Institut für Höhere Studien
Institute for Advanced Studies
Vienna

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Continuing Education Center

MSc Economics

Testing Efficiency of Stock Markets and Their Implications for Predictability of Stock Returns

A Master's Thesis submitted for the degree of
"Master of Science"

supervised by

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Vienna, June 2017

MSc Economics

Affidavit

I, Shahabeddin Gharaati

hereby declare

that I am the sole author of the present Master's Thesis,

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25 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

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Abstract

The purpose of this paper is testing efficiency of the Germany and UK stock market. German DAX and FTSE 100 stock market data show that the stock returns tend to have a leptokurtic probability distribution rather than normal distribution. Moreover, the stock returns first and second serial correlation tends to increase during the crisis which implies that when the market is not efficient the predictability of stock return increase, and investors get the opportunities to gain profits. Applying a test proposed by Gibbons et al. (1989) on all assets of Germany DAX with 110 months return data over the period August 1999 to April 2017 shows that the null hypothesis of efficiency is rejected at the periods of the crisis. Furthermore, there is a weak positive correlation between a twelve-month moving average P-Values of the test and excess returns of long/short equity strategies over the period of January of 2006 to December of 2011. On the other hand, Applying a test proposed by Pesaran and Yamagata (2012) on all assets of FTSE 100 with 60 months return data over the period February 2000 to April 2017 shows that the null hypothesis of efficiency rejected at the periods of the crisis. Furthermore, there is a weak positive correlation between a twelve-month moving average P-Values of both tests and excess returns of long/short equity strategies over the period of January of 2001 to December of 2006.
Keywords: Efficient Market Hypothesis, Predictability of stock markets, long/short equity strategies, moving average P-Values, German DAX, FTSE 100, leptokurtic distribution

1 Introduction

First of all, this paper is concerned with testing the efficiency of the Germany and UK stock markets by testing the time series implication of capital asset pricing model (CAPM) proposed by Sharpe (1964) and Lintner (1965) and Fama-French three factors model. The Sharp-Lintner CAPM demonstrates that if the market is efficient expected excess return of an individual asset only depends on excess market return and just coefficient of the excess market return is non-zero. In the second stage, the paper considers the relationship between the Long/Short trading strategies returns and market strategies. One of the empirical properties of the stock returns is the increasing predictability during the crisis periods, so it is predicted that there is an inverse relationship between the efficiency of the market and returns of the Long/Short Strategies, which means that if the market is not efficient the Long/Short strategy returns relative to the stock market return are higher, however, inefficient market there is no significant difference between market return and Long/Short strategy returns.

The paper contains seven sections. In the second section, the paper proposes some stylized fact about the stock market returns. Financial economists have long been interested in variations of the stock market returns. Fama (1970) suggesting that the random walk model can explain the behavior of the stock prices, and thus the stock returns are not predictable. In the second section we started with a statistical model with stock returns with standard normal innovation. Then we capture some statistical properties of the stock returns. First, the stock returns tend to have a leptokurtic probability distribution rather than normal distribution. Second, the stock returns first and second serial correlation tends to increase during the crisis. It means that the stock returns are more predictable during the crisis periods, or intuitively when the market is not efficient the predictability of stock return increase, and investors get the opportunities to gain profits. To test this hypothesis we need a theoretical framework for market efficiency and in the third section, we introduce efficient market hypothesis.

The efficient market hypothesis (EMH) was introduced by Samuelson (1965) through his works on random walk theory of asset prices. Samuelson proposed that when the market is efficient informationally then market returns are unenforceable. There are many versions of the efficient market hypothesis through a literature, so we use the simple framework proposed by Pesaran (2010). The main conclusion of this framework is that "market efficiency could coexist with heterogeneous beliefs and individual irrationality, so long as individual errors are cross sectionally weakly dependant." (Chudik et al. (2011))

In the fourth section, we introduce the empirical methods for testing efficiency of stock markets. In order to testing efficiency, we stick to the Sharp-Lintner CAPM model and Fama-French model for an individual portfolio. There is a large literature in empirical asset pricing for testing various implications of these models. Jensen (1968) was propose the t-statistics to test the null hypothesis that the intercept of the OLS regression of CAPM is zero which is the fine test for individual assets and portfolios. However, when the larger number of the assets are considered at the same time the interpretation of the result is hard. Moreover, when some error terms are dependent, so sometimes the size of the overall test in presence of individual correlated t-statistics is high and out of control. Gibbons et al. (1989) proposed a test in order to deal with this problem. This test, which is called GRS test, is one of the most common tests in the empirical asset pricing literature, and it is based on the assumptions the CAPM error terms are normal ant the number of the asset is lower than the period of the data that used for the regression. However, the GRS has some disadvantages. First, it is applicable for the small number of the individual of portfolios, typically between 20 to 30 portfolios, and it needs a long period observed data for regression. In order to solve these problems, Pesaran and Yamagata (2012) present multivariate Jensen statistics for a large number of the assets which can apply on relative short period.

In this paper we use German DAX and FTSE 100 stock returns. DAX and FTSE 100 data are downloaded from data stream database, and In section 5 the data and the method of calculation of returns is explained in details. Section 6 of the paper presents the empirical results. applying GRS test on all assets of Germany DAX with 110 months return data over the period August 1999 to April 2017 shows that the null hypothesis of efficiency rejected at the periods of the crisis. Furthermore, there is a weak positive correlation between a twelve-month moving average P-Values of GRS test and excess returns of long/short equity strategies over the period of January of 2006 to December of 2011. On the other hand, Applying the test proposed by Pesaran and Yamagata (2012) on all assets of FTSE 100 with 60 months return data over the period February 2000 to April 2017 shows that the null hypothesis of efficiency rejected at the periods of the crisis. Furthermore, there is a weak positive correlation between a twelve-month moving average P-Values of the test and excess returns of long/short equity strategies over the period of January of 2001 to December of 2006. In the lats section concluding remarks are stated.

2 Stylized facts about stock returns

2.1 Statistical Model of Returns

Suppose that the information set of $\Omega_t = \{R_1, \ldots, R_t\}$ is given. The log returns of an asset can be written as follows,

$$R_{t+1} = \Delta ln(P_{t+1}) = p_{t+1} - p_t = \mu_t + \sigma_t \epsilon_{t+1} \qquad t = 1, 2, , T$$

where μ_t and σ_t^2 is conditional mean and variance of returns. Moreover, ϵ_{t+1} is a random part of the returns and a distribution can be assigned to the ϵ_{t+1}. There are two famous distribution assigned to ϵ_{t+1} for modeling stock returns, standard normal distribution and t student distribution which can be written as follow,

$$\epsilon_{t+1}|\Omega_t \sim IID\, Z \qquad Z \sim N(0, 1),$$

$$\epsilon_{t+1}|\Omega_t \sim (\sqrt{\frac{\nu - 2}{\nu}}) IID\, T_\nu, \qquad T_\nu \sim \text{Student's t-distribution},$$

where T_ν has ν degree of freedom. If returns density is normal distribution then the probability density function can be stated as

$$f(r_{t+1}) = (2\pi\sigma_t^2)^{-1/2} exp[-\frac{1}{2\sigma_t^2}(r_{t+1} - \mu_t)^2]$$

where $\mu_t = E(r_{t+1}|\Omega_t)$ and $\sigma_t^2 = E[(r_{t+1} - \mu_t)^2|\Omega_t]$ are conditional mean and variance. Where the return process is stationary, then $\mu = E(r_{t+1})$ and $\sigma^2 = E[(r_{t+1} - \mu_t)^2]$. Skewness and kurtosis estimators

$$\hat{Skewness} = \sqrt{b_1} = \hat{m}_3 / \hat{m}_2{}^{3/2}$$

$$\hat{Kurtosis} = b_2 = \hat{m}_4 / \hat{m}_2{}^2$$

where

$$\hat{m}_j = \frac{\sum_{t=1}^{T} (r_t - \bar{r})^j}{T}, \qquad j = 2, 3, 4$$

If a distribution is normal then $\sqrt{b_1} = 0$, and $b_2 = 3$. For testing normality, the Jarque-Bera statistics proposed by Jarque and Bera (1980) can be used. This statistic given by

$$JB = T\{\frac{1}{6} b_1 + \frac{1}{24}(b_2 - 3)^2\}.$$

The joint null hypothesis is $b_1 = 0$ and $b_2 = 3$. The asymptotic distribution of the JB statistics is a chi-square with 2 degrees of freedom, χ_2^2. Thus, if a value of the statistics

Table 1: Descriptive statistics for monthly returns on FTSE 100 and German DAX

Variables	FTSE 100	DAX
Maximum	13.83	17.7
Minimum	-23.68	-29.16
Mean	0.45	0.79
S.D	4.58	6.16
Skewness	-0.76	-1.0
Kurtosis	5.64	6.05
JB Statistics	127.19	182.6
JB P-Value	2.2e-16	2.2e-16

is exceed 5.99 the null hypothesis will be rejected since, the JB would be statistically significant at the 95 percent confidence level.

2.2 Empirical Properties of Returns

In order to consider empirical properties of the returns in this paper *FTSE 100* and *DAX* stock retruns will be considered. The first two graphs of Figure 2.2 shows price index of the *FTSE 100* and German *DAX*, the third and fourth graphs of Figure 2.2 shows *FTSE 100* and German *DAX* stock returns.

