargements (See Figure 4.1). The founding members of the ECSC established two more communities in 1957: the European Economic Community (EEC) and the European Atomic Energy Community (EURATOM). By 1967, the ECSC, the EEC, and EURATOM constituted the European Community (EC).

The first enlargement took place in 1973 with the entry of the United Kingdom, Ireland, and Denmark. Further, Greece joined the EC in 1981 and Spain and Portugal joined in 1986. The Single European Act (SEA) was signed in 1986 and came into force in 1987. The SEA was an answer to the situation of ‘Eurosclerosis’, in which Europe experienced economic stagnation in the period from the early 1970s up to the early 1980s ((Carchedi, 2001, p.12)). Additionally, it created new institutional reforms for political cooperation.

The name ‘EU’ was used for the first time in the Maastricht Treaty (or the Treaty on European Union (TEU)) in 1992. The EU consisted of three pillars ((Carchedi, 2001, p.8)). In addition to the EC, a Common Foreign and Security Policy (CFSP) and Justice and Home Affairs (JHA) constituted the EU. The Maastricht Treaty also outlined an Economic and Monetary Union (EMU), including a common European currency (the Euro).

In 1995, Austria, Finland and Sweden joined the EU. With this enlargement, the number of member states increased to fifteen.

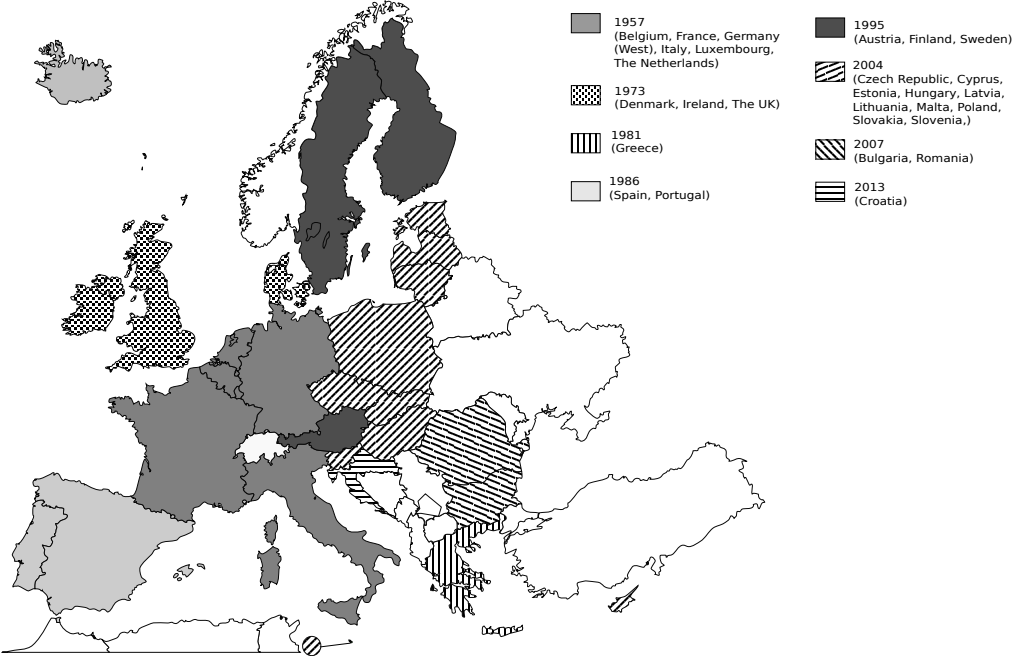
After the fall of the Soviet bloc, an opportunity was given to Central and Eastern European countries to join the EU. The criteria for membership were decided at the Copenhagen Summit in 1990. The Czech Republic, Estonia, Cyprus, Slovakia, Latvia, Lithuania, Hungary, Malta, Poland and Slovenia became new members of the EU in 2004. In 2007, Romania and Bulgaria were accepted. The most recent enlargement was the membership of Croatia in 2013.

4.2 Coalition Formation in the EU

Usually, more than one country joined the EU at the same time. However, independent negotiations took place for each country in order to gradually bring the potential candidate

¹For a detailed history, see the EU website: <http://europa.eu/about-eu/eu-history/>

Fig. 4.1 Enlargement of EU 1957–2013



Source: reproduced from [Pinder and Usherwood \(2013\)](#)

The EU and International Coalitions

countries closer to the EU. Negotiations for and the acceptance of new members widened and deepened the EU. The progression of admissions can be seen as a sequential process in which only one country could join the existing coalition. This line of reasoning applies to the incremental expansion, but another critical issue is how the founding states got together in the first place.

It can be hard to form a large coalition in the beginning of the process due to several reasons. A common start is a coalition of few players. The initial members can stay together and no expansion takes place. For example, the North American Free Trade Agreement (NAFTA) did not experience expansion until recently. Or, as in the case of European Free Trade Association (EFTA), even the founding members can leave. There are many other international coalitions (for example, the North Atlantic Treaty Organization (NATO), the Association of Southeast Asian Nations (ASEAN), the World Trade Organization (WTO)) following a sequential pattern in which they start as a small group of countries and continue to accept new members.

One reason to start with a small group is that as the number of members increases, a stable state becomes harder to achieve. Therefore one option is to assume that the initial state of a coalition is taken as exogenously given². Afterward, expansions can be modeled as a process in a sequential game. Our approach differs in that we approach the creation problem as well as the expansion problem. To reduce the complexity of the problem we omit, for now, many other factors such as cultural heterogeneity, a specification of trade, etc.

Despite the large literature on coalition formation and accession games, formalizing them, particularly in the case of the EU, as a process and including the creation problem seems to be less developed. There are few models providing useful insight to our sequential coalition formation game. I have mentioned some of them in the introduction. The intention here is not to repeat all these models, but rather to review the most relevant ones.

Grofman (1982), Downs et al. (1998), Brams et al. (2002) study coalition formation as a process. Their models follow a dynamic approach. In Grofman's model of Proto-coalition Formation, players form pairwise coalitions in each step depending on their pairwise distance. The model best describes the process of coalition formation in parliaments.

²See for example Downs et al. (1998). They start with an exogenously given group of players for one-step and multi-step coalition formation. Implicitly, these approaches assume that the formation of the initial group needs a different kind of model.

[Brams et al. \(2002\)](#) introduce two different processes. The first one is the *Build-Up* (BU) process in which players form pairwise coalitions. This is close to the idea of [Grofman \(1982\)](#). Players merge with their closest neighbors until a majority coalition is formed. The difference is that a majority coalition can be disconnected in one dimension. The second one is *Fall-Back* (FB) process in which players look for their potential partners by descending lower and lower in their preference rankings. Two different procedure allow us to explain varying coalition formation situations that are different from empirically observed life. For example, while coalitions in legislatures reflect a BU-like process, political parties in parliament follow an FB process when they form a government.

Both models share the winning coalition principle. However, this can be inadequate for some coalition formations as was discussed in Section 1.1.

Adopting two different procedures, [Downs et al. \(1998\)](#) analyze the evolution of multilateralism. Particular attention is paid to the EU. They discuss whether to form a multilateral sequentially or inclusively.

Another formal model to analyze coalition formation as a process is introduced by [Bloch \(1996\)](#). His model captures the effects of externalities among coalitions. In the process, players take the reaction of external players into account with the presence of externalities. There are two models which explicitly model EU integration: [Kóczy \(2009\)](#) and [Morelli \(2012\)](#). Both models propose a sequential admission game with externalities.

I have already mentioned that physical models can be used as templates to explain social aggregations. [Vinogradova and Galam \(2013\)](#) use a theoretical framework, namely the Ising Model of Spin Glass in Statistical Physics, to analyze the countries' decision-making in coalition formation. In this model, stability can be achieved as a result of the maximization process. They apply their model to explain the current (in)stability of the Eurozone.

4.3 Data

This section outlines the data used for application of sequential coalition formation in the next section. Furthermore, it discusses why the characteristics in the distance function are chosen in Section 6 and 5. In order to do that, we take a closer look at particular coalitions in Europe.

The EU and International Coalitions

After World War II, Europe segregated into two groups: the Western and Eastern Bloc. Moreover, there were three main economic groups. Communist countries³ formed the COMECON (Council for Mutual Economic Assistance) in 1949 under the leadership of the Soviet Union. The ECSC was formed in 1951 (see Section A Brief History of the EU). As a reaction to the establishment of the ECSC, the UK together with Norway, Sweden, Denmark, Austria and Switzerland formed the EFTA (European Free Trade Association) in 1960.

There are similarities and differences in terms of structure and evolution of these coalitions. At the time of EFTA's creation, Western Europe could be described as two non-overlapping circles (Baldwin, 1994, p.16). In the late 1980s together with COMECON there were three disconnected sets of European countries⁴. What factors influenced actors' decisions and led to this partition? Of course, taking into account all factors is not possible. Some factors, however, have more importance in the model than others.

The first factor considered here is geographical location – a quantity often used in international trade theory. Geographically close countries are more likely to cooperate (Deardorff (1998)). This is studied often in the gravity model of trade. The model predicts the bilateral trade flows depending on the economic sizes and distance between two countries.

The gravity model of trade is an analogy to Newton's law of gravity (gravitational attraction between two bodies). In the simplest specification of the gravity model, trade between two countries is proportionate to the product of their economic sizes (GDPs) and inversely related to the distance between them (Frankel et al., 1998, p.93)⁵. Krugman (1991) makes a similar argument and empirically tests whether geographical distance matters in forming trading blocs.

Economic strength and population are the additional factors in the presented model. Here, GDP is considered to represent economic strength. As the gravity model predicts, bilateral trade increases with the increase in GDPs of the trading partners. Following this, in the sequential coalition formation model, population and GDP are critical parameters in the pairwise distance function considered by prospective coalition partners in the environment.

³The founder members were the Soviet Union, Bulgaria, Czechoslovakia, Hungary, Poland, and Romania.

⁴Yugoslavia was not a member of any of three groups. It had only an observer status in COMECON.

⁵See (Baldwin, 1994, p.70–71) for theoretical foundations. Populations of two countries are also considered in the model.

The factors GDP and population have two sides. First, they are important within the coalition both in creation phase and afterwards in the decision making process. For example, countries can have more weights on policy decision depending on their GDP or population. Second, they play a role in competition with other coalitions. But these concerns are not in the scope of this work.

One of the motivations for building the ESCS was reaching more economic strength to be able to compete with the global hegemony of the US (Carchedi (2001)). The reason for creating EFTA was similar. It was a rival free trade area against the ESCS for non-EEC member states (Dinan (2004)). Although the number of countries was six in the ESCS and seven in EFTA, total GDPs significantly differed. In the 1970s, the GDP of the EEC nations was more than twice the size of the GDP of the EFTA nations (Baldwin, 1994, p.14).

The last factor is regime type. The cooperation between the Western European countries can be seen as the formation of a bloc against the expansion of the Soviet Union. No coalition was formed across bloc boundaries. Spain, Portugal and Greece were either too poor or not democratic (McCormick, 2015, p.82). Therefore, countries are categorized according to their regime types: liberal, socialist and military states. Among the Western European countries Spain and Portugal were only military dictatorships in the 1950s (see Table 4.1).

This thesis uses Maddison Project⁶ data on population and GDP. The focus is on 28 European countries. Table 4.1 gives populations, GDPs, per capita GDPs and regime types in 1950. This gives us an overview of Europe before the establishment of the ECSC.

As discussed in Section 5, countries tend to merge with similar countries with respect to GDP, population, regime and location. Now, we take a closer look at the founder states of three trading blocs.

European countries differ in terms of the discussed characteristics. The most clear division is visible in terms of regime types. While democratic countries are either members of the ESCS or EFTA, communist countries are members of COMECON. Spain and Portugal are excluded because of their military regimes.

Among the Western European countries West Germany, France, UK, and Italy have large populations and GDPs. The Western European countries have higher per capita GDP relative

⁶The data set “Historical Statistics of the World Economy: 1-2008 AD”
<http://www.ggd.net/maddison/oriindex.htm>

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to the Eastern European countries with the exception of geographically far Greece, Portugal, Spain and Ireland.

The six founder members of the ECSC were three big economies and three small/medium economies. They also shared borders so that no state was disconnected. Since the creation of the EU, no country has left and it has gradually expanded. Early enlargements were the accession of former EFTA members and later enlargements took place after the collapse of Soviet Bloc.

The seven EFTA countries were not as strong as the ECSC countries economically. Finland (1970), Iceland (1986) and Lichtenstein (1991) were three enlargements. But as the economic integration advanced, the inner six's economic performance improved. This motivated the UK to join the EEC in 1961 and led other countries follow (Ireland, Denmark and Norway⁷). In 1986 Portugal and in 1995 Austria, Finland and Sweden left EFTA and joined the EU⁸. Currently, only Switzerland, Norway, Liechtenstein and Iceland are EFTA members.

COMECON collapsed in 1991. It had the largest population among the three coalitions. Later expansions increased the diversity of member states in terms of geographical location⁹, economic strength and size.

⁷Norway was accepted, but EEC membership were rejected in referendum twice

⁸In the literature one explanation for EFTA countries joining to the ESCS/EEC/EU sequentially is "Domino effect". See [Baldwin \(1993\)](#) for more details.

⁹Three non European countries were Mongolia (1962), Cuba (1972), and Vietnam (1978).

Table 4.1 Europe 1950

Country ¹	Population ²	GDP ³	Per Capita GDP	Regime ⁴
Albania	1227	1229	1001	Soc.
Austria	6935	25702	3706	Lib.
Belgium	8639	47190	5462	Lib.
Bulgaria	7251	11971	1651	Soc.
Czechoslovakia	12389	43368	3500	Soc.
Denmark	4271	29654	6943	Lib.
Finland	4009	17051	4253	Lib.
France	42518	220492	5185	Lib.
East Germany ⁵	18388	51412	2795	Soc.
West Germany ⁶	50958	213942	4198	Lib.
Greece	7566	14489	1915	Lib.
Hungary	9338	23158	2479	Soc.
Iceland	143	762	5328	Lib.
Ireland	2963	10231	3452	Lib.
Italy	47105	164957	3502	Lib.
Luxembourg	296	2481	8381	Lib.
Netherlands	10114	60642	5996	Lib.
Norway	3265	17728	5429	Lib.
Poland	24824	60742	2446	Soc.
Portugal	8443	17615	2086	Dic.
Romania	16311	19279	1181	Soc.
Soviet Union	179571	510243	2841	Soc.
Spain	28063	61429	2188	Dic.
Sweden	7014	47478	6769	Lib.
Switzerland	4694	42545	9063	Lib.
United Kingdom	50127	347850	6939	Lib.
Yugoslavia	16298	25277	1550	Soc.

¹ Microstates such as Liechtenstein and Monaco are excluded.

² Population is given in '000 at mid-year.

³ GDP is given in million 1990 International Geary-Khamis dollars

⁴ There are three main regime types; Soc.: socialist, Lib.: liberal democracy, Dic.: military dictatorship

^{5,6} GDPs and Populations are from <http://dx.doi.org/10.1787/486663055853>

The Dynamics of Europe's Political Economy: a game theoretic analysis

This part introduces two models of coalition formation with special emphasis on the coalition formation process. While the first model uses a sequential approach in which players iteratively form subcoalitions, the second model uses a simultaneous approach in which players form coalitions at once with all members. These models attempt to account for international coalition formation. To illustrate the predictions made by these models, I will examine the formation of the European Coal and Steel Community (ECSC). Finally, I will compare the two models in terms of their outputs and mechanisms.

Chapter 5

Sequential Coalition Formation

The related literature is reviewed and a number of issues in the formation of coalitions are discussed. In light of these, the key result is that the process is as important as the reasons for cooperation and the resulting coalitions. This chapter presents a model of sequential coalition formation model.

The sequential coalition formation model studies the process of coalition formation in a game theoretical framework. A common discussion in game theory concerns how coalitions are formed and how coalitions distribute payoffs among their members. Here, the main challenge is describing how coalitions can be formed by using players' preferences on their potential coalition partners. This offers the motivation for building a model to establish a way of viewing coalition formation as a process which is sensitive to different procedures.

One main property of the sequential coalition formation game is that once a coalition is formed, players are restricted to remaining in that coalition (See [Bloch \(1996\)](#)). If two players join a coalition, they act as a single player. The evolution of the EU provides a relevant example. Moreover, the model guarantees that a subcoalition is formed at each step. (See [Grofman \(1982\)](#), [Brams et al. \(2002\)](#)¹). In a different perspective, players can be partitioned simultaneously but this will be examined later.

The algorithm of sequential coalition formation provides a set of rules for grouping players into coalitions. The idea is to repeatedly combine the closest pair of players according to some distance measure. To do so, one needs to define a distance or dissimilarity function

¹Sequential games can be seen as a solution to the myopia of simultaneous games. However, [Brams et al. \(2002\)](#) argue that this setting can also lead a myopic outcome as in their Build-Up model.

