General Management MBA

# A game theory model on authority delegation

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# Abstract


This thesis develops a model for allocation of effort in principal-agent relations for two special situations. In the first case the principal hired an agent but is not sure about the skillfulness of the agent. In the second case the principal is risk averse. We illuminate how the different situations influence the amount of control of the principal and the initiative of the agent.

Companies significantly depend on the output and therefore effort of their employees. Initiative of their employees is the force that drives their success. The supervision of employees and the resulting effort the agents will put into fulfilling their tasks are described by a game theoretical principal-agent model. If we take for instance a research and development department, the challenge of the company's corporate management is to balance the control over the output of the agents and on the other hand leave the agent enough room for their decisions. Too much surveillance can jeopardize the motivation of the employee to create valuable results.

We base our investigation on the game theoretical model of the principal-agent relation from Aghion and Tirole (Aghion & Tirole, 1997). In their paper they developed a theory of allocation of authority where they could show the balance of loss of control of the principal versus initiative of the agent. Starting from their viewpoint we modify the model and add additional parameters to see how this tradeoff of supervision of principal and initiative of the agent changes.

Two situations are analyzed in detail:

1) The ability of the agent is not known

Even though a company may have a very sophisticated recruiting process, the project manager is never 100% sure about the abilities of her project members. Some members are perfectly suited for the given task, but others may, for whatever reason, be "at the wrong place".  The project manager (principal) has only a probability that her agent is skilled. She has to choose the optimal allocation of supervision depending on the possible types of the agent.

2) The principal is risk averse

The ability of the agent is known, but the principal is risk averse with regard to her net benefit. We will see how her cautious behavior influences the effort of the agent.

Expectation and behavior of the principal directly influences the agent who reacts on the given situation. We investigate the mechanism how the agent responds and describe a way to avoid demotivation. In the last chapter, we present three cases where companies were able to raise the motivation of their employees and at the same time raise the success of the company.

Key words: Principal – Agent Theory, game theory, ability of agent, risk averse, cost of effort, disutility, congruence of preferences, authority

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## Executive Summary

This thesis focuses on the delegation problem in the principal-agent interaction. It deals with the tradeoff between a principal's supervision effort and the de/motivation of the agent. A principal hires an agent to fulfill a certain task. The interests and motivations of the agent do not completely match with the one of the principal. Therefore the principal is never sure if the agent acts in her best interest.

In particular the effort the agent puts into a task is unobservable or non-contractable. Given all the information, the principal can anticipate what agent will do but she never can measure it. The fulfillments of complex jobs depend on the initiative of the agent that depends on the behavior of the principal. Two specific aspects of the principal agent problem will be illuminated within the game theoretical framework introduced by Aghion and Tirole (Aghion & Tirole, 1997). We will have a closer look on the influence of the ability or skillfulness of an agent and the risk averseness of the principal.

First an introduction of the basic game theoretical model of Aghion and Tirole is given. It is an incentive model that shows well the tradeoff between supervising effort from the principal and the effort invested by the agent. The principal overrules the agent if she found out her best solution due to her supervising effort. It lowers the initiative of the agent to be overruled too often, thus he invests less effort. The mechanisms of the model are visualized to get a feeling for the parameters used.

Subsequently the framework for two extensions of the model is developed. One important characteristic in the way a principal supervises an agent is the ability of the agent to fulfill the given task. Frequently the principal is not sure about the agent's skillfulness. The agent could be the wrong person in that situation, so there is just a probability that the agent is skilled. This situation is modeled as a Bayesian game, where the principal has incomplete information about the agent's skills. The question is: How does the effort of the principal and the agent invested in fulfilling the task change when the ability of the agent for this task is not known to the principal with certainty? We find that the principal will raise her effort in supervision compared to the model with complete information about the ability. Due to the higher supervision the performance (effort) of the skilled agent declines. He gets demotivated from being overruled by the principal.

Next the original model is modified in a way that the principal is risk averse with regard to her net payoff. The utility function of the principal was chosen to be the natural logarithm to reflect risk averseness. The utility function of the agent is the same as in the original model. The system of reaction curves is solved graphically, as the reaction curve of the principal cannot be expressed in explicit form regarding her effort. We find that the agent puts more effort into his work for a risk neutral principal than for a risk averse principal. This is due to the fact that the risk averse principal supervises more.

Both situations had a negative impact, as the effort of the principal rises and at the same time the effort of the agent declines. As a cautious conclusion, for companies which depend on motivating their top performers (skilled agents) as much as possible, we favor the strategy that the principal commits herself to lower her supervising effort.

As our mathematical model is very limited in its complexity, we need to emphasize that lowering the supervision effort is only one measure embedded in a set of corporate rules.

Nevertheless, when combining with other measures, it turns out to be a very powerful tool for successful business. Three examples were shown. Google has drastically reduced supervision and gives their researchers the freedom to use a percentage of their time to pursue whatever they want. They publish hundreds of research papers each year and bring most innovative products to the market. The next example illuminates that even in the world of Formula 1 where the pressure is extreme it makes sense to interfere not too much with the car-developers. And last but not least, we introduce Semco, a Brazilian company which got around bankruptcy by totally changing their hierarchy model. The employees can decide about their working hours, they have access to the company books and the majority vote on many important corporate decisions.

# 1 Introduction

In this thesis two specific aspects of the principal agent problem will be illuminated within the game theoretical framework introduced by Aghion and Tirole (Aghion & Tirole, 1997). We will have a closer look on the influence of the ability or skillfulness of an agent and the risk averseness of the principal.

The first chapter gives the motivation, an overview and the formulation of the research question. The second chapter gives a short summary of the mathematical tools that game theory provides for this kind of strategic interactions. The third chapter presents the model described by Aghion and Tirole. Additionally we introduce a quadratic form of the cost of effort (disutility) instead of the original one. In the fourth chapter we implement two new aspects to the model: first, the ability of the agent can be skilled or inept; second the principal is risk averse. The fifth chapter gives a discussion on the results and in the sixth chapter summarizes the thesis.

## 1.1 Principal Agent problem

The principal-agent theory models the strategies of two subjects within a hierarchy. The principal hires an agent to fulfill a certain task. The agent is then authorized to make certain decisions on behalf of the principal to complete the task. In general, the aims and motivation of the agent do not totally match with the needs of the principal. The agent is tempted to act in his own interest. Therefore the interaction between the principal and her agent shows lots of costly frictions.

Examples for this kind of relationship are:

- company – employee
- shareholder – corporate management
- trading company – sales man
- house owner – plumber

and others.

In these systems, the agent can make decisions in the name of the principal. Not necessarily his decisions are ideal for the principal. The deviation to the optimal benefit of the principal is called "**agency costs**". It is in the principal's interest to minimize agency costs.

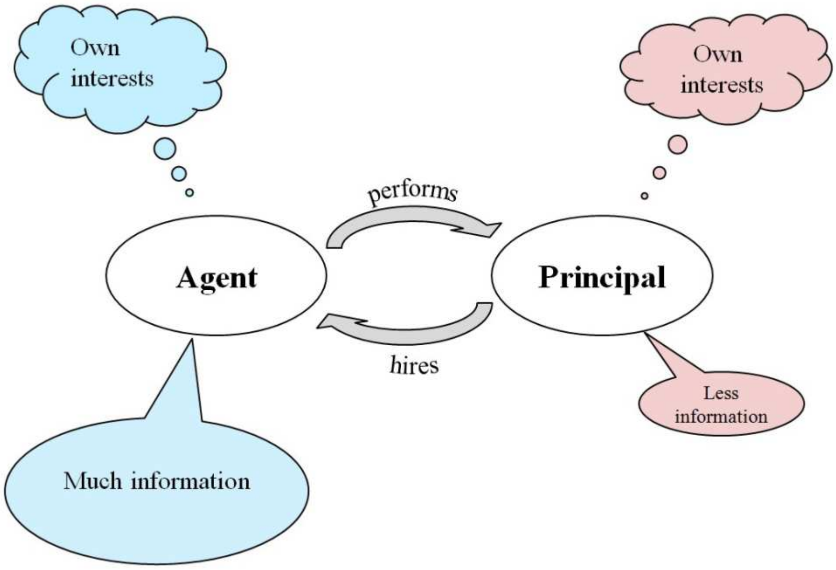



**Fig. 1 Visualization of Principal – Agent interaction. Both parties have their own interest. In most situations there is a dissymmetry of information. adapted from (Wikipedia, 2016)**

One of the major sources of agency costs is an asymmetry in information (Fig. 1). Usually the agent is better informed than the principal. For instance the corporate management directly runs the company and therefore knows the situation much better than the shareholders. The shareholders cannot really judge the situation inside the company. And inside the employee is better informed about the part of the project that he is working on than his boss.

We will investigate one of the hidden characteristics of the agent, more specifically the ability or skillfulness of the agent. When the principal and the agent meet the first time, it might not be clear to the principal if the new agent is suited for his job or not.

We will further investigate how the players change their behavior when the principal is risk averse. The property we focus on is the effort. Effort is not directly observable to the principal. She can expect a certain effort, but she cannot write a contract for its delivery.

## 1.2 Motivation and research question

**Initial situation and problem statement**

The interaction between a principal and her agent has been widely investigated in literature. It is of great economic interest for the principal to optimize her benefit and avoid too many costly frictions called "agency costs" when hiring an agent.
One of the most recognized approaches is the model of Aghion and Tirole (Aghion & Tirole, 1997). It shows a very intuitive relation between balancing
*"loss of control" of the principal* versus *"initiative" of the agent*
and is therefore well suited as a starting point for further investigations.

The issues to be addressed are:

- Demonstrate the mechanisms involved in the principal agent interaction using the game theoretical model of Aghion and Tirole as basis and extend them.
- Develop the awareness that the principal's expectation of the agent's ability and the principal's risk attitude influences the effort of the agent, and thereby his success at work.
- Present the results in which direction the effort of the players changes when the parameters of the model or the model itself is modified.

**Objects of investigation**

In this work we investigate the influence of

- not knowing the **ability** of the agent and
- what happens when the principal is **risk averse**

We further introduce a quadratic cost of effort (disutility) function instead of the original disutility by Aghion and Tirole, in order to be able to visualize the effects. The aim is to get a better feeling for the model.

**Research questions**

This work frames two concrete problems within the principal agent theory:

1. In the original model of Aghion and Tirole the principal is sure that the agent is skilled. Now we ask: How does the effort of the principal and the agent invested in finding relevant information change when the ability of the agent for this task is not known to the principal with certainty?
2. How is the principal agent interaction change when the principal is risk averse instead of risk neutral?

## 1.3 Game theory as the research method

This thesis uses GAME THEORY for the investigation of the principal-agent interactions.

The results of the work are based on a theoretical model. Mathematical methods are used to get results describing the reaction of a principal to different agent-behavior and vice versa.

Proceeding from a well-established model (Aghion & Tirole, 1997), additional scenarios are added to further refine the mechanisms.

**Why do we use game theory for the analysis of the relations between the principal and the agent?**

The principal and the agent act in a strategic way. Both of them act according some utility and will then try to develop a strategy to get the best out of the situation taking into account the other person’s strategy.

# 2 General summary of game theory

Game theory helps to understand the outcome of the strategic interaction of two or more individuals. It provides a logical analysis how players should rationally make their decisions. Mathematical tools are used to model the strategic situations.

The players act in a certain environment with given rules and their preferences of outcomes. The strategy one player chooses defines his/her own and the other players outcome and vice versa. The players may be in a certain conflict, as different

players may value different outcomes differently. This can be easily seen in a zero-sum game where the gain of one player is the loss of the other.
Game theory assumes that the individuals act in their own interest when facing different choices. During the process of analyzing the game we are forced to step into the shoes of both, the principal AND the agent. (Polak, 2007)

**Examples of applications of game theory**

Amongst the numerous application fields of game theory there are economic applications, military strategy, political sciences, biology and others.
Game theory is widely used in economics. This is very natural, as we normally consider more dollars to be better than fewer dollars, which means we optimize our payoff in dollars.
Therefore the mathematical tools are well suited for the investigation of challenges in economics.

## 2.1 Ingredients of Game Theory

"*Game theory is the name given to the methodology of using mathematical tools to model and analyze situations of interactive decisions. These are situations involving several decision makers (called players) with different goals.*" (Maschler, Solan, & Zamir, 2013, p. xxiii)

According to (Straffin, 1993, p. 3) a game is a situation in which

1. There are at least two **players** (individual, company,…)
2. Each player has a number of possible **strategies**
3. The strategies chosen by the players determine the **outcome** of the game
4. Associated to each possible outcome is a collection of numerical **payoffs**, one to each player. These outcomes represent the value of the outcome to the different players.

### 2.1.1 Utility

(Straffin, 1993, pp. 49-55), (Maschler, Solan, & Zamir, 2013, pp. 9-30) The starting point for the analysis is to know “Who can gain/lose how much?”.

Definition A **utility function** u is a function associating each outcome *x* with a real number *u(x)* in such a way, that the more an outcome is preferred, the larger is the real number associated with it.

How is the utility assigned?
(Straffin, 1993, pp. 49-55), For being able to put sentences like:”I like tea more than coffee.” into a mathematical form, and to draw conclusions using mathematical analysis, we need to find a procedure to handle that information. And that is not very obvious, if we look at the above sentence.

1. Preference relation

In the first step we need to setup a preference relation between the possible outcomes of the game.
For instance we can state: “I like 100€ more than 80€ and that more than 50€” and so on.
Using the ranking operator ≻ we write:

$$100€ \succ 80€ \succ 50€ \quad (1)$$

2. Ways of ranking

Any ranking puts more value to more desirable outcome and less value to less desirable outcome. Additional to that, we would like to make statements how much more desirable the result is.

**Ordinal utility**
An ordinal scale tells us what is preferred but not how much it is preferred.
tea ≻ coffee

makes not clear if one would answer:

"Ok, if you don't have tea I will take coffee." or

"No thanks, I will get serious heart problems, if I drink coffee."

To overcome this shortcoming we use the

**Cardinal utility**

Using cardinal utility the difference between two utilities has an interpretation in a similar way as:

"I like tea twice as much as coffee."

A scale, where not only the order, but also the ratio of the differences between two numbers is meaningful is called **interval scale**. Numbers reflecting preferences on an interval scale are called cardinal utilities.

The cardinal utility is the basis to conduct analysis over it and is used in further investigations.

3. Difficulties in finding a utility

3.1. No Completeness

"Do you prefer a bus ticket or a glass of water?"

You might say: "It depends. I cannot compare these two outcomes".

Hence the act of ranking is meaningless. The outcomes are so different, that it is not possible to compare them. Thus the ranking is not complete.

3.2. No transitivity

Somebody might rank tea $\succ$ coffee and coffee $\succ$ milk

but he might also state: milk $\succ$ tea

No ordering would be possible. It is not transitive.

3.3. Reflexivity

Every outcome is weakly preferred to itself. tea $\succcurlyeq$ tea. Thus, I like tea as much as tea.

To build a meaningful utility function we shall make sure that we avoid inconsistencies:

**If we manage to setup a cardinal scale of outcomes that is complete, reflexive and transitive, we can call this ranking a utility function!**

4. How do we obtain the utility function?

Now we know what properties the utility function should have and shall find a way to construct the utilities.

Following the procedure described in (Straffin, 1993, pp. 49-55), we ask our player questions about **lotteries.**

First the player tells us that the outcome is ranked $x \succ y \succ z$

We want to put *y* at the right place to construct a cardinal utility.

We ask:

- "Would you prefer *y* for certain or
- a lottery that gives you *x* with probability ½ and *z* with probability ½ ?"

If the player would be indifferent between the sure outcome and the lottery, we would place *y* exactly in the middle of *x* and *z*. If not, we must ask other proportionalities.

5. Remarks on properties of the utility function

We cannot assign absolute values to utility functions. And we cannot compare the utility 50 of player A with the utility 100 of player B and say "B likes the outcome twice as much as A".

This is not possible, as we can always perform a **linear transformation** between two utility functions *u(x)* and *v(x)* of one player.

$$v(x) = au(x) + b \quad (2)$$

6. Utility and payoff

Throughout this work we will use the word utility and the word payoff with a different meaning.
We say, that payoff is “What you get on your hand”, e.g. in € or $.
The payoff will be a parameter of the utility function and might have a different cardinal ranking, as the proportion of differences in payoffs may not be linear.
In the easiest case the relation is linear: You value 10€ twice as much as 5€. But when we come to “risk”, we will modify this relation.

7. Rationality

We try to predict the outcome of the game, given the players decide in a rational way.
**Rationality** implies that every player is driven by maximizing his/her own utility.

### 2.1.2 How to take risk into account?

(Umbhauer, 2016, pp. 47-49), (Maschler, Solan, & Zamir, 2013, pp. 23-26) Human beings behave differently when exposed to risk. Risk can concern monetary benefits, quality of a product or service, stock investments and others.
Game theory uses the following classification for the human’s attitude towards risk:

- Risk seeking
- Risk neutral
- Risk averse

In chapter 2.1.1 sub point “4 How do we obtain the utility function?”, we introduced a way to construct the utility function using a lottery. We asked the question:”What do you prefer? A certain outcome for sure, or the lottery?” Now we elaborate this concept further to introduce the notion of risk.

Suppose we asked the question: “Do you prefer 50€ for sure or a lottery where a coin is flipped and you get either 100€ or nothing?”
We can look at three different answers:

- “I prefer the lottery, because I am interested in the 100€”

- “I am indifferent”
- “I would even prefer 40€ for sure than the lottery”

In the following we will classify the person who prefers the lottery as risk seeking, the one who is indifferent as risk neutral and the person who will take the sure payoff as risk averse.