Table 1 demonstrates descriptive statistics of the two stock returns. The kurtosis of two stock returns are over 3 and suggest that the distribution of the returns are not normal. The skewness of both returns are close to zero, however, there are pieces of evidence of negative kurtosis for both stock returns. The large values of excess kurtosis reflected in the JB statistics and the P-Value of the JB statistics are statistically significant at the 95% confidence level, which means hypothesis of normality will be rejected. Moreover, the assumption of the normality of returns implied that the maximum and minimum of the monthly returns with 99% confidence level in the region of $\pm 2.33 \times S.D$ which is [-10.22,11.13] for FTSE 100 and [-13.5,15.14] for German DAX. However, it can be seen that maximum and minimum of returns of both stock markets are not in the regions. (Figure 2.2 and Table 1) Regarding the above observations, the stock returns tend to departure from normality.

The stock returns which are uncorrelated to each other has two characteristics, they have fat-tailed distribution, and they are difficult to predict. However, the absolute value of the stock returns have a higher serial correlations. Figure 2.2 shows the first and second order of serial correlation over the time period of Jan-1990 to May-2017. As it can be seen the first order and second order correlation of the FTSE 100 over the period of Jan-100 to Jan2000 are -0.001 and -0.1765 respectively. It increases to the -0.1264 for the first order serial correlation and decreases to the -0.0993 for the second order of serial correlation. On the other hand, German Dax stock markets first order serial correlation is volatile and shows increases after the dot-com bubble and also there is increasing in absolute value

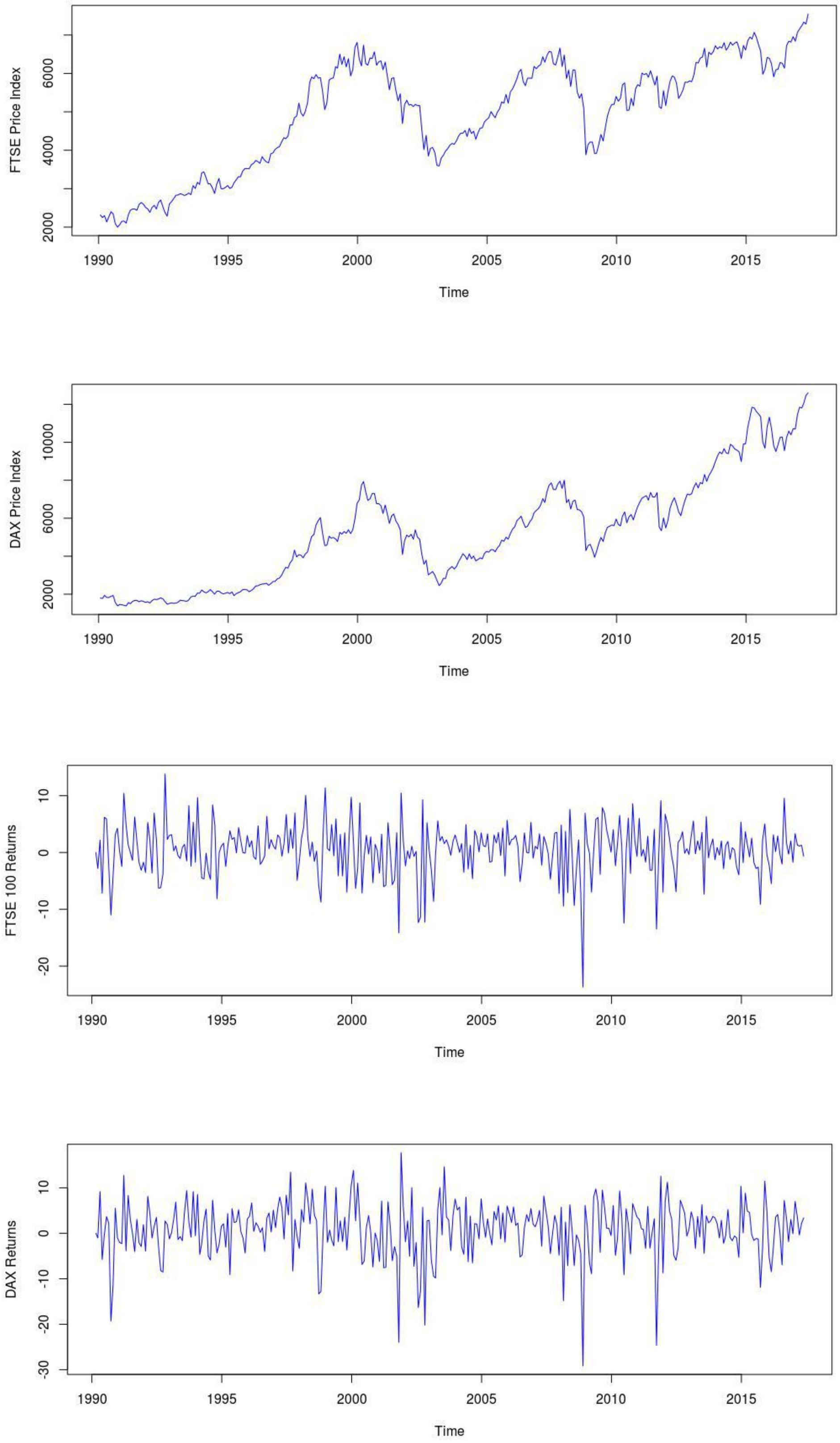


Figure 1: Monthly price index and monthly returns on FTSE 100 and German DAX over the period Jan-1990 to May-2017

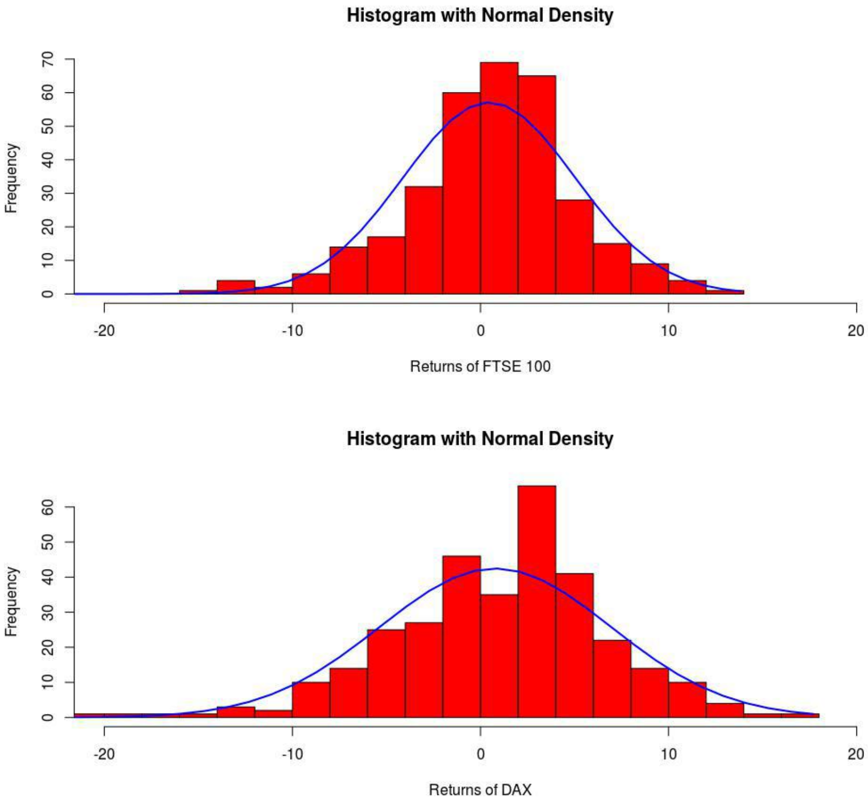


Figure 2: Histograms and normal density (blue solid line) for monthly returns on FTSE 100 and German DAX over the period Jan-1990 to May-2017.

of first order correlation of Dax after 2008 credit crunch. Therefore, increasing in serial correlation of the FTSE 100 and German DAX demonstrate in the crisis period the stock returns are mush more predictable.