Sequential Coalition Formation

between players. A distance function can be chosen from a rich set of functions that measure different characteristics of elements.

A distance function can involve one or more parameters that indicate how close two players are. For example, physical distance can be the only parameter, and GDP, population or regime type can be used as additional parameters. Furthermore, a rule governs how to combine the properties of two previously separate players into the properties of a coalition. The simplest rule is to take the average of these characteristics. Taken together, the algorithm with its parameters and a combination rule for these parameters form the sequential coalition formation model.

Using a distance measure will enable the model to have endogenous preferences on coalition partners. This makes it different to most models of coalition formation in which preferences are exogenously given. In hedonic games, players have exogenous preferences over coalitions, e.g. [Bogomolnaia and Jackson \(2002\)](#), [Banerjee et al. \(2001\)](#). Indeed, exogenously given preferences are one of the problems in the game theoretic study of coalition formation in general (see [Deemen, 2013](#), p.27)). The formation of preferences is as important as coalition preferences are in the coalition formation process.

Once parameters are determined for specific economic analysis tasks, the final question that needs to be addressed is that of determining the partition of players across coalitions. As previously discussed, many coalition formation models predict one winning coalition if one exists. This might be a suitable prediction, for example, for party coalitions in governments after political elections. However, many coalition formations cannot be formulated such that resulting coalitions are either winning or losing. This model makes a crucial structural difference. Similar to hedonic games, it predicts a set of coalitions.

In the model, players merge step-by-step. Now the question arises of when this process stops. When analyzing a sequential formation, the formation of a winning coalition can be used as a stopping condition. Since the model predicts no winning coalition, a different solution is needed. A possible approach is to define a threshold value, for example, for the size of a coalition, the utility of players or the distance measure. Or one can decide where to stop depending on the length of the coalition formation sequence. However, this can lead to somewhat ambiguous boundaries.

Both attempts have advantages and disadvantages. Ideally, the stopping condition should be defined without introducing an exogenous threshold value which varies. The reason for this is that the value of the threshold can affect the result dramatically. Thus, a rather simple solution is considered to determine a stopping condition: If players do not merge in early iterations then they cannot be in the same coalition later. This approach might be more relevant to the coalition formation process than specifying a threshold value for the size, utility or distance. I will discuss this condition in more detail in later sections.

A partition of coalitions that seems to be unstable or unreachable can indeed be stable or reached if players are allowed to form coalitions not only once and for all, but sequentially. Both Nash Equilibrium and the core can be guaranteed under certain restrictions on the preferences of players (Banerjee et al. (2001) and Bogomolnaia and Jackson (2002), see Section 2.4). Otherwise, they might fail to exist. I do not impose any of these restrictions here (See Section 2.2 for the arguments of these restrictions). Therefore, it is very likely that a stable solution cannot be reached. However, the model can predict a partition in the absence of a stable outcome.

A sequential recombination algorithm is simpler to state and study relative to a simultaneous algorithm. Additionally, it provides more information than just finding a partition. It describes a clear sequence of steps towards resulting coalitions, which might be closely connected to the dynamics of coalition formations, expansions, and modifications found in actual political life.

5.1 Sequential Coalition Formation

This section attempts to formalize a sequential coalition formation. It encompasses three essential elements: (i) the distance function, (ii) partner preferences and (iii) the coalition formation. These will be studied in details.

Different models have been discussed in the domains of economics and political science. Despite the fact that players (in our discussion countries) are not physical objects and their interactions are not governed by laws of physics, tools from physics and mathematics can contribute to the understanding of economic and political events. Surprisingly, in view of the mechanics of the process, there are great similarities in the sense that the formation

Sequential Coalition Formation

of international coalitions and the study of jets in physics can exhibit similar patterns. Furthermore, different tools can overcome challenges such as computational complexity. Particle physics methods are also designed to reduce the computing time. This is very important practical consideration regarding events with large numbers of elements because computing time and space are bounded. Therefore, producing efficient algorithms should be of an interest in this work.

To examine and better understand these similarities, I begin with a short overview of jet clustering in particle physics.

Sequential Jet Clustering Algorithms

The sequential algorithm for country clustering is inspired by the study of jets in high energy physics experiments such as CMS and ATLAS at the LHC at CERN²³. The LHC accelerates protons and other subatomic particles to large energies. Inside the ATLAS and CMS experiments these particles are brought to collisions. *Jets* are collimated showers resulting from the decay of strongly interacting particles which are in turn a product of these collision events. These collimated showers can be reconstructed as so-called jets in the detector⁴. Thus, jets are physical structures which are observable.

The aim of jet clustering algorithms is to find the original particle that initiated the shower. Jets are built from measurements of the energy deposited in parts of a detector. A detector is designed to observe decay particles resulting from the colliding beam of, for example, protons. In such a detector, there is a calorimeter which slows down or stops the particles and measures their energy. The jet clustering algorithm is a set of rules that uses the distance between particles on a cylinder (the shape of the detector) and their energy (measured by calorimeters) to predict the original particle.

Jet clustering algorithms can be divided into two categories: cone and sequential recombination algorithms. Cones are circles in a plane with a fixed radius. In cone based algorithms, cones are placed around seeds (particles with high energy) and the momenta

²The Large Hadron Collider (LHC) is a particle collider at European Organization for Nuclear Research (CERN). ATLAS (A Toroidal LHC Apparatus) and CMS (Compact Muon Solenoid) are two general purpose experiments at the LHC.

³The main reference for this section is [Salam \(2010\)](#).

⁴See [Salam \(2010\)](#) for an introduction to jets and jet algorithms.

of all the particles are calculated within the cone. This determines the new center, and the direction of the resulting sum is then used as a new seed direction. The procedure is iterated until the direction of the resulting cone is stable.

Cone based algorithms follow a top-down approach. They are not used in this work. Here, I consider the latter category – sequential recombination jet algorithms which use a bottom-up approach.

Sequential recombination algorithms repeatedly combine pairs of elements that are similar. This is a process of working backwards, from the resulting particles, to find the original particle. Two points are important when designing such algorithms: firstly, how one chooses which pair of particles to combine, and secondly, when the algorithm stops combining particles.

The first question is addressed by defining a distance measure. Typically, a distance measure uses the transverse momentum and direction of the jets. Finding a stopping condition is harder to ascertain and a radius parameter of jets is used for this purpose. Sequential recombination algorithms differ in their choice of the distance measure and the numerical value of the radius parameter.

The distance measure can be based on different properties of particles. For example, the Cambridge/Aachen algorithm only considers the spatial distance between particles without their momenta. Similarly, in Section 5.2.1, countries are clustered exclusively based on their geographical locations. Other algorithms can use distance measures that include a combination of energy and angle. In Section 5.2.2, in addition to geographical locations, GDPs and populations are used in the distance function.

The choice of radius parameter is also not a trivial task. Currently, the most relevant way to determine this choice is to systematically examine different values and decide which value is the best. This is also true for choosing a jet algorithm. There is no best choice for all jet studies. Most suitable jet clustering algorithm (and its respective radius parameter) depends on where it is applied and what it aims to explain.

The three commonly most used sequential recombination algorithms are the k_t , Cambridge/Aachen, and anti- k_t algorithms. The details of algorithms are neglected here. In the following, a general distance measure is given. The distance measure d_{ij} between two particles is defined as

Sequential Coalition Formation

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{R_{ij}^2}{R^2}, \quad R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2 \quad (5.1)$$

where k_{ti} denotes the momentum (a 4-vector that describes the direction, energy and mass of a particle), R_{ij} is the geometrical distance on the surface of a cylinder (the geometry of the detectors is typically cylindrical), R is the distance parameter and p is a parameter usually set to 0, 1 or -1 for different algorithms.

Each particle is assumed as a proto-jet. The distance d_i for each proto-jet is defined as

$$d_i = k_{ti}^{2p}. \quad (5.2)$$

A sequential recombination algorithm first finds the smallest distance of all the d_{ij} and d_i . If the smallest distance is d_{ij} , two particles are combined by adding their momenta. This step is repeated until there is no particle pair to satisfy the threshold condition. This means that d_i is the smallest distance. Then the object is removed from the process and taken as a jet. Clustering terminates when no particles are left.

Using an analogy to the jet algorithms, the model describes the pattern of coalition formation among countries as a result of pairwise combination depending on the distance between them. Therefore, the model follows an algorithmic similarity-based approach. Similarly, the distance functions considered here have geometrical and non-geometrical components.

Sequential Coalition Formation Models in Game Theory

The sequential algorithm is in the line of modeling processes of coalition formation coming from the *BU* model ([Brams et al. \(2002\)](#)) and protocoalition model ([Grofman \(1982\)](#)). It is important to predict which coalition will be formed, however, this cannot be isolated from its process. Therefore, I will follow the approach in those kinds of models and study the process of coalition formation.

There are considerable similarities and differences across sequential coalition formation models. The model presented in this chapter also adopts a step-by-step formation. Similarly, the model uses a clustering algorithm. One difference is the winning coalition condition. In those models, a winning coalition serves as a stopping condition of the process. Instead, in

the model presented in this chapter, a partition of players is predicted. Another difference is the distance function. The distance between two players is not ‘typical’. Moreover, an abstract distance function can have different realizations. This gives us the opportunity to compare different clustering results in terms of emerging player pairs. This is discussed more in detail in Section 5.2.

This model can be easily tested and the numerical evaluation is fast as the jet clustering algorithms were designed to be executed on a huge amount of data. In Section 5.3, the results with only about thirty players are presented. Computational complexity is not critical with this number of players. However, the model is applicable to a large range of situations in which the number of players can be numerous.

The model aims to study the process in coalition formation among countries in which actors behave rationally and aim to maximize their individual benefits differently than particles. I do not explicitly aim to study the stability or stabilization in this problem. Countries will be used throughout the discussion but the model also holds for other players from a multitude of economic, political or social situations.

5.2 Model

Recall that there are a finite number of n countries, $N = \{1, 2, \dots, n\}$. A partition of countries is a specification of m groups of countries, such that each country belongs to one, and only one, group. Thus, a partition $S = (S_1, S_2, \dots, S_m)$ describes a particular allocation of countries. One country is assumed to be a coalition with one member.

Given the complexity of the issue, I will simplify the set-up of the problem. In order to focus on our basic question of process, many interesting details and also some equally important factors or characteristics will be abstracted from this model.

Each country $i \in N$ is identified by $K \geq 1$ characteristics. A number of standard characteristics are considered. Namely, they are the geographical location, population, GDP and regime type. Thus, country i is denoted by the vector $C_i = (\lambda_i, \phi_i, p_i, g_i, r_i)$ where λ_i and ϕ_i are longitude and latitude, p_i is population, g_i is GDP and r_i is regime type of country i . Section 4.3 describes these characteristics and gives numerical values.

Sequential Coalition Formation

I will use a clustering solution for coalition formation. The utility of a country depends on the distance, with utility equaling the inverse of the distance. Thus, smaller distance means higher utility.

Distance function

The distance between two coalitions depends on these characteristics. Consider two coalitions i and j . The distance between two coalitions d_{ij} is a measure of how different they are from one another. Hence, according to the previous discussion, a coalition consists of mutually similar players, and therefore if two coalitions i and j belong to the same coalition then they have a low value of d_{ij} . Although the terms of distance and similarity are not equivalent, both are referred to as distances in this model. A small distance means a large similarity.

The concept of distance should be clarified. It is used to calculate the difference between players. Traditionally coalition formation models use Euclidean distance ([Boekhoorn et al. \(2006\)](#)). However, ([Grofman and Straffin, 1984](#), p.272) point out that other metrics can be used for equal or better measurements. Indeed, they use a distance measurement which combine the Euclidean distance and players' weight. In this model, the distance function is a combination of physical distance and certain properties of countries. It can have many variants with situations where the distance function is pure geometrical or non-geographical or both.

The reason why this is done is because of the results different distances have for clustering. Since the distance function determines the closeness between two players, which in turn determines the partition of players, the choice of distance function is crucial. This gives us the opportunity to compare different distance functions and study their results. This comparison helps to find a good measurement even if there is probably no unique one.

Different clustering results might also be important in terms of country pairs. For example, when we compare two clustering results there are four scenarios. Two countries can be members of the same cluster in both results. They can belong to different clusters in both results. They can be in the same cluster in the first case, but they can be in different clusters in the second case. They can belong to different clusters in the first case, but they can belong to the same cluster in the second case.

The geographical distance between two countries is defined by the distance between their capital cities and denoted by Δ^5 . Note that it is a non-Euclidean distance since it depends on the earth's curvature and is not a straight line distance.

A general distance function is given by

$$d_{ij} = \begin{cases} f(p_i, p_j, g_i, g_j)\Delta_{ij} & : r_i = r_j \\ \infty & : r_i \neq r_j. \end{cases} \quad (5.3)$$

The distance function is defined as a product of the distance in population and GDP space and the distance in physical space. This form is motivated by the fact that the distance which determines which countries should be clustered first can correspond to their power (for example GDP) or geographical region. For now $f(p_i, p_j, g_i, g_j)$ is only in an abstract form. Section 5.3 discusses different concrete realizations.

Populations and GDPs are in some intervals. It can be difficult to combine numerical characteristics with regime type. On the one hand, countries geographically or economically close to each other are likely to form a coalition. On the other hand, countries with different regime types are unlikely to form a coalition. The distance between two countries is set to infinity if they have different regime types. Thus, countries can merge only within the same regime type. This reduces the set of possible coalitions. (See Section 4.3 for descriptions, motivations and discussions of these parameters.)

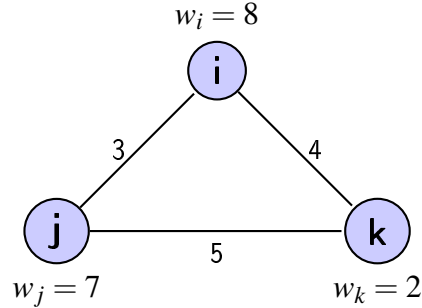
An important property of the distance measure is (non)symmetry. Symmetry means that the distance between i and j is equal to the distance between j and i . Typically it is assumed that $d_{ij} = d_{ji}$. Do countries want each other as a coalition partner at the same degree? Of course, geometrical distance alone is always symmetric. There are possible scenarios in which the distance measure is not necessarily symmetric. If players are concerned about their weight in the coalition, their perception on the distance can be different. For example,

⁵In Euclidean plane the distance between two points is given by

$$\Delta_{ij}^2 = (\lambda_i - \lambda_j)^2 + (\phi_i - \phi_j)^2.$$

Since the distance between countries is not an Euclidean distance, a geographical distance is used instead. In order to measure distance between two points on the earth, the inverse geodesic problem is solved. (A python package, `geographiclib`, is used to compute geodesic distances.)

Fig. 5.1 Three countries triangle



consider a case in which the distance between players is a combination of physical distance and the ratio of their weights to total weight,

$$d_{ij} = f(w_i, w_j)\Delta_{ij}. \quad (5.4)$$

where $f(w_i, w_j) = \frac{w_i}{w_i + w_j}$. The weight of player i and j are w_i and w_j respectively. The physical distance is denoted by Δ_{ij} . Grofman and Straffin (1984) use this particular distance function in the proto-coalition formation model. Obviously $d_{ij} \neq d_{ji}$ if $w_i \neq w_j$. Thus, it is a non-symmetric distance measure.

Preferences are not exogenously given. Now the question is determining how countries shape their partner preferences. The simplest way is that country i prefers being with country j to being with country k if and only if $d_{ij} \leq d_{ik}$. Thus, countries prefer to cooperate with closer potential partners. In other words, countries' preferences based on the distances.

Example 5.2.1 *Non-transitive and non-symmetric distance function* Note that the distance function is not necessarily transitive or symmetric. Consider the following three countries i , j and k with their pairwise distance and weights given in Figure 5.1.

- Non-Symmetry*: First distance measure is $d_{ij} = \frac{w_j}{w_i + w_j}\Delta_{ij}$ (Grofman and Straffin (1984)). The distance between i and j is not symmetric. The idea is that if the weight of player j , w_j , is bigger than the weight of player i , w_i , player j has more impact in the coalition. Therefore, player i perceives a bigger distance. In other words, a weak player has to move more than a strong player. Thus, it is not symmetric.

- b. *Non-Transitivity*: Second distance measure is $d_{ij} = \min(w_i, w_j)$. This distance is non-geometrical part of k_t distance. This distance function is symmetric but geometrically closed countries are not necessarily close to each other. Non-transitivity is a well-known phenomenon in rational choice theory. The distance transitivity requires triangle inequality (Watts, 1999, p.11). However, unlike the physical systems, social systems are not necessarily follow this property. It is easy to verify this. Assume that $w_k < w_j < w_i$ as in Example 5.1. Then, distances must obey the triangle inequality : $d_{ij} < d_{jk} + d_{ik}$ but they do not.