Of course the answer depends on the situation, on the individual and can also change over time.

**Risk neutral**

Maximizing the expected value of your payoff is equivalent to maximizing your expected utility for your payoff exactly when your utility function is linear.

A risk neutral party is indifferent between choices with equal expected payoffs shall it be for sure or the lottery. So there is a **linear relation** between expected payoff and utility.

example:

When the payoffs are ranked in the following way $100€ \succ 80€ \succ 50€$ and the utility function is $u(x) = 2.58\,x$ you are risk neutral. The constant 2.58 has no meaning as we are allowed to perform a linear transformation to $\mathrm{u(x) = x}$ if we want. ( see chapter 2.1.1 point 5 Remarks on properties of the utility function)

**Risk aversion**

Taking the lottery of flipping the coin and gain 100€ or nothing means being exposed to uncertainty. To avoid this uncertainty, a risk averse person is rather willing to take a payoff of $x$ = 40€ for sure.

For a risk averse person, his feeling of what utility he gets does not rise linearly with $u(x) = k\ x$. The extra he gains by winning the lottery is valued less than the sure payoff. On the other side, losing everything in the coin flipping game really makes the person unhappy.

Risk aversion is synonymous to a diminishing marginal utility. The more I get, the less valuable it is for me, but the more I loose, the more I am unhappy.

In economy, risk averse investors will choose the safer options more likely than risking to lose all their money. A risk averse investor requires a higher marginal reward to accept additional risk.

People care more about the sure lower payoff they get than a high payoff with less extra advantage. (Geanakoplos, 2009)

A risk averse utility function is concave with respect to the expected payoff. It lowers the marginal utility with rising payoff.

**Risk seeking**

The risk seeker has a convex utility. She values the high payoffs in the gambling more than the moderate payoff of the risk neutral or risk averse person. Her marginal utility rises with the higher payoff.

The following graph Fig. 2 shows the dependence of the utility on the payoff the person gets for the different character of players.

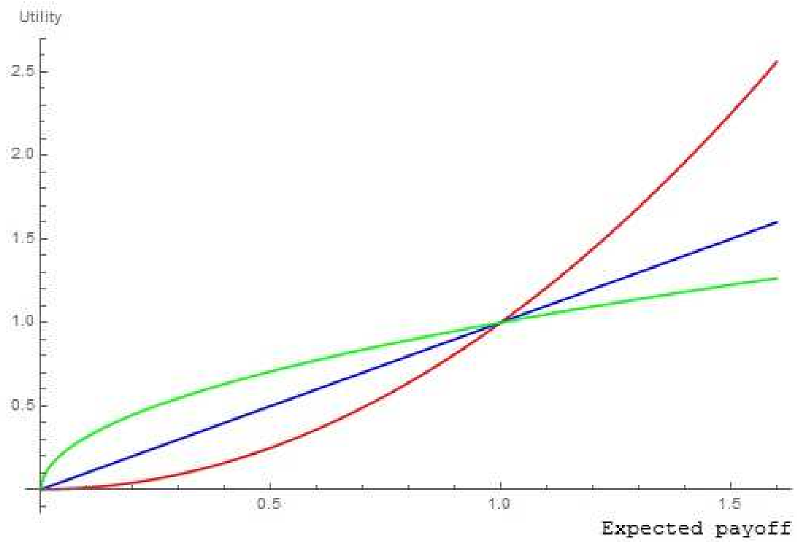



**Fig. 2 Green line: risk averse, blue line risk neutral, red line: risk seeking**

We see in Fig. 2 that the risk seeking person (red) is not so happy with low payoffs and feels a higher degree of marginal utility when the payoff becomes big, whereas the risk averse person is already happy with smaller payoffs.

### 2.1.3 Nash equilibrium

(Polak, 2007), (Maschler, Solan, & Zamir, 2013, pp. 95-97), (Vassili N Kolokoltsov, 2010, pp. 16-21) What strategy should we use, once we know the utilities of all players? Game theory recommends a rational approach.

Every player tries to maximize his/her utility given the rules of the game and the decisions of the other players, i.e. his/her own and the other's strategies.

After knowing ones own utilities one must now take into account the payoffs/utilities of the other players.

The thinking steps for this approach are:

1. First we figure out what would be the best response of player A to each possible strategy of player B.

All of these best responses together are called reaction function or reaction curve in the continuous case.

The reaction function can be regarded as a list: "If player A would play strategy $s_1$, the best answer for player B is written as $bA(s_1)$" and so on for all possible strategies.

2. Next, we determine the reaction curve for all players.

3. If there is an intersection of the reaction curves, it is called Nash equilibrium. There, the players are playing a best responses to each other.

We assume that rational players will play the Nash equilibrium, because there is no strategy that a player could play that would yield a higher utility, given the strategies of the other players.

It is notable that the Nash equilibrium does not provide the highest payoff one can get at all, but only the best one can get taking into account the best responses of the other players, given that one knows all their equilibrium strategies.

### 2.1.4 Example: Cournot duopoly competition

Game theory uses a wide spectrum of mathematical methods and throughout this work we need only a specific part out of it suited to our class of problems.
We introduce the methods used in this work by presenting one classical example that is representative for this kind of modeling: The Cournot duopoly (following Maschler, Solan, & Zamir, 2013, pp. 99-100)

The Cournot duopoly model describes market behavior. The topic itself is different from the *principal –agent* issue. It is presented here as the most well known role model of games using the same mathematical tools as the model that we use in later chapters.

In the Cournot duopoly competition, two manufacturers produce the same good for the market. The customers are completely indifferent buying from manufacturer A or B.
The selling price is also identical.

The production costs are:
$c_A$ for manufacturer A and
$c_B$ for manufacturer B.

The total demand of the product in the market is

$$q_A + q_B \tag{3}$$

where $q_A$ and $q_B$ are the quantities produced from A and B.

The price gets lower the higher the quantities produced. Assume that the selling price of each item is $2 - \mathrm{q_A} - \mathrm{q_B}$.

Each player can choose to produce between zero and an infinite number of items. So the strategy set for each player is: $s(q_i) \in [0,\infty)$.

## 1. Deriving the utility function

We consider a risk neutral firm, so the utility is proportional to the payoff.

If A chooses to play strategy $q_A$ and B chooses to play (produce) $q_B$, the payoffs are:

$$payoff = price - cost = u_{A/B}(q_A, q_B) \quad (4)$$

$$u_A(q_A, q_B) = q_A(2 - q_A - q_B) - q_A * c_A = q_A(2 - q_A - q_B - c_A) \quad (5)$$

$$u_B(q_A, q_B) = q_B(2 - q_A - q_B) - q_B * c_B = q_B(2 - q_A - q_B - c_B) \quad (6)$$

$u_A(q_A, q_B)$ is called the utility function of player A.
$u_B(q_A, q_B)$ is called the utility function of player B.

In this game the utility is directly given by the profit in $ or €. Not in all games, the utility has the dimension of a currency, but here it is very convenient.

We summarize the ingredients in the following table:

| **Initial Situation** | **Manufacturer A** | **Manufacturer B** |
|---|---|---|
| **cost** of production for each item | $c_A > 0$ | $c_B > 0$ |
| **quantity** to produce | $q_A$ | $q_B$ |
| selling **price** | $2 - q_A - q_B$ | |
| In this model $q_A + q_B < 2$ to obtain a positive selling price | | |
| total **demand** of the product in the market | $q_A + q_B$ | |
| price - cost = **payoff** for each player | $u_A = q_A(2 - q_A - q_B - c_A)$ | $u_B = q_B(2 - q_A - q_B - c_B)$ |

| | | |
|---|---|---|
| (= utility for risk neutral firm) | | |
| In this model $q_A + q_B - c_{A/B} < 2$ to obtain a positive payoff | | |

**Table 1: Cournot duopoly**

## 2. Deriving the reaction curves

The reaction curve is the best answer one player can play to the possible strategies of the other player.

To find her best answer player A looks for the **maximum utility** that she can get if player B plays a certain strategy.
We therefore perform the derivation to find the maximums and set it to zero.

$$\frac{\partial u_A(q_A, q_B)}{\partial q_A} = 2 - 2q_A - q_B - c_A = 0 \quad (7)$$

$$\frac{\partial u_B(q_A, q_B)}{\partial q_B} = 2 - q_A - 2q_B - c_B = 0 \quad (8)$$

The second derivatives are both negative, so both are in a maximum.
reformulated:

$$q_A = \frac{2 - c_A - q_B}{2} \quad (9)$$

$$q_B = \frac{2 - c_B - q_A}{2} \quad (10)$$

Each player will adjust his/her production rate according to the strategy of the other player. The strategy of the other player is public knowledge. It is a game of perfect information. So each player will choose his production quantity where the two reaction curves cross.
This is the point of the **Nash equilibrium.** None of the players can improve his/her situation by deviating from that point.

## 3. Finding the Nash equilibrium

Solving the system of equations ( 9 )and ( 10 ) with respect to $q_A$ and $q_B$ leads to the Nash equilibrium.

$$q_A = \frac{2 - 2c_A - c_B}{3} \tag{11}$$

$$q_B = \frac{2 - 2c_B - c_A}{3} \tag{12}$$

To calculate the profit of each firm, we look at the payoffs in equation ( 5 )and ( 6 ):

$$u_A(q_A, q_B) = 2q_A - {q_A}^2 - q_A q_B - q_A c_A = \left(\frac{2 - 2c_A - c_B}{3}\right)^2 = {q_A}^2 \tag{13}$$

$$u_B(q_A, q_B) = 2q_B - {q_B}^2 - q_A q_B - q_B c_B = \left(\frac{2 - 2c_B - c_A}{3}\right)^2 = {q_B}^2 \tag{14}$$

## 4. Graphical solution of the Nash equilibrium

The Nash equilibrium is found where the two reaction curves cross

For different production costs, e.g. $c_A = 0.7 \text{ and } c_B = 0.5$ the plot looks like.

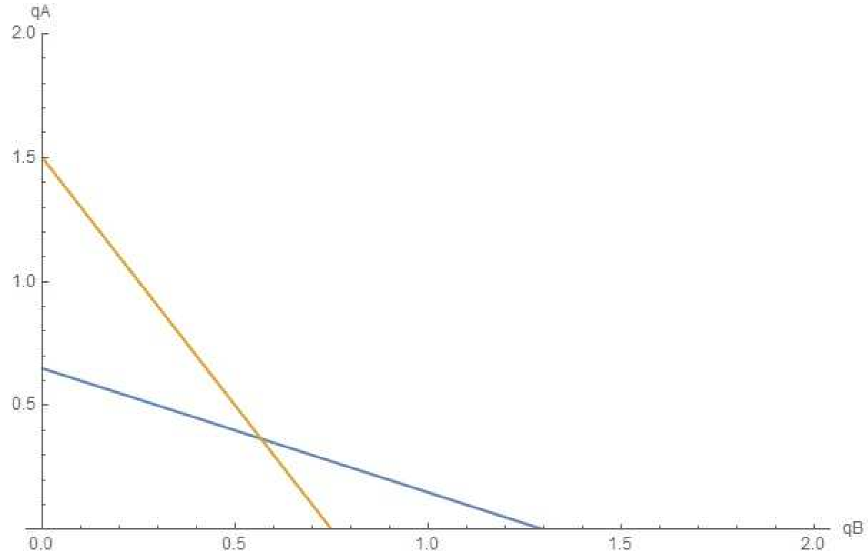



**Fig. 3 Graphical solution of the Cournot duopoly competition with cost of goods $c_A$ =0.7 and $c_B$ = 0.5**

The special case, where both companies have the same costs $c_A = c_B = 1$, the production rate will be 1/3 each.
The payoff of each player is $(1/3)^2$ = 1/9.

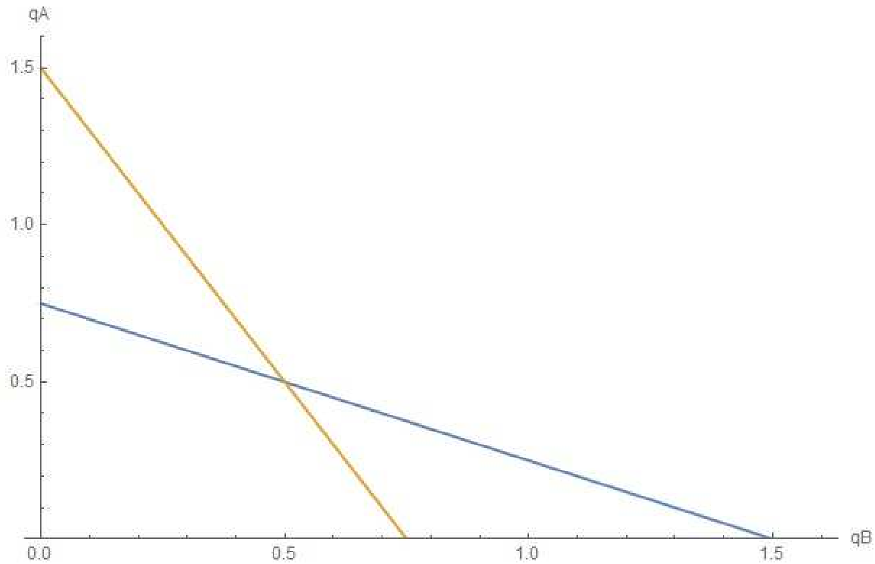



**Fig. 4 Graphical solution of the symmetric Cournot duopoly competition with cost of goods $c_A$ = $c_B$ = 1**

The Cournot model uses only easily quantifiable parameters like cost and quantity. Everything can be measured in Dollars, Euro or any currency and kilograms or pieces. All parameters are perfectly known to both players. Each player can guess very well the thoughts and strategy of the other player. A game where everything is known to everyone is called “**game with complete information**”.

Summarizing the steps:

1) Defining the utility functions
2) Calculating the reaction curves by setting the derivative of the utility function to zero
3) Finding the Nash equilibrium by solving the system of reaction curves

In the following we need to introduce another method used in game theory, as we will modify the model by introducing some uncertainty. Games involving an uncertainty for the players concerning which environment they face are called Bayesian Games.

### 2.1.5 Incomplete information - Bayesian Games

Bayesian games are games with incomplete information. The utility of player A depends on the type of player B. For instance, the utility of the boss depends on the skills of her employee, but she might not know if the employee is skilled or not. So she has **incomplete information**. Her utility is not fixed, but will occur with a certain probability.

If $P_i$ is the probability that the utility is $u_i$ , we can calculate the **expected value of the utility** $u(a_j)$in the following way:

$$u\left(a_1,..a_j\right) = \sum_{i=1}^{n} P_i u_i(a_1,...a_j) \qquad (15)$$

$a_j$ are the possible actions of the players. n is the number of different payoffs (Maschler, Solan, & Zamir, 2013, p. 353).

### 2.1.6 Example: Bayesian Cournot duopoly competition

(Maschler, Solan, & Zamir, 2013, pp. 358-361) In case of the Cournot duopoly competition of chapter 2.1.4 the production cost of the products were perfectly known to everybody. It was a game of complete information.

One implementation of a Bayesian game would be that the costs of player A is known to both players, but player A knows only that the costs $c_B$ is either $c_B^{high}$ or $c_B^{low}$ with probabilities $P^{high}$ and $P^{low}$ .
In that case the utility functions are modified taking into account the uncertainty that manufacturer A has about the production cost of her competitor.

As A does not know what quantity B will produce according to his production costs, equation ( 5 ) gets modified to:

$$u_A(q_A, q_B{}^{high}, q_B{}^{low}) = \\ = P_1 q_A(2 - q_A - q_B{}^{high}) + P_2 q_A(2 - q_A - q_B{}^{low}) - q_A * c_A \quad (16)$$

From A's point of view, there are two possible types of manufacturer B. In this method, both types are treated as they would be different players.
Each one has a different utility function depending on his type

$$u_B{}^{high}(q_A, q_B{}^{high}, q_B{}^{low}) = q_B{}^{high}(2 - q_A - q_B{}^{high}) - q_B{}^{high} * c_B{}^{high} \quad (17)$$

$$u_B{}^{low}(q_A, q_B{}^{high}, q_B{}^{low}) = q_B{}^{low}(2 - q_A - q_B{}^{low}) - q_B{}^{low} * c_B{}^{low} \quad (18)$$

There are now three reaction curves, due to the asymmetry of information. **Asymmetric information** means that player B knows something that player A does not know (his own production costs). Player A has therefore incomplete information.
From this point on, the calculation follows the same path as described for the game with fixed strategies, with the only difference, that now we have three utility functions, three reaction curves, and therefore we need to solve a system of three equations.

# 3 Presentation of the basic model from Aghion and Tirole

There have been several attempts to model the principal – agent interaction. One of the most recognized paper is the work from Aghion and Tirole (Aghion & Tirole, 1997) (more than 3.000 citations). We will take their approach as a starting model.

In this chapter, the ideas presented by Aghion and Tirole in their famous paper are briefly summarized. Then, in subchapter 3.4 we alter the original paper in a way to be able to visualize the utility functions and reaction curves. This allows for qualitative visual discussion.

**Initial situation**

A principal needs to find out which of a set of projects gives her the most benefit. She hires an agent to collect information which of the possible projects (or decision within a project) is the best. She also controls his result by supervision, so that she could overrule his decision if she knows the most favorable project for her.

Each potential project is associated with a certain benefit for each player.