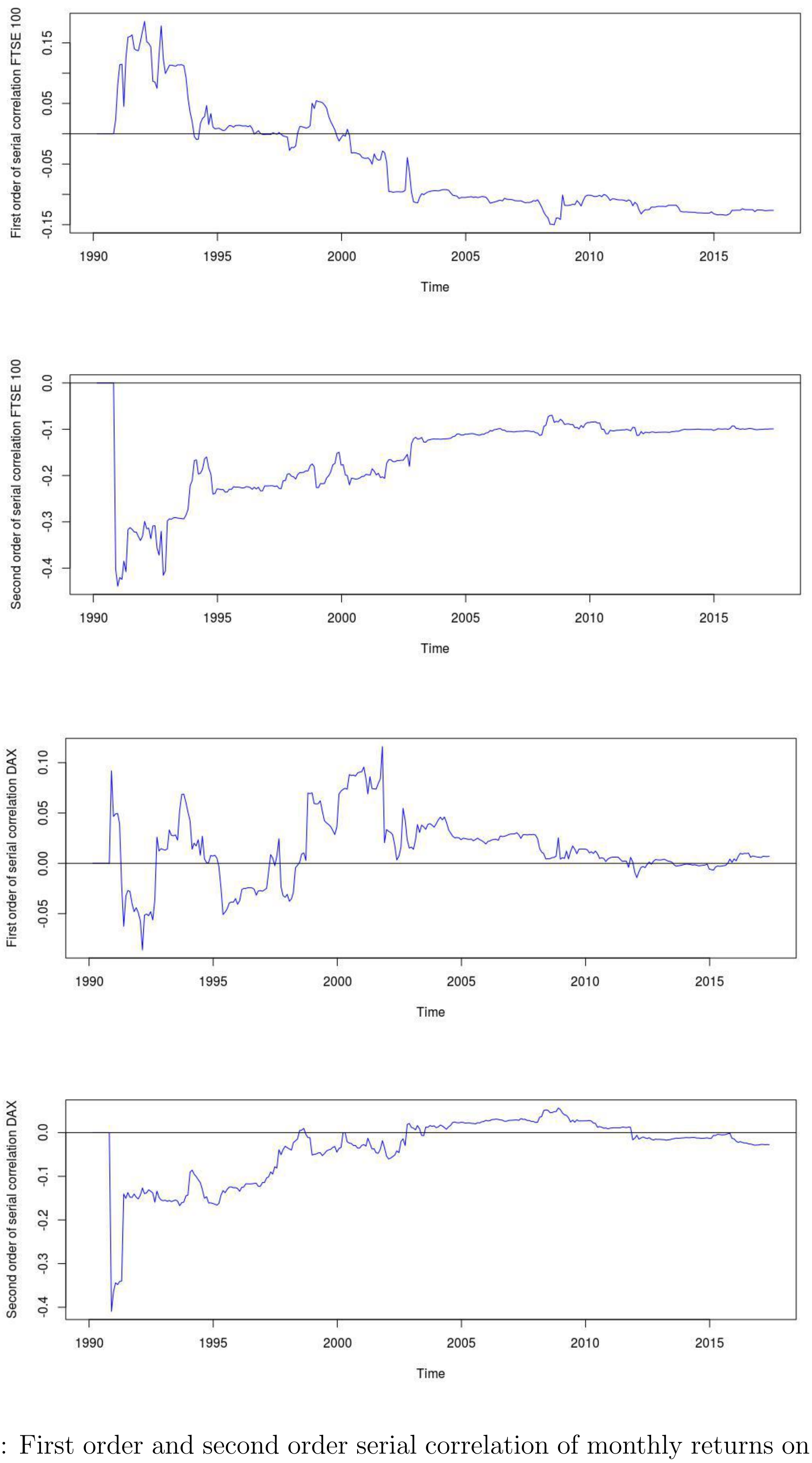


Figure 3: First order and second order serial correlation of monthly returns on FTSE 100 and German DAX over the period Jan-1990 to May-2017.

3 The concept of efficient market hypothesis(EMH)

The efficient market hypothesis is based on several assumptions. First, it is assumed that investors are rational and they permanently update their beliefs with newly available information. Second, Investors make their decision by calculation of subjective expected utility(Pesaran (2010, p.24)). Third, the difference in beliefs across investors cancel out in the market(Pesaran (2010, p.24)).

In order to construct a theoretical model for the efficient market hypothesis, assume that we start at a beginning of period t with N_t investors. Moreover, suppose that R_{t+1} is return of an asset and r_i^f is risk-free rate. The following condition hold for investor i, which is implied by the consumption-saving model of the risk-averse investor.

$$\hat{E}_i(R_{t+1} - r_t^f|\Omega_{it}) = \lambda_{it} + \delta_{it} \tag{1}$$

In above formula, $\hat{E}_i(R_{t+1} - r_t^f|\Omega_{it})$ is a subjective expectation of excess return, $R_{t+1} - r_t^f$, with respect to the information set

$$\Omega_{it} = \Psi_{it} \cup \Phi_t$$

where Φ_t is a pubic information and availble for every investors. The expectation of excess return is different among investors since the perceived conditional distribution of the $R_{t+1} - r_t^f$ is different. Moreover, the information set of investors, Ω_{it}, and risk preferences are different among the investors. Thus, we denote the expected return of investor i with the subjective expectation. Furthemore $\lambda_{it} > 0$ is a investor risk premium, and δ_{it} is trading cost per units of fund invested. If there is no trading cost in the model, then λ_{it} can be written in term of the utility function

$$\lambda_{it} = \hat{E}_i(R_{t+1} - r_t^f|\Omega_{it}) = \frac{-\hat{C}ov_i(m_{i,t+1}, R_{t+1}|\Omega_{it})}{\hat{E}_i(m_{i,t+1}|\Omega_{it})}$$

where $\hat{C}ov_i(\cdot|\Omega_{it})$ is a subjective conditional covariance on information set of the ith investor, and $m_{i,t+1} = \beta_i u_i'(c_{i,t+1})/u_i'(c_{i,t})$ is *stochastic discount factor*. In the stochastic discount factor the $u_i'(\cdot)$ is the firts derivative of the utility function, c_t is a real consumption expenditures during the periodt to t+1, and β_i is investor discount factor.

According to above explanation, since the difference in the conditional probability distribution, the difference in the information sets and difference in the risk preferences subjective expected excess return is different across the investors(Pesaran (2010, p.24)). However, by imposing rational expectation hypothesis it can be implied

$$\hat{E}_i(R_{t+1} - r_t^f|\Omega_{it}) = E(R_{t+1} - r_t^f|\Omega_{it})$$

where $E(R_{t+1} - r_t^f|\Omega_{it})$ is the objective conditional expectation. Thus, the rational expectation or the objective conditional expectation do not differ systematically from equilibrium results. In other words, people do not make systematic errors when predicting the future, and by the rational expectation hypothesis in our model, it assumed that expected value of a variable is equal to the expected value predicted by the model. Moreover, it can be inferred

$$E[\hat{E}_i(R_{t+1} - r_t^f|\Omega_{it})|\Phi_t] = E[E(R_{t+1} - r_t^f|\Omega_{it})|\Phi_t] \tag{2}$$

and since $\Phi_t \subset \Omega_{it}$ then we have

$$E[\hat{E}_i(R_{t+1} - r_t^f|\Omega_{it})|\Phi_t] = E(R_{t+1} - r_t^f|\Phi_t). \tag{3}$$

Therefore, under the rational expectation condition (4) we have

$$E(R_{t+1} - r_t^f|\Phi_t) = E(\lambda_{it} + \delta_{it}|\Phi_t) \tag{4}$$

then $E(\lambda_{it} + \delta_{it}|\Phi_t)$ is the same across the investors and it can be written

$$E(R_{t+1} - r_t^f|\Phi_t) = E(\lambda_{it} + \delta_{it}|\Phi_t) = \rho_t \qquad \forall i$$

where the ρ_t is an average market measure of the risk premia and transaction cost. Thus the rational expectation condition plus the premises that we assumed such as a rationality of investors ensure that the different investors have the same expectation. Moreover, the condition shows that in this setting the prediction of excess return depends on relationships of the risk premium with macro and business cycle indicators.

One of the things that lead to departure from the RE equilibrium solution is herding and correlated behaviour across some of the investors.(Pesaran (2010, p.26)) Each investor has its own subjective estimation $\hat{E}_i(R_{t+1} - r_t^f|\Phi_t)$. If investor has a ω_{it} market share the departures of the $\sum_{i=1}^{N_t} \omega_{it}\hat{E}_i(R_{t+1} - r_t^f|\Phi_t)$ from $E(R_{t+1} - r_t^f|\Phi_t)$ is a source of the stock market predictability. we can write

$$\bar{\xi}_{\omega t} = \sum_{i=1}^{N_t} \omega_{it}\hat{E}_i(R_{t+1} - r_t^f|\Phi_t) - E(R_{t+1} - r_t^f|\Phi_t)$$

where $\sum_{i=1}^{N_t} \omega_{it} = 1$. Also, we can write

$$\bar{\xi}_{\omega t} = \sum_{i=1}^{N_t} \omega_{it}\xi_{it}$$

where

$$\xi_{it} = \hat{E}_i(R_{t+1} - r_t^f|\Phi_t) - E(R_{t+1} - r_t^f|\Phi_t).$$

In the above formula, ξ_{it} shows the difference between individual expectation and the unobserved correct expectation. This difference can be nonzero due to the irrationality of investor, uncertainty, costly information and disparity of information across investors. However, if N_t is sufficiently large, and so long as $\xi_{it}, i = 1, 2, \cdots, N_t$ are not cross-sectionally strongly dependent[1], and no investor or group of investors dominate the market which means $\omega_{it} = O(N_t^{-1})$ at any time, then the average expected excess returns across the individual investors converge in quadratic means to the expected excess return of a representative investor, and we have

$$\sum_{i=1}^{N_t} \omega_{it} \hat{E}_i(R_{t+1} - r_t^f | \Omega_{it}) \xrightarrow{q.m} E(R_{t+1} - r_t^f | \Phi_t) \qquad N_t \to \infty$$

where, despite the individual deviation market is collectively efficient, and representative agent paradigm would be applicable and predictability of excess return will be governed soley by changes in business cycle conditions and available information(Pesaran (2010, p.27)).

[1]Concepts of weak and strong cross section dependence are defined and explained in Chudik, Pesarsn, and Tosetti (2010).