The distance functions presented in this work are symmetric. Furthermore, they have a geometric part. Therefore, they are transitive as well. This has a particular importance in finding nearest neighbors of a given point which has significant help to reduce the time complexity of the algorithms. Next, a set of rules is defined which describe the combination of characteristics when two coalitions merge.

Stability

This game can be consider as *social distance game*(Brânzei and Larson (2011)). The utility of player i in coalition S can be defined as

$$u_i(S) = \frac{1}{|S|} \sum_{s_i=S_j} d_{ij}^{-1} \quad (5.5)$$

The inverse distance function captures the *closeness* of the players. That means adding a close player increases the utility. The utility of players also depends on the size of the coalition. Therefore, an additional player increases utility with a decreasing rate.

I do not focus on stability for this game. Trivially, the grand coalition is the only core stable partition. As we have discussed, the focus will be on the coalition formation process.

Combination Rules

When two coalitions find each other acceptable, they form a single coalition. This new coalition needs to inherent the geographical location, population, GDP and regime from its two *parents*. Since it is assumed that the first requirement to form a coalition is having same

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type of regime, there is no need to define a combination rule. The following rules show how to combine the characteristics of two coalitions into one.

At each stage when two coalitions merge, the GDP and population of new coalition are simply the sum of two GDPs and populations.

$$g_{\{i,j\}} = g_i + g_j \quad \text{and} \quad p_{\{i,j\}} = p_i + p_j \quad (5.6)$$

Thus the GDP and population of a final coalition S are given by

$$g_S = \sum_{i \in S} g_i \quad \text{and} \quad p_S = \sum_{i \in S} p_i. \quad (5.7)$$

The new geographical location of two coalitions is

$$\begin{aligned} \lambda_{\{i,j\}} &= \frac{w_i \lambda_i + w_j \lambda_j}{w_i + w_j} \\ \phi_{\{i,j\}} &= \frac{w_i \phi_i + w_j \phi_j}{w_i + w_j}. \end{aligned} \quad (5.8)$$

where w_i and w_j are the weights of coalitions i and j . For example, if it is assumed that each player has equal weight then the new location is simply the mid point of two coalitions. The weight of a coalition can be its population, GDP or GDP per capita.

5.2.1 Coalition formation depends on only geographical location and regime type

The first algorithm describes the coalition formation among countries which can have different regime types. The distance function $f(\cdot)$ takes a simple form, i.e. it is a constant. Thus, countries form coalitions depending on only geographical distance and regime type.

Initially, the number of coalitions is equal to the number of countries. First, all distances between capital cities are calculated. If two countries have different regime types, for example, country i is a communist state and country j is a democracy, then the distance between them set to infinity $d_{ij} = \infty$. Otherwise, the distance between two coalitions is the geographical distance between them, $d_{ij} = \Delta_{ij}$. Next, the closest pair is merged to form a new coalition. This reduces the number of coalitions by one as the two previous coalitions

are removed from the list.⁶ The location of new coalition is simply the population, GDP or GDP per capita weighted average distance or midpoint of two capital cities. Distances between the new coalition and the others are calculated. This iteration repeats until each country is attached to a coalition. Algorithm 4 describes these steps.

Algorithm 1: Coalition Formation with Pairwise Geographical Distance

Data: Locations of capital cities and regime types

1 For each pair of countries define

$$d_{ij} = \begin{cases} \Delta_{ij} & : r_i = r_j \\ \infty & : r_i \neq r_j \end{cases}$$

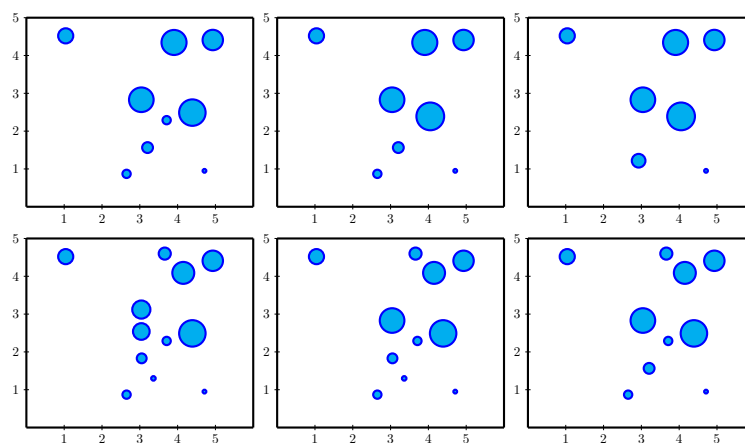
2 Find the smallest d_{ij}

3 Merge i and j into a new coalition k with new location at the weighted midpoint of i and j 's locations

4 repeat until no country is left

The geometric distance measure $d_{ij} = \Delta_{ij}$ is symmetric. That is $d_{ij} = d_{ji}$. It is also transitive: if i is closer to j and j is closer to k , then i is closer to k . Note that while this distance measure is transitive, a more general distance measure need not to be transitive (See Example 5.2.1).

Fig. 5.2 Steps of coalition formation with geographical distance and regime



⁶There can be pairs with the same distance. In the presence of ties, a rule is used to choose from the cases. However, the data used in applications is finely measured so that there is no tie.

Sequential Coalition Formation

Before we proceed to define a general distance measure, we take a look at Figure which illustrates coalition formation according to Algorithm 4. There are countries with different weights and randomly distributed on the plane. They are represented by circles. According to the Euclidean distances, the closest elements are merged together. Here I assume that the center of a new circle is the middle point of these elements (one can also consider the weighted average point) and the area is the sum of two areas. This iteration continues with updated distances. If we determine a threshold value (in which two points merge only if they are in given radius), clustering is not possible after certain iteration and multiple clusters may appear. If we do not have any threshold value, at the last stage there will be two clusters and no alternative but to merge these two.

5.2.2 A general coalition formation

Now consider a more general distance function. There are other characteristics besides geographical location and regime types, namely population and GDP of countries. Of course, the most critical point is how to choose $f(p_i, p_j, g_i, g_j)$. A set of different functions of populations and GDPs can be used. Section 5.3 presents the results of different functions.

The general case is almost identical to the previous algorithm except introducing non-geometric parts (populations and GDPs) in the distance measure. First, the pairwise distances are calculated. Countries look for the closest coalition partner. The closest country pair merges to a new coalition, i.e single player. Again, the new coalition's location is defined as the weighted midpoint of two capital cities. When two countries merge, new coalition's population and GDP are simply the sums of populations and GDPs of two countries. The process continues until all countries are in some coalitions. Algorithm 4 describes the coalition formation process.

Figure shows the steps of this algorithm. Here y coordinate is assumed to be zero for all countries. The process starts with finding the smallest distance on the x -axis. Then two countries are merged according to combination rules given in equations 5.6 and 5.8. Weights can be any characteristics of countries. In this particular example, weights represent GDP per capita.

Algorithm 2: A General Sequential Coalition Formation**Data:** Location of capital cities, regime type, population, GDP

- 1 Choose a function, $f(p_i, p_j, g_i, g_j)$, and for each pair of countries define the distance function

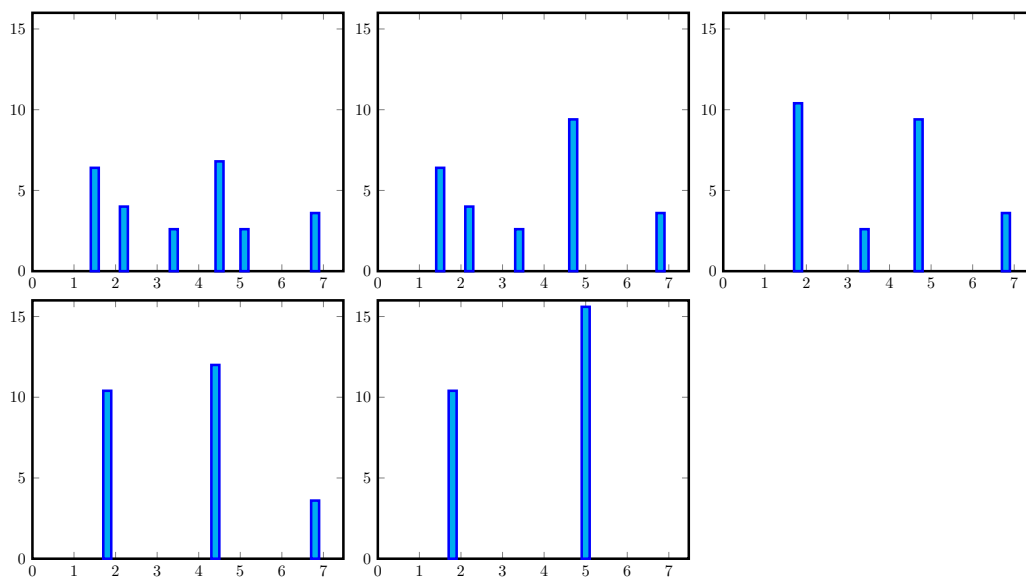
$$d_{ij} = \begin{cases} f(p_i, p_j, g_i, g_j)\Delta_{ij} & : r_i = r_j \\ \infty & : r_i \neq r_j. \end{cases}$$

- 2 Find the smallest d_{ij}
- 3 Merge i and j into a new coalition k with new location at the weighted midpoint of i and j 's locations. New population and GDP are given by

$$p_k = p_i + p_j \text{ and } g_k = g_i + g_j.$$

- 4 repeat until no country is left

Fig. 5.3 Steps of General Coalition Formation

**5.2.3 Computational Complexity**

The term computational complexity can be broadly used in two contexts. It is then often not clear which usage is meant. It might be used to represent either *time or space efficiency* of a given algorithm or *traceability* of a given problem. In this subsection, we will focus on the first usage.

Before investigating the time and space complexity of the algorithms presented in Subsection 5.2.2, it is useful to be more specific about the second usage. In computational

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complexity theory, there is a main distinction among problems. They belong to the complexity classes such that P (Polynomial Time) and NP (Nondeterministic Polynomial Time).

In P , computational problems are solvable by a polynomial time algorithm. The polynomial time complexity is denoted by $\mathcal{O}(n^k)$ where n is the complexity of input and k is a non-negative integer. In NP , solutions of computational problems can be approved in polynomial time. In simple words, problems in P can be decided quickly and in NP can be checked quickly. Clearly, NP problems contain P problems. Then, the question is if the answer to a problem is easy to check, is the problem itself easy to solve? This is the most famous and unsolved question in computer science, so called *The P versus NP problem*.

How does the computational complexity relate to game theory? In the context of games, there are some interesting questions concerning complexity. For example, consider hedonic games outlined in Section 2.1. There are two main algorithmic problems around core stability (Woeginger, 2012, p.3). The first question is the existence of a core stable partition of N players and the second question is the verification of whether a given partition is core stable. The existence and verification problems are NP – complete. This means that these decision problems are in NP and at least as hard as the hardest problems in NP . Equivalently we can say that there is not an (immediate) efficient algorithm for these decision problems and they fall into “hard” problems category. Another question related to the core stable partition is verification – whether a partition is blocked by any other coalition. The latter question turns out as difficult as the former one. Thus, the verification problem is also NP – hard. (See Ballester (2004) and Sung and Dimitrov (2010) for computational complexity in hedonic games.).

We now examine the efficiency of a given algorithm. Obviously, algorithms in this thesis perform in polynomial time. Then, the question is how many steps are needed to reach a solution. When we know that we can calculate the clustering problem in polynomial time why we should care about how much time it takes. After all, computers will do the job for us. However, the running time might be crucial to some problems.

In the following, we will study the time complexity of Algorithm 4. This simply means the amount of time taken by an algorithm. In each iteration, Algorithm 4 calculates distances among all pairs and determines the smallest one. Therefore, the first step to determine the

time complexity is finding the smallest distance. Then, the second step is to decide how many times we should find the smallest distance.

The first step is called *The Closest pair of points problem*. This is the problem of finding the closest pair of $n \geq 2$ points in a set Q by using the euclidean distance. Consider a brute force approach: first calculate the distance of every pair of points then pick the pair with the smallest distance. There are $\frac{n(n-1)}{2}$ pairs of points. Thus, the closest pair of points can be computed in $\mathcal{O}(n^2)$ time.

Then, the closest pair is merged to one point and the same procedure is applied until no player is left. The minimum distance is searched in $\mathcal{O}(n)$ time and there can be at most $n - 1$ clustering. This makes the total time of $\mathcal{O}(n^3)$. These steps are shown in Algorithm 16. Recall that every country is a coalition and *coalitions* is the set of countries.

Can we further reduce the time complexity? *The Closest pair of points problem* is a geometric problem. There are methods in computational geometry in which we can find the closest pair of points in a set of n points in the plane in $\mathcal{O}(n \log n)$ time. Furthermore, the total time of Algorithm 16 can be reduced to $\mathcal{O}(n \log n)$.

The difference between $\mathcal{O}(n^3)$ and $\mathcal{O}(n \log n)$ is not significant when the application includes only 15 European countries. But if one works with numerous players as in the case of networks or jet clustering, the time complexity becomes important. One structure which can be used is the Voronoi diagram⁷. Skiena (2009) suggests to use Voronoi diagram for $n \geq 100$). Figure shows various running times. [p. 581]

A Voronoi diagram partitions a plane into sites based on distances. In this diagram, every point is assigned to the nearest site. The advantage of the Voronoi diagram is that we can efficiently calculate the nearest neighbors of given points. Thus, we do not have to calculate all pairwise distances to find the minimum distance. It is enough to calculate the nearest neighbors. The Figure gives an intuitive picture. Blue lines represent Voronoi Diagram and red lines represent its dual Delaunay Triangulation.

A point has its site and any point within the site is closest to that point than any other points. In this example, people prefer to get their goods at the nearest site. Every region is attained to a capital.

The time and space complexity to construct such structure as follows.

⁷See De Berg et al. (2008) for formal introduction.

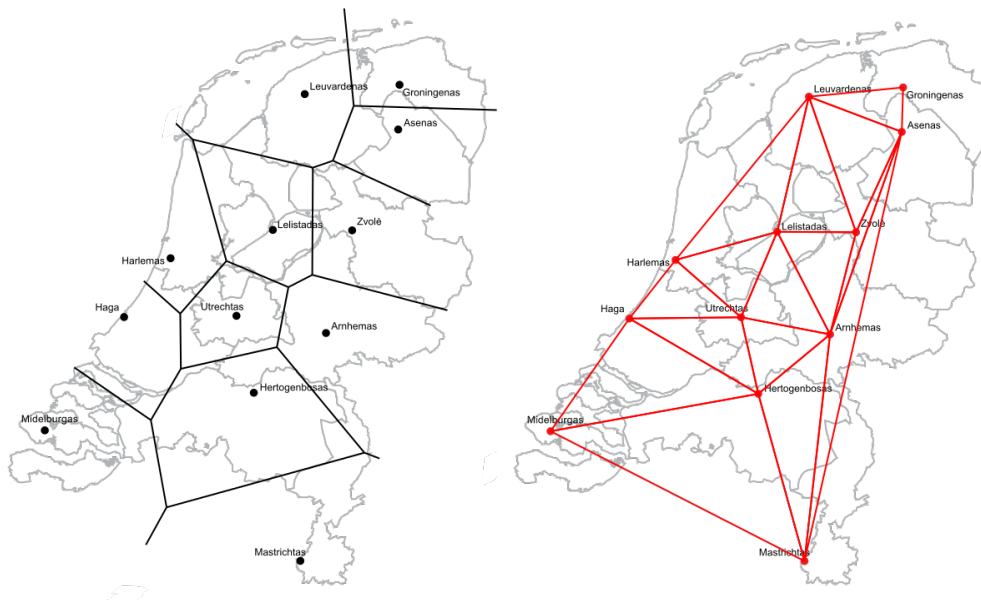
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Algorithm 3: Coalition Formation

```
input : Countries on a plane
output : A partition of countries

1 default distance =  $\infty$ ;
2 while length of coalition > 1 do
3   minimum distance = default distance;
4   first coalition = None;
5   second coalition = None;
6   for  $i$ , coalition  $i$  in coalitions do
7     for  $j$ , coalition  $j$  in coalitions do
8       if  $i \geq j$  then
9         continue
10      if distance(coalition  $i$ , coalition  $j$ ) < minimum distance then
11        minimum distance = distance(coalition  $i$ , coalition  $j$ );
12        first coalition = coalition  $i$ ;
13        second coalition = coalition  $j$ ;
14   merge (first coalition and second coalition) and append to coalitions;
15   remove first coalition from coalitions;
16   remove second coalition from coalitions;
```

Fig. 5.4 The trading areas of the capitals of the twelve provinces in the Netherlands, as predicted by the Voronoi assignment model



Source: reproduced and modified from (De Berg et al., 2008, p.147)

Theorem 5.2.2 (De Berg et al., 2008, p.159) *The Voronoi diagram of a set of n point sites in the plane can be computed with a sweep line algorithm in $\mathcal{O}(n \log n)$ time using $\mathcal{O}(n)$ storage.*

Now, the question is that how we can use Voronoi Diagram to find the minimum distance. We can construct a list of size n for the nearest neighbors. That means instead of calculating all d_{ij} for i (takes n operations), we only calculate one distance, i.e. distance between i and its geometrical nearest neighbor. The minimum distance can then be identified among elements of the list with $\mathcal{O}(n)$ operations.