**Payoffs:**

| | |
|---|---|
| $B_k$ [$] $\in \mathbb{R}$ | Net payoff/benefit that the principal gets. E.g. Benefit minus Price she pays; or her monetary gain or profit[1] |
| B [$] >0 | The principal's most profitable project, or the solution with the highest payoff for her. |
| $b_k$ $\in \mathbb{R}$ | Private benefit of the agent (psychological factors like appreciation or promotion). His wage is constant and can therefore be set to zero. |
| b [$] >0 | The project with best benefit for the agent. |
| $\alpha \in [0,1]$ | Congruence parameter seen from the principal's perspective. If the agent's most preferable decision is realized, she gets $\alpha B$ |
| $\beta \in [0,1]$ | Congruence parameter seen from the agent's point of view. If the principal's most preferable decision is realized, he gets $\beta b$ |

They both have a certain motivation, as there is at least one project for each of them that is a loss. If no project is implemented, the payoffs are all zero.

[1] Remark for the wording: We use "payoff" for B and b as defined in "point 6 Utility and payoff" on page 9. Alternatively we may use "benefit".

Before the game starts all possible projects look the same. This is similar to a brainstorming board where all possible solutions are written down, but without having started to analyze them. When the game starts the parties collect information to be able to make decisions.

Simplification No pre-knowledge or experience exists. That is why all projects look completely the same at the beginning.

Both, the principal and the agent acquire information to find her/his best project.

Simplification The information is digital, either 100% knowledge or none about the projects is acquired. It is not possible, that the person knows only the payoff of half of the projects.

The principal is risk neutral.

**Effort and cost of effort (disutility functions)**

| | |
|---|---|
| e<br>[0,1] | "e" can be seen as the effort the agents puts into the game, normalized to one,<br>alternatively, it can be seen as the probability to acquire the required knowledge (the more effort, the more probable to acquire knowledge; probability $\propto$ effort). With probability e he learns the payoff of all candidate projects (decisions) With probability (1-e) he acquires no knowledge at all. (digital information). |
| $g_A(e)$<br>> 0 | $g_A(e)$ is called "disutility function" as it reduces the benefit. At private cost $g_A(e)$ he learns everything about the projects. |
| E<br>[0,1] | Probability that the principal acquires all knowledge about the candidate projects (decisions).<br>Can be also seen as the normalized effort E to acquire complete knowledge. |
| $g_P(E)$ | Disutility function of the principal, corresponding to the benefit reduction of |

| | |
|---|---|
| >0 | the principal by supervising or controlling the agent.<br>She has to invest her private cost / cost of effort $g_P$(E)to acquire the knowledge about the candidate projects. |

The mechanism of acquiring information is the following: The person puts in a certain amount of effort, and either stays completely uninformed with probability (1-E) respectively (1-e), or she/he “suddenly” finds the whole information and gets completely informed about everything with probability E respectively e.

Disutility can be interpreted as the cost of effort of a person. In order to reach a certain goal, one has to sacrifice some comfort (e.g. writing a report instead of playing golf).

The disutility functions g(E) and g(e) are increasing and strictly convex.

Aghion and Tirole impose the following constraints:

$$g(0) = 0 \qquad (19)$$

The marginal costs are zero at zero effort

$$g'(0) = 0 \qquad (20)$$

and the marginal costs are infinite at an effort of one.

$$g'(1) = \infty \qquad (21)$$

## 3.1 Forms of authority

Aghion and Tirole regard two types of authority. They distinguish between the person who is the “formal boss” and the person who “makes the decision”.

Definition **P-formal authority (integration):** The principal has the formal authority. She can always overrule the decision of the agent. In case she knows her best choice she will use her authority to dismiss the agent’s decision. In case she did not find out the payoff structure of the candidate projects she will rubber-stamp his suggestion (as the congruence parameter α is >0). In our system the agent's suggested project is positive for her so she can allow for its implementation.

Some systems have a different decision structure, like A-formal authority.

Definition **A-formal authority (delegation):** The decision of the agent cannot be overruled by the principal. This might be realized in a system where for example the head of the purchasing department has a budget for which he can make his own decisions. He has this right per contract.

We note the A-formal authority for completeness, but will use the P-formal authority later, as the principal normally keeps the authority for very important decisions.

## 3.2 Utility functions under P-formal authority

The utility function ansatz from Aghion and Tirole consists of three contributions. Two components have positive impact on the utility function, as either the principal or the agent makes a decision that brings positive benefit to both. The third part is the disutility function that reduces the utility because effort and "pain" is needed for aquiring information.

**The utility function of the principal** is setup in the following way:

$$u_P = EB + (1-E)B\alpha e - g_P(E) \qquad (22)$$

1. The first term reflects the situation where the principal invests the effort E to acquire the full information about the payoffs of all candidate decisions and chooses her most profitable project.
2. The second term reflects the situation where she did not find out the information about the projects (1-E), but the agents became informed (e). She then receives the payoff $B\alpha$. If the congruence parameter $\alpha$ is small, and she has to rubber stamp the decision, her remaining utility will be small.
3. The third term is the disutility function which reflects the fact, that the more effort E she puts into the surveillance of the agent, the less net-utility will be left for her.

With the boundary constraints from equation ( 21 ) an effort E of one yields to an infinite disutility. So in that model she never wants to put too much effort in the game.

**The utility function of the agent** is setup in the following way:

$$u_A = E\beta b + (1 - E)be - g_A(e) \qquad (23)$$

1. The first term reflects the situation where the principal is informed and overrules the agent. She makes the final decision regardless of his decision, where he still gets the payoff $\beta b$ out of it.
2. The second term reflects the situation where she did not succeed to find out the information and the agent can make the decision.
3. The third term is the disutility function which reflects the fact, that also the agent will restrict his effort e. Also his disutility function yields to an infinite disutility if he puts too much effort into finding the solution.

## 3.3 Reaction curves under P-formal authority

The key mechanism of game theory is:

**EVERY PLAYER MAXIMIZES HIS/HER UTILITY!**

Maximizing the utility function leads to the “reaction curve”. The reaction curve represents the best respond of the player to each strategy of the other players.
The maximum of the utility function is determined by calculating the derivative and setting it to zero.

**The reaction curve of the principal** can be written in the following way:

$$\frac{\partial u_P}{\partial E} = B - \alpha Be - g'_P(E) = 0 \qquad (24)$$

with the condition for a maximum:

$$\frac{\partial^2 u_P{}^2}{\partial E^2} = -g''_P(E) < 0 \tag{25}$$

**The reaction curve of the agent** can be written in the following way:

$$\frac{\partial u_A}{\partial e} = (1 - \mathrm{E})\mathrm{b} - g'_A(e) = 0 \tag{26}$$

with the constraint:

$$\frac{\partial^2 u_A{}^2}{\partial e^2} = -g''_A(e) < 0 \tag{27}$$

This is a game of complete information, so each player knows the utility of the other player and will behave in the following way:

## THE PLAYERS PLAY THE BEST RESPONSE TO EACH OTHER!

If a Nash equilibrium exists in the game, then no player wants to deviate from it, as this would lower his/her utility.

The two reaction curves form the set of two equations to be solved to find the Nash equilibrium:

$$(1 - \alpha e)B = g'_P(E) \tag{28}$$

$$(1 - E)b = g'_A(e) \tag{29}$$

In the next chapter we will visualize the solution by choosing an explicit disutility function.

## 3.4 Choosing parabolic disutility functions for visualization

Originally, the disutility functions g(E) and g(e) have to obey the constraints eq.( 19 ), ( 20 ) and ( 21 ), which were:

$$g(0) = 0$$
$$g'(0) = 0 \qquad (30)$$
$$g'(1) = \infty$$

In order to be able to explicitly calculate the results we use a modified disutility function in this sub-chapter for the agent and the principal. The disutility function approximating the above one is set to

$$g(E) = cE^2 \qquad (31)$$

for the principal and

$$g(e) = de^2 \qquad (32)$$

for the agent

These disutility functions violate the 3rd constraint. Instead of being asymptotic at E/ e= 1 it is set up with a quadratic parabola.

The effect of the modification is that for e = 1 or E = 1 the disutility is not infinite. It has the property, that for small c and d the parabola is very flat and therefore far away from being asymptotic at the point e/E=1. For higher parabola parameters c and d, the parabola gets steeper and the derivative at g(1) becomes bigger.

The parabolic utility function has therefore the property that we will get results with e>1 and E>1 when we solve the system of reaction curves.

The persons cannot work more than a certain amount of hours a day. So later on we need to impose constraints on our solution.

### 3.4.1 Plotting the utility function of the principal

Under p-formal authority (integration), the utility function of the principal is:

$$\mathrm{u_P} = B\mathrm{E} + \alpha B\mathrm{e}(1 - \mathrm{E}) - c\mathrm{E}^2 \tag{33}$$

The derivation of the utility function of the principal is:

$$\frac{\partial \mathrm{u_P}}{\partial \mathrm{E}} = B - \alpha B\mathrm{e} - 2c\mathrm{E} \tag{34}$$

We visualize the utility of the principal as a function of her effort E.
The payoff of the principal and her disutility parabola factor are held constant and the parameters α and e are varied with a slider depicted in each screenshot.

B = 30; (*Net benefit of principal*)
c = 20; (*disutility parabola factor of principal c>0*)

The blue line is the utility function and the yellow line is the derivative of it.
To get some feeling for the behavior of the model, five scenarios are plotted.

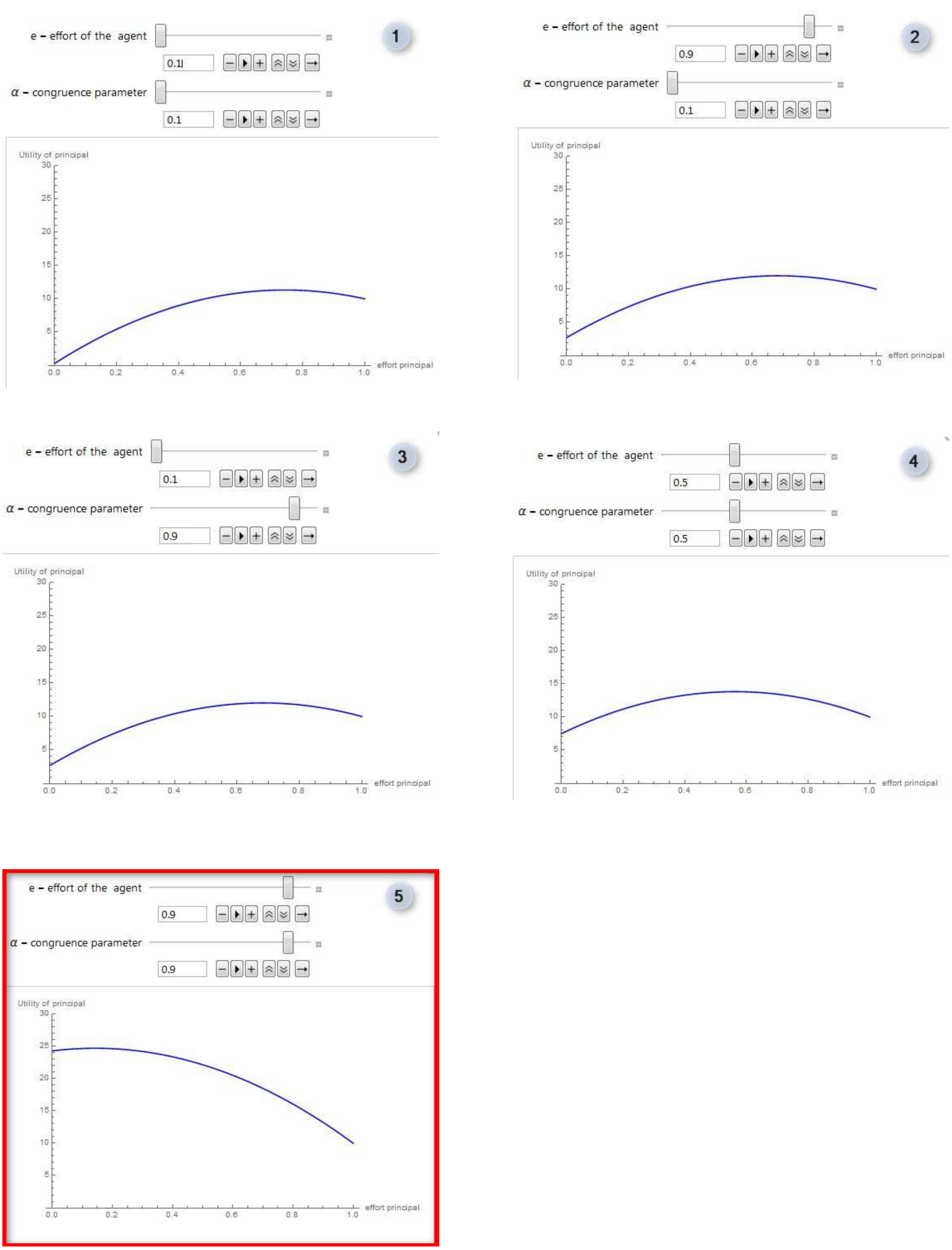



**Fig. 5 Plot of the utility function of the principal using different congruence parameter and effort of the agent. The framed plot shows the win-win situation, where the utility of the principal is highest for small E.**

Discussion of the influence of the parameters:

The Net benefit “B” and the disutility “c” are held constant for the visualization.
We look at the cases of strong and weak congruence “$\alpha$” in the aims of principal and agent.
Further we visualize the dependence of the principal’s utility function from the effort of the agent “e”.

1. The first four plots are very similar, as either the low congruence or the low agent’s effort and even the medium ones lead to a low utility if she does not invest much time and a high utility when she does everything herself. Only for very high effort the disutility term lowers the utility. It can be seen, that for low congruence parameter a small effort of the principal always leads to a small resulting utility. Of cause, the case high congruence but small effort of both also leads to a small resulting utility. The medium congruence case with medium motivated agent shows the same tendencies.

This models clearly the fact that no congruence and no motivation of the agent leads to the situation that the principal gets most benefit by doing all herself. It is therefore not worth hiring anybody who is either demotivated or has completely other ambitions.

2. The case of high congruence and motivated agent shows the best situation. The maximum utility for the principal is located where she puts in just a little effort. Putting in more effort reduces her utility, as the agent stops acquiring information if he gets overruled too often.

Small $\alpha$ occurs for instance in the cases of exploitation or slavery. Exploitation of workers seems to be profitable on first glance. One may think: “The less I pay for wages, the more I get for myself”. This is not a zero sum game. With small wages the agent might feel not much congruence. Exploitation may also lead to a positive utility for the principal. But if the principal wants to <u>optimize</u> the utility for her, it is clearly better to set up an infrastructure and environment that shows high congruence, so that both parties can expect a high payoff.

This effect will be seen again when analyzing the reaction curve of the agent.

### 3.4.2 Plotting the reaction curve of the principal

The reaction curve of the principal is the respective best response of the principal to the strategies of the agent. We get it by setting the derivative to zero.

$$B(1 - \alpha e) - 2cE = 0 \qquad (35)$$

The congruence parameter plays a key role in the model. For visualization of its strong influence, we plot the resulting effort of the principal versus the effort of the agent. We show the plots for two different congruence parameters α.

The parameter set:

B=30; (*Benefit of principal.*)
c=20; (*Disutility parabola factor of principal: c>0*)

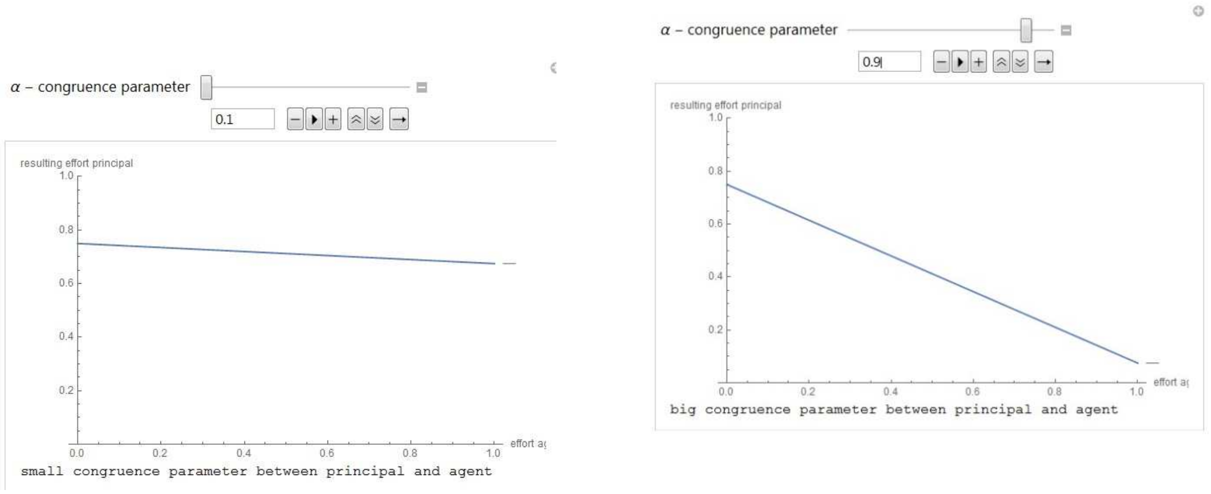



**Fig. 6 Effort of the principal in dependence on the effort of the agent. The influence of the congruence parameter on the effort of the principal.**

For a small congruence parameter, the best response of the principal is to invest a lot of effort for supervision regardless of the effort of the agent, whereas for a high congruence parameter the effort of the principal reduces significantly with rising effort of the agent. She can trust the agent much more.