4 Testing CAPM and Factor Models

For the individual stock portfolio, we use capital asset pricing model (CAPM), with augmented with potential predictors,

$$R_{i,t+1} = a_i + b_{1i}x_{1t} + b_{2i}x_{2t} + \cdots + b_{ki}x_{kt} + \beta_i R_{t+1} + \epsilon_{i,t+1}$$

where $x_{ji}, j = 1, 2, \cdots, k$ are potential predictors, $R_{i,t+1}$ is excess return of the ith protfolio and R_{t+1} is excess return of the market portfolio. If CAPM holds, we have

$$a_i = 0, b_{1i} = b_{2i} = \cdots = b_{ki} = 0$$

the only variable β_i would be significantly different from zero, and this variable denotes the risk of holding the asset i with respect to the market portfolio.(Pesaran (2010, p.14))

4.1 The Panel Regression Model and GRS Test

The individual return regressions can be written as the following panel regressions,

$$y_{it} = \alpha_i + \beta_i' f_t + u_{it}, \quad for \quad i = 1, 2, \cdots, N; t = 1, 2, \cdots, T, \tag{5}$$

where f_t is $m \times 1$ matrix of observed factors. When we stacking above the formula by the time series observations then we have

$$\mathbf{y}_i = \alpha_i \tau_T + \mathbf{F}\beta_i + \mathbf{u}_i, \tag{6}$$

where $\mathbf{y}_i = (y_{i1}, y_{i2}, \cdots, y_{iT})', \tau_T = (1, 1, \cdots, 1)'$, and $\mathbf{u}_i = (u_{i1}, u_{i2}, \cdots, u_{iT})'$. Stacking by cross sectional observation we have

$$\mathbf{y}_t = \alpha + \mathbf{B} f_t + \mathbf{u}_t \tag{7}$$

where $\mathbf{y}_t = (y_{1t}, y_{2t}, \cdots, y_{Nt})', \alpha = (\alpha_1, \alpha_2, \cdots, \alpha_N)', \mathbf{B} = (\beta_1, \beta_2, \cdots, \beta_N)'$, and $\mathbf{u}_t = (u_{1t}, u_{2t}, \cdots, u_{Nt})'$.

Assumption 1: The common factor f_t are distributed independently of the error terms, u_{it}' for all i, t and $t', T^{-1}G'G$, with $G = (F, \tau_T)$, is a positive definite matrix for all T, and as $T \to \infty$, and $\tau_T' M_F \tau_T > 0$, where $M_F = I_T - F(F'F)^{-1}F'$. (Pesaran and Yamagata (2012, p.4))

Assumption 2: $\mathbf{u}_t \sim IID\ N(\mathbf{0,V})$, where $\mathbf{V}$ is an $N \times N$ symmetric positive definite matrix.(Pesaran and Yamagata 2012)

We need these two assumptions for setting up our models the first assumption is more likely satisfied while i is a single asset and not a portfolio. The implication of the second assumption is that error terms are not serially correlated with each other, or $E(u_{it}u_{jt'}) = 0$ for all i, j, and $t \neq t'$.

In order to construct the GRS statistic, we need to estimate α_i which can be obtained by the OLS regression. We have

$$\hat{\alpha}_i = \mathbf{y}_i' \Big(\frac{M_F \tau_T}{\tau_T' M_F \tau_T} \Big)$$

by using equation (2) we have

$$\hat{\alpha}_i = (\alpha_i \tau_T' + \beta_i' \mathbf{F}' + \mathbf{u}_i') \Big(\frac{M_F \tau_T}{\tau_T' M_F \tau_T} \Big)$$

$$= \alpha_i + \mathbf{u}_i' \mathbf{c}, \quad for \quad i = 1, 2, \cdots, N,$$

while

$$c = \frac{M_F \tau_T}{\tau_T' M_F \tau_T}.$$

Then we can write

$$\hat{\alpha} = \alpha + \begin{pmatrix} u_1'.c \\ u_2'.c \\ \vdots \\ u_N'.c \end{pmatrix},$$

where $u_i'.c = \sum_{t=1}^{T} u_{it} c_t$, and c_t is a t^{th} element of $\mathbf{c}$. Then

$$\hat{\alpha} = \alpha + \sum_{t=1}^{T} u_t c_t,$$

where $\mathbf{u}_t = (u_{1t}, u_{2t}, \cdots, u_{Nt})'$, Under the Assumptions 1 and 2 we have

$$\hat{\alpha} \sim N\Big(\alpha, \frac{1}{\tau_T' M_F \tau_T} \mathbf{V}\Big).$$

It can be proved that if $T \geq N+m+1$ then $\Big(\frac{T}{T-m-1}\Big)\hat{\mathbf{V}}$ would be a invertible unbiased estimator of $\mathbf{V}$ (Pesaran and Yamagata (2012, p.5)), where $\hat{\mathbf{V}}$ is a sample covariance matrix estimator

$$\hat{\mathbf{V}} = T^{-1} \sum_{t=1}^{T} \hat{u}_t \hat{u}_t'$$

and $\hat{u}_t = (\hat{u}_{1t}, \hat{u}_{2t}, \cdots, \hat{u}_{Nt})'$. If Assumptions 1 and 2 hold, then $\hat{u}_t$ have a multivariate normal distribution with zero means, and hence by multivariate analysis (look at theorem

5.5.2 of Anderson (2003)).

$$\hat{\mathbf{W}}_\alpha = \frac{T-N-m}{N}\Big(\frac{\tau_T' M_F \tau_T}{T}\Big)(\hat{\alpha}-\alpha)'\hat{\mathbf{V}}^{-1}(\hat{\alpha}-\alpha)$$

has a non-central distribution **F** with (T-N-m) and N degrees of freedom, and non-centrality parameter is $\mu_\alpha^2 = \frac{T-N-m}{N}\Big(\frac{\tau_T' M_F \tau_T}{T}\Big)\alpha'\hat{\mathbf{V}}^{-1}\alpha$. Under $H_0 : \alpha = 0$,

$$\hat{\mathbf{W}}_0 = \frac{T-N-m}{N}\Big(\frac{\tau_T' M_F \tau_T}{T}\Big)\hat{\alpha}'\hat{\mathbf{V}}^{-1}\hat{\alpha} \tag{8}$$

is the Gibbons, Ross, and Shanken(GRS) statistics(Gibbons et al. (1989)). According to the Pesaran and Yamagata (2012, p.5) we have, $T^{-1}\Big(\frac{\tau_T' M_F \tau_T}{T}\Big) = \Big(1+\bar{f}'\hat{\Omega}^{-1}\bar{f}\Big)^{-1}$, where $\bar{f} = T^{-1}\sum_{t=1}^{T} f_t$, and $\hat{\Omega} = T^{-1}\sum_{t=1}^{T}(f_t-\bar{f})(f_t-\bar{f})'$. Hence we can write GRS test as

$$GRS = \hat{\mathbf{W}}_0 = \frac{T-N-m}{N}\Big(1+\bar{f}'\hat{\Omega}^{-1}\bar{f}\Big)^{-1}\hat{\alpha}'\hat{\mathbf{V}}^{-1}\hat{\alpha}. \tag{9}$$

4.2 J_α Test for Large N Assets

There are several reasons which explain why the GRS test cannot be used for a stock market with a large number of the assets. First, the most important limitation of the GRS test is that the T must be larger than N, and if we grouped the assets into a portfolio then GRS test practically applicable to 20-30 portfolio over long periods, and grouping portfolio lead to loss of power because of the following reason. Suppose that N_p of N assets are grouped by using portfolio weight w_p such that $\tau_N' w_p = 1$ for $p = 1, 2, \cdots, P$, and $w_p' w_s = 0$ for $p \neq s$, with $\sum_{p=1}^{P} N_p = N$. The GRS test then applied to the P portfolio excess return defined by $w_p'\mathbf{y}_t$ for $p = 1, 2, \cdots, P$ where P is small fraction of N. The null hypothesis is

$$H_0^p : w_p'\alpha = \mathbf{0}, p = 1, 2, \cdots, P.$$

In the case of $H_0 : \alpha = \mathbf{0}$ when the null hypothesis is not rejected, then it is clear $w_p'\alpha = \mathbf{0}$ because all the companies are considered in the test. However, when H_0^p fails to reject, H_0 may be rejected. (Pesaran and Yamagata (2012, p.6))

The second problem is that, when there is a p such that $\frac{N_p}{N_m} \to c_p$ and $c_p > 0$ it means that the excess return on the portfolio p will be a non-negligible component of the excess return on the market portfolio and the regression of the former on the latter may lead to the endogeneity problem. (Pesaran and Yamagata (2012, p.6))

Third, entry and exit of the firms in the market and possible structural change in the market when T is large can lead to the new type of bias and unpredictable consequence for the test outcomes. Hence, the construction of the test for efficiency which can be applicable for a large number of the assets over the short periods of time is essential.(Pesaran and

Yamagata (2012, p.6)) propose J_α test for large number of the assets when $T < N$. In this section, we introduce the J_α test. For introducing the test we need further assumptions. (Pesaran and Yamagata (2012, p.6))

Assumption 3 : $\mathbf{u}_t \sim IID(\mathbf{0}, \mathbf{V})$, where $\mathbf{V}$ is an $N \times N$ symmetric positive definite matrix, such that $V = QQ'$, and $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \cdots, \epsilon_{Nt})' = \mathbf{Q}^{-1}\mathbf{u}_t$. ϵ_{it} is an IID process over i and t, with means zero and unit variances, and for some $\epsilon > 0$, $E(|\epsilon_{it}|^{4+\epsilon})$ exists, for all i and t. (Pesaran and Yamagata (2012, p.7))

Assumption 4: Q and Q^{-1} are non-singular lower triangular matrices with bounded absolute maximum column and row sum matrix norms.(Pesaran and Yamagata (2012, p.7))