Theorem 5.2.3 (Preparata and Shamos, 2012, p.220) *The all nearest neighbors problem is linear-time transformable to Voronoi Diagram and thus can be solved in $\mathcal{O}(\log n)$ time, which is optimal.*

A special data structure can be used to store the distances: a priority queue. A priority queue is often implemented with a heap which is a nearly complete binary tree. Therefore, any operation such as adding an element in the tree takes $\mathcal{O}(\log n)$ time (the depth of the tree).

Finally, I present the Voronoi implementation in sequential jet clustering algorithms in [Cacciari and Salam \(2006\)](#):

1. Construct the Voronoi diagram of the n particles. $\mathcal{O}(n \log n)$
2. Find the Geometric Nearest Neighbor of each of the n particles. Construct the d_{ij} distances, store the results in a priority queue. $\mathcal{O}(n \log n)$
3. Merge/eliminate particles appropriately. Update Voronoi diagram and distances' map. $\mathcal{O}(\log n)$

Thus, the running time for the implementation is $\mathcal{O}(n \log n)$.

5.3 Application

In this section, I will test various distance functions discussed in Chapter 5.2 and give results. This will examine the theoretical model in explaining empirically observed formation of

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the ESCS. The section concludes with an overview of advantages and disadvantages of step-by-step clustering in international coalitions.

The sequential coalition formation procedure is simple but still captures important properties of the coalition formation process. The order of merging two coalitions depends on how the underlying quantities are combined in the distance function. Thus, this section is an investigation of whether the combination of parameters in the distance functions are valid from an empirical standpoint. Initially, for simplicity, I assume that players have equal weights. Later, different weights and combination rules will be discussed.

5.3.1 Geographical pattern of coalition formation

I begin with the simplest distance function: the physical distance. The distance measure between countries is physical distance. In the literature of spatial coalition formation models, various distance functions are used i.e. Euclidean distance, Squared Euclidean distance, Manhattan distance. Even if, the main idea of the general distance function given in equation (5.3) is to abstract the distance between countries and not to restrict it to the physical location, geography plays still an important role. There are two motivations for using Euclidean distance.

The first motivation is that geographically close countries are more likely to cooperate in areas of trade, military or environment. There are two immediate reasons for that. Firstly, if the distance is smaller, the gain is larger from reduced transport and communication costs (Krugman, 1991, p.19). These cost were even more important in the 1950s than they are today. Secondly, the geographical distance can be a proxy for cultural distance (Desmet et al., 2006, p.27) which potentially plays a role in coalition formation. In this work, the properties like culture, language or in more general identity of a country are not explicitly considered but the geographical distance implicitly brings these characteristics in the picture.

The second motivation is technical. One immediate advantage is its computational tractability. Furthermore, Euclidean distance satisfies the triangle inequality therefore significantly reduced the time complexity of clustering algorithms.

If countries consider only the regime type and geographical distance, they follow Algorithm 4. It is assumed that all countries start as single coalitions and have identical weights. Then, the algorithm looks for similarities. In this case only the geographical distance is used

since countries with different regime types are not allowed to unite. A coalition formation is seen as a process including a series of stages. In each stage, the closest coalition pair is merged to a single coalition.

Table 5.1 gives the output of this process. Each line includes first and second coalition which are merged at that stage and the distance between them in kilometers.

When countries form coalitions step-by-step according to geographical distances and regime types, there are three final coalitions, namely; {Spain and Portugal}, {Soviet Union, Hungary, Yugoslavia, Albania, Bulgaria, Romania, Poland, Czechoslovakia, East Germany} and {Greece, Iceland, Finland, Sweden, Denmark, Norway, Ireland, Switzerland, Netherlands, Belgium, West Germany, Luxembourg, France, United Kingdom, Austria, Italy}.

Since there are three regime types, three coalitions are formed in the end. This resulting partition is trivial. The interesting point is the merging order or substructure. West Germany, Luxembourg, Belgium and Netherlands form a coalition in the first three steps. This is followed by merging with France and the United Kingdom. Five of the founder states of the EU cluster rather quickly. Later, middle and south European countries merge. The Northern bloc joins towards the end. Finally, this large coalition expands by Iceland joining last. Among the Eastern European countries, coalitions merge relatively quickly and in the last step the Soviet Union joins to the rest of Eastern countries. Spain and Portugal merge to one coalition since both are ruled by the military regimes.

Table 5.1 Coalition Formation – Geographical Distance

First Coalition	Second Coalition	Distance
West Germany	Luxembourg	123
Belgium	Netherlands	173
West Germany, Luxembourg	Belgium, Netherlands	200
Czechoslovakia	East Germany	281
Bulgaria	Romania	296
Hungary	Yugoslavia	317
France	United Kingdom	343
West Germany, Luxembourg, Belgium, Netherlands	France, United Kingdom	389

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Finland	Sweden	397
Bulgaria, Romania	Hungary, Yugoslavia	466
Albania	Bulgaria, Romania, Hungary, Yugoslavia	456
Denmark	Norway	484
Poland	Czechoslovakia, East Germany	500
Portugal	Spain	503
Switzerland	West Germany, Luxembourg, Belgium, Netherlands, France, United Kingdom	507
Finland, Sweden	Denmark, Norway	559
Ireland	Switzerland, West Germany, Luxembourg, Belgium, Netherlands, France, United Kingdom	738
Austria	Italy	767
Albania, Bulgaria, Romania, Hungary, Yugoslavia	Poland, Czechoslovakia, East Germany	882
Greece	Austria, Italy	1054
Ireland, Switzerland, West Germany, Luxembourg, Belgium, Netherlands, France, United Kingdom	Greece, Austria, Italy	1259
Finland, Sweden, Denmark, Norway	Ireland, Switzerland, West Germany, Luxembourg, Belgium, Netherlands, France, United Kingdom, Greece, Austria, Italy	1340
Soviet Union	Albania, Bulgaria, Romania, Hungary, Yugoslavia, Poland, Czechoslovakia, East Germany	1451

Iceland	Finland, Sweden, Denmark, Norway, Ireland, Switzerland, West Germany, Luxembourg, Belgium, Netherlands, France, United Kingdom, Greece, Austria, Italy	2395
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5.3.2 General Case

As a result of Algorithm 4, countries are clustered according to richer distance functions. There are a wide range of distance functions. However, I start by examining few building blocs.

$$d_{ij} = |p_i - p_j| \Delta_{ij}, \quad (5.9)$$

$$d_{ij} = |g_i - g_j| \Delta_{ij}, \quad (5.10)$$

$$d_{ij} = \left| \frac{g_i}{p_i} - \frac{g_j}{p_i} \right| \Delta_{ij}. \quad (5.11)$$

Population and GDP are normalized to 1 by dividing each quantity by the maximum values. All three distance functions given above include also geographical distance. In this section, only liberal democracies are considered.

Firstly, GDP distance is examined. GDP is chosen to represent economical state of a country. Secondly, population distance is considered. Population is a measure for the size of a country. Lastly, GDP per capita distance is evaluated. Tables 5.2, 5.3 and 5.4 present the outcomes respectively. They provide an initial hint on some possible coalitions.

Table 5.2 Coalition Formation – GDP and Geographical Distance

First coalition	Second coalition	distance
Belgium	Sweden	1.06
Finland	Norway	1.54
France	West Germany	7.54
Denmark	Finland, Norway	9.13

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Netherlands	Denmark, Finland, Norway	10.80
Belgium, Sweden	Netherlands, Denmark, Finland, Norway	10.21
Iceland	Luxembourg	11.37
Ireland	Iceland, Luxembourg	8.25
Greece	Ireland, Iceland, Luxembourg	8.26
Austria	Greece, Ireland, Iceland, Luxembourg	3.35
Switzerland	Austria, Greece, Ireland, Iceland, Luxembourg	22.72
Italy	Switzerland, Austria, Greece, Ireland, Iceland, Luxembourg	91.38
United Kingdom	France, West Germany	98.77
Belgium, Sweden, Netherlands, Denmark, Finland, Norway	Italy, Switzerland, Austria, Greece, Ireland, Iceland, Luxembourg	164.48
United Kingdom, France, West Germany	Belgium, Sweden, Netherlands, Denmark, Finland, Norway, Italy, Switzerland, Austria, Greece, Ireland, Iceland, Luxembourg	646.44

In each case, the formation process is completed in fifteen steps. I will focus on the most likely merging in the beginning and the resulting blocs towards the end of the process. All three distance measures have two properties in common. First, now non-contiguous neighbor states can form coalitions. Second, the merging of two coalitions can lead to a decrease in the distance: the distances are not monotonically increasing anymore. Thus, the absence or addition of some players can change the process. Furthermore, the algorithm still allows for merging small and big countries.

Table 5.3 Coalition Formation – Population and Geographical Distance

First coalition	Second coalition	distance
Austria	Sweden	1.93

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Denmark	Finland	4.55
Belgium	Netherlands	5.01
Iceland	Luxembourg	6.91
Ireland	Norway	7.52
West Germany	United Kingdom	8.36
Iceland, Luxembourg	Ireland, Norway	31.19
Denmark, Finland	Iceland, Luxembourg, Ireland, Norway	31.36
Austria, Sweden	Denmark, Finland, Iceland, Luxembourg, Ireland, Norway	11.71
Greece	Switzerland	93.75
France	Italy	99.75
Belgium, Netherlands	West Germany, United Kingdom	151.92
France, Italy	Belgium, Netherlands, West Germany, United Kingdom	435.14
Austria, Sweden, Denmark, Finland, Iceland, Luxembourg, Ireland, Norway	Greece, Switzerland	525.49
France, Italy, Belgium, Netherlands, West Germany, United Kingdom	Austria, Sweden, Denmark, Finland, Iceland, Luxembourg, Ireland, Norway, Greece, Switzerland	2547.97

In the first six steps of Table 5.2, there are two coalitions: {France, Germany} and {Belgium, Netherlands, Denmark, Norway, Finland, Sweden,}. The last two coalitions are three biggest economies {France, Germany, United Kingdom} and the rest of Europe.

In Table 5.3, according to population distance, the last two coalition are {France, Italy, Belgium, Netherlands, West Germany, United Kingdom} and the rest of Europe. The first coalition is close to six founder members of the ESCS in which Luxembourg is replaced by the United Kingdom.

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Both GDP and population distances might produce historically significant results such as {France, West Germany}, {Belgium, Netherlands} and clusters of the Scandinavian countries. After examining GDP and population distance, a natural step is to examine the GDP per capita distance. Table 5.4 shows the result.

Table 5.4 Coalition Formation – GDP per capita and Geographical Distance

First coalition	Second coalition	distance
Denmark	United Kingdom	0.52
Belgium	Norway	5.21
Netherlands	Belgium, Norway	10.15
Finland	West Germany	12.30
Ireland	Italy	13.56
Austria	Ireland, Italy	21.98
Luxembourg	Switzerland	33.30
Sweden	Denmark, United Kingdom	34.06
Netherlands, Belgium, Norway	Sweden, Denmark, United Kingdom	35.58
France	Iceland	46.80
Finland, West Germany	France, Iceland	72.94
Netherlands, Belgium, Norway, Sweden, Denmark, United Kingdom	Finland, West Germany, France, Iceland	91.14
Greece	Austria, Ireland, Italy	269.98
Luxembourg, Switzerland	Netherlands, Belgium, Norway, Sweden, Denmark, United Kingdom, Finland, West Germany, France, Iceland	273.34
Greece, Austria, Ireland, Italy	Luxembourg, Switzerland, Netherlands, Belgium, Norway, Sweden, Denmark, United Kingdom, Finland, West Germany, France, Iceland	397.12

5.3 Application

This first attempt at formulating the non-geometrical part of the distance function as the difference of GDP per capita has not provided historically relevant coalitions. Now, I propose another measure which is the ratio of GDP per capita of paired countries. The distance function takes the following form:

$$d_{ij} = \frac{\max(\frac{g_i}{p_i}, \frac{g_j}{p_j})}{\min(\frac{g_i}{p_i}, \frac{g_j}{p_j})} \Delta_{ij}. \quad (5.12)$$

The resulting merging process is given in Table 5.5. The five of the founder states, Belgium, Netherlands, Luxembourg, France and West Germany, and northern countries, Denmark, Sweden, Finland and Norway, merge in the early stages. Iceland and Greece join the grand coalition in the end of the process.

In both distance measures including GDP per capita given in Equations 5.11 and 5.12, the distance increases monotonically.

Table 5.5 Coalition Formation

First coalition	Second coalition	distance
Belgium	Netherlands	190.16
West Germany	Luxembourg	246.85
Belgium, Netherlands	West Germany, Luxembourg	273.26
France	Belgium, Netherlands, West Germany, Luxembourg	422.73
Norway	Sweden	520.84
Denmark	Norway, Sweden	517.52
United Kingdom	France, Belgium, Netherlands, West Germany, Luxembourg	544.40
Austria	Italy	812.56
Switzerland	United Kingdom, France, Belgium, Netherlands, West Germany, Luxembourg	838.30
Finland	Denmark, Norway, Sweden	936.63

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Ireland	Switzerland, United Kingdom, France, Belgium, Netherlands, West Germany, Luxembourg	1195.56
Finland, Denmark, Norway, Sweden	Ireland, Switzerland, United Kingdom, France, Belgium, Netherlands, West Germany, Luxembourg	1375.26
Austria, Italy	Finland, Denmark, Norway, Sweden, Ireland, Switzerland, United Kingdom, France, Belgium, Netherlands, West Germany, Luxembourg	1852.77
Iceland	Austria, Italy, Finland, Denmark, Norway, Sweden, Ireland, Switzerland, United Kingdom, France, Belgium, Netherlands, West Germany, Luxembourg	2428.38
Greece	Iceland, Austria, Italy, Finland, Denmark, Norway, Sweden, Ireland, Switzerland, United Kingdom, France, Belgium, Netherlands, West Germany, Luxembourg	5046.66

The UK Case

The UK presents a complicated case because even though its initial high potential of being a part of the coalition, accession occurred only in 1973. One reason was its transatlantic and Commonwealth trade links, particularly the special relation with the US (Carchedi (2001)). Another reason was its strong concern about sovereignty. The idea of supranationalism prevented the UK to participate to the ESCS. This was also related to the first reason: supranationalism would weaken its transatlantic and Commonwealth links (Dinan, 2004, p.45). The UK initiated less strict economic cooperation, the EFTA. However, shortly after

the establishment of EFTA, the UK gained interest in joining the union which evolved to the ECC by that time.

I will not discuss the UK case in terms of political reasons here. The results in Table 5.5 suggest that the UK is part of the coalitions from early stages. The participation delay of qualified countries is an interesting research topic but in the present chapter, the focus is the creation stage.⁸

⁸For example see [Konstantinidis \(2015\)](#), [Kóczy \(2009\)](#).

Chapter 6

Simultaneous Coalition Formation

In this chapter, a simultaneous coalition formation model is considered. The correlation clustering problem seems to be a natural formulation for a simultaneous coalition formation game¹. Therefore, it can be called a correlation clustering game. In the first part of the chapter, I introduce and discuss correlation clustering and define the correlation clustering game. Additionally, various solution concepts and a range of distance functions are considered. The second part of chapter presents the outcomes of the model by using the data given in section 4.3 and selected distance functions to illustrate different scenarios for the formation of the ESCS.

6.1 Correlation Clustering

A correlation clustering algorithm partitions elements into clusters based on their similarities. The idea is to find the best partition of elements depending on their pairwise similarity measures. To do so, similar elements are allocated in the same cluster and dissimilar elements are allocated in different clusters. (This is the main objective of similarity based clustering approaches in general.) Initially, correlation clustering is motivated to solve document clustering problems, which aims to cluster textual documents according to, for example, topics (see [Becker \(2005\)](#)).

As discussed in section 3, it is not always possible or desirable to determine the number of clusters in advance. Correlation clustering does not require a prior knowledge of number of

¹See [Becker \(2005\)](#) for a survey on Correlation clustering.

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clusters. Additionally, it does not require a threshold value that specifies a distance threshold for clustering. This makes correlation clustering a natural formulation in clustering analysis and particularly useful in coalition formation theory.