Given the assumption of the model that the “congruent agent” and the “not-congruent agent” earn the same wage, the supervision of the agent with low congruence makes him overall more expensive.

### 3.4.3 Plotting the utility function of the agent

The utility function of the agent under p-formal authority is:

$$u_A = Eb\beta + (1 - E)eb - de^2 \tag{36}$$

As before, we investigate the utility of the player in dependence on his effort and the effort of the other player. We plot the utility function of the agent and its derivative for the following parameter set:

b=20; (*private benefit b of the agent *)
d=15; (*disutility parabola factor of one agent d>0*)
β =0.1 (*β congruence parameter of agent *)
the slider regulates “E”, the effort of the principal

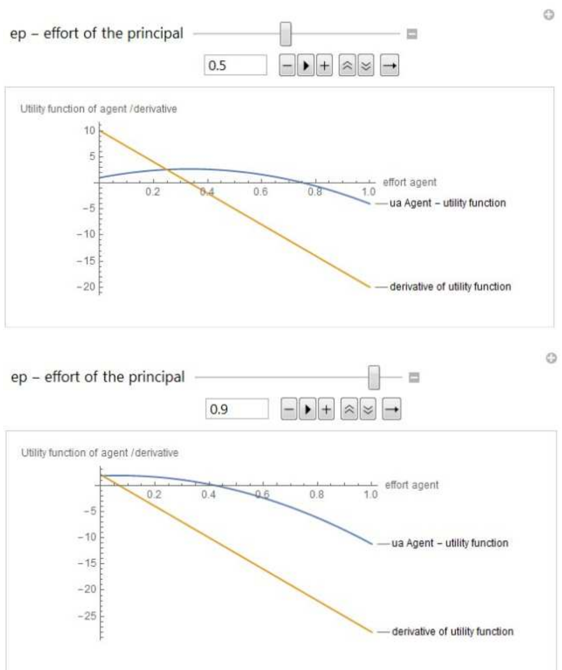



**Fig. 7 Utility function of the agent in dependence on the effort of the agent (blue) and the derivative (yellow).**
**Left: Medium effort of the principal. Right: high effort of the principal**

The blue curve in Fig. 7 shows, as discussed before, when the principal invests a moderate amount of effort (E=0.5; left plot), the utility of the agent is positive for a moderate effort of the agent and shows a maximum at reasonable effort e.

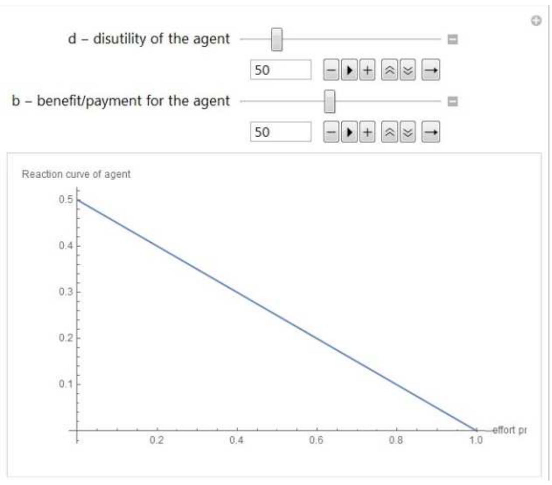
In the right plot we can see, if the principal interferes too much with her effort (E=0.9), the utility of the agent becomes soon negative. The maximum of the agent's utility is at nearly zero effort e.

The utility functions of the players have hence been very well tuned by Aghion and Tirole with everyday's experience of a big class of principal –agent systems in their model.

### 3.4.4 Plotting the reaction curve of the agent

The reaction curve of the agent in p-formal scenario (integration) is:

$$e = \frac{b(1-\mathrm{E})}{2d} \qquad (37)$$




The downward sloping of the reaction curve means that the agent becomes less active, the more the principal interferes. Some people think, that the less you surveil the agent, the less he will perform. This might be representative in systems like the government and the tax payers. Aghion and Tirole model the opposite hypothesizing that the performance of the agent is better if he is not too much supervised. The agent is driven by initiative.

### 3.4.5 Nash equilibrium of the principal – agent game under integration

The Nash equilibrium can be found where the best answer of the principal to the agent equals to the best answer of the agent to the principal. In this case, none of the two persons have a reason to change her/his behavior.

We need to solve the system of equations ( 35 ) and ( 37 ).

$$B(1-\alpha \mathrm{e})-2c\mathrm{E}=0 \qquad (38)$$

$$\mathrm{e}=\frac{b(1-\mathrm{E})}{2d} \qquad (39)$$

This gives us the explicit solutions:

$$E=-\frac{B(2d-\alpha \mathrm{b})}{\alpha B\mathrm{b}-4cd} \qquad (40)$$

$$\mathrm{e}=\frac{b(B-2c)}{\alpha B\mathrm{b}-4cd} \qquad (41)$$

### 3.4.6 Constraints on the solutions

As the efforts of the two players are interpreted as the probability to find out the necessary information, we need to impose the constraints of a probability to them, i.e. E $\in$ [0,1] and e $\in$ [0,1].

In contrast to the original paper of Aghion and Tirole, we introduced a quadratic disutility function which does not follow the property $g'(1)=\infty$ . This leads to the necessity to put restrictions onto the parabolic disutility factors.

1. The effort of the principal is normalized to one, i.e. $E \leq 1$.

Starting from ( 40 ) we impose the following constraint:

$$\frac{-2Bd + \alpha B\mathrm{b}}{\alpha B\mathrm{b} - 4cd} \leq 1 \qquad (42)$$

from this we can deduct:

$$B \leq 2c \qquad (43)$$

for $E \leq 1$

and $e \geq 0$

The role of the disutility parabola factor *c* is to limit the effort of the principal. If she would feel no disadvantages, she would supervise the whole time.

$e \geq 0$: Equation ( 41 ) shows that at B = 2c the effort of the agent vanishes. If B is too high and there is too much on stake for the principal, the agent does not even start to put effort in finding the information.

2. The effort of the principal is positive, i.e. $E \geq 0$

Starting from ( 40 ) we impose the following constraint:

$$\frac{-2Bd + \alpha B\mathrm{b}}{\alpha B\mathrm{b} - 4cd} \geq 0 \qquad (44)$$

we can therefore deduct:

$$\alpha\mathrm{b} \leq 2d \qquad (45)$$

for $E \geq 0$

If the disutility of the agent would be very small, the congruence and private benefit high, the principal would not need to supervise any more.

This is a very important and intuitive lesson!

It has cost-saving implications. Working conditions should be adjusted in a way that the necessity, and therefore the costs, of supervision can be reduced.

3. The effort of the agent is normalized to one, i.e. $e \leq 1$.

$$\frac{b(B-2c)}{\alpha B\text{b}-4cd} \leq 1 \qquad (46)$$

With ( 43 ) the numerator is negative. We get

$$\alpha B\text{b} \leq 4cd \qquad (47)$$

for $e \leq 1$

4. The effort of the agent is positive, i.e. $e \geq 0$

$$\frac{b(B-2c)}{\alpha B\text{b}-4cd} \geq 0 \qquad (48)$$

This coincides with inequality ( 43 ).

### 3.4.7 Discussion of the result – influence on principal's behavior

The model has a lot of parameters influencing the behavior of the two players. In the following we verify the plausibility of the influence of each parameter on the utility graphically. We add a short discussion of the tendencies of the players that the model predicts.

How does the principal behave? We are interested in which direction a change in the parameters changes the behavior of the principal according to equation ( 40 ). As the results are mostly not strictly linear or proportional to 1/x, we depict one or two example plots to show the tendency that we can expect.

**Model with perfect information:** The setup is in a way that all parameters are known to both players. So here, the benefits and disutilities of one player are perfectly known to the other player.

For visualization, we use the following parameter set:

α = 0.5; (*alpha congruence parameter of principal*)

B = 50; (*Benefit of principal*)

b=30; (*private benefit b of the agent*)

c = 30; (*disutility parable factor of principal c>0*)

d=20; (*disutility parabola factor of one agent d>0*)

(* constraints: b <= 2c and

αb <= 2d and

αbB <= 4cd *)

1. **How is the effort of the principal influenced by the congruence parameter alpha?**

As expected, the effort of the principal is less when the congruence between the two parties is high. The principal trusts the agent more if their motives are aligned.

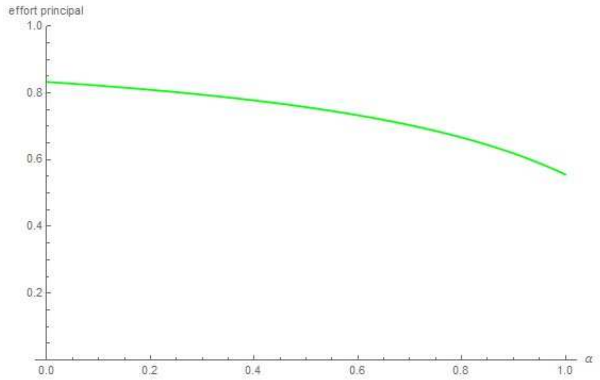



**Fig. 8 Effort of the principal in dependence on the congruence parameter α**

2. **How is the effort of the principal influenced by her disutility c?**

As we would intuitively expect, the higher the disutility, the less the principal supervises. Her motivation to supervise drops with 1/c. We further see the influence of the constraint ( 50 ). As B = 50, we would get an effort bigger than one for c < 25.

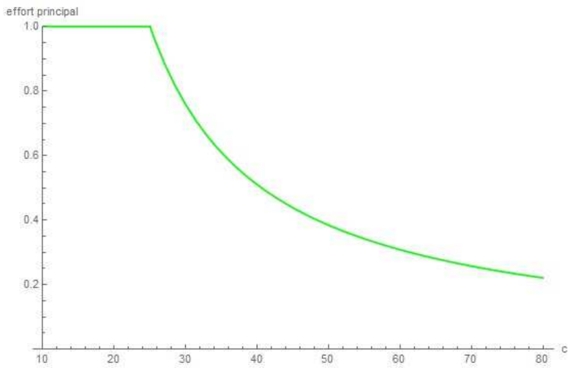



**Fig. 9 Effort of the principal in dependence on her disutility c**

### 3. How is the effort of the principal influenced by the agent's disutility d?

The model has perfect information, so the disutility of the agent is known to the principal.

As we can see from Fig. 10 this assumption has not too much effect, as the effort of the principal stays quite constant for a reasonable high disutility of the agent. Only in the region of higher congruence of 0.7 and low disutility of the agent the principal stops her surveillance completely, as constraint ( 45 ) is not fulfilled.
She trusts an agent who is highly motivated (small d) and aligned with her aims (high α).

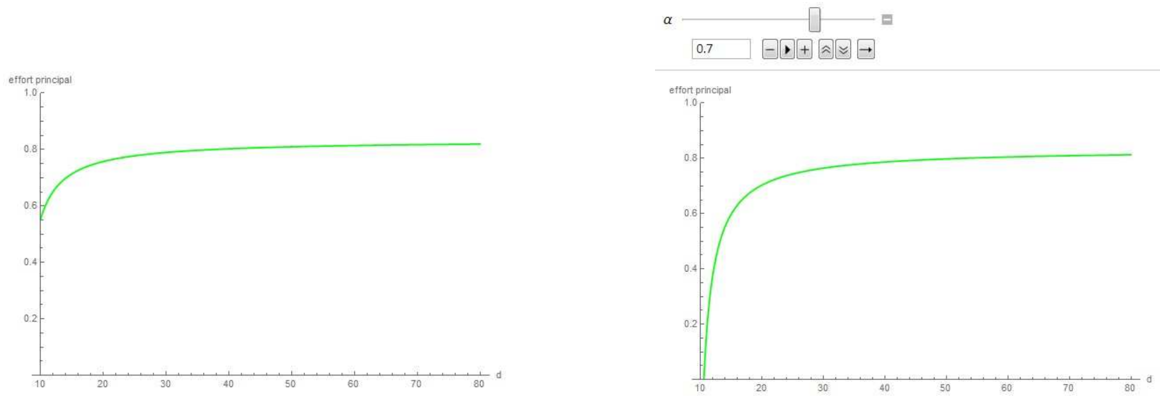



**Fig. 10 Effort of the principal in dependence on the agent's disutility d. left: For a medium congruence of 0.5 , the principal supervises quite constantly. right: for high congruence and very low disutility of the agent, the principal supervises very little.**

4. **How is the effort of the principal influenced by the private benefit of the agent b?**

Here again we note the assumption, that the principal has perfect information about the agent's private benefit.
The effort of the principal depends very strongly on the private benefit of the agent.

As we see from Fig. 11 constraint ( 45 ) is satisfied at $b = 80 = 2\mathrm{d}/\alpha$
From that on, the principal stops supervision, as for higher benefit her effort would become negative. This shows very well the potential to reduce supervision by augmenting the private benefit of the agent.

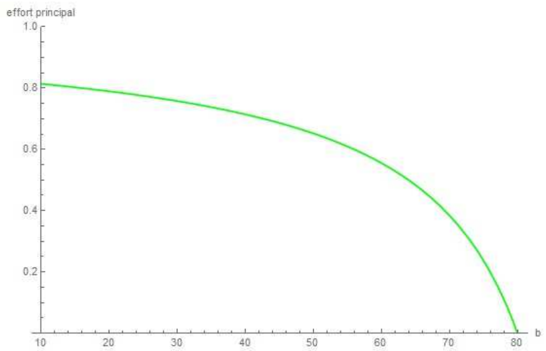



**Fig. 11 Effort of the principal in dependence on the private benefit of the agent.**

5. **How is the effort of the principal influenced by her own benefit B?**

Fig. 12 shows the fact that the principal supervises a lot for decisions that are very important (bearing high benefit) to her. She supervises less if her own benefit is low.
We can see again the effect of constraint ( 43 ). She basically does the work herself if her benefit is very high.

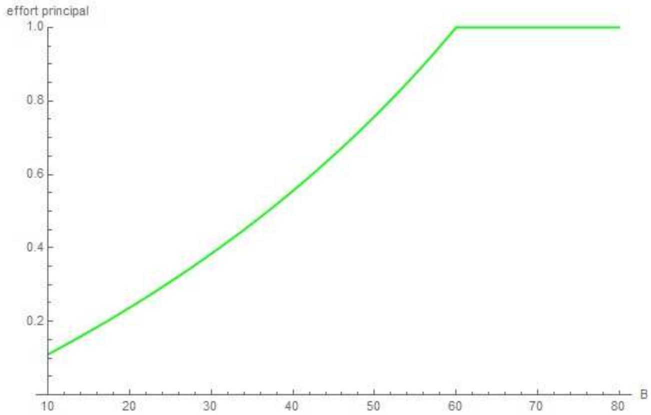



**Fig. 12 Effort of principal in dependence on her benefit B**

**Summary:**

As a summary of the investigated parameters we can say that the behavior of the principal is modeled with reasonable accordance of what we experience in cases where for instance a project manager is supervising her team.

### 3.4.8 Discussion of the result – influence on agent's behavior

How does the agent behave? We analyze the influence of the model parameters on the effort of the agent according to the solution of equation ( 41 ).

We recall that the agent's congruence parameter $\beta$ does not influence his effort, as the model is based on P-formal integration and his congruence parameter in the term $E\beta b$ vanishes with the derivative with respect to e when we calculate the reaction curve.

We use the same parameter set as for the visualization of the effort of the principal (chapter 3.4.7 )

1. **How is the effort of the agent influenced by the congruence parameter alpha?**

As one would expect, the effort of the agent rises with the congruence. But we recall that $\alpha$ is the congruence from the principal's perspective.

From equation ( 41 ) we can see that the effort rises with $1/\alpha Bb$. This is shown in Fig. 13 as we see a stronger rising of the curve for higher b.

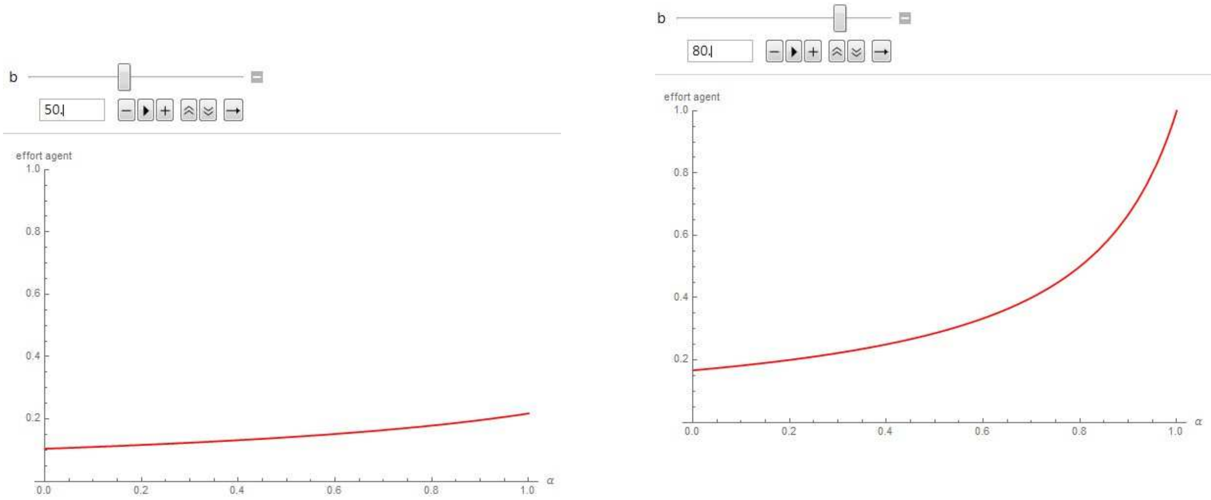



**Fig. 13 Effort of the agent in dependence of the congruence parameter alpha. left: private benefit b=50. right: private benefit b=80.**

## 2. How is the effort of the agent influenced by the principal's disutility c?

Following constraint ( 43 ), the agent does not start working unless the principal's disutility is high enough that she reduces her supervision. For rising principal's disutility her effort becomes reduced, and the effort of the agent rises. He has then more chance that he will not be overruled by the principal. Again we see a big effect of the possibility to discourage the agent. He does not contribute much to solving the principal's task, but the wages are still paid.