Theorem 1. *Consider the CAPM regressions, (1), suppose Assumptions 1 and 3 hold, and* $\boldsymbol{V}$ *is known. Then under* $H_0 : \alpha_i = 0$ *for all i; and as* $N \to \infty$, *for any* $T > m + 1$

$$J_\alpha(V) = \frac{N^{-1/2}\Big[\Big(\tau_T' M_F \tau_T\Big)\hat{\alpha}' \hat{\boldsymbol{V}}^{-1} \hat{\alpha} - N\Big]}{\sqrt{2 + \bar{\gamma}_{2,\epsilon} q_T}} \to_d N(0,1), \tag{10}$$

where $\hat{\alpha}' = (\hat{\alpha}_1', \hat{\alpha}_2', \cdots, \hat{\alpha}_N')$, $\hat{\alpha}_i'$ *is the OLS estimator of* α_i *in (1),* $\bar{\gamma}_{2,\epsilon} = E(\gamma_{2,\epsilon i}) > 0$, $\gamma_{2,\epsilon i} = E(\epsilon_{it}^4) - 3$ $\epsilon_t = Q^{-1}\boldsymbol{u}_t$, *and* $q_T = O_p(T^{-1})$ *is defined by*

$$q_T = \Big(\sum_{t=1}^{T} c_t^4\Big) / \Big(\sum_{t=1}^{T} c_t^2\Big)^2 \tag{11}$$

(Pesaran and Yamagata (2012, p.8))

It can be shown that the off-diagonal elements of $\mathbf{V}$ are less important when $N \to \infty$ and so we can replace the full covariance matrix with $N \times N$ diagonal matrix $\mathbf{D} = (\sigma_1^2, \sigma_2^2, \cdots, \sigma_N^2)$ with σ_i^2. Then we have

Theorem 2. *Consider the CAPM regressions, (1), suppose Assumptions 1, 3 and 4 hold, and* $\boldsymbol{V}$ *is known. Then under* $H_0 : \alpha_i = 0$ *for all i; and as* $N \to \infty$, *for any* $T > m + 1$

$$J_\alpha(\boldsymbol{D}) = \frac{N^{-1/2}\Big[\Big(\tau_T' M_F \tau_T\Big)\hat{\alpha}' \boldsymbol{D}^{-1} \hat{\alpha} - N\Big]}{\sqrt{2N^{-1}Tr(R^2) + \Big(N^{-1}\sum_{i=1}^{N} a_{ii}^2 \gamma_{2,\epsilon i} q_T\Big)}} \to_d N(0,1), \tag{12}$$

where $\boldsymbol{V} = \boldsymbol{D}^{-1/2}\boldsymbol{R}\boldsymbol{D}$, *and as in theorem 1* $\hat{\alpha}' = (\hat{\alpha}_1', \hat{\alpha}_2', \cdots, \hat{\alpha}_N')$, $\hat{\alpha}_i'$ *is the OLS estimator of* α_i *in (1),* $\bar{\gamma}_{2,\epsilon} = E(\gamma_{2,\epsilon i}) > 0$, $\gamma_{2,\epsilon i} = E(\epsilon_{it}^4) - 3$ $\epsilon_t = Q^{-1}\boldsymbol{u}_t$, *and* $q_T = O_p(T^{-1})$ *is defined by (7). Moreover,* $\boldsymbol{A} = Q'\boldsymbol{D}^{-1}Q$ *which* a_{ii} *is the* i^{th} *diagonal element of* $\boldsymbol{A}$. *It is easily seen that* $Tr(\boldsymbol{A}) = Tr(\boldsymbol{R})$ *and* $Tr(\boldsymbol{A}^2) = Tr(\boldsymbol{R}^2)$, *where* $\boldsymbol{R} = (\rho_{ij})$. *(Pesaran and Yamagata (2012, p.12))*

For constructing a simple applicable statistic suppose the error terms are Gaussian and uncorrelated and consider the following standardized test statistics

$$\hat{J}_{\alpha,1} = \frac{N^{-1/2}\sum_{i=1}^{N}\left(t_i^2 - \dfrac{\nu}{\nu-2}\right)}{\left(\dfrac{\nu}{\nu-2}\right)\sqrt{\dfrac{2(\nu-1)}{(\nu-4)}}} \tag{13}$$

then we have following theorem.

Theorem 3. *Consider the regression model (1), and suppose that Assumptions 1, 3 and 4 hold. Further assume that: (i)* $f_t' f_t \leq K < \infty$ *for all t, and; (ii)* $E(|\epsilon_{it}|^{8+\epsilon}) < K < \infty$ *for some* $\epsilon > 0$*. Consider the statistic,* $\hat{J}_{\alpha,1}$*, defined by (9). Then, under* $H_0 : \alpha_i = 0$ *for all i,*$\hat{J}_{\alpha,1} \to_d N(0,1)$*, if* $N/T^3 \to 0$*, as* $N \to \infty$ *and* $T \to \infty$*, jointly. (Pesaran and Yamagata (2012, p.15))*

5 Data Description

In this paper we consider the application of the GRS test on the assets of the German DAX index, besides considering the application of the $\hat{J}_\alpha$ for the assets of FTSE 100 in the UK, and all data are given from *Data Stream* The German DAX contains 30 securities, so GRS test can be used for applying to these securities. We use monthly data for DAX from June 1990 to April 2017, and due to the entry and exit the companies and not the availability of replaced data we just use 23 of securities in our analysis. The FTSE 100 contain approximately 100 securities and so GRS test could not be used for the assets and we use $\hat{J}_\alpha$ test for these securities. We use monthly data for DAX from June 1990 to April 2017, and due to the entry and exit the companies and not the availability of data we just use 71 of securities in our analysis.

In order to compute the asset i return in the month t we use $r_{it} = 100(P_{i,t} - P_{i,t-1})/P_{i,t-1} + DY_{it}/12$. Where $P_{i,t}$ is the end of the month price of the asset, and DY_{it} is per cent per annum dividend yield on the asset.

The time series of the risk-free rate of return and market return and other market factors are obtained by the Ken French data library web page[2]. The monthly US Treasury bill rate is used for the risk-free rate of return which can be shown by (r_{ft}), for the market return since we use French/Fama 3 Factor and French/Fama 5 Factor for developed markets (European) the region's value-weight market portfolio used as a proxy, and it can be shown by (r_{mt}). The equal-weighted average of the returns on the three small stock portfolios for the region minus the average of the returns on the three big stock portfolios is used for (SMB_t). The equal-weighted average of the returns for the two high B/M portfolios for a region minus the average of the returns for the two low B/M portfolios denotes by (HML_t), and all data are monthly percentages. In 5 factors model, RMW (Robust Minus Weak) is the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios, and CMA (Conservative Minus Aggressive) is the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios.

[2]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html

CAPM			Fama-French 3		Fama-French 5	
	P-Value of GRS	P-Value of $\hat{J}_{\alpha,1}$	P-Value of GRS	P-Value of $\hat{J}_{\alpha,1}$	P-Value of GRS	P-Value of $\hat{J}_{\alpha,1}$
Mean	0.09	0.01	0.11	0.02	0.16	0.01
Median	0.01	0.00	0.02	0.00	0.01	0.00
Min	0.00	0.00	0.00	0.00	0.00	0.00
Max	0.52	0.7	0.68	0.8	0.85	0.74
SD	0.13	0.09	0.18	0.12	0.26	0.09

Table 2: Summary statistics for P-Values of GRS and $\hat{J}_{\alpha,1}$ tests

6 Empirical Results

6.1 Test Results

In order to test CAPM and factor models, we use the GRS test for the German DAX index and use $\hat{J}_{\alpha,1}$ for The FTSE 100. $\hat{J}_{\alpha,1}$ test can be used for Gaussian and uncorrelated securities of FTSE 100. In this paper, the test is applied from August 1999 to April 2017 to the assets at the end of the each month. For the German DAX the sample period of GRS test, ($\mathbf{T}$), is 110 months and for FTSE 100 the sample period of $\hat{J}_{\alpha,1}$ is 60 months, which reduces the effect of the structural change in the market as soon as possible. We estimate the CAPM regression below for the test

$$r_{i,\tau t} - r_{f,\tau t} = \alpha_{i\tau} + \beta_{i\tau}(r_{m,\tau t} - r_{f,\tau t}) + u_{i,\tau t} \tag{14}$$

and we estimate Fama-French (FF) three-factor regression as below,

$$r_{i,\tau t} - r_{f,\tau t} = \alpha_{i\tau} + \beta_{1,i\tau}(r_{m,\tau t} - r_{f,\tau t}) + \beta_{2,i\tau}SMB_{t\tau} + \beta_{3,i\tau}HML_{t\tau} + u_{i,\tau t} \tag{15}$$

and we estimate Fama-French (FF) three-factor regression as below,

$$\begin{aligned} r_{i,\tau t} - r_{f,\tau t} = \alpha_{i\tau} + \beta_{1,i\tau}(r_{m,\tau t} - r_{f,\tau t}) + \beta_{2,i\tau}SMB_{t\tau} + \beta_{3,i\tau}HML_{t\tau} \\ + \beta_{4,i\tau}RMW_{t\tau} + \beta_{5,i\tau}CMA_{t\tau} + u_{i,\tau t} \end{aligned} \tag{16}$$

for $t = 1, 2, \cdots, 110$ (or $t = 1, 2, \cdots, 60$), $i = 1, 2, \cdots, N_\tau$, and the end of the month $\tau = 1999M8, 1999M9, \cdots, 2017M4$. For the German DAX the asset with less than 110 months returns and for FTSE 100 the assets with less than 60 months returns are eliminated.

Table 2 reports the descriptive statistics of the P-Value of the GRS test and $\hat{J}_{\alpha,1}$ test for both CAPM and FF model. As it can be seen, the P-Value range from 0 to 1, and for the German DAX the median and mean of the P-Values for the CAPM is 0.01 and 0.09 and for the Fama-French model of the DAX index, they are 0.02 and 0.11 respectively.