Correlation clustering can be applied a wide range of problems in game theory. For example, consider the following problem (Demaine et al. (2006)): there is a set of people who are invited to an event. Every guest has a preference list that shows with whom they want to interact and whom they want to avoid. Is there a stable setting in which we assign all guests to a number of tables so that nobody wants to change their seats? This problem is called the *Stable Invitation Problem* and correlation clustering can be used to solve it.

Correlation clustering was introduced by Bansal et al. (2004). There are two variants: minimizing disagreement and maximizing agreement. Agreements and disagreements are determined by pairwise distances. Two variants are equivalent. The authors show that these decision problems are *NP-complete*. Of course, computational complexity is crucial but since the number of countries is small, it is not critical to this chapter. Therefore, a brute force algorithm will be used to predict the partition of countries. Section 6.3.3 presents the computational complexity of the correlation clustering game more in detail.

Bansal et al. (2004) consider the following clustering problem: on a complete graph² $G = (V, E)$ with n vertices each edge $(u, v) \in E$ is labeled with $+$ or $-$ indicating whether two the vertices $u, v \in V$ are similar or different. I focus on the problem of minimizing disagreements³. Then, the question is how to partition the vertices such that the number of $-$ edges between clusters' members and the number of $+$ edges between disjoint clusters is minimized. The number of clusters is not given as a parameter of the minimization problem. It is determined by the resulting clusters and can be between 1 and n .

Assuming that C is the collection of the disjoint clusters, the minimizing disagreements problem can be formulated as

²A complete graph is an undirected graph in which each pair of nodes are connected by an edge.

³These two are equivalent at optimality but, as usual, differ from the point of view of approximation (Bansal et al. (2004))

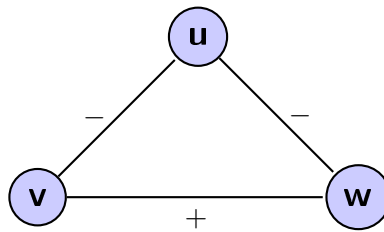
$$\min_C \sum_{u,v} c_{u,v}(1 - e(u,v)) + (1 - c_{u,v})e(u,v)$$

$$c_{u,v} = \begin{cases} 1 & \text{if } u \text{ and } v \text{ are in the same cluster} \\ 0 & \text{otherwise} \end{cases} \quad (6.1)$$

$$e_{u,v} = \begin{cases} 1 & \text{if it is a + edge} \\ 0 & \text{otherwise} \end{cases}$$

The idea of this optimization problem is essentially to count the number of disagreements in all possible partitions and determine the minimum one. Figure 6.2 illustrates a three nodes example. Straightforward calculation of the correlation clustering calculation yields a cluster including v and w together and u as a separate cluster. This is a perfect clustering in which all the edges are positive within the cluster and all the edges are negative between the clusters. That is we can simply delete $-$ edges in order to obtain a partition.

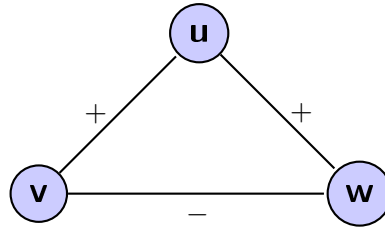
Fig. 6.1 Correlation Clustering: a perfect cluster



In general, however, a graph may not have a perfect clustering. For example, given nodes a, b, c such that a, b and a, c are similar while b, c are dissimilar, a perfect clustering is not possible. Then, one approach is to cluster elements in order to maximize the number of agreements or minimize the number of disagreements.

In Chapter 1, pointed out that predicting a partition of players is essential to the coalition formation in this work. Correlation clustering partitions elements into clusters regarding to their similarities therefore, the resulting partition greatly depends on the distance measure. Then, we have to answer the question of how to define the similarity function between

Fig. 6.2 Correlation Clustering: not a perfect cluster



countries. As in Chapter 5, a general distance function will be introduced (see Section 6.3) in order to measure the similarities.

6.2 Clustering Games

Games are categorized into sequential games in which players move in turns and simultaneous games in which players move simultaneously. In Chapter 5, we have studied a sequential game of country clustering. In this chapter, the correlation clustering game falls into category of simultaneous games.

Although correlation clustering can be seen as a natural formulation of the coalition formation problem, we have to translate the problem into game theory. [Hoefler \(2007\)](#) describes the translation as follows. Vertices represent players, $N = \{1, \dots, n\}$. Strategies are clusters, i.e. each player chooses a cluster. Utility for players is benefit of being in the same cluster with similar players. In a given partition, the utility for player i is

$$u_i = \sum_j c_{i,j}(1 - d(i, j)) + (1 - c_{i,j})d(i, j). \quad (6.2)$$

where $d(i, j)$ denotes the distance between players i and j . The objective is to minimize the disagreements within the clusters. These define a simultaneous game, namely a correlation clustering game.

Equation 6.2 has two properties. The first part increases the sum by similarity within the clusters. That is, players maximize their utility by clustering together with similar players. The second part decreases the sum by dissimilarity within the clusters. If players are not similar, they maximize their utility by avoiding each other. Clusters are formed as a result

of the minimizing disagreements which are according to pairwise distances. Each player simultaneously chooses a cluster and receives a payoff according to the similarity in this cluster. Players are rational and have complete information.

The above game has been introduced and analyzed in a few works. Particularly, [Feldman et al. \(2012\)](#) explicitly model correlation clustering in a game theoretic setting. Authors study hedonic clustering games and investigate how to apply different clustering methods, fixed clustering (k-median, k-center) and correlation clustering in order to study a non-cooperative game. They adopt the Nash stability as a solution concept in additively separable hedonic games. The focus is on providing upper and lower bounds on the price of stability and the price of anarchy which are defined as follows ([Feldman et al. \(2012\)](#)): “*The price of stability is defined as the ratio between the social welfare/cost of the best Nash equilibrium and the social optimal solution, while the price of anarchy is defined as the ratio between the social welfare/cost of the worst Nash equilibrium and the social optimal solution.*”

Clustering analysis is a recently developing subject and is gaining much attention in game theory. Though not the main subject in this work, it is worth mentioning one more approach of clustering in game theory. [Pelillo and Bulò \(2014\)](#) study the clustering problem in a different environment. They use evolutionary game theory to analyze the clustering problem. Clustering is defined as a non-cooperative game and they show that finding clusters turns out to be equivalent to the equilibrium concept in evolutionary game theory.

6.3 Model

This section describes how I integrate the correlation clustering of [Bansal et al. \(2004\)](#) and the correlation clustering game of [Hoefler \(2007\)](#) into a simultaneous country formation game. Countries’ preferences as well as distance functions are introduced and some intermediate results are presented. Comparing this model with sequential coalition formation will yield a fruitful discussion for coalition formation in general and coalition formation in the EU in particular.

There are a finite number of n countries, $N = \{1, 2, \dots, n\}$. A partition of countries is a specification of m groups of countries, such that each country belongs to one, and only one, group. Thus, a partition $S = (S_1, S_2, \dots, S_m)$ describes a particular allocation of countries.

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Each country $i \in N$ is identified by $K \geq 1$ characteristics. A number of standard characteristics are considered. Namely, they are the geographical location, population, GDP and regime type. Thus, country i is denoted by the vector $C_i = (\lambda_i, \phi_i, p_i, g_i, r_i)$ where λ_i and ϕ_i are longitude and latitude, p_i is population, g_i is GDP and r_i is regime type of country i . Section 4.3 describes these characteristics and gives numerical values.

Finding a meaningful partition is a general problem. In this model, countries form coalitions according to a correlation clustering algorithm with respect to the distances between them. The algorithm evaluates every possible coalitions and finds the *best* one. Due to the complexity, I focus on subsets of Western democracies. A relatively small number of countries allows the respective codes for algorithms to run in reasonable time.

There is no limitation on the number of clusters and no limitation on their sizes. For example: the best solution could be one giant cluster or n singletons. In our formulation, the input is naturally a complete graph. But each edge (distance between two countries) d_{ij} is still required to be normalized to interval $[0, 1]$.

The distance d_{ij} is a measure of similarity between two countries. For now we consider a general and abstract metric. In the following section possible realizations of this metric are presented. Additionally, numerical values can be assigned to countries as weights. The country $i, j \in N$ has the weight w_{ij} .

The correlation game is formulated as an n -players game with complete information. The objective is to

$$\begin{aligned}
 &\text{minimize} && \sum_{S_i=S_j} w_{ij}d_{ij} + \sum_{S_i \neq S_j} w_{ij}(1 - d_{ij}) \\
 &\text{subject to} && d_{ij} \in [0, 1] \\
 &&& d_{ij} + d_{jk} \geq d_{ik} \\
 &&& d_{ij} = d_{ji}
 \end{aligned}$$

The intuition underlying the minimization formulation is that if two countries i and j share the same cluster they pay the price for their dissimilarity, which is measured by the

distance d_{ij} while if they are in different clusters they pay the price of their similarity which is measured by $1 - d_{ij}$.

The weights are assumed to be equal to one. There are two reasons for this. First, I want to focus on the distances and simplify the optimization problem. Second, the correlation clustering game is equivalent to an additively-separable hedonic game (Feldman et al. (2012)).

To examine the equivalence, assume that P is the partition of the minimization problem given in equation 6.2. Then, for player i the cost is less than any other partition P' . Formally,

$$c_i(P) = \sum_{S_i=S_j} d_{ij} + \sum_{S_i \neq S_j} 1 - d_{ij} \leq c_i(P') = \sum_{S'_i=S'_j} d_{ij} + \sum_{S'_i \neq S'_j} 1 - d_{ij} \quad (6.3)$$

Recall that a game is additively separable if $\forall i \in N$ there exists a function $v : N \rightarrow R$ such that

$$S \succeq_i T \Leftrightarrow \sum_{j \in S} u_i(j) \geq \sum_{j \in T} u_i(j).$$

Equivalently, if we consider cost as negative utility

$$S \succeq_i T \Leftrightarrow \sum_{j \in S} c_i(j) \leq \sum_{j \in T} c_i(j). \quad (6.4)$$

The cost c_i is $-u_i$ for player i and it is given by

$$c_i(P) = \sum_{S_i=S_j} d_{ij} + \sum_{S_i \neq S_j} 1 - d_{ij}$$

Distance

The key factor determining a partition of elements is the distance measure. As before, the general distance between two countries is defined as a product of the distance in population and GDP space and the distance in geographical space. The distance in regime space is omitted. Recall that

- p_i is the population of country i
- g_i is the GDP of country i

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- Δ_{ij} is the Euclidean distance between countries i and j .

The general distance between two countries is

$$d_{ij} = f(p_i, p_j, g_i, g_j) \Delta_{ij}. \quad (6.5)$$

The pairwise distances d_{ij} will be computed and then normalized to $[0, 1]$. There are two reasons for using this general form for distance measure. First, it allows us to select any characteristics of countries using the non-geometrical part of the distance function. Second, the geometrical part of the distance function ensures that computational geometry can be used to design algorithms.

Stability

When we search for a stable outcome, we can examine either individual deviations or group deviations. A partition of players is called Nash stable if players have strategies such that players cannot increase their payoff by unilaterally changing their strategy, namely their clusters.

Definition 6.3.1 A partition P is Nash stable if $\forall i \in S_P(i) \succeq_i S_j \cup \{i\}$ for all $S_j \in P \cup \{\emptyset\}$.

Corollary 6.3.1 A Nash stable solution exists in the clustering game.

Proof

Let P be the partition that is the outcome of the minimization problem. Assume that the list of coalitions in P is $\{S_1, \dots, S_l, S_m, \dots, S_k\}$, $k \leq n$ and i belongs to coalition S_m . Recall that if partition P is the solution of the minimization problem, then sum of cost functions of all players is minimized and equal to $\sum c_i(P)$.

Player i prefers another coalition over the current coalition if her cost is lower there. Let P' be the partition that results from i moves to another coalition. Assume that the new list of coalitions is $\{S_1, \dots, S_l \cup \{i\}, S_m \setminus \{i\}, \dots, S_k\}$, $k \leq n$ and now i belongs to coalition S_l . Her cost in the new coalition should be lower.

$$\begin{aligned} c_i(P') &= \sum_{j \in S_l} d_{ij} + \sum_{j \in S_m} (1 - d_{ij}) + \sum_{j \in \cup_{k=1} S_k \setminus \{S_l, S_m\}} (1 - d_{ij}) < c_i(P) \\ &= \sum_{j \in S_m} d_{ij} + \sum_{j \in S_l} (1 - d_{ij}) + \sum_{j \in \cup_{k=1} S_k \setminus \{S_l, S_m\}} (1 - d_{ij}) \end{aligned}$$

implies that

$$0 < \sum_{j \in S_m} d_{ij} + \sum_{j \in S_l} (1 - d_{ij}) - \sum_{j \in S_l} d_{ij} - \sum_{j \in S_m} (1 - d_{ij}). \quad (6.6)$$

Let the right part of inequality be equal to M . After player i moves to S_l , the cost of partition P' can be written as

$$\sum c_i(P') = \sum c_i(P) + \frac{M}{2} < \sum c_i(P).$$

This is a contradiction. Therefore, partition P is Nash stable. This completes the proof.

More generally every symmetric and additively separable hedonic game admits a Nash stable partition ([Bogomolnaia and Jackson \(2002\)](#)). A Nash stable partition implies other individual stability concepts. Nash stability \Rightarrow individual stability \Rightarrow contractual individual stability. Thus, the correlation clustering game is individual and contractually individually stable.

If we want to examine group behavior we can look at a core stable partition. A partition of the players is called core stable if players have strategies such that no group of players can increase their payoff by unilaterally changing their strategy.

Definition 6.3.2 A coalition partition P is core stable if $\nexists T \subset N$ such that $T \succ_i S_p(i)$ for all $i \in T$.

Definition 6.3.3 A game satisfies the common ranking property if and only if there exists an ordering \succeq over $2^N \setminus \{\emptyset\}$ such that for any $i \in N$ any coalitions $i \in S$ and $i \in T$ it holds that

$$S \succeq_i T \Leftrightarrow S \succeq T. \quad (6.7)$$

Theorem 6.3.2 ([Farrell and Scotchmer \(1988\)](#)) The common ranking property guarantees the existence of a core stable partition.

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Corollary 6.3.3 *A core stable solution exists in the correlation clustering game.*

Proof.

It is enough to show that the correlation clustering game satisfies the common-ranking property. Player i prefers being in coalition S to being in coalition T if and only if the cost of being in coalition S is lower:

$$c_i(S) = \sum_{S_i=S_j} d_{ij} + \sum_{S_i \neq S_j} (1 - d_{ij}) < c_i(T) = \sum_{T_i=T_j} d_{ij} + \sum_{T_i \neq T_j} (1 - d_{ij}) \quad (6.8)$$

Equivalently,

$$S \succeq_i T \Leftrightarrow -c_i(S) \geq -c_i(T) \quad (6.9)$$

Equation 6.9 implies that there is a linear ordering over coalitions consistent with players' preferences. It means that preference profiles satisfy the common ranking property. This completes the proof.

Note that the top-coalition property is also satisfied since the common ranking property implies the top-coalition property (the other direction does not necessarily hold).

The idea behind the theorem is the common ranking property allows us to find a coalition S that maximizes the value for all its members. In coalition S , all players prefer S to any other coalition, therefore they do not attempt to deviate. We can repeat this process for the remaining players until no players are left. Note that common ranking property is not a necessary but sufficient condition for a non-empty core.

A Nash stable and core stable coalition exist in the correlation clustering game. However, these two solution concepts do not have an implication relationship.

$$Nash \not\Rightarrow Core, \quad Core \not\Rightarrow Nash$$

[Bogomolnaia and Jackson \(2002\)](#) demonstrate this statement with the following examples.

Example 6.3.4 (Core stable but not Nash stable)

$$\text{Player 1} \quad \{1,2\} \succ \{1\} \succ \{1,2,3\} \succ \{1,3\}$$

$$\text{Player 2} \quad \{1,2\} \succ \{2\} \succ \{1,2,3\} \succ \{2,3\}$$

$$\text{Player 3} \quad \{1,2,3\} \succ \{2,3\} \succ \{1,3\} \succ \{3\}$$

Partition $\{\{1,2\}, \{3\}\}$ is core stable because no other coalition is more preferable for player 1 and 2. However, since player 3 prefers $\{1,2,3\}$ to being alone it is not Nash stable.

Example 6.3.5 (Nash stable but not core stable)

$$\text{Player 1} \quad \{1,2\} \succ \{1,3\} \succ \{1,2,3\} \succ \{1\}$$

$$\text{Player 2} \quad \{2,3\} \succ \{1,2\} \succ \{1,2,3\} \succ \{2\}$$

$$\text{Player 3} \quad \{1,3\} \succ \{2,3\} \succ \{1,2,3\} \succ \{3\}$$

Partition $\{\{1,2,3\}\}$ is Nash stable because no player wants to deviate and join another coalition. However, there is no core stable partition: being alone is preferred to a grand coalition, a grand coalition is preferred to any pair, any pair and one singleton is preferred to any other pair and singleton.