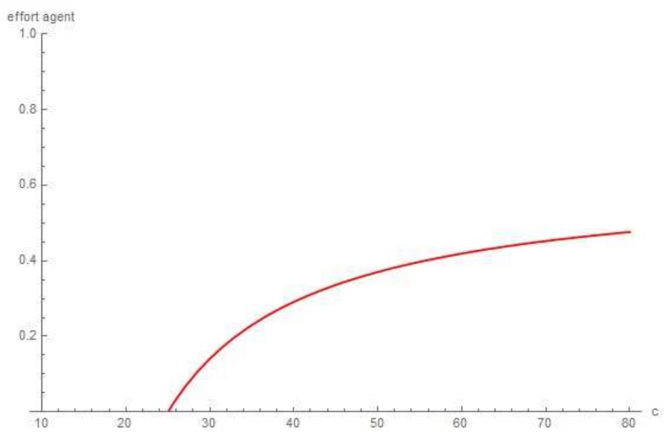


Fig. 14 Effort of the agent in dependence on the principal's disutility c

### 3. How is the effort of the agent influenced by the agent's disutility d?

The effort of the agent decreases with 1/d. A higher disutility makes the agent want to put less effort into learning.

When he feels no discomfort he puts maximum effort into finding the solution. He is then on ease with his work. His maximum working effort is restricted by inequality ( 47 ).

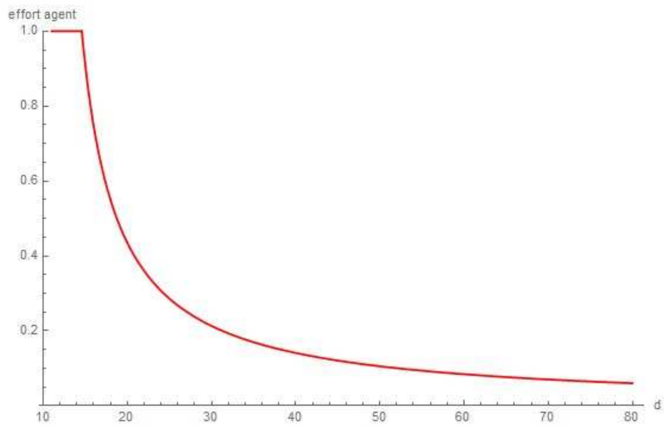



**Fig. 15 Effort of the agent in dependence on the agent's disutility d**

### 4. How is the effort of the agent influenced by the private benefit of the agent b?

Here again, the influence of the congruence is very high.

As can be seen from equation ( 41 ), for higher congruence $\alpha$, the effort rises faster.

For $\alpha$ =0 it rises linearly and for $\alpha$ =1 it rises as $\frac{-b}{b-const}$.

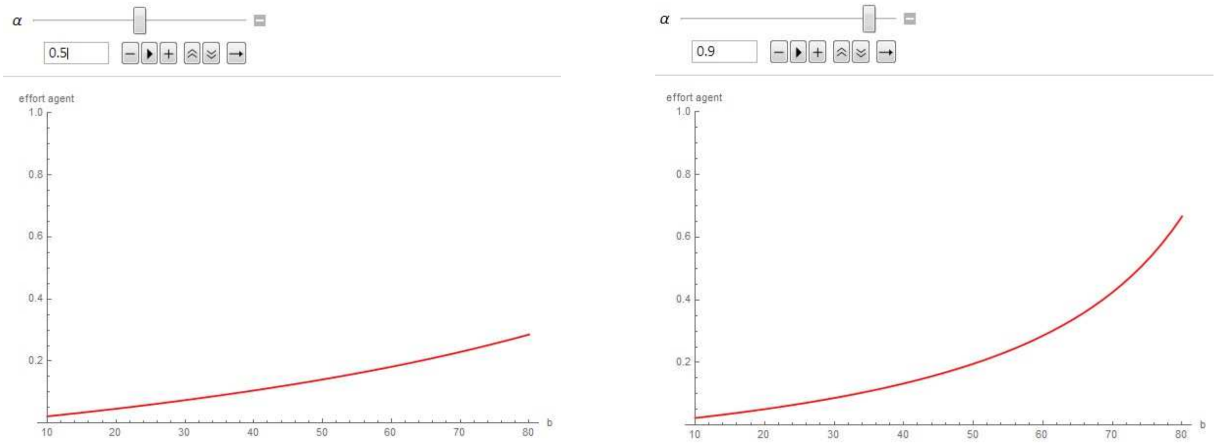



**Fig. 16 Effort of the agent in dependence on his private benefit for α =0.5 and α =0.9**

### 5. How is the effort of the agent influenced by the principal's benefit B?

In equation ( 41 ) the term (B-2c) dominates the behavior. At B=2c the effort of the agent becomes zero.

In the special case of α =0 the effort of the agent falls linearly with B.

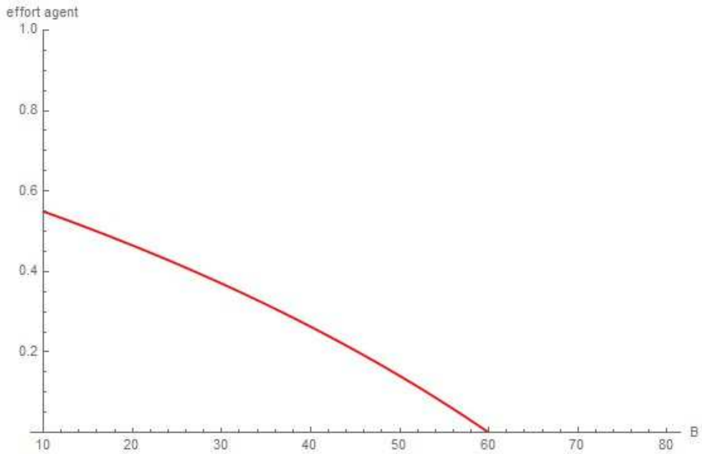



**Fig. 17 Effort of the agent in dependence on the benefit B of the principal**

The effort of the agent becomes less when the benefit of the principal rises. This is due to the fact, that the principal supervises much more when her benefit is big.

## 3.4.9 Graphical solution of the Nash equilibrium

The Nash equilibrium is found where the two reaction curves ( 38 )and ( 39 ) cross.

As they sometimes cross in one of the corners, the Nash equilibrium is not very meaningful (corner solutions). This can be avoided when we choose parameter sets that obey the constraints ( 43 ), ( 45 ) and ( 47 ).

For the following parameter set the curves cross and we find a Nash equilibrium

| | |
|---|---|
| α= 0.3; | (*alpha –congruence parameter of principal *) |
| B=5; | (*Benefit of principal.*) |
| b= 7; | (*Private benefit b of the agent *) |
| c=6; | (*disutility parabola factor of principal c>0*) |
| d=10; | (*disutility parabola factor of agent d>0*) |

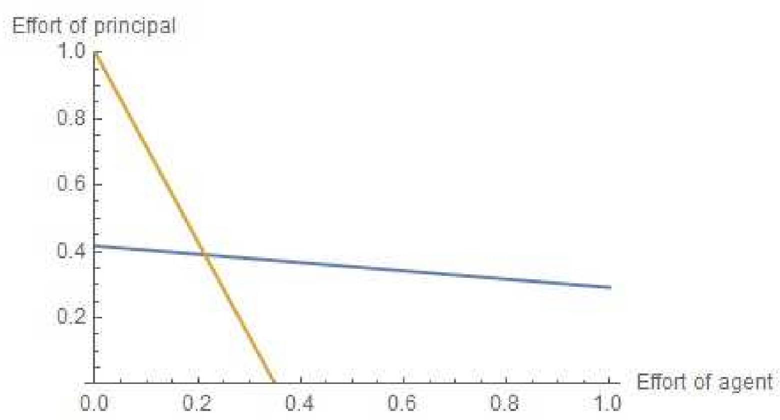



**Fig. 18: Graphical Nash Equilibrium**

Example of a “corner solution”:

| | |
|---|---|
| α= 0.3; | (*alpha –congruence parameter of principal *) |
| **B=13;** | (* Benefit of principal.*) |
| b= 7; | (* Private benefit b of the agent *) |
| c=6; | (*disutility parabola factor of principal c>0*) |
| d=10; | (*disutility parabola factor of agent d>0*) |

remark: The only parameter that was changed is the benefit of the principal
from B = 5 to B = 13, therefore violating constraint ( 43 ).

The curves cross where the agent is not involved in finding the information. He puts in zero effort (e=0).

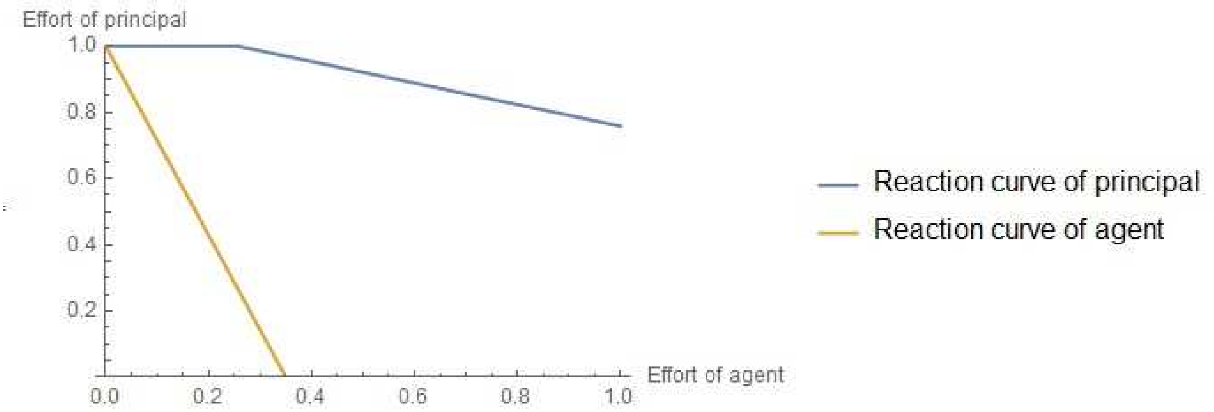



**Fig. 19: Example of a corner solution. For certain combinations of parameters, the principal supervises with E=1**

So here, there was so much on stake for the principal (high benefit B) combined with too less disutility d. She does all the work herself.

If her disutility rises from c=6; to c = 15 they will find an equilibrium again, as she has less motivation to work too much:

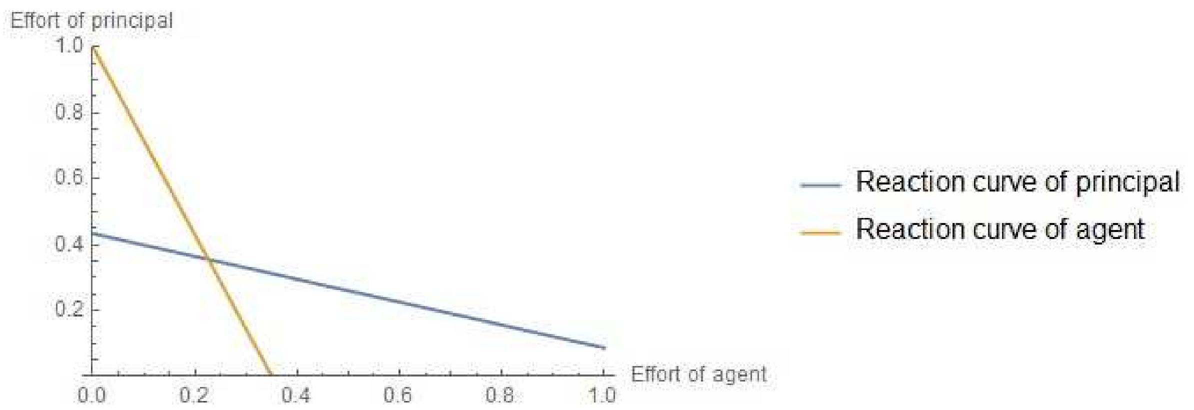



**Fig. 20 Nash equilibrium**

This introduced model using P-formal authority will be our starting point for extending the model to take into account the ability of the agent and the risk aversion of the principal.

## 3.5 Utility functions under A-formal authority

For completeness we mention the delegation model of Aghion and Tirole and show the differences and similarities to P-formal authority.

In the delegation scenario (A-formal authority), the agent has the right to make the decision if he is able to get informed. If not and the principal is informed then she will make the decision. This is quite the opposite from the p-formal mechanism, and we will work out the mathematical symmetry of the two.

The **utility function of the principal** can be written in the following way:

$$u_P = e\alpha B + (1-e)EB - cE^2 \quad (49)$$

1. The first term reflects the situation where the agent was able to acquire the necessary information with effort/probability *e*. He will choose his preferred project and the principal will get $\alpha B$ according to her congruence parameter α.
2. The second term reflects the situation, that with probability (1-e) he was not able to become informed. So she will make the decision if she manages to get informed with effort/probability E.
3. The third term is the disutility function which reflects the fact, that the more effort E she puts into finding the information, the less net-utility will be left for her.

The **utility function of the agent** can be written in the following way:

$$u_A = eb + (1-e)E\beta b - de^2 \quad (50)$$

1. The first term reflects the situation where the agent is informed and overrules the principal.
2. The second term reflects the situation where he did not get informed, and the principal makes the decision.
3. The third term is the disutility function which reflects the fact, that also the agent will restrict his effort e. Also his disutility function yields to a low benefit if he puts too much effort into finding the solution.

There is symmetry in the equations ( 49 ) and ( 50 ) to equations ( 33 ) and ( 36 ) of p-formal authority. This can easily be seen when we write both systems of utility functions in a table:

| | | |
|---|---|---|
| P-formal | $u_P = BE + (1-E)\alpha Be - cE^2$ | $u_A = E\beta b + (1-E)eb - de^2$ |
| A-formal | $u_A = be + (1-e)\beta bE - de^2$ | $u_P = e\alpha B + (1-e)EB - c\mathrm{E}^2$ |

**Table 2: Comparison of P-formal and A-formal authority**

The formulas of A-formal authority are mathematically obtained by interchanging all parameters, variables and subscripts of the agent with the ones of the principal. It is like changing the roles.

We therefore do not need to go through the mathematics again and can restrict our self to the p-formal model.

# 4 Investigation on agent’s ability and risk averse principal

This chapter investigates two aspects that are modified with regard to the original model from Aghion and Tirole (Aghion & Tirole, 1997).

We will illuminate the influence if **the ability of the agent is not known** and the influence of a **risk averse principal**.

The original model extended with quadratic disutilities is taken as the basic scenario.

## 4.1 Delegation and agent's ability

### 4.1.1 Motivation

In many companies the research and development cost consist mostly of the wages of the developers. It is vital for the continuation of existence of high tech companies to have effective teams of highly skilled employees. We examine mechanisms that determine the effort and success that the individual members of the team display in dependence on the beliefs about skills and the resulting behavior of the principal.

Aghion and Tirole show in their model that the effort of the agent declines when the principal puts more effort in surveillance of the agent. They state that it can be seen

as an advantage if the principal is not too much informed herself. This would lead to overruling the agent frequently. If the boss always makes everything herself: "Why should I struggle?"
It is in the interest of the principal that the agent gets informed. The informed agent will suggest/perform projects with positive benefit for the principal. We will see how the extend of supervision needs to be changed optimizing utility when taking into account different level of skills of the employees.

### 4.1.2 Modeling background

As we have now a game with incomplete information with regard to ability, we use the method of chapter 2.1.5 and setup a **Bayesian game**. For this purpose, we need to model the two types of agents and introduce the probability that the agent is less talented.

**Modeling two types of agents:**

We have two types of agent that the principal can face:

- The skilled agent
- The inept agent

The distinguishing feature in our model between the two types of agents is their disutility.
We determine the disutility of the inept agent to be higher than the disutility of the skilled agent, as the required task will be more difficult, unpleasant and unsatisfactory for the inept agent. Inept does not mean "not intelligent", but "not suited for this special task".
One could argue that also the private benefit of the two agents should be different. We leave the private benefit the same for both types, as factors like appreciation promotion opportunities and so forth are regarded equal for each employee.

| | |
|---|---|
| Skilled agent | The disutility parameter $d_{skilled}$ of the skilled agent is low.<br>The desired task is easy to fulfill and fun for this agent. It is his professional specialty.<br><br>Private benefit of agent is b. |
| Inept agent | The disutility parameter $d_{inept}$ of the inept agent is high.<br>The desired task is not exactly inside the competency range of the agent, it is difficult and painful.<br>$d_{inept} > d_{skilled}$<br><br>Private benefit of agent is the same b as for the skilled agent. |

**Table 3: Distinction between skilled and inept agent**

In this scenario the principal faces the situation where she does not know the ability of the agent to fulfill his task.
She faces an agent, maybe for the first time, and does not know if he is qualified (skilled) or not qualified (inept) to fulfill the job for which she hired him.
How will she change her behavior when we introduce a probability that the agent is less skilled? What will she do to maximize her utility?