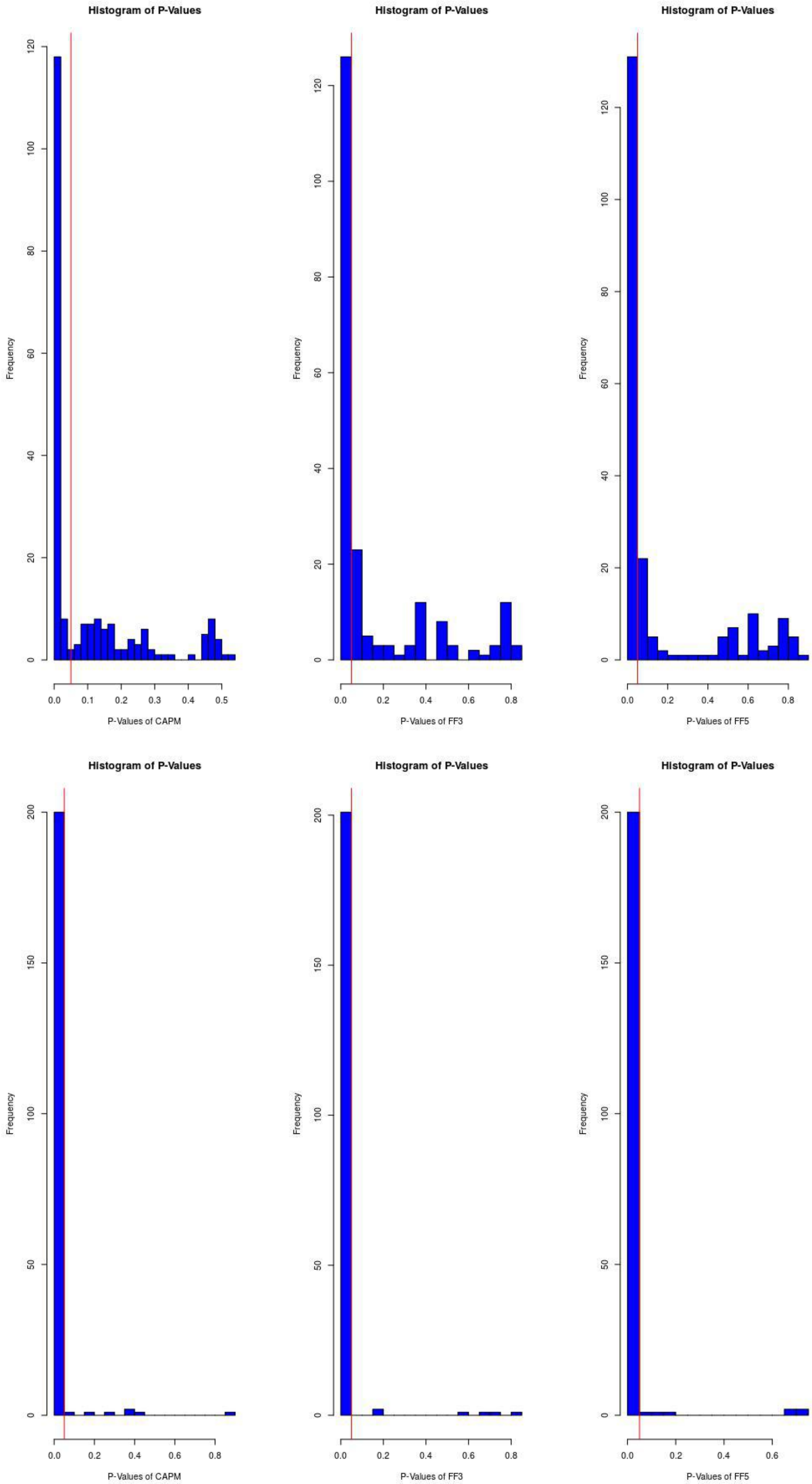


Figure 4: Reported histogram of the p-values of GRS $\hat{J}_{\alpha,1}$ tests, which are computed using CAPM regressions, French-Fama 3, and French-Fama 5 factors regressions

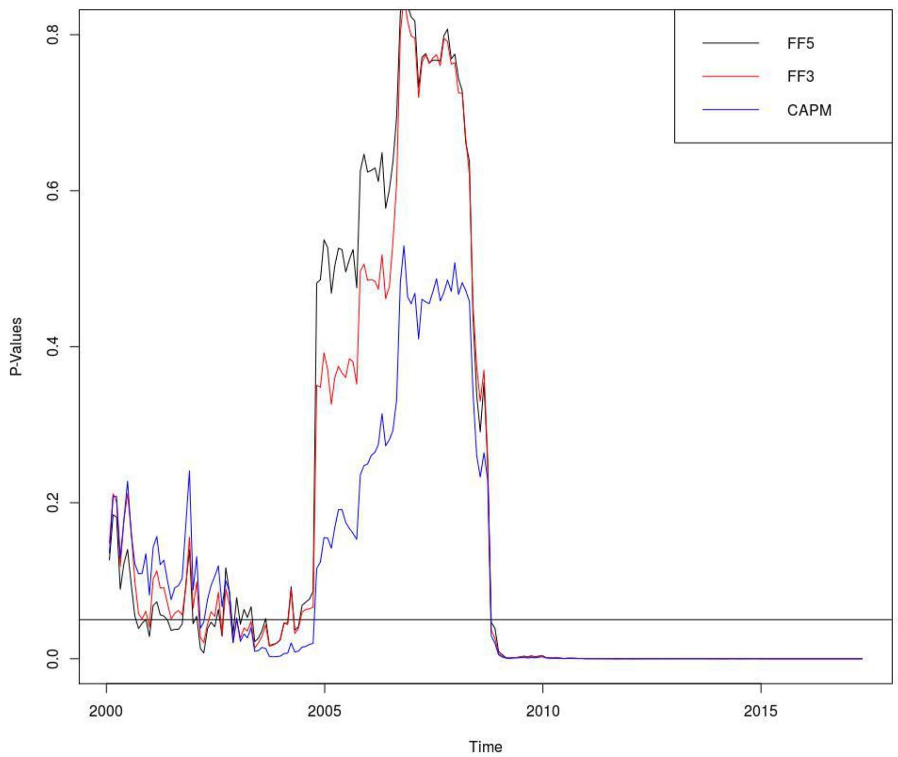
For FTSE 100 the median and mean of the P-Value for CAPM is 0.00 and 0.01, and for the Fama-French model of the FTSE 100 index, they are 0.00 and 0.02 respectively.

The top graph of the Figure 6.1 present plots of the evolution of P-Values of GRS test for German DAX index based on the CAPM and FF regressions from the end of the month of 1999M08 to 2017M04. As it can bee seen in the graph during the end of the dot-com bubble the market efficiency rejected and after the second period of the rejection of market efficiency in the German DAX index in coinciding with the financial crisis 2008 and after that. The bottom graph of the figure 6.1 is the evolution of P-Values of the $\hat{J}_{\alpha,1}$ test. It can be seen that after the dot-com bubble and bubble collapse the market efficiency null hypothesis is rejected.

6.2 Long/Short Equity Returns and P-Values

Figure 6.1 shows how the P-Values of a the CAPM, French-Fama 3 factors, and French-Fama 5 factors vary through the time, especially on the crisis period the null hypothesis of CAPM and factor models is rejected in both markets. It is also interesting to see the relationship of this variation with trading strategies. The trading strategies target is exploited profit of the nonzeros α from the portfolios of assets. One of the most common strategies is the Long/Short equity strategy. In this strategy, assets are ordered based on their predicted returns. Thus, the investor takes a long position which the alpha are positive and take the short position for the negatively estimated alpha. What we are interested in here is a relationship between the evidence of inefficiencies and return of Long/Short strategies. Theoretically, while $\alpha_i = 0$ for asset i, then L/S strategy might not perform better than the market return. However, we could expect a higher return on L/S strategies relative to the market, when alphas are not zero and investor can exploit profit. Therefore, *a priori* there is an inverse relation between P-Value and L/S strategies returns.

For L/S strategies returns monthly data of Dow Jones Credit Suisse Core Long/Short Equity Hedge Fund Index is used in this paper. This index present aggregate performance of long/short equity funds. The monthly return in this index is shown by r_{ht} and we denote performance of the L/S strategies return relatve to the market return by $\tilde{r}_{ht} = r_{ht} - r_t$, where r_t are returns of German DAX and FTSE 100 index, and monthly P-Value of $\hat{J}_{\alpha,1}$ denotes by $\hat{\pi}_t$. Figure 6.2 and figure 6.2 denotes twelve-month moving averages of returns and P-Values which are calculated by $\tilde{r}_{ht}(12) = \frac{1}{12}\sum_{j=0}^{11} \tilde{r}_{h,t-j}$, and $\hat{\pi}_t(12) = \frac{1}{12}\sum_{j=0}^{11} \hat{\pi}_{t-j}$. Figures 6.2 and 6.2 shows the relationship and Long/Short strategies returns for German DAX index. The weakly negative correlation between two variables show that our supposition of the inverse relationship is true since the correlation between the variables in CAPM model is -0.008, in the French-Fama 3 factors model is -0.08, and in the French-Fama 3 factors model is -0.19 Figures 6.2 and 6.2 demonstrates