We now discuss the coalition formation in two scenarios. I propose brute force algorithms that solve the optimization problem. These algorithms will be applied to the data given in Section 4.3 using a range of distance functions.

6.3.1 Coalition formation depends on location

The first algorithm uses the distance between capital cities of countries which is only a function of geographical location, $d_{ij} = \Delta_{ij}$. As pointed out, the correlation clustering is *NP-hard* problem. However, the focus is not on the computational issues but is on the process of different approaches. Thus, I propose a straightforward calculation for only a limited number of countries. In this calculation, the partitions of an n -element set are first listed. Then, the cost of each partition is calculated as the sum of each player's cost. They prefer being in the same coalition with similar countries and in a different coalition than dissimilar countries. The following brute force algorithm defines this process.

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Algorithm 4: A Geographical Simultaneous Coalition Formation

Data: Geographical locations of capital cities

- 1 Calculate pairwise distances for each pair of countries as

$$d_{ij} = \Delta_{ij}.$$

- 2 List all partitions of countries.
- 3 Calculate the cost of every partition such that

$$c(S) = \sum_{S_i=S_j} d_{ij} + \sum_{S_i \neq S_j} (1 - d_{ij}), \forall i \in N$$

- 4 Choose the partition with the minimum cost.
-

We can observe countries following such strategies when the cost of the coalition is linear to the geographical distance. For example, countries can form a coalition in order to reduce the transportation cost of goods only which linearly increases with geographical distance. Then, this algorithm gives us the network of countries which minimizes the overall cost.

6.3.2 A general coalition formation

Now we turn our attention to a richer distance measure. As we have discussed, one aim of this thesis is also to examine a general definition of distance function and not be stuck on the Euclidean space. To be able to be consistent I will examine the same distance measures that are defined in Section 5.

A general coalition formation differs from a geographical coalition formation only in terms of the distance measure. The distance between two countries is a combination of a geometrical element (geographical distance) and one or more non-geometrical elements (population, GDP, regime type, etc.). The combination of these elements is normalized to $[0, 1]$ interval. In the following, the algorithm for general coalition formation is given.

This algorithm is a base model underlying the idea of a compound distance measure. Coalitions can have various concerns: trade, public health, innovation, education, transportation, energy and so on. Indeed, formation requests are not limited to only a few factors.

Algorithm 5: A General Simultaneous Coalition Formation

- 1 Choose a function, $f(p_i, p_j, g_i, g_j)$, and for each pair of countries calculate the distance as

$$d_{ij} = \begin{cases} f(p_i, p_j, g_i, g_j)\Delta_{ij} & : r_i = r_j \\ \infty & : r_i \neq r_j. \end{cases}$$

- 2 List all partitions of countries.
- 3 Calculate the cost of every partition such that

$$c(S) = \sum_{S_i=S_j} d_{ij} + \sum_{S_i \neq S_j} (1 - d_{ij}), \forall i \in N$$

- 4 Choose the partition with the minimum cost.
-

However, we can always add these considerations in the distance measure. Before presenting results of these algorithms, we will take a closer look at the computational aspects.

6.3.3 Computational Complexity

In correlation clustering games the time taken to reach a stable partition depends strongly on the number of players. On the one hand, the game presented in this section is an additively separable hedonic game; on the other hand, it is a reformulation of the correlation clustering problem. Thus, in this subsection, I will review the computational complexity of both additively separable hedonic games and correlation clustering. As we discussed, the problems of the existence of a stable outcome and the decision of whether a given partition is stable are at the heart of the computational aspect of game theory. We will restrict complexity results for Nash and core stable outcomes.

The problem of deciding whether a Nash or a core stable partition exists in a hedonic game is NP-hard in general (Ballester (2004)). Even if we restrict our attention to additively separable hedonic games NP-hardness is inherited.

Theorem 6.3.6 (Sung and Dimitrov (2010)) *In the class of additively separable hedonic games checking whether a core stable or Nash stable outcome exists is NP-hard.*

The existence of a Nash stable outcome can be guaranteed by restricting arbitrary preferences. It has been shown that in the class of symmetric additively separable hedonic games

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the existence of a Nash partition is guaranteed. If that is so, then how hard is it to compute this outcome?

Theorem 6.3.7 (Gairing and Savani (2010)) *In the class of symmetric additively separable hedonic games, the problem of computing a Nash-stable outcome, is PLS-complete*

PLS (polynomial local search) is a complexity class and PLS-complete problems in which a local optimum solution to an optimization problem can be verified in polynomial time. For details and algorithms see [Gairing and Savani \(2010\)](#).

Trivially, when the common-ranking property is satisfied, the question of existence is in P. Otherwise, computing this partition can still be done in polynomial time for special cases. For example,

Theorem 6.3.8 (Dimitrov et al. (2004)) *In the class of additively separable games, if preferences are based on the appreciation of friends, a core stable coalition structure can be found in polynomial time.*

The correlation game fulfills the common-ranking property. However, when we know that a core stable partition exists, how to find it is not always an easy task.

Theorem 6.3.9 (Sung and Dimitrov (2010)) *The verification problem for core stability is strongly NP-complete under enemy-oriented preferences.*

The problem of correlation clustering is also NP-hard. Even for small size n (in our case, sixteen countries), the running time can be impractical. [Demaine et al. \(2006\)](#) reformulate the minimizing disagreement as a linear program and provide approximation algorithm which has a complexity of $O(\log n)$. The exact complexity of a correlation clustering game depends on the problem definition and the method used to evaluate the quality of partitions.

Algorithm 1 and Algorithm 2 are intended to be used for calculations with a very limited number of countries. In the brute force, all partitions are first listed and evaluated, then a comparison is performed. Listing all partition traps the implementation in a exponential running time. There are $O(2^N)$ possible subsets which are used to list all partitions for each element $O(N)$. Evaluating the distance $O(N^2)$. Then finding the minimum takes $O(N)$ time. The following demonstrates the severity of impracticality. If clustering 10 countries takes 1 second, for 30 countries it takes about 36 days and for 100 countries about 10^{20} years. Therefore, in the next section only a few countries are used.

6.4 Application

In this section, I present the implementation of the two algorithms given in Section 6.3.1 and 6.3.2. Here, applications are not considered to illustrate the predictive power of the theoretical model but to study a special problem, discover new questions and obtain feedback to the theory. In economics, this type of communication is uncommon but models can benefit from their applications.

Given a set of players (countries) and properties (geographical location, GDP and population) implementations provide a stable partition of countries. Brute force algorithm is given above.

6.4.1 Coalition formation depends on only location

First, only geographical location is considered. Therefore, the distance matrix contains the geographical distance between countries. It is given in Table 6.1.

Countries form two groups. The best score is 69.88.

- Iceland
- Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Norway, Sweden, Switzerland, UK

Table 6.1 Geographical distance between countries

	Austria	Belgium	Denmark	Finland	France	Germany	Greece	Iceland
Austria	0.0	0.22	0.21	0.35	0.25	0.17	0.31	0.69
Belgium	0.22	0.0	0.18	0.4	0.06	0.05	0.5	0.51
Denmark	0.21	0.18	0.0	0.21	0.25	0.16	0.51	0.51
Finland	0.35	0.4	0.21	0.0	0.46	0.37	0.59	0.58

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France	0.25	0.06	0.25	0.46	0.0	0.1	0.5	0.54
Germany	0.17	0.05	0.16	0.37	0.1	0.0	0.46	0.54
Greece	0.31	0.5	0.51	0.59	0.5	0.46	0.0	1.0
Iceland	0.69	0.51	0.51	0.58	0.54	0.54	1.0	0.0
Ireland	0.4	0.19	0.3	0.49	0.19	0.23	0.69	0.36
Italy	0.18	0.28	0.37	0.53	0.27	0.26	0.25	0.79
Luxembourg	0.18	0.04	0.19	0.4	0.07	0.03	0.46	0.55
Netherlands	0.23	0.04	0.15	0.36	0.1	0.06	0.52	0.48
Norway	0.32	0.26	0.12	0.19	0.32	0.25	0.63	0.42
Sweden	0.3	0.31	0.13	0.1	0.37	0.28	0.58	0.51
Switzerland	0.16	0.12	0.25	0.45	0.1	0.1	0.4	0.63
UK	0.3	0.08	0.23	0.44	0.08	0.12	0.57	0.45

	Ireland	Italy	Luxembourg	Netherlands	Norway	Sweden	Netherlands	UK
Austria	0.4	0.18	0.18	0.23	0.32	0.3	0.16	0.3
Belgium	0.19	0.28	0.04	0.04	0.26	0.31	0.12	0.08
Denmark	0.3	0.37	0.19	0.15	0.12	0.13	0.25	0.23
Finland	0.49	0.53	0.4	0.36	0.19	0.1	0.45	0.44
France	0.19	0.27	0.07	0.1	0.32	0.37	0.1	0.08

West Germany	0.23	0.26	0.03	0.06	0.25	0.28	0.1	0.12
Greece	0.69	0.25	0.46	0.52	0.63	0.58	0.4	0.57
Iceland	0.36	0.79	0.55	0.48	0.42	0.51	0.63	0.45
Ireland	0.0	0.45	0.23	0.18	0.3	0.39	0.29	0.11
Italy	0.45	0.0	0.24	0.31	0.48	0.47	0.17	0.34
Luxembourg	0.23	0.24	0.0	0.07	0.28	0.31	0.08	0.12
Netherlands	0.18	0.31	0.07	0.0	0.22	0.27	0.15	0.09
Norway	0.3	0.48	0.28	0.22	0.0	0.1	0.35	0.28
Sweden	0.39	0.47	0.31	0.27	0.1	0.0	0.37	0.34
Switzerland	0.29	0.17	0.08	0.15	0.35	0.37	0.0	0.18
UK	0.11	0.34	0.12	0.09	0.28	0.34	0.18	0.0

6.4.2 A general coalition formation

In the following, I will present three results depending on the distance function defined as

$$d_{ij} = \frac{\max(\frac{g_i}{p_i}, \frac{g_j}{p_j})}{\min(\frac{g_i}{p_i}, \frac{g_j}{p_j})} \Delta_{ij} \quad (6.10)$$

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Algorithm 6: Coalition Formation

```
input :Countries on a plane
output :A partition of countries

1 List all partitions given list l and number of clusters K, clusters(l, K)
2 Choose non-empty clusters, neclusters(l, K)
3 Define the minimization function Phi(partition)
4 distance = 0
5 for coalition in partition do
6     for member in coalition do
7         for other member in coalition do
8             if member == other member then
9                 | continue
10            else
11                | distance += distance(member, other member)
12            for other coalition in partition do
13                if other coalition == coalition then
14                    | continue
15                else
16                    for other member in other coalition do
17                        | distance += 1-distance(member, other member)

18 return distance
19 Set best partition = None and set best score =  $\infty$ 
20 for number of clusters in range(1, number of countries) do
21     Find neclusters(countries, K)
22     while True do
23         try;
24         current partition = clust.next()
25         score = Phi(current partition)
26         if score  $\leq$  best score then
27             | best score = score
28             | best partition = current partition
29     except StopIteration
30     break
```

Table 6.2 Geographical distance between countries

	Austria	Belgium	Denmark	Finland	France	Germany	Greece	Iceland
Austria	0.	0.12	0.14	0.14	0.13	0.07	0.21	0.36
Belgium	0.12	0.	0.08	0.18	0.02	0.02	0.51	0.19
Denmark	0.14	0.08	0.	0.12	0.12	0.09	0.67	0.24
Finland	0.14	0.18	0.12	0.	0.2	0.13	0.47	0.26
France	0.13	0.02	0.12	0.2	0.	0.04	0.49	0.2
West Germany	0.07	0.02	0.09	0.13	0.04	0.	0.37	0.25
Greece	0.21	0.51	0.67	0.47	0.49	0.37	0.	1.
Iceland	0.36	0.19	0.24	0.26	0.2	0.25	1.	0.
Ireland	0.16	0.11	0.22	0.22	0.1	0.1	0.44	0.2
Italy	0.07	0.16	0.26	0.23	0.14	0.11	0.17	0.43
Luxembourg	0.15	0.02	0.08	0.28	0.04	0.02	0.72	0.31
Netherlands	0.13	0.02	0.06	0.18	0.04	0.03	0.58	0.2
Norway	0.17	0.09	0.05	0.09	0.12	0.12	0.64	0.15
Sweden	0.2	0.14	0.05	0.05	0.17	0.16	0.73	0.23
Switzerland	0.14	0.07	0.12	0.34	0.07	0.08	0.68	0.38
United Kingdom	0.2	0.04	0.08	0.26	0.04	0.07	0.75	0.21

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	Ireland	Italy	Luxembourg	Netherlands	Norway	Sweden	Netherlands	UK
Austria	0.16	0.07	0.15	0.13	0.17	0.2	0.14	0.2
Belgium	0.11	0.16	0.02	0.02	0.09	0.14	0.07	0.04
Denmark	0.22	0.26	0.08	0.06	0.05	0.05	0.12	0.08
Finland	0.22	0.23	0.28	0.18	0.09	0.05	0.34	0.26
France	0.1	0.14	0.04	0.04	0.12	0.17	0.07	0.04
West Germany	0.1	0.11	0.02	0.03	0.12	0.16	0.08	0.07
Greece	0.44	0.17	0.72	0.58	0.64	0.73	0.68	0.75
Iceland	0.2	0.43	0.31	0.2	0.15	0.23	0.38	0.21
Ireland	0.	0.17	0.2	0.11	0.17	0.28	0.27	0.08
Italy	0.17	0.	0.21	0.19	0.27	0.33	0.15	0.25
Luxembourg	0.2	0.21	0.	0.04	0.15	0.14	0.03	0.05
Netherlands	0.11	0.19	0.04	0.	0.09	0.11	0.08	0.04
Norway	0.17	0.27	0.15	0.09	0.	0.04	0.21	0.13
Sweden	0.28	0.33	0.14	0.11	0.04	0.	0.18	0.13
Switzerland	0.27	0.15	0.03	0.08	0.21	0.18	0.	0.08
United Kingdom	0.08	0.25	0.05	0.04	0.13	0.13	0.08	0.

Countries form two groups. The best score is 43.62.

- Greece
- Austria, Belgium, Denmark, Finland, France, Germany, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Sweden, Switzerland, UK

Next, I consider the founding members of the ECSC and the UK.

Table 6.3 Geographical distance between countries

	Belgium	France	Germany	Italy	Luxembourg	Netherlands	UK
Belgium	0.	0.1	0.09	0.64	0.09	0.07	0.14
France	0.1	0.	0.17	0.58	0.17	0.17	0.16
Germany	0.09	0.17	0.	0.45	0.09	0.12	0.3
Italy	0.64	0.58	0.45	0.	0.85	0.78	1.
Luxembourg	0.09	0.17	0.09	0.85	0.	0.15	0.2
Netherlands	0.07	0.17	0.12	0.78	0.15	0.	0.15
UK	0.14	0.16	0.3	1.	0.2	0.15	0.

Countries form two groups. The best score is 7.74.

- Italy
- Belgium, France, Germany, Luxembourg, Netherlands, UK

Last, only the group of founding members is examined. Again countries form two groups. The best score is 5.08.

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- Italy
- Belgium, France, Germany, Luxembourg, Netherlands

The results are consistent with the results in table 5.5.

Table 6.4 Geographical distance between countries

	Belgium	France	Germany	Italy	Luxembourg	Netherlands
Belgium	0.	0.12	0.1	0.76	0.11	0.08
France	0.12	0.	0.2	0.68	0.2	0.21
Germany	0.1	0.2	0.	0.53	0.1	0.14
Italy	0.76	0.68	0.53	0.	1.	0.92
Luxembourg	0.11	0.2	0.1	1.	0.	0.17
Netherlands	0.08	0.21	0.14	0.92	0.17	0.

Chapter 7

Conclusion

The point of departure of this thesis is the process of coalition formation. The focus is on the emergence and evolution of coalitions, in particular the creation of the ECSC. For this purpose, I use methods from game theory and beyond to construct two theoretical frameworks and apply these to the ECSC.

The recent political and economical crisis urges a better understanding of national and international coalitions. Its urgency was recently demonstrated by the Brexit. Game theory is a very strong tool to tackle such problems. However, the equilibrium centered part of economic game theory has shortcomings in particular with regard to the aspects of political economy (see for example [Hanappi \(2013b\)](#)). In this thesis tools from physics and computer science were combined with game theory in an attempt to investigate coalition formation and its process.