We have to model a game with incomplete information as introduced in chapter 2.1.5 “Incomplete information - Bayesian Games“.
The agent’s cost of effort (disutility) is now not known with certainty to the principal. So her expected benefit is not known with certainty.
The information is asymmetric, as the agent himself knows his type (ability).

A department head in an enterprise knows the probability of how the new employee fits to the job he is hired for. She has statistics or judgment based on experience about that. But as they do not know each other beforehand she does not know which type she is facing in the concrete situation.

### 4.1.3 Utility function of the principal not knowing the ability of the agent

For setting up the new utility functions we first introduce the new parameters:

| | |
|---|---|
| $d_{skilled}$ > 0 | Disutility parabola factor of the skilled agent: $g_A$ ($e_{skilled}$) = $d_{skilled}$ $e^2_{skilled}$ At private expenses $g_A$ ($e_{skilled}$) the agent learns the complete information needed. |
| $d_{inept}$ > 0 | Disutility parabola factor of the inept agent: $g_A$ ($e_{inept}$) = $d_{inept}$ $e^2_{inept}$ At private expenses $g_A$ ($e_{inept}$) the agent learns the complete information needed. |
| | $d_{inept}$ > $d_{skilled}$ |
| $e_{skilled}$ $\in$ [0,1] | $e_{skilled}$ can be seen as the effort a skilled or talented agents dedicates to finding the solution normalized to one. Alternatively, it can be interpreted as the probability that he acquires the needed knowledge. |
| $e_{inept}$ $\in$ [0,1] | $e_{inept}$ is the effort the less qualified agent is willing to invest to find the answer. |
| P $\in$ [0,1] | P= $P_{skilled}$ is the probability that the principal faces a skilled agent. Respectively with $(1-P)$ she faces an inept agent. |

Following the Bayesian procedure, our new utility function for the scenario where the agent can be skilled or not-skilled becomes:

$$u_P = EB + (1-E)B\alpha\left[Pe_{skilled} + (1-P)e_{inept}\right] - cE^2 \tag{51}$$

Only the second term is modified with regard to equation ( 33 ), where the agent is able to find a solution, but the principal could not get informed.

We presume the skilled and inept agent need to put different effort into finding the information. This is justified as we expect that somebody called “skilled” will come up with the solution easier than an inept person.

**Remark:**

Here, we cannot use an interpretation of *e* that says: “e is proportional to the hours worked”. Both types of agent get the same salary. Both work 40 hours a week, so we can subtract the working time of 40h/week from the “effort”, as it is a constant for both.

The interpretation of “e” is therefore reduced to non-observables.

For example, the skillful agent puts more “heart blood” into the work and succeeds better each time, or he can do all the work in 5 hours a day and then enjoy chatting with his colleagues or browsing web.

### 4.1.4 Reaction curve of the principal

The principal maximizes her utility. The reaction curve is the derivative of her utility function with respect to her effort E.

With a general disutility function $g_P(E)$ the reaction curve is written as:

$$g'_P\,(E) = B - \alpha B\left[P e_{skilled} + (1-P) e_{inept}\right] \qquad (52)$$

In our case of a parabolic disutility function, $g_P(E) = cE^2$ the reaction curve becomes:

$$\frac{\partial u_P}{\partial E} = B - \alpha B\left[P e_{skilled} + (1-P) e_{inept}\right] - 2cE = 0 \qquad (53)$$

For a maximum the second derivative has to be negative:

$\frac{\partial^2 u_P{}^2}{\partial E^2} = -2c = \text{neg.}$ -> it is a maximum, as c > 0.

Equation ( 53 ) can be brought into an explicit form.

$$E = \frac{B\,(1 - e_{inept}\,\alpha\,(1-P) - P\,\alpha\,e_{skilled})}{2c} \qquad (54)$$

We see that the disutility limits the effort of the principal with 1/c as in the case of one agent.

### 4.1.5 Utility function of the two types of agents

Each of the two possible types of agents has his own utility function, which has the same structure as the utility function of one type of agent described in chapter (3.4.3 Plotting the utility function of the agent). We added subscripts for accuracy of distinction.

Utility function of skilled agent:

$$u_{A\_skilled} = E\beta b + (1-E)be_{skilled} - d_{skilled}{e_{skilled}}^2 \quad (55)$$

Respectively, the utility function of the inept agent:

$$u_{A\_inept} = E\beta b + (1-E)be_{inept} - d_{inept}{e_{inept}}^2 \quad (56)$$

The two types of agents have different disutility parameters. For this reason they will invest different amount of effort into the task.

### 4.1.6 Reaction curve of the two types of agents

From the utility function we get the reaction curves by setting the first derivative to zero. The best answer to the principal's behavior is where the utility has a maximum.

Reaction curve of skilled agent:

$$\frac{\partial u_{A\_skilled}}{\partial e_{skilled}} = (1-\mathrm{E})\mathrm{b} \; - 2d_{skilled}e_{skilled} \; = 0 \quad (57)$$

$\frac{\partial^2 {u_{A\_skilled}}^2}{\partial {e_{skilled}}^2} = -2d_{skilled} = \mathrm{neg.}$ -> it is a maximum

In the same manner the reaction curve of the inept agent leads to:

$$\frac{\partial u_{A\_inept}}{\partial e_{inept}} = (1-\mathrm{E})\mathrm{b} \; - 2d_{inept}e_{inept} = 0 \quad (58)$$

$\frac{\partial^2 {u_{A\_inept}}^2}{\partial {e_{inept}}^2} = -2d_{inept} = \mathrm{neg.}$ -> it is a maximum

The disutility parameter of the inept agent is higher than the one for the skilled agent. The explicit form of the reaction curve is:

$$e_{skilled/inept} = \frac{(1-\text{E})\text{b}}{2d_{skilled/inept}} \quad (59)$$

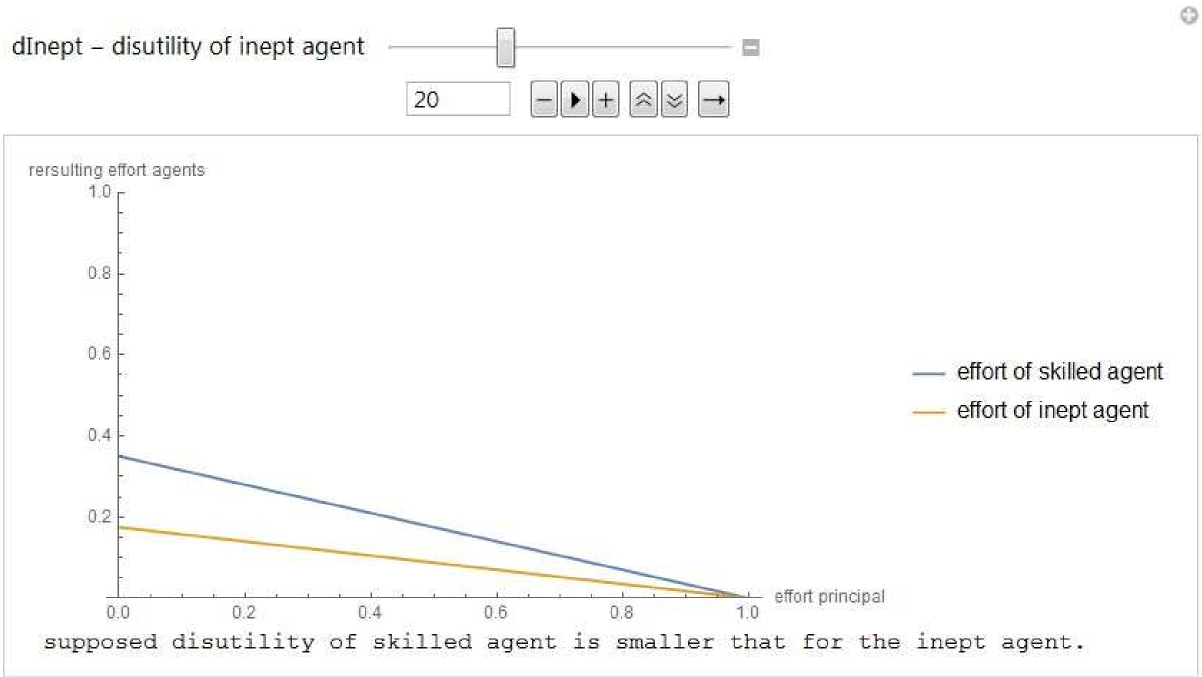
The slope of the straight line ( 59 ) is given by

$$slope = -\frac{\text{b}}{2d_{skilled/inept}} \quad (60)$$

We choose the following parameter set for visualization:

b = 7; (*Private benefit b of the agents –supposed to be equal for both types.*)
dSkilled = 10; (*disutility parabola factor of skilled agent d>0*)

The plot of equation ( 59 ) shows, that the inept agent, who has a higher disutility will put less effort into the game, according to the different slope ( 60 ) of the reaction curve.



The higher the disutility the less the motivation to put effort into finding the information.

### 4.1.7 Nash equilibrium

To find the Nash equilibrium, the system of the three reaction curves needs to be solved. The system of equations is:

$$B - \alpha B\left[P e_{skilled} + (1 - P) e_{inept}\right] - 2cE = 0 \qquad (61)$$

$$e_{skilled} = \frac{(1 - E)b}{2d_{skilled}} \qquad (62)$$

$$e_{inept} = \frac{(1 - E)b}{2d_{inept}} \qquad (63)$$

Solving the system of equations ( 61 ), ( 62 ) and ( 63 ), we get:

$$E = \frac{B\mathrm{d}_{\mathrm{inept}}(b\alpha P - 2\mathrm{d}_{\mathrm{skilled}}) - B\ b\ \alpha\ \mathrm{d}_{\mathrm{skilled}}(\mathrm{P} - 1)}{\alpha\mathrm{Bb}\left[\mathrm{d}_{\mathrm{skilled}} + \mathrm{Pd}_{\mathrm{inept}} - \mathrm{Pd}_{\mathrm{skilled}}\right] - 4\mathrm{cd}_{\mathrm{skilled}}\mathrm{d}_{\mathrm{inept}}} \qquad (64)$$

$$e_{skilled} = \frac{b(\alpha B\mathrm{d}_{\mathrm{inept}} - B\mathrm{d}_{\mathrm{inept}} + 2c\mathrm{d}_{\mathrm{inept}})}{\alpha bB\mathrm{P}\left(\mathrm{d}_{\mathrm{skilled}} - \mathrm{d}_{\mathrm{inept}}\right) + 4c\mathrm{d}_{\mathrm{inept}}\mathrm{d}_{\mathrm{skilled}}} \qquad (65)$$

$$e_{inept} = \frac{b(\alpha B\mathrm{d}_{\mathrm{skilled}} - B\mathrm{d}_{\mathrm{skilled}} + 2c\mathrm{d}_{\mathrm{skilled}})}{\alpha bB\mathrm{P}\left(\mathrm{d}_{\mathrm{skilled}} - \mathrm{d}_{\mathrm{inept}}\right) + 4c\mathrm{d}_{\mathrm{inept}}\mathrm{d}_{\mathrm{skilled}}} \qquad (66)$$

For convenience we combine the disutilities by a factor q, that determines how much more disutility the inept agent has to face as a multiplication factor.

$$d_{inept} = q\ d_{skilled} \text{ with q > 1} \tag{67}$$

and get the set of results

$$E = \frac{\mathrm{Bqd_{skilled}(b\ \alpha\ P-2\ d_{skilled}\ )-B\ b\ \alpha\ d_{skilled}(P-1)}}{\mathrm{B b\alpha(q\ P\ d_{skilled}-P\ d_{skilled}+d_{skilled})-4cqd_{skilled}}^2} \tag{68}$$

$$e_{skilled} = \frac{qb\ \mathrm{d_{skilled}}(\alpha B\ - B\ + 2c)}{\alpha bBP\mathrm{d_{skilled}}(1-\mathrm{q}) + 4cq\mathrm{d_{skilled}}^2} \tag{69}$$

$$e_{inept} = \frac{b\mathrm{d_{skilled}}(\alpha B - B + 2c)}{\alpha bB\mathrm{Pd_{skilled}}(1-\mathrm{q}) + 4c\mathrm{qd_{skilled}}^2} \tag{70}$$

As the salary “b” is the same for the skilled and inept agent, we get:

$$e_{skilled} = \mathrm{q}\ e_{inept} \tag{71}$$

For P = 1 and q=1 equations ( 68 ) ( 69 )and( 70 ) evolve into the solutions ( 40 ) and ( 41 ) where the type of the agent is known.

To see how the behavior of the principal changes, when she presumes that an inept agent will appear we need to compare equation ( 68 ) with ( 40 ).The effort that a principal puts into a project where an inept agent is likely to appear will be different from the effort when she knows the type of the agent:

The effort of the principal knowing that the agent is skilled (one type) was ( 40 ):

$$E_{type\ of\ agent\ known} = -\frac{B(2d - \alpha \mathrm{b})}{\alpha B\mathrm{b} - 4cd} \tag{72}$$

This needs to be compared to the effort when she is not sure about the skills of the agent (equation ( 68 )):

We set d= $d_{skilled}$ and therefore assume that the agent in the original model ( 72 ) is equal to the skilled agent.

$$E_{2\ types\ of\ agent} = \frac{\mathrm{Bqd(b\,\alpha\,P-2\,d\,)-B\,b\,\alpha\,d(P-1)}}{\mathrm{Bb\alpha(q\,P\,d-P\,d+d)-4cqd^2}} \qquad (73)$$

The difference of equation ( 72 ) and equation ( 73 ) amounts to:

$$E_{2\ types\ of\ agent} - E_{type\ of\ agent\ known} = \frac{2\,b\,B\,d\,\alpha(B\ -\ 2\,c)\ \ (P-1)\,(q-1)}{(4\,c\,d\ -\ b\,B\,\alpha)\,(4\,c\,d\,q\ +\ b\,B\,\alpha\,(P-1\ -\ P\,q))} \qquad (74)$$

Building the quotient of ( 72 ) / ( 73 ):

$$\frac{E_{2\ types\ of\ agent}}{E_{type\ of\ agent\ known}} = \frac{(2\,d\ -\ b\,\alpha)\,(4\,c\,d\,q\ +\ b\,B\,\alpha\,(P\ -1\ -\ P\,q)\,)}{(4\,c\,d\ -\ b\,B\,\alpha)\,(2\,d\,q\ +\ b\,\alpha\,(P\ -1\ -\ P\,q)\,)} \qquad (75)$$

For P = 1 and q=1 it amounts to one.

### 4.1.8 Constraints on the solutions

In chapter 3.4.6, “Constraints on the solutions” we derived inequalities that the parameters have to fulfill in order to restrict the effort of the principal and the agent between zero and one.

We recall constraint ( 43 ):

$$B < 2c \qquad (76)$$

For B=2c we get E= 1 in equation ( 72 ) AND ( 73 ), hence this inequality has to be imposed also for the case of two agents.

The role of the disutility parabola factor *c* is to limit the effort of the principal. If she would feel no disadvantages, she would supervise the whole time. The same holds if the benefit *B* is so high that she has too much on stake. Equation ( 76 ) limits her effort for the case of equation ( 72 ) and ( 73 ).

### 4.1.9 How does the effort of the principal change with the probability that the agent is skilled?

Depending on her assumed probability of meeting the agent, the principal will change her effort to supervise the agent and the Nash equilibrium changes.
To see how the effort changes, we perform the derivative of equation ( 73 ) with respect to *P*:

$$\frac{dE_{2\,types\,of\,agent}}{dP} = \frac{2\ \mathrm{b\ B\ d\ q}\ \alpha\ (\mathrm{B} - 2\ \mathrm{c})\ (\mathrm{q}-1)}{[4\ \mathrm{c\ d\ q} + \mathrm{b\ B}\ \alpha(\mathrm{P} - 1 - \mathrm{P\ q})]^2} \qquad (77)$$

Using inequality ( 76 ) and the fact that b, B, d and α > 0 and q > 1 the numerator of equation ( 77 ) gets negative. The denominator is positive as it is a squared number. So we arrive at:

$$\frac{dE_{2\,types\,of\,agent}}{dP} \leq 0 \qquad (78)$$

This is reasonable, as we expect that the principal puts less effort into the project, when the probability is high that the agent is skilled.

We visualize the dependence of the principal's effort on the probability to meet a skilled agent for the following parameter set ( 73 ):

| | |
|---|---|
| b=20; | (*Private benefit b of the agent *) |
| B=50; | (*Benefit of principal*) |
| α=0.8; | (*alpha congruence parameter of principal*) |
| PSkilled=0.5; | (*Probability to face a skilled agent*) |
| dSkilled=d=30; | (*disutility parabola factor of skilled agent d>0*) |
| dInept = 90; | (*disutility parabola factor of inept agent d>0*) |
| q=3; | (*multiplication factor dInept=q dSkilled*) |
| c=30; | (*disutility parabola factor of principal c>0*) |

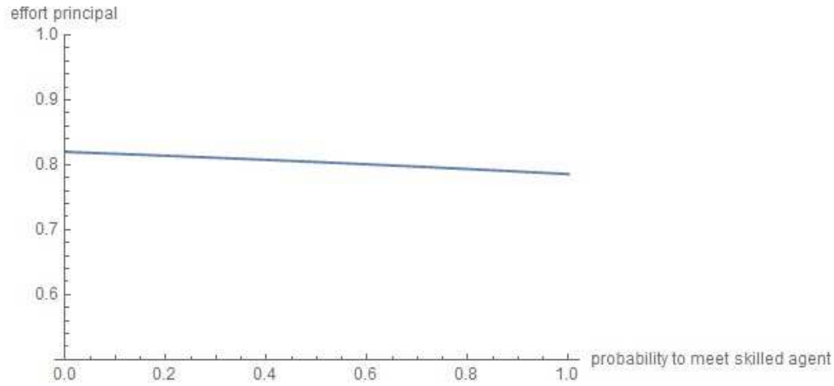



**Fig. 21 Visualization of equation ( 73 ): The effort of the principal is less when it is likely to meet a skilled agent**

The effort of the principal falls linearly with the probability to meet a skilled agent. Equation ( 77 ) denotes the slope in Fig. 21 of equation ( 73 ).