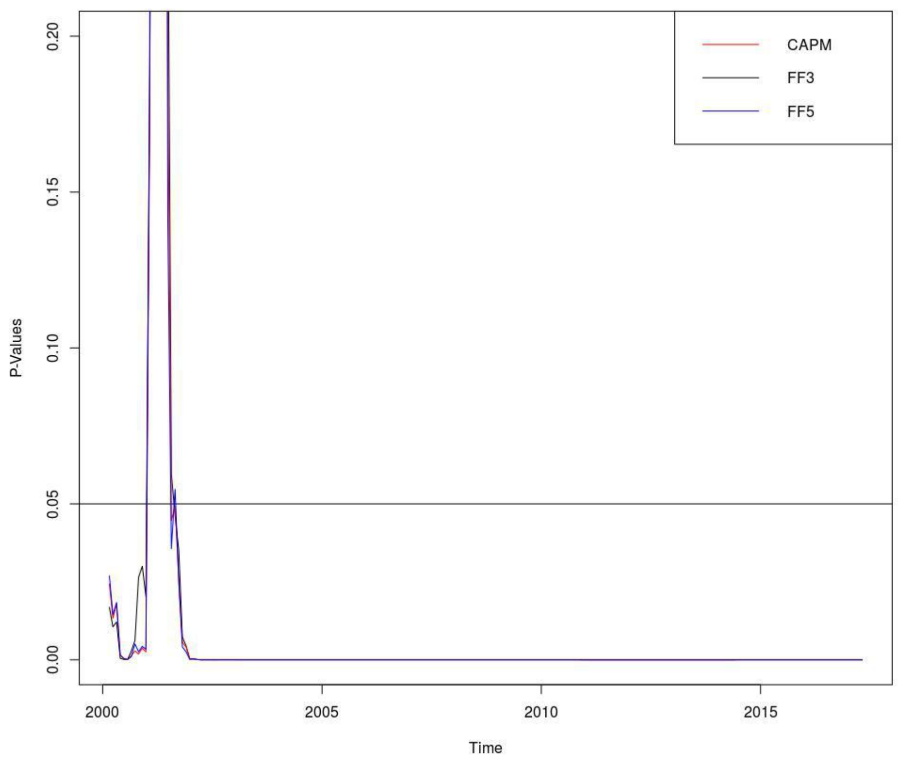


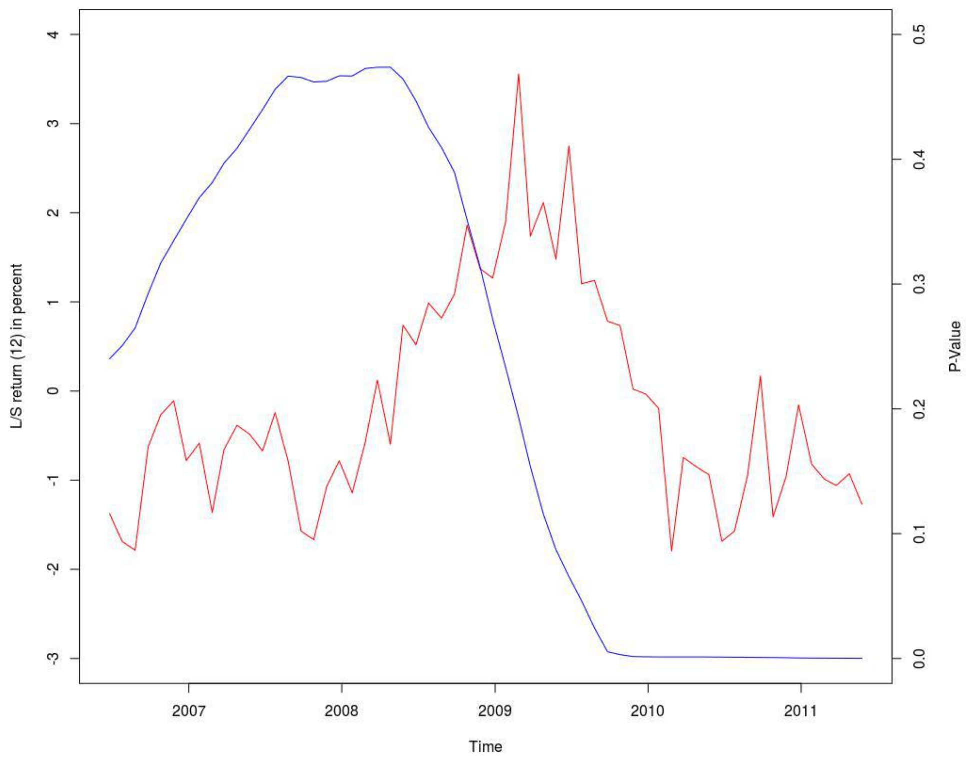
Figure 5: The top graph is plots of the evolution of p-values of GRS test base on CAPM and FF regressions of assets in the German DAX using 110 month estimation at the end of months 1999M08-2017M04. The bottom graph is plots of the evolution of p-values of $\hat{J}_{\alpha,1}$ test base on CAPM and FF regressions of assets in the German DAX using 60 month estimation at the end of months 2000M01-2017M04

the relationship and Long/Short strategies returns for FTSE 100 index. The positive relationship between two variables show that there is no inverse relationship since the correlation between the variables in Fama-French 3 factors model is 0.19, in Fama-French 5 factors model is 0.20, and in CAPM model is 0.20.

7 Conclusion

In this paper first, we considered the empirical properties of the stock returns. There is strong evidence of deviation from normality besides fattailness of the returns. Moreover, increasing the first or second order cross-correlation of assets lead to the hypothesis that during the crisis periods predictability of assets increases. In order to test this hypothesis, first, we use efficient market hypothesis (EMH) concept proposed by Pesaran (2010) to explain a market efficiency. Using an asset regression models, such as CAPM and FF model, and applying GRS and $\hat{J}_{\alpha,1}$ tests which proposed by Gibbons, Ross and Shaken (1989), and Pesaran and Yamagata (2012) on German DAX and FTSE 100 we tested efficiency of market on these stock markets.

Applying GRS test on all assets of Germany DAX with 110 months return data over the period August 1999 to April 2017 shows that the null hypothesis of CAPM test rejected at the periods of the crisis. Furthermore, there is a weak negative correlation between a twelve-month moving average P-Values of GRS test and excess returns of long/short equity strategies over the period of January of 2006 to December of 2011. On the other hand, Applying $\hat{J}_{\alpha,1}$ test on all assets of FTSE 100 with 60 months return data over the period February 2000 to April 2017 shows that the null hypothesis of CAPM rejected at the periods of the crisis. Furthermore, there is a weak positive correlation between a twelve-month moving average P-Values of $\hat{J}_{\alpha,1}$ test and excess returns of long/short equity strategies over the period of January of 2001 to December of 2006.



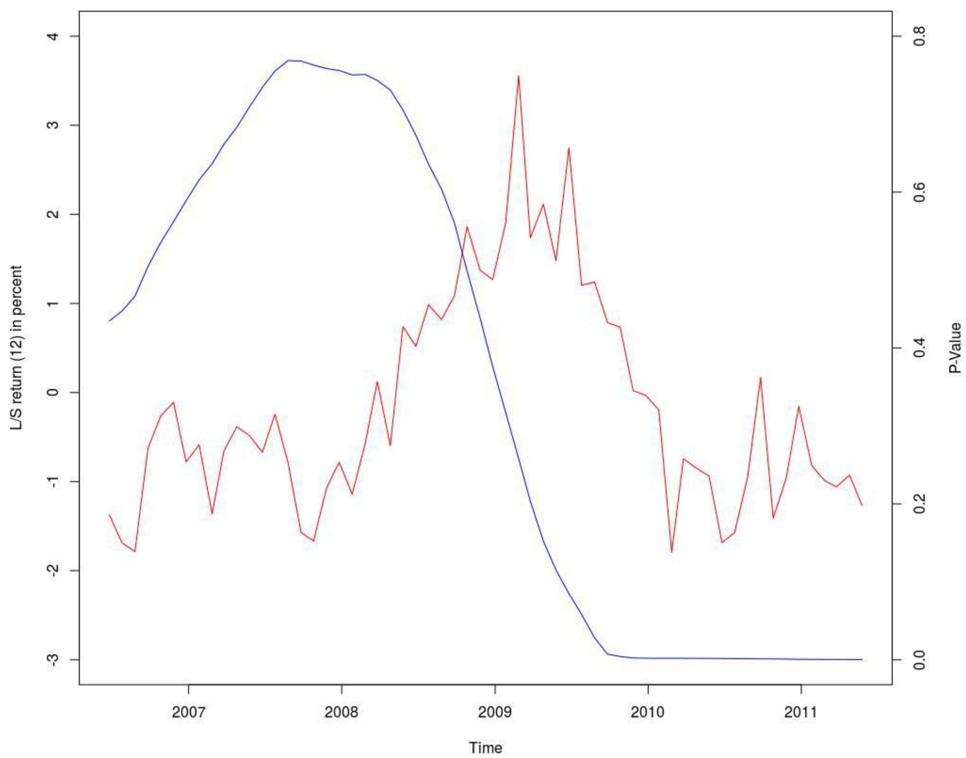
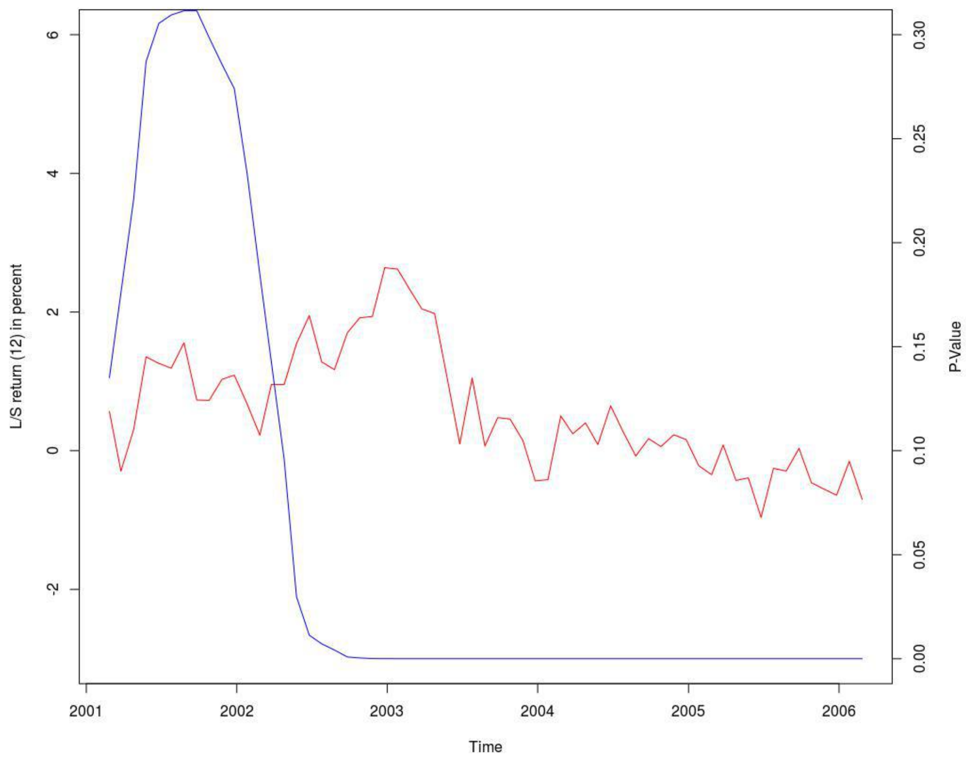