First, theoretical models of coalition formation with a focus on hedonic games in which players have preferences over which group they belong to were reviewed. Although these models have a simple setting they have a wide range of applications and very interesting features. One of the important features of these games is that the number of coalitions is not fixed, i.e. the number number of coalitions can lie between and the total number of players. The main focus is on the restrictions of preferences required to guarantee a solution as well as the computational complexity of finding a solution. Several restrictions, solution concepts and computational issues are discussed. It was concluded that the definitions of similarity, and of the process need further attention in order to make the theory more applicable in such setting.

Conclusion

Cluster analysis has been successfully used in many fields and has attracted attention as an important tool in economics. Particularly, in the modeling of coalition formation processes clustering algorithms can be used. In chapter 3, clustering techniques and algorithms were discussed. Then, the history of the EU was reviewed and the relevant data from the 1950s for the application of these models were presented. In the application GDP and population size were considered measures for economic strength. Furthermore, government type, a further variable, was classified in three types.

In light of these, two country coalition formation games were studied. Two points are important when designing such coalition formation games: (i) determining the players' preferences and (ii) the procedure. The first point is addressed by defining a distance measure. The distance between countries i and j is measured by a distance function containing geometrical and non-geometrical elements. Thus, the distance between two countries is defined as a product of the distance in population, GDP and political regime space and the distance in physical space.

The second point is addressed by the use of clustering algorithms. In the first model a sequential procedure is used in which players iteratively form subcoalitions. This is a clustering methods borrowed from high energy physics. In the second model a simultaneous procedure is used in which players form coalitions at once. This correlation clustering algorithm is a well studied method in computer science. These algorithms are tailored to the coalition formation process by subjecting them to the constraints required by the game theoretical setting..

To illustrate the predictions made by this model, the formation of the ECSC is examined. Concrete realizations of the general distance function are presented. The results produced using data from 28 European countries illustrate the impact of the distance function in the process. If the distance is defined as a ratio of countries' GDP per capita in combination with the geographical distance, the sequential algorithm produces a coalition of five of the six founder states of the ESCS in the first four steps; namely *France, Belgium, the Netherlands, West Germany and Luxembourg*. Due to the computational complexity, if one restricts the number of players only to the founder states in the simultaneous coalition formation game using the same distance function exactly the same outcome is produced. Many other results of both algorithms using different realizations of the distance function were presented.

Furthermore, the computational complexity of the algorithms was examined. Since the simultaneous coalition formation problem is $NP - hard$, only a limited number of players were selected for the application. The running time is n^3 for the naive sequential algorithm. However, a special class of sequential clustering algorithms proposed in physics improves the running time to $n \log n$. Given the rising importance of computational matters in game theory, this type of tool can provide a significant contribution.

Coalition formation is a difficult task and there is no unique method for solving this problem. However, in my opinion three common aspects are necessary for a coalition formation process in the context of international coalitions. These are geographic, economic and social elements. Two models studied in this work contain several ideas with jet clustering from particle physics and correlation clustering from computer science. This brings an algorithmic approach to coalition formation processes.

This thesis should be seen as a step towards developing new frameworks in game theory by using different techniques and improvements. This stream of research is crucial and essential for understanding the coalition formation process in political and economical environments.

Looking forward, although these models bring a new perspective and attempt to deal with technical matters such as defining a distance function and computing and reducing the complexity of the solution method, coalition formation processes still need a considerable effort in order to be considered as being well understood.

A natural extension to the sequential coalition formation model would be to develop a dynamic model in which the evolution of a coalition can be studied. The EU has been expanded gradually and several structural changes took place. If such model is developed it could be tested on the data by simulations. Furthermore, within this dynamic model many interesting problems can be studied such as fairness and power.

In both models a fundamental challenge is to find the distance function. In this thesis a general distance function was proposed but only a small number of possible realizations were presented. The elements in the distance function and their relevant importance should be studied in more details. The distance function is a measure for similarity. The concepts of similarity and distance are crucial for the theory.

Conclusion

Another important point is determining a threshold for the pairwise distance between countries in the the sequential coalition formation model. This allows more precise analysis using the algorithm proposed in this thesis. Any stability depends on the choice of the threshold value.

References

- Acemoglu, D., Egorov, G., and Sonin, K. (2008). Coalition formation in non-democracies. *Review of Economic Studies*, 75(4):987–1009.
- Alcalde, J. and Revilla, P. (2001). Tops Responsiveness, Strategy-Proofness And Coalition Formation Problems. *Working Papers. Serie AD*.
- Alcalde, J. and Romero-Medina, A. (2006). Coalition Formation and Stability. *Social Choice and Welfare*, 27(2):365–375.
- Aumann, R. J. and Dreze, J. H. (1974). Cooperative games with coalition structures. *International Journal of Game Theory*, 3(4):217–237.
- Axelrod, R. and Bennett, D. S. (1993). A Landscape Theory of Aggregation. *British Journal of Political Science*, 23(02):211—233.
- Axelrod, R. M. (1970). *Conflict of interest: a theory of divergent goals with applications to politics*. Markham Pub. Co.
- Baldwin, R. (1993). A Domino Theory of Regionalism. *National Bureau of Economic Research Working Paper Series (Cambridge)*, 4465(4465):1–39.
- Baldwin, R. E. (1994). *Towards an Integrated Europe*. Centre for Economic Policy Research.
- Baldwin, R. E. and Wyplosz, C. (2012). *The Economics of European Integration*. McGraw-Hill Education.
- Ballester, C. (2004). NP-completeness in hedonic games. *Games and Economic Behavior*, 49(1):1–30.
- Banerjee, S., Konishi, H., and Sönmez, T. (2001). Core in a simple coalition formation game. *Social Choice and Welfare*, 18(1):135–153.
- Bansal, N., Blum, A., and Chawla, S. (2004). Correlation Clustering. *Machine Learning*, 56(1-3):89–113.
- Barberà, S., Bevia, C., and Ponsati, C. (2013). Meritocracy, Egalitarianism and the Stability of Majoritarian Organizations. *SSRN Electronic Journal*.
- Barberà, S. and Gerber, A. (2003). On coalition formation: Durable coalition structures. *Mathematical social sciences*, 45(2):185–203.

References

- Barbera, B., Bossert, W. and Pattanaik, P. (2004). Ranking Sets of Objects. In Salvador Barbera, Peter Hammond, C. S., editor, *Handbook of Utility Theory: Volume 2: Extensions*, chapter Chapter 17, page 626. Springer Science & Business Media.
- Becker, H. (2005). A Survey of Correlation Clustering. *Advanced Topics in Computational Learning Theory.*, pages 1–10.
- Black, D. (1948). On the Rationale of Group Decision-making. *Journal of Political Economy*, 56(1):23.
- Bloch, F. (1996). Sequential Formation of Coalitions in Games with Externalities and Fixed Payoff Division. *Games and Economic Behavior*, 14(1):90–123.
- Bloch, F. and Diamantoudi, E. (2011). Noncooperative formation of coalitions in hedonic games. *International Journal of Game Theory*, 40(2):263–280.
- Boekhoorn, V., van Deemen, A. M. A., and Hosli, M. O. (2006). *The European Union Decides*. Cambridge University Press, Cambridge.
- Bogomolnaia, A. and Jackson, M. O. (2002). The Stability of Hedonic Coalition Structures. *Games and Economic Behavior*, 38(2):201–230.
- Brams, S. J., Jones, M. A., and Kilgour, D. M. (2002). Single-Peakedness and Disconnected Coalitions.
- Brânzei, S. and Larson, K. (2011). Social distance games. In *IJCAI International Joint Conference on Artificial Intelligence*, pages 91–96.
- Breton, M. L., Ortuño-Ortín, I., and Weber, S. (2007). Gamson’s law and hedonic games. *Social Choice and Welfare*, 30(1):57–67.
- Bronstein, M. and Mumford, D. (2009). Discrete geometry - Lecture 2. In *Numerical geometry of non-rigid shapes*. Stanford University.
- Burani, N. and Zwicker, W. S. (2003). Coalition formation games with separable preferences. *Mathematical social sciences*, 45(1):27–52.
- Cacciari, M. and Salam, G. P. (2006). Dispelling the the N^3 myth for the Kt jet-finder. *Physics Letters B*, 641(1):57–61.
- Carchedi, G. (2001). *For Another Europe: A Class Analysis of European Economic Integration*. Verso.
- Chwe, M. S.-Y. (1994). Farsighted Coalitional Stability. *Journal of Economic Theory*, 63:299–325.
- Darmann, A., Elkind, E., Kurz, S., Lang, J., Schauer, J., and Woeginger, G. (2012). Internet and Network Economics. *Lecture Notes in Computer Science*, 7695:156–169.
- De Berg, M., Cheong, O., Van Kreveld, M., and Overmars, M. (2008). *Computational Geometry: Algorithms and Applications*, volume 17.

- de Swaan, A. (1973). *Coalition theories and cabinet formations: a study of formal theories of coalition formation applied to nine European parliaments after 1918*. Jossey-Bass.
- Deardorff, A. V. (1998). Determinants of Bilateral Trade: Does Gravity Work in a Neoclassical World? *NBER Working Paper No. W5377*, (January):pp. 7–32.
- Deemen, A. M. V. (1997). *Coalition Formation and Social Choice*. Springer US.
- Deemen, A. M. V. (2013). *Coalition Formation and Social Choice*. Springer Science & Business Media.
- Demaine, E. D., Emanuel, D., Fiat, A., and Immorlica, N. (2006). Correlation clustering in general weighted graphs. *Theoretical Computer Science*, 361(2-3):172–187.
- Desmet, K., Le Breton, M., Ortuño Ortín, I., and Weber, S. (2006). Nation Formation and Genetic Diversity. *SSRN Electronic Journal*.
- Diamantoudi, E. and Xue, L. (2003). Farsighted stability in hedonic games. *Social Choice and Welfare*, 21(1):39–61.
- Dimitrov, D., Borm, P., Hendrickx, R., and Sung, S. C. (2006). Simple Priorities and Core Stability in Hedonic Games. *Social Choice and Welfare*, 26(2):421–433.
- Dimitrov, D., Borm, P. E. M., Hendrickx, R., and Sung, S. C. (2004). Simple Priorities and Core Stability in Hedonic Games. *SSRN Electronic Journal*.
- Dinan, D. (2004). *Europe Recast: A History of European Union*. Lynne Rienner Publishers.
- Downs, A. (1957). *An Economic Theory of Democracy*. Harper.
- Downs, G. W., Roche, D. M., and Barsoom, P. N. (1998). Managing the Evolution of Multilateralism. *International Organization*, 52(02):397–419.
- Drèze, J. H. and Greenberg, J. (1980). Hedonic Coalitions: Optimality and Stability. *Econometrica*, 48(4):987–1003.
- Farrell, J. and Scotchmer, S. (1988). Partnerships. *Quarterly Journal of Economics*, 103(2):279–297.
- Feldman, M., Lewin-Eytan, L., and Naor, J. S. (2012). Hedonic clustering games. *Proceedings of the 24th ACM symposium on Parallelism in algorithms and architectures - SPAA '12*, page 267.
- Frankel, J. A., Stein, E., and Wei, S.-J. (1998). Continental Trading Blocs: Are They Natural or Supernatural? In *The Regionalization of the World Economy*, pages 91–120. University of Chicago Press.
- Gairing, M. and Savani, R. (2010). Computing Stable Outcomes in Hedonic Games. pages 174–185. Springer Berlin Heidelberg.
- Galam, S. (2008). SOCIOPHYSICS: A REVIEW OF GALAM MODELS. *International Journal of Modern Physics C*, 19(03):409–440.

References

- Gale, D. and Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*, 69(1):9–15.
- Greenberg, J. (1994). Coalition structures. In Hart, R. A. and S., editors, *Handbook of Game Theory with Economic Applications*, vol 2, volume 2, pages 1305–1337. Elsevier.
- Grofman, B. (1982). A dynamic model of proto-coalition formation in ideological space. *Behavioral Science*, 27(1):77–90.
- Grofman, B. and Straffin, J. P. D. (1984). Parliamentary Coalitions: A Tour of Models. *Mathematics Magazine*, Vol. 57(No. 5):259–274.
- Hajdukov, J. (2004). On Coalition Formation Games. *International Game Theory Review*, 8(4):613–641.
- Hanappi, H. (2013a). Can Europe Survive? Ten Commandments for Europe’s Next Ten Years. In *Growth Perspectives in Europe*, pages pp. 27–92. Polish Economic Society, Torun.
- Hanappi, H. (2013b). The Neumann-Morgenstern Project – Game Theory as a Formal Language for the Social Sciences. In *Game Theory Relunched*. InTech.
- Hanappi, H. (2016). Shangri-La Governance. A Sketch of an Integral Solution for European Economic Policy Based on a Synthesis of Europe’s Problems. In Savvas Katsikides, H. H., editor, *Society and Economics in Europe*, pages 1–18. Springer International Publishing, Cham.
- Hart, S. and Kurz, M. (1983). Endogenous Formation of Coalitions. *Econometrica*, 51(4):1047–1064.
- Hoefer, M. (2007). *Cost Sharing and Clustering under Distributed Competition*. PhD thesis.
- Hotelling, H. (1929). Stability in competition. *The Economic Journal*, 39(153):41–57.
- Iehlé, V. (2007). The core-partition of a hedonic game. *Mathematical social sciences*, 54(2):176–185.
- Jackson, M. O. (2008). *Social and Economic Networks*. Princeton university press.
- Kalai, E. and Schmeidler, D. (1977). Aggregation Procedure for Cardinal Preferences: A Formulation and Proof of Samuelson’s Impossibility Conjecture. *Econometrica*, 45(6):1431–1438.
- Kóczy, L. Á. (2009). Sequential coalition formation and the core in the presence of externalities. *Working Paper Series*.
- Konishi, H. and Ray, D. (2002). Coalition Formation as a Dynamic Process. *Boston College Working Papers in Economics*.
- Konstantinidis, N. (2015). On the Political Geometry of International Unions: A Coalition-Theoretic Approach. *Research in Economics*.

- Krugman, P. (1991). Increasing Returns and Economic Geography. *Journal of Political Economy*, 99(3):483–99.
- Lazarova, E. and Dimitrov, D. (2012). Status-seeking in hedonic games with heterogeneous players. *Social Choice and Welfare*, 40(4):1205–1229.
- Leiserson, M. A. (1968). *The study of coalition behavior: theoretical perspectives and cases from four continents*. Holt, Rinehart and Winston.
- McCormick, J. (2015). *European Union Politics*. Palgrave Macmillan.
- Milchtaich, I. and Winter, E. (2002). Stability and Segregation in Group Formation. *Games and Economic Behavior*, 38(2):318–346.
- Moran, T. H. (1981). Modeling OPEC behavior: economic and political alternatives. *International Organization*.
- Morelli, M. (2012). Union Integration Paths. *Rivista di Politica Economica*, (3):7–25.
- Pápai, S. (2004). Unique stability in simple coalition formation games. *Games and Economic Behavior*, 48(2):337–354.
- Peleg, B. (1981). Coalition formation in simple games with dominant players. *International Journal of Game Theory*, 10(1):11–33.
- Pelillo, M. and Bulò, S. R. (2014). Clustering games.
- Perry, M. and Reny, P. J. (1994). A Noncooperative View of Coalition Formation and the Core. *Econometrica*, 62(4):795–817.
- Pinder, J. and Usherwood, S. (2013). *The European Union: A Very Short Introduction*. OUP Oxford.
- Ray, D. and Vohra, R. (1999). A Theory of Endogenous Coalition Structures. *Games and Economic Behavior*, 26(2):286–336.
- Riker, W. H. (1960). *The Theory of Political Coalitions*. New Haven: Yale University Press.
- Salam, G. P. (2010). Towards jetography. *The European Physical Journal C*, 67(3-4):637–686.
- Scarf, H. E. (1967). The core of an N person game. *Econometrica*, 35(1):50–69.
- Schelling, T. C. (1971). Dynamic models of segregation†.
- Schofield, N. (1993). Political competition and multiparty coalition governments. *European Journal of Political Research*, 23(1):1–33.
- Seidmann, D. J. and Winter, E. (1998). A Theory of Gradual Coalition Formation. *Review of Economic Studies*, 65(4):793–815.
- Skiena, S. S. (2009). *The Algorithm Design Manual*. Springer Science & Business Media.

References

- Sung, S. C. and Dimitrov, D. (2010). Computational complexity in additive hedonic games. *European Journal of Operational Research*, 203(3):635–639.
- Tan, J. J. (1991). A necessary and sufficient condition for the existence of a complete stable matching. *Journal of Algorithms*, 12(1):154–178.
- Tilly, R., Welfens, P. J. J., and Heise, M., editors (2007). *50 Years of EU Economic Dynamics*. Springer Berlin Heidelberg, Berlin, Heidelberg.
- Vinogradova, G. and Galam, S. (2013). Rational instability in the natural coalition forming. *Physica A: Statistical Mechanics and its Applications*, 392(23):6025–6040.
- Von Neumann, J. and Morgenstern, O. (1944). *Theory of Games and Economic Behavior*, volume 2. Princeton University Press.
- Watts, D. J. (1999). *Small Worlds: The Dynamics of Networks Between Order and Randomness*. Princeton University Press.
- Woeginger, G. J. (2012). Core stability in hedonic coalition formation. page 18.