#### 4.1.10 How does the effort of the principal change when the disutility of the inept agent gets higher

The ability of the agent is defined by his disutility. The higher the disutility of the agent the less qualified the agent is.

We re-formulated the solution ( 64 ) of the principal's effort using $d = \frac{d_{inept}}{q}$ with q > 1 to arrive at equation ( 68 ), respectively ( 73 ). The higher the disutility, the less likely it is that the agent will succeed. He will put less effort into learning about the project and that shall affect the principal's behavior.

We will now investigate how the effort of the principal changes, if she expects the disutility of the inept agent to be high, i.e. if q is rising.

The derivative of equation ( 73 ) with respect to q leads to:

$$\frac{dE_{2\ types\ of\ agent}}{dq} = \frac{2\ b\ B\ \alpha\ d\ (B-2\ c)\ (P-1)}{[4\ c\ d\ q + b\ B\ \alpha\ (P-1-P\ q)]^2} \qquad (79)$$

Using again $B < 2c$ P $\in$[0,1], and the fact that the quadratic denominator is positive leads to:

$$\frac{dE_{2\ types\ of\ agent}}{dq} \geq 0 \qquad (80)$$

This is also reasonable, as we expect that the principal will put more effort in supervising an agent who does not like his work. She will not really trust an un-motivated agent.

We visualize this fact for the following parameter set:

| | |
|---|---|
| b=20; | (* private benefit b of the agents *) |
| B=50; | (*Benefit of principal*) |
| α=0.8; | (*alpha congruence parameter of principal*) |
| P=0.1; | (*Probability to face a skilled agent*) |
| dSkilled=d=30; | (*disutility parabola factor of skilled agent d>0*) |
| c=30; | (*disutility parabola factor of principal c>0*) |

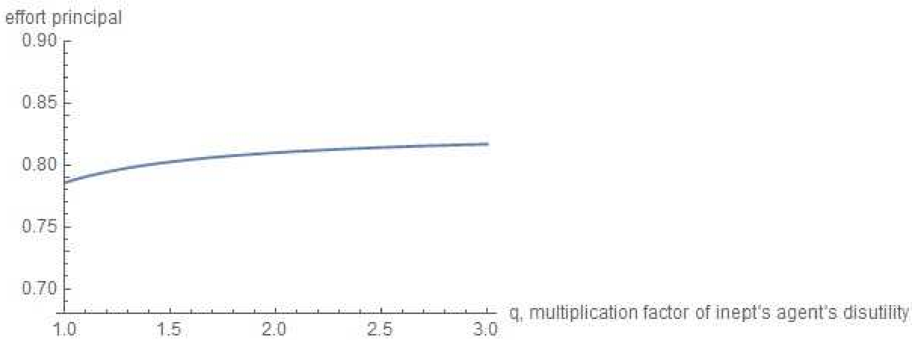



**Fig. 22 Visualization of equation ( 73 ): Effort of principal in dependence of q, which is the multiplication factor of the disutility of inept agent. dInept = q dSkilled**

We see in Fig. 22 that the principal supervises more when the disutility of the agent is high.

The inept agent, due to his higher disutility puts less effort into the project (equation ( 71 )). As a reaction to that the principal has to put more effort herself to get informed.

### 4.1.11 Influence on the effort of the agent

The effort of the skilled agent is:

$$e_{skilled} = \frac{(1 - \mathrm{E})\mathrm{b}}{2d} \qquad (81)$$

We see in equation ( 81 ), that when E rises due to the doubts of the principal as derived above, the skilled agent reduces his effort in the Nash equilibrium.
So only the principal's knowledge of maybe meeting an inept agent with probability *P* influences the skilled agent to lessen his effort!

The inept's players best response taking into account the principals reaction is ( 71 ):

$$e_{inept} = \frac{e_{skilled}}{q} \qquad (82)$$

As q>1 the effort of the inept agent is smaller than the one of the skilled agent.

We can write the effective value of the effort the principal expects as:

$$\begin{aligned} \bar{e} = Pe_{skilled} + (1 - P)e_{inept} &= \frac{(1 - \mathrm{E})\mathrm{b}}{2qd}[1 - \mathrm{P} + \mathrm{Pq}] \\ &= e_{skilled}\frac{[1 - \mathrm{P} + \mathrm{Pq}]}{\mathrm{q}} \end{aligned} \qquad (83)$$

The expectation value rises linearly with P, the probability that the agent is skilled and declines with 1/q compared to the case where the agent is skilled for sure.

So the effective value the effort the principal expects from the agent gets smaller due to the two effects:

1. The supervision of the principal E gets higher and
2. The disutility $\mathrm{d}_{\mathrm{inept}}$ of the inept agent is bigger by a factor q.

If we denote the effort of the agent in the original setup, where it is sure that the agent is skilled with $e_{OneType}$, we get the important result:

$e_{OneType} > e_{skilled} > e_{inept}$ ( 84 )

Even though, the first two types are both the skilled one and have all parameters in common, the fact that the boss monitors more when she expects to meet an inept agent makes the skilled agent perform worse.

### 4.1.12 Visualization of the comparison Agent's ability is known vs. agent's ability is not sure

In the following we visually compare the case where the principal knows the ability of the agent with our modified model where the ability of the agent is not sure.

The two cases to compare are:

case 1. One type of agent appears with disutility *d*. (skilled agent)

case 2. It is not sure which type of agent will appear. Additionally to the skilled agent with the same disutility *d* than in the first case, a second agent is likely to appear with probability P and disutility $d_{inept} = q\ d_{skilled}$ with q>1.

We visualize the result in a plot Fig. 23, where we depict

1. The reaction curve of the principal when she does not know the ability of the agent (case 2, solid red line)
2. The reaction curve of the principal when she knows the agent is skilled (case 1, dashed red line)
3. The reaction curve of the skilled agent (same for both cases, green line)

Remark: The reaction curve of the skilled agent is independent of the possibility that an inept agent can appear, so it is the same line in both cases. This is because equation ( 62 ) and equation ( 39 ) are the same.

We depict this result in Fig. 23 where both graphical Nash equilibriums are overlaid in one graph.

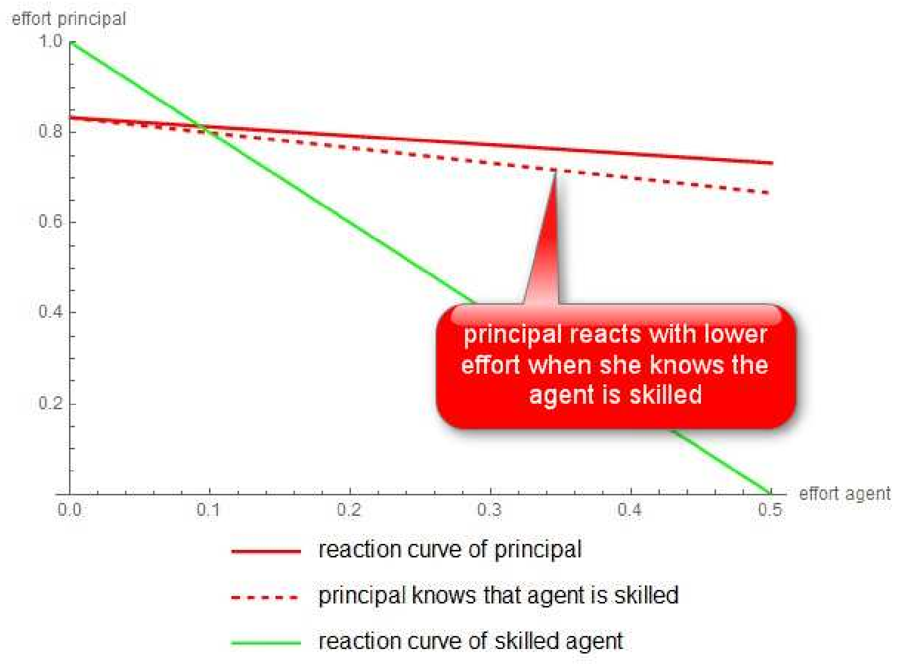



Fig. 23 When the principal knows that the agent is skilled her reaction curve (dashed red) lies below the reaction curve where she takes into account that the agent could be inept (solid red).

In Fig. 23 we see clearly how the intersection of the Nash equilibria changes from the situation where the principal knows the agent is skilled (dashed line – green line) to the situation where she does not know the ability of the agent she is facing (solid red line – green line).

**Chapter summary:**

Answer to research question 1

Aghion & Tirole  (Aghion & Tirole, 1997) calculated the case where the type of the agent is known to the principal. We extended the model in a way that a less suited or inept agent could appear and the principal has an assumption of the probability that the agent is skilled or not. She will therefore change her behavior and supervise more than when she was sure to meet a skilled agent.

We investigated the parameters and could show that she rises her effort when there is a non-zero probability P that an inept agent will appear and she rises her effort when the disutility of the inept agent rises.  Comparing the case where it is sure that the agent is skilled to the case where is likely that the agent is inept, we find that also the performance (effort) of the skilled agent declines due to more supervision effort of the principal.

## 4.2 Delegation by risk-averse principal

Risk averse principals behave more cautious than the risk neutral ones. They tend to secure their payoffs even when their benefit becomes lower. Their marginal utility is diminishing, meaning that the first unit of benefit is valued more than the subsequent. Following the theory in chapter 2.1.2, "How to take risk into account?" we need to change the utility function of the risk neutral principal to the concave utility function of a risk averse principal.

**Introducing a logarithmic utility function**

A popular form of modeling risk averseness is the logarithmic function as it shows a constant relative risk aversion. The relative risk aversion is the same at every wealth level.

$$u_c(x) = const * ln(x) \qquad (85)$$

For the risk averse principal we need to modify the two cases of finding the solution herself and the agent finding the solution according to equation ( 85 ).

### 4.2.1 Utility function of the principal

The original utility function of the principal ( 33 ):

$$\mathrm{u_P} = B\mathrm{E} + \alpha B\mathrm{e}(1 - \mathrm{E}) - c\mathrm{E}^2 \qquad (86)$$

is composed of the part where she finds the solution herself and gets her prefered payoff B, the part where the agent finds the solution and she gets αB and the "disutility" which stems from the effort she has to invest into the project herself and is deducted from the gross payoff.

case 1) The principal's net-payoff *x* when she finds the solution herself is:

$$x = B - c\, E^2 \quad (87)$$

E is the probability that she finds the solution herself and also her own effort put into the project normalized to one.
c is the "disutility parameter" of the principal

The utility of the risk averse principal for case 1 where she finds the solution herself is:

$$u_1(E) = \ ln(B - c\, E^2) \quad (88)$$

case 2) Her net-payoff when the agent finds the solution is:

$$x = \alpha B - c\, E^2 \quad (89)$$

The utility of the risk averse principal for case 2 where the agent finds the solution is:

$$u_2(E) = \ ln(\alpha B - c\, E^2) \quad (90)$$

The overall utility is the composition

$$u = \ probability_{case1} * \ utility_{case_1} + \\ + probability\_case2 \ * \ utility\_case\_2 \quad (91)$$

The probability that she finds the solution is proportional to her effort and is
$probability_{case1} = E$
In the second case we have the joint probability that the principal does not find the solution herself with (1-E) and that the agent gets informed with probability e.
$probability_{case2} = \ \ e(1 - E)$

which leads us to the utility function of the risk averse principal:

$$u_P(E) = E\, ln(B - c\, E^2) + e(1 - E)\, ln(\alpha B - c\, E^2) \quad (92)$$

The principal is risk averse to income loss. To get a feeling about the behavior of the utility we visualize how the utility function depends on the gross payoff B and the net payoff ($B\text{-}cE^2$; in case of congruence $\alpha = 1$).

First we elicit that the utility function is concave also with respect to the gross benefit B:

B = 40; (*Benefit of principal *)
α = 0.5; (*alpha congruence parameter of principal*)
c = 20; (*disutility parable factor of principal c>0*)
ep = 0.5 ; (* effort of principal E *)
ea = 0.7 (* effort of agent e *)

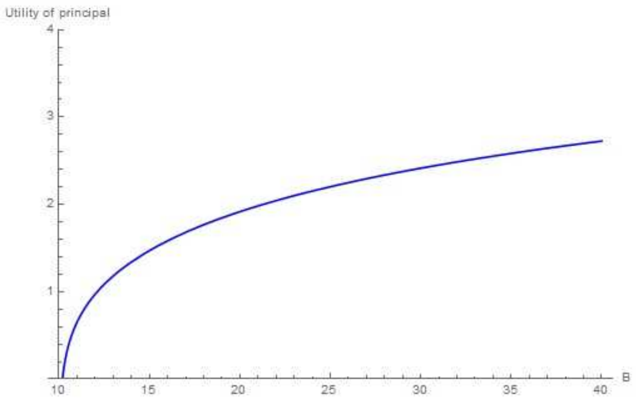



**Fig. 24 Utility function of the risk averse principal in dependence on the principal's benefit B. (concave)**

With regard to the net benefit $B\text{-}cE^2$ of the principal, in case $\alpha = 1$, the plot looks like:

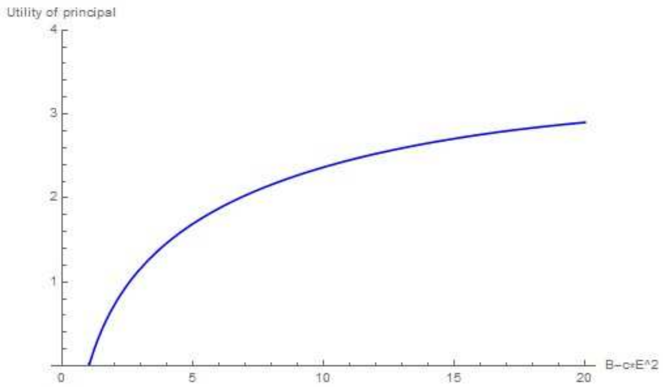



**Fig. 25 Utility function of the risk averse principal in dependence on the principal's net benefit $B\text{-}c{*}E^2$ , with $\alpha = 1$.**

Thus, the utility function equation ( 92 ) has the desired characteristics of a utility function of a risk averse person.

### 4.2.2 Reaction curve of the principal

To get the reaction curve we need to find the maximum of the utility function ( 92 ).

$$\frac{\partial \mathrm{u_P}}{\partial \mathrm{E}} = ln(B - c\,\mathrm{E}^2) - \frac{2cE^2}{B - cE^2} - e\,ln(\alpha B - c\,\mathrm{E}^2) + \frac{2\,c\,e\,E(E-1)}{\alpha B - c\,\mathrm{E}^2} = 0 \qquad (93)$$

For a maximum the second derivative has to be negative:

$$\frac{\partial^2 {u_P}^2}{\partial E^2} = \frac{-4\,c^2E^3}{(B - c\,E^2)^2} - \frac{6\,\mathrm{c\,E}}{\mathrm{B - c\,E^2}} + \frac{4\,\mathrm{c^2 e\,(E-1)E^2}}{(\mathrm{c\,E^2 - B}\,\alpha)^2} - \frac{2\,\mathrm{c\,e\,(E-1)}}{\mathrm{c\,E^2 - B}\,\alpha} - \frac{2\,\mathrm{c\,e\,E}}{\mathrm{c\,E^2 - B}\,\alpha} + \frac{2\,\mathrm{c\,e\,E}}{-\mathrm{c\,E^2 + B}\,\alpha} = \mathrm{neg.}$$

-> it is a maximum, as E-1 <0 and B<2c (equation ( 76 ))

### 4.2.3 Utility function and reaction curve of the agent

The agent has the same utility function equation ( 36 ) and the same reaction curve, equation ( 37 ) as in the original model of chapter 3.

### 4.2.4 Graphical solution of Nash equilibrium

For plotting the numerical solution, we need to bring equation ( 93 ) into an explicit form. This time we cannot find an explicit representation for E, but we can find one for e. So the plot for the Nash equilibrium is just rotated by 90° in comparison to the Nash equilibrium we found for the original model depicted in Fig. 20.

The system of the reaction curves to graphically solve is:

Risk averse principal (equation ( 93 )):

$$e = \frac{(c\,E^2 - B\,\alpha)\,[-2\,c\,E^2 + (B - c\,E^2)\ln(B - c\,E^2)]}{(B - c\,E^2)\,[-2\,c\,E + 2\,c\,E^2 + (c\,E^2 - B\,\alpha)\ln(-c\,E^2 + B\,\alpha)]} \qquad (94)$$

Agent’s reaction curve (equation ( 37 )):

$$\mathrm{e} = \frac{b(1 - \mathrm{E})}{2d} \qquad (95)$$

For comparison, we recall the reaction curve ( 35 ) of the risk neutral principal:

$$e = \frac{(\mathrm{B} - 2\,\mathrm{c}\,\mathrm{E})}{\alpha\mathrm{B}} \qquad (96)$$

Fig. 26 shows the comparison of the graphical solution of the risk neutral principal and the risk averse principal’s Nash equilibrium.