Figure 6: Monthly rate of returns of Dow Jones Credit Suisse Core Long/Short Equity Hedge Fund Index relative to German DAX returns (red line), and p-values of GRS test (blue line) based on CAPM (top graph) and Ferenc-Fama 3 factors (bottom graph) regressions over the period September 2006 to December 2011



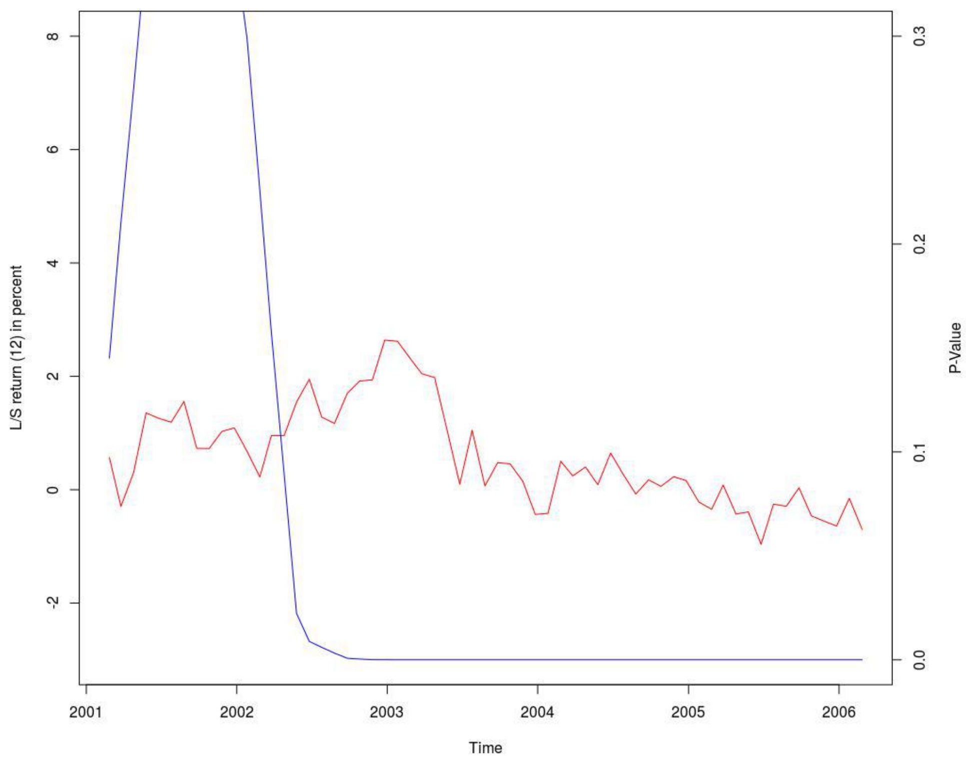
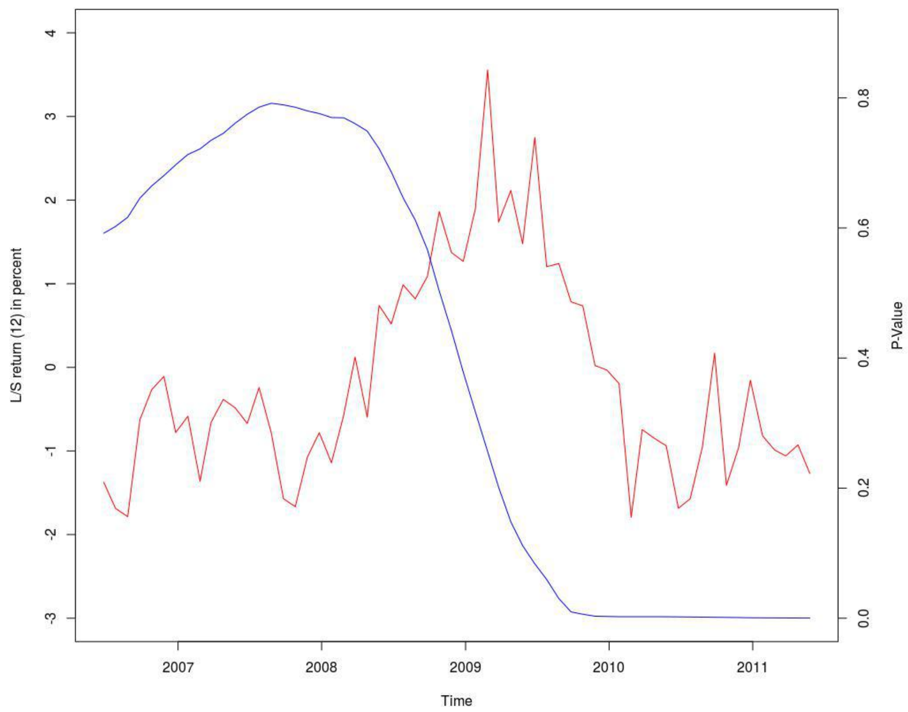


Figure 7: Monthly rate of returns of Dow Jones Credit Suisse Core Long/Short Equity Hedge Fund Index relative to FTSE 100 returns (red line), and p-values of $\hat{J}_{\alpha,1}$ test (blue line) based on CAPM (top graph) and French-Fama 3 factors (bottom graph) regressions over the period February 2001 to February 2006



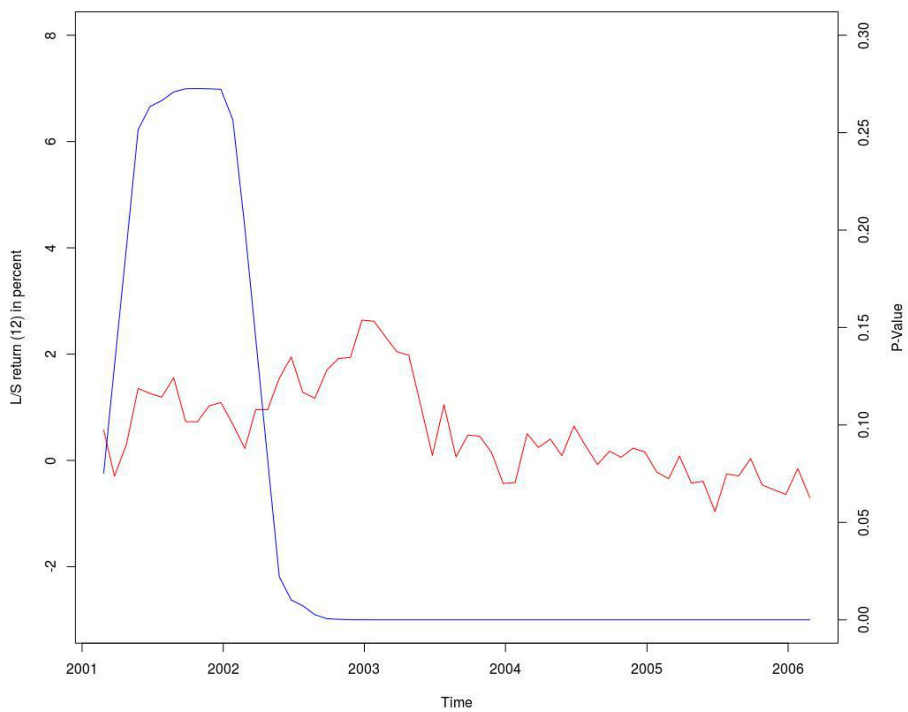


Figure 8: the top graph is monthly rate of returns of Dow Jones Credit Suisse Core Long/Short Equity Hedge Fund Index relative to German DAX returns (red line), and p-values of GRS test (blue line) based on Ferenc-Fama 5 factors regressions over the period September 2006 to December 2011, and bottom graph is Monthly rate of returns of Dow Jones Credit Suisse Core Long/Short Equity