Appendix A

Computer Codes

A.1 Sequential Coalition Formation

```
import math
import sys
sys.path.append("/usr/local/lib/python/site-packages")
from geographiclib.geodesic import Geodesic
import time
default_dummy_distance = float("inf")
class coalition:
    def __init__(self,
                 name,
                 lat,
                 lng,
                 pop,
                 gdp,
                 regime):
        self.name = name
        self.lat = lat
        self.lng = lng
        self.pop = pop
        self.gdp = gdp
```

Computer Codes

```
        self.regime = regime
# End of constructor
# Create a dictionary of distances
distances = {}
def distregime(A, B):
    if A.regime == B.regime:
        return 1
    else:
        return default_dummy_distance
def distloc(A, B):
    distloc = Geodesic.WGS84.Inverse(A.lat, A.lng, B.lat, B.lng)
    return (distloc['s12']/1000)*distregime(A, B)
# End of location distance - dist['s12'] is in meters
distances["distregime"] = distregime
distances["distloc"] = distloc
# And for the rest, use lambda function
# geocode distance combination with pop
distances["distpop"] = lambda A,B: distloc(A, B)*abs(A.pop-B.pop
)
# geocode distance combination with gdp
distances["distgdp"] = lambda A,B: distloc(A, B)*abs(A.gdp-B.gdp
)
# geocode distance combination with per capita gdp
distances["distpercapitagdp"] = lambda A,B: distloc(A, B)*abs(A.
    gdp/A.pop-B.gdp/B.pop)
# geocode distance combination with max/min per capita gdp ratio
distances["distmaxminpercapitaratio"] = lambda A,B: distloc(A, B
) * max((A.gdp/A.pop), (B.gdp/B.pop)) / min((A.gdp/A.pop), (B.gdp/B
    .pop))
def midpoint(A,B):
    d = Geodesic.WGS84.Inverse(A.lat, A.lng, B.lat, B.lng)
    h = Geodesic.WGS84.Direct(A.lat, A.lng, d['azi1'], d['s12'
] / 2)
```

```

    return h['lat2'], h['lon2']
def merge(A, B):
    new_name = "{0},{1}".format(A.name, B.name)
    new_lat, new_lng = midpoint(A, B)
    new_pop = A.pop + B.pop
    new_gdp = A.gdp + B.gdp
    new_regime = A.regime
    C = coalition(new_name,
                  new_lat,
                  new_lng,
                  new_pop,
                  new_gdp,
                  new_regime)

    return C
# Choose the distance function here
if len(sys.argv) == 2:
    distance_name = sys.argv[1]
else:
    print "Invalid number of command line arguments"
    print "Usage: program.py distance_function_name"
    print sys.exit()
distance = distances[distance_name]
in_f = open("input_europe.txt", "r")
coalitions = []
for line in in_f:
    # Allow comments in the text file
    if "#" in line:
        continue
    # split by "," and remove extra whitespace
    [name, lat, lng, pop, gdp, regime] = [x.strip() for x in
        line.split(",")]
    # Create coalition object and add to list
    coalitions.append(coalition(name,

```

Computer Codes

```
        float(lat),
        float(lng),
        float(pop),
        float(gdp),
        int (regime)))

# End of reading coalitions from file
outfile_name = "out_"+distance_name+"_"+time.strftime("%Y_%m_%d_%H%M")+".odt"
out_f = open(outfile_name, "w")
m = [[distance_name], ['First coalition', 'Second coalition', 'distance']] # python nested list
n = [[distance_name], ['Country', 'pop', 'gdp']] # python nested list
while len(coalitions)>1:
    min_distance = default_dummy_distance
    first_coalition = None
    second_coalition = None
    for i, coalition_i in enumerate(coalitions):
        for j, coalition_j in enumerate(coalitions):
            if i >= j:
                continue
            if distance(coalition_i, coalition_j) < min_distance:
                :
                min_distance = distance(coalition_i, coalition_j)
            )
            first_coalition = coalition_i
            second_coalition = coalition_j

    out_f.write(first_coalition.name)
    out_f.write(" - ")
    out_f.write(second_coalition.name)
    out_f.write("{:.2f}".format(distance(first_coalition,
        second_coalition)))
```

```
out_f.write("\n")
coalitions.append(merge(first_coalition, second_coalition))
coalitions.remove(first_coalition)
coalitions.remove(second_coalition)
m.append([first_coalition.name, second_coalition.name, "
        {:.2f}".format(distance(first_coalition, second_coalition
        ))])
t = matrix2latex(m)
out_f.write(t)
```

Computer Codes

Country Data:

```
# input_europe.txt
# Name, lat, lng, pop, gdp, regime
Albania, 41.3275459, 19.8186982, 1227.0, 1229.0, 0
Austria, 48.2081743, 16.3738189, 6935.0, 25702.0, 1
Belgium, 50.8503396, 4.3517103, 8639.0, 47190.0, 1
Bulgaria, 42.6977082, 23.3218675, 7251.0, 11971.0, 0
Czechoslovakia, 50.0755381, 14.4378005, 12389.0, 43368.0 ,0
Denmark, 55.6760968, 12.5683371, 4271.0, 29654.0, 1
Finland, 60.17332440000001, 24.9410248, 4009.0, 17051.0, 1
France, 48.856614, 2.3522219,42518.0, 220492.0, 1
East Germany, 52.5200066, 13.404954, 18388.0, 51412.0, 0
West Germany, 50.73743, 7.0982068, 50958.0, 213942.0, 1
Greece, 37.983917, 23.7293599, 7566.0, 14489.0, 1
Hungary, 47.497912, 19.040235, 9338.0, 23158.0, 0
Iceland, 64.133333, -21.933333, 143.0, 762.0, 1
Ireland, 53.3498053, -6.2603097, 2963.0, 10231.0, 1
Italy, 41.8723889, 12.4801802, 47105.0, 164957.0, 1
Liechtenstein, 47.14137, 9.5207, 14.0, 159.0, 1
Luxembourg, 49.815273, 6.129583, 296.0, 2481.0, 1
Monaco, 43.73841760000001, 7.4246158, 18.0, 158.0, 1
Netherlands, 52.3702157, 4.895167900000001, 10114.0, 60642.0, 1
Norway, 59.9138688, 10.7522454, 3265.0, 17728.0, 1
Poland, 52.2296756, 21.0122287, 24824.0, 60742.0, 0
Portugal, 38.7222524, -9.1393366, 8443.0, 17615.0, 1
Romania, 44.4325, 26.103889, 16311.0, 19279.0, 0
Soviet Union, 55.755826, 37.6173, 179571.0, 510243.0, 0
Spain, 40.4167754, -3.7037902, 28063.0,61429.0, 2
Sweden, 59.3293235, 18.0685808, 7014.0, 47478.0, 1
Switzerland, 46.9479222, 7.4446085, 4694.0, 42545.0, 1
United Kingdom, 51.5073509, -0.1277583, 50127.0, 347850.0, 1
Yugoslavia, 44.816667, 20.466667, 16298.0, 25277.0, 0
```

Shell Program:

A.1 Sequential Coalition Formation

```
#!/run_all.sh -- in the console
python program.py distloc
python program.py distpop
python program.py distgdp
python program.py distpercapitagdp
python program.py distmaxminpercapitaratio
```

A.2 Simultaneous Coalition Formation

```
import copy
import sys
import time
import itertools as itt
import numpy as np
from geographiclib.geodesic import Geodesic
from scipy.spatial import distance
```

```
class coalition:
    def __init__(self,
                 name,
                 lat,
                 lng,
                 pop,
                 gdp,
                 regime):
        self.name = name
        self.lat = lat
        self.lng = lng
        self.pop = pop
        self.gdp = gdp
        self.regime = regime
    # End of constructor
```

```
# Read text file
```


A.2 Simultaneous Coalition Formation

```
in_f = open("input_western_europe.txt", "r")

coalitions = []

for line in in_f:

    # Allow comments in the text file
    if "#" in line:
        continue

    # split by "," and remove extra whitespace
    [name, lat, lng, pop, gdp, regime] = [x.strip() for x in
        line.split(",")]

    # Create coalition object and add to list
    coalitions.append(coalition(name,
                                float(lat),
                                float(lng),
                                float(pop),
                                float(gdp),
                                int (regime)))

# End of reading coalitions from file

i=0
for country in coalitions:
    print i, country.name
    i=i+1

# Calculate Partition
```

Computer Codes

```
def all_partitions(l):
    try:
        newitem = l[0],
        except IndexError:
    yield ()
    else:
    for part in all_partitions(l[1:]):
        yield (newitem,) + part
        for i, cluster in enumerate(part):
            yield part[:i] + (newitem + cluster,) + part[i+1:]

def powerset(s):
    s = tuple(s)
    return itt.chain.from_iterable(itt.combinations(s, i)
                                   for i in range(1, len(s)+1))

def phi_for_cluster(indices, dist):
    mask = np.zeros(dist.shape[0], bool)
    mask[list(indices)] = True
    dist_part = dist[mask, :]
    return dist_part[:,mask].sum() + (1 - dist_part[:,~mask]).
        sum()

def best_partition(dist):
    num_countries = dist.shape[0]
    countries = tuple(range(num_countries))
    cluster_list = np.array([set(indices) for indices
                              in powerset(countries)])
    phic = {tuple(sorted(cluster)): phi_for_cluster(cluster,
        dist)
    for cluster in cluster_list}
```

A.2 Simultaneous Coalition Formation

```
phi = lambda p: sum(philc[cluster] for cluster in p)
best = min(all_partitions(countries), key=phi)
return best, phi(best)

def distloc(A, B):
    distloc = Geodesic.WGS84.Inverse(A.lat, A.lng, B.lat, B.lng)
    return (distloc['s12']/1000)

def max_geodist(l):
    max_geodist = 0
    for i in coalitions:
        for j in coalitions:
            if distloc(i,j) > max_geodist:
                max_geodist = distloc(i,j)
    return max_geodist

print "max geodist", max_geodist(coalitions)

def max_popdist(l):
    max_popdist = 0
    for i in coalitions:
        for j in coalitions:
            if distloc(i,j)*abs(i.pop-j.pop) > max_popdist:
                max_popdist = distloc(i,j)*abs(i.pop-j.pop)
    return max_popdist

print "max popdist", max_popdist(coalitions)

def max_gdpdist(l):
    max_gdpdist = 0
```

Computer Codes

```
    for i in coalitions:
for j in coalitions:
    if distloc(i,j)*abs(i.gdp-j.gdp) > max_gdpdist:
        max_gdpdist = distloc(i,j)*abs(i.gdp-j.gdp)
return max_gdpdist

print "max geodist", max_gdpdist(coalitions)

def max_gdppercapitadist(l):
    max_gdppercapitadist = 0
    for i in coalitions:
for j in coalitions:
    if distloc(i,j)*abs((i.gdp/i.pop)-(j.gdp/j.pop)) >
        max_gdppercapitadist:
            max_gdppercapitadist = distloc(i,j)*abs(i.gdp/i.pop
                -j.gdp/j.pop)
return max_gdppercapitadist

print "max gdpdist", max_gdppercapitadist(coalitions)

def max_maxmingdppercapitadist(l):
    max_maxmingdppercapitadist = 0
    for i in coalitions:
for j in coalitions:
    if distloc(i,j)*max((i.gdp/i.pop),(j.gdp/j.pop))/min((i.gdp
        /i.pop),(j.gdp/j.pop)) > max_maxmingdppercapitadist:
            max_maxmingdppercapitadist = distloc(i,j)*max((i.
                gdp/i.pop),(j.gdp/j.pop))/min((i.gdp/i.pop),(j.
                gdp/j.pop))
```

A.2 Simultaneous Coalition Formation

```
    return max_maxmingdppercapitadist

print "max maxmindist", max_maxmingdppercapitadist(coalitions)

geodist = []

#normalized distance
geodist = [[round(distloc(i,j)/max_geodist(coalitions),2) for i
            in coalitions] for j in coalitions]

geodist = np.array(geodist)

print "geodist matrix"
print geodist

best, phi = best_partition(geodist)

print "Best partition:", best
print "Score:", phi

popdist = []

#normalized distance
popdist = [[round((abs(i.pop-j.pop)*distloc(i,j))/max_popdist(
            coalitions),2) for i in coalitions] for j in coalitions]

popdist = np.array(popdist)

print "popdist matrix"
print popdist
```

Computer Codes

```
best, phi = best_partition(popdist)

print "Best partition:", best
print "Score:", phi

gdpdist = []

#normalized distance
gdpdist = [[round((abs(i.gdp-j.gdp)*distloc(i,j))/max_gdpdist(
    coalitions),2) for i in coalitions] for j in coalitions]

gdpdist = np.array(gdpdist)

print "gdpdist matrix"
print gdpdist

best, phi = best_partition(gdpdist)

print "Best partition:", best
print "Score:", phi

gdppercapitadist = []

#normalized distance
gdppercapitadist = [[round((abs(i.gdp/i.pop-j.gdp/j.pop)*distloc
    (i,j))/max_gdppercapitadist(coalitions),2) for i in
    coalitions] for j in coalitions]
```

```
gdppercapitadist = np.array(gdppercapitadist)

print "gdppercapita matrix"
print gdppercapitadist

best, phi = best_partition(gdppercapitadist)

print "Best partition:", best
print "Score:", phi

maxmingdppercapitadist = []

#normalized distance
maxmingdppercapitadist = [[round(distloc(i,j)*max((i.gdp/i.pop)
,(j.gdp/j.pop))/min((i.gdp/i.pop),(j.gdp/j.pop)))/
max_maxmingdppercapitadist(coalitions),2) for i in coalitions
] for j in coalitions]

maxmingdppercapitadist = np.array(maxmingdppercapitadist)

print "maxmingdppercapitadist matrix"
print maxmingdppercapitadist

best, phi = best_partition(maxmingdppercapitadist)

print "Best partition:", best
print "Score:", phi
```


Appendix B

CV

MAG. GIZEM YILDIRIM

PERSONAL DETAILS

Date of Birth: February 1, 1985 Address: Hohlweggasse 10/10, Vienna

Place of Birth: Ankara, Turkey Email: gizem.yildirim@student.tuwien.ac.at

EDUCATION

- present** Doctoral Program in Social and Economic Sciences,
Vienna University of Technology, Austria
- Title: **The Dynamics of Europe's Political Economy:
a game theoretical analysis**
 - Advisors: Professor Hardy Hanappi, Professor Pasquale Tridico
- 2012** Master Program in Economics, University of Vienna, Austria
- Title: **Game Theoretical Models of Revolution and Democratization**
 - Advisor: Professor Gerhard Sorger
- 2009** German Courses, University of Vienna, Austria
- 2008** B.Sc Mathematics and Computer Science, Cankaya University, Turkey
- 2008** Bachelor Minor Program in Economics, Cankaya University, Turkey
- 2006** Erasmus Program in Mathematics, Vienna University of Technology, Austria

REFERENCES

- Prof. Hardy Hanappi** Department of Economics, Vienna University of Technology
Phone: +43 1 58801 10535 Email: hanappi@tuwien.ac.at
- Prof. Pasquale Tridico** Department of Economics, University Roma Tre
Phone: +39 339 2794393 Email: tridico@uniroma3.it
- Dr. Elisabeth Beckmann** Economic Studies Division, Oesterreichische Nationalbank
Phone: +43 1 404 205252 Email: Elisabeth.Beckmann@oenb.at

EXPERIENCE

- 2014/present** Oesterreichische Nationalbank, Foreign Research Division
- Data processing and geocoding of the OeNB Euro Survey data using Stata, Python and Google Maps API in various projects
- May/Dec 2016** Oesterreichische Nationalbank, Economic Studies Division
Project “*Payment behavior in Austria*”
- Data processing and validation Statistical analysis using Stata and reporting Presentation of tables and graphs
- Oct 2007/Feb 2008** Mathematics Laboratory, Cankaya University
- Jun/Oct 2005** Work and Travel Program in Panama City, Florida, USA

SCHOLARSHIPS

- 2003–2008 Scholarship of Cankaya University
Success in university entrance examination

SKILLS

- Programming** C++, Java, Python
- Applications** Office, L^AT_EX, Stata, E-views, Gauss, Mathematica
- Operating Systems** Linux, Microsoft Windows
- Languages** Turkish (Mother Tongue), English (Fluent), German (Intermediate)

Vienna, 1 May 2017