Parameter set

b = 40; (*Private benefit b of the agent *)

B = 40; (*Benefit of principal *)

α = 0.5; (*alpha congruence parameter of principal*)

c = 30; (*disutility parable factor of principal c>0*)

d = 25; (*disutility parable factor of agent d>0*)

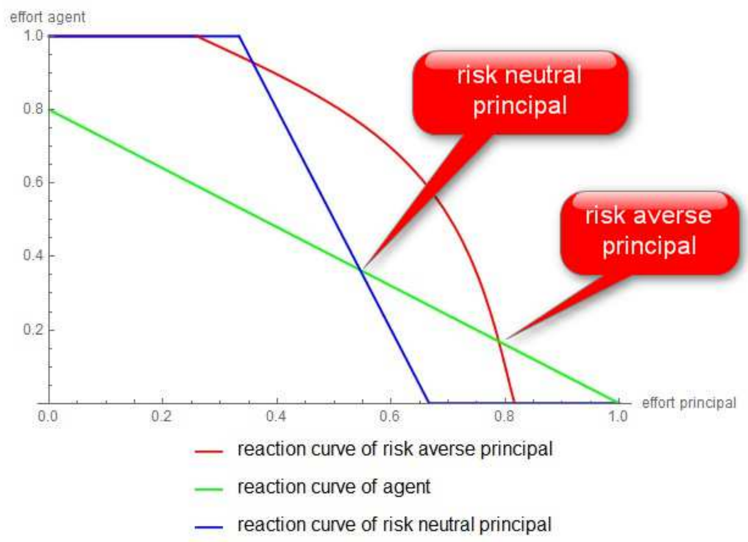



**Fig. 26 comparison of Nash equilibrium in case of risk neutral principal and risk averse principal.**

We see that the effort of the risk averse principal is higher than for the risk neutral principal. This leads to a reduction in the effort of the agent.

The work performance of the agent goes down.

**Chapter summary:**

Answer to research question 2

We modified Aghion and Tirole's model in a way that the principal is risk averse with regard to her net payoff. The utility function of the principal was chosen to be the natural logarithm. The utility function of the agent is the same as in the original model. The system of reaction curves is solved graphically, as the reaction curve of the principal cannot be expressed in explicit form regarding her effort E. Fig. 26 shows the graphical solution of the Nash equilibrium.

The agent puts more effort into his work for a risk neutral principal than for a risk averse principal. This is due to the fact that the risk averse principal supervises more.

# 5 Considerations on applications of the model

Our model has many simplifications with regard to real-world situations. Nevertheless we can show trends that are well realized in several principal agent interactions. We will show examples where companies base their success on the mechanism identified in this work.

First we put our results into an overview:

| **Ability of agent not known** | **Principal is risk averse** |
|---|---|
| Probability that agent is inept ↑ | same parameter set as for the risk neutral principal |
| effort of principal E ↑ | effort of principal E ↑ |
| effort of agent:<br>$e_{OneType} > e_{skilled} > e_{inept}$ | effort of agent: e ↓ |
| remark: all parameters other that the probability P are the same for $e_{OneType}$ and $e_{skilled}$. | |

**Table 4: Overview of results**

legend: ↑ is rising / goes up

↓ is declining / goes down

$e_{OneType}$ is the effort when the agent is skilled for sure

$e_{skilled}$ is the effort when the agent is skilled with probability P

$e_{inept}$ is the effort when the agent is inept with probability (1-P)

The mechanism is the following: In a game where the agent is skilled for sure, the effort of the agent in the Nash equilibrium is $e_{OneType}$. When the game changes and the principal counts on a non-zero probability that the agent is inept, the principal raises her effort. The skilled agent knows that in that case that he gets overruled more often. This lowers his initiative. The Nash equilibrium shifts. This is reasonable for the principal with regard to supervising the inept agents, as his effort is only $e_{inept} = \frac{e_{skilled}}{q}$ . But the high potential employees will bring worse results as before.

The same happens when the principal becomes risk averse. Her employee lowers his effort.

As the employee has a fixed salary, lowering his effort is equivalent to raising the agency costs. Especially for high innovative firms the most skilled employees are driving their success. Their effort is vital for being leader in technology.

In this model effort is unobservable or non-contractable. Given all the information, the principal can anticipate what agent will do but she never can see it directly. So it is

useless to write a contract to claim a certain effort, as the agent will act according to the Nash equilibrium anyway, regardless of the contract.
If we think about the prisoner's dilemma as an example, it can be rational and wealth-improving to deviate from the Nash equilibrium. The problem is how to sustain a better situation?

**Lowering supervision effort**

For companies which depend on motivating their top performers as much as possible, we favor the following strategy:
The principal can commit herself to lower her supervising effort.
One crucial feature of an incentive model is that when the principal raises her effort, the reaction of the agent is always to lower his effort. This can be seen in the following way: In all three models (original model, ability not known, risk avers principal) the reaction curve of the principal is downward sloping in the e - E diagram. The reaction curve of the agent is exactly the same (equation ( 37 )) in all three cases and also downward sloping. If the principal puts more effort E into supervising, the slope of her reaction curve gets smaller; the curve is shallower and therefore hits the reaction curve of the agent at a point where the agent invests less effort. (Fig. 18, Fig. 23, Fig. 26). In the reaction curve of the agent, equation ( 37 ), it can be directly seen that the effort of the agent rises when the effort of the principal is lowered.
In all three models the utility of the principal, equation ( 33 ), ( 51 ) and ( 92 ) rise linearly with the effort of the agent. So the principal is better off in committing to lower her supervision effort.
The advantage is that the decrease in supervising effort of the principal is observable to the agent. So this contract is meaningful as it is verifiable and credible to the agent.

There are examples of top companies following this strategy with success. We mention three outstanding examples:

## 1. Google Research

Google is one of the most innovative companies at the present time. They develop leading edge technology products as autonomous cars, the Google watch, internet-connected eyeglasses, and others. Google publishes hundreds of research papers each year. And last but not least, they are the standard internet search machine.

Google does not only limit the supervision of researcher, but gives them the freedom to use a percentage of their time to pursue whatever they want.

When it comes to concrete task assignments, Eric Schmidt the CEO states: “Let your followers own the problems you want them to solve” (Human Resource Management Issues at Google, 2011)

The credo at Google is: "Before joining, the advantages of Google Research were obvious in terms of scope and scale. But once here, you realize there's way more to it: you get to influence great products by working with great engineers, while having the freedom to do research alongside some of the world experts in machine learning." (Google, 2016).

Does this strategy work?

Google publishes hundreds of research papers each year. The list of Google products is very long and getting longer every year.

The financial situation of Google Inc. shows a positive trend in sales and profit:

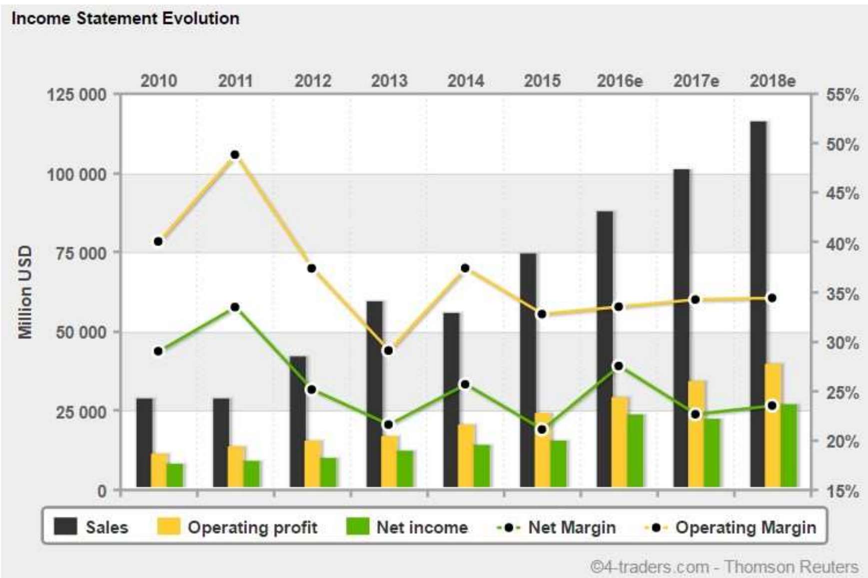



**Fig. 27 Income Statement Evolution of Google, retrieved from 4-traders.com**

## 2. Formula 1

The technical performance of the Formula 1 racing car is the crux of the matter for winning the race. In a Business Evening "Formula 1 - what's the story behind the design?" with Mr. Willem Toet, motorsport, F1 and aerodynamics specialist, we had the pleasure to look behind the scenes of success in motorsport. (Toet, 2016)

Mr. Toet gave an excellent technical overview which was supplemented by the appreciation of chief designer Rory Byrne. “Every time one of the managers called Rory totally upset due to the latest loss, Rory would just absorb all the screaming and shouting and would not pass this pressure and interference over to the development team.”

Car development in Formula 1 is a high pressure task. Nevertheless, the success of the team is not boosted by increasing the pressure with short term instructions, but by focusing on the long run perspective and giving the team the freedom to develop excellent cars rather than making them feel guilty about the latest loss.

## 3. Semco

Semco is a Brazilian manufacturing company. They are in the field of development and manufacturing of agitators, mixers, dispersers and mills, providing solutions for the mining, chemical, petrochemical, pulp, paper, sugar, alcohol, food, beverage and other industries.

They fosters individual involvement of their employees to a very great extent.

In the eighties Semco was already on the verge of a financial disaster.

Then they drastically changed their way of management and supervision. The employees can decide about their working hours, they have access to the company books and the majority vote on many important corporate decisions. (Charness, Cobo-Reyes, & Jiménez, 2012)

Semco has three fundamental values: democracy, profit sharing, and information

Ricardo Semler , CEO of Semco explains the reason: (Semler, 1989)

“In an immense production unit, people feel tiny, nameless, and incapable of exerting influence on the way work is done or on the final profit made. This sense of helplessness is underlined by managers who, jealous of their power and prerogatives, refuse to let subordinates make any decisions for themselves—sometimes even about going to the bathroom."

And he explains the strategy:
"We are very, very rigorous about the numbers. We want them in on the fourth day of the month so we can get them back out on the fifth. And because we're so strict with the financial controls, we can be extremely lax about everything else. Employees can paint the walls any color they like. They can come to work whenever they decide. They can wear whatever clothing makes them comfortable. They can do whatever the hell they want. It's up to them to see the connection between productivity and profit and to act on it."

Does this strategy work?
According to Ricardo Semler: "Close to financial disaster in 1980, Semco is now one of Brazil's fastest-growing companies, with a profit margin in 1988 of 10% on sales of $37 million."
Semco has a constant grow until now.

We could add more examples where the management is committed to reduce the supervision and monitoring of teams in a way that the collective performance of the employees rises drastically. As our mathematical model is very limited in its complexity, we need to emphasize that lowering the supervision effort is only one measure embedded in a set of corporate provisions. Each company acts in a different environment with many more parameters and has to setup their company policy in accordance with their specific situation safeguarding their financial success.
Generally we see the trend that giving up control can lead to a great win-win situation.

# 6 Summary

This thesis investigates the consequences in the principal-agent relationship if a principal does not know the ability of her agent, and secondly investigates a situation with a risk averse principal.

In the case where the principal is not sure to meet a skilled agent, we model the ability of the agent in a way that the skilled agent feels little disutility for his work whereas for the inept agent the disutility is significantly higher.

Our results indicate that not knowing the ability of the agent increases the effort of control from the principal, thereby decreasing the effort of the skilled agent to fulfill his task. The skilled agent gets demotivated from being overruled too often from his principal.

The risk averse principal is modeled with a logarithmic utility function. As a result of her risk averseness the principal raises her supervision with regard to the risk neutral principal. As a consequence the agent lowers his effort invested in trying to find the required task-information.

Both situations had a negative impact, as the effort of the principal rises and at the same time the effort of the agent declines.

As a possible improvement we favor that the principal commits herself to decrease her supervision, leading to a more profitable situation for both.

The success of using the method of reducing supervising effort is described in three examples of Google, Formula 1 and Semco.

Managing innovation is an important challenge of modern business. One of the greatest challenge is to provide the environment for the best people to perform even better.

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# 8 Appendix

## 8.1 Appendix A - Variables and their interpretation

| | |
|---|---|
| $u_P$ | Utility function of the principal |
| $u_A$ | Utility function of the agent |
| $B_k$ [$] $\in \mathbb{R}$ | payoff/benefit that the principal gets for each project k. |
| B [$] >0 | The principal's most profitable project, or the solution with the highest payoff for her. |
| $b_k$ $\in \mathbb{R}$ | Private benefit of the agent (psychological factors like appreciation, perks or promotion). His wage is constant and can therefore be set to zero. |
| b [$] >0 | The project with best benefit for the agent. |
| $\alpha \in [0,1]$ | Congruence parameter seen from the principal's perspective. If the agent's most preferable decision is realized, she gets $\alpha B$ |
| $\beta \in [0,1]$ | Congruence parameter seen from the agent's point of view. If the principal's most preferable decision is realized, he gets $\beta b$ |
| e $\in [0,1]$ | "e" can be seen as the effort the agents puts into the game, normalized to one. Alternatively, it can be seen as the probability to acquire the required knowledge (the more effort, the more probable to acquire knowledge; probability $\propto$ effort). With probability e he learns the payoff of all candidate projects (decisions) With probability (1-e) he acquires no knowledge at all. (digital information). |
| $g_A(e)$ > 0 | $g_A(e)$ is called "disutility function" or cost of effort as it reduces the benefit. At private cost $g_A(e)$ he learns everything about the projects. |
| E $\in [0,1]$ | Probability that the principal acquires all knowledge about the candidate projects (decisions). Can be also seen as the normalized |

| | |
|---|---|
| | effort E to acquire complete knowledge. |
| $g_P(E)$<br>$>0$ | Disutility function / cost of effort of the principal, corresponding to the benefit reduction of the principal by supervising or controlling the agent.<br>She has to invest her private cost of effort $g_P(E)$ to acquire the knowledge about the candidate projects. |
| c<br>$> 0$ | parameter for parabolic disutility function of principal $g_P(E) = cE^2$ |
| d<br>$> 0$ | parameter for parabolic disutility function of agent<br>$g_A(e) = de^2$ |
| | **Ability of agent is not certain and can be can be skilled or inept:** |
| $u_{A\_skilled}$<br>$u_{A\_inept}$ | Utility function of the skilled / inept agent |
| P<br>$\in [0,1]$ | $P = P_{skilled}$ is the probability that the principal faces a skilled agent. Respectively with $(1 - P)$ she faces an inept agent. |
| $e_{skilled}$<br>$\in [0,1]$ | $e_{skilled}$ can be seen as the effort a skilled or talented agents dedicates to finding the solution normalized to one. Alternatively, it can be interpreted as the probability that he acquires the needed knowledge. |
| $e_{inept}$<br>$\in [0,1]$ | $e_{inept}$ is the effort the less qualified agent is willing to invest to find the answer. |
| $d_{skilled}$<br>$> 0$ | Disutility parabola factor of the skilled agent: $g_A(e_{skilled}) = d_{skilled}\, e^2_{skilled}$<br>At private expenses $g_A(e_{skilled})$ the agent learns the complete information needed. |
| $d_{inept}$<br>$> 0$ | Disutility parabola factor of the inept agent: $g_A(e_{inept}) = d_{inept}\, e^2_{inept}$<br>At private expenses $g_A(e_{inept})$ the agent learns the complete information needed. |
| | $d_{inept} > d_{skilled}$ |

## 8.2 Appendix B - Wording

As different literature uses different words with slightly different meaning, we give a table of the wording used in this thesis.

| Term | Usage in this thesis |
|---|---|
| player | examples: person, company, government |
| strategy | Courses of actions which the player may choose to follow (according to the rules of the game) |
| outcome | examples are:<br>I get 100€ or 200€;<br>I get coffee or tea;<br>I go to jail 5 years or 7 years;<br>The set of outcomes is denoted *O.* |
| payoff<br>(benefit, gain) | "profit" B for the principal<br>"benefit" b for the agent |
| preference relation | example player *i*:<br>I prefer tea over coffee:<br>tea $\succ_i$ coffee |
| utility function | A utility function u is a function associating each outcome x with a real number u(x) in such a way, that the more an outcome is preferred, the larger is the real number associated with it. |
| best response | The best response is the strategy that gives the best outcome to one player taking the other players' strategies as given |
| reaction curve | All of these best responses together are called reaction function or reaction curve in the continuous case. |
| Nash equilibrium | In the Nash equilibrium the players are playing a best responses to each other. No player can gain by deviating |

| | |
|---|---|
| disutility | = cost of effort<br><br>The “discomfort” for reaching the goal to acquire the information |
| complete information | A game where everything is known to everyone is called “game with complete information”. |
| Asymmetric information | Asymmetric information means that player B knows something that player A does not know. Player A has incomplete information. Player B has complete information. |
| P-formal authority (integration) | The principal has the formal authority. She can always overrule the decision of the agent. |
| A-formal authority (delegation) | The decision of the agent cannot be overruled by the principal. He has the right to decide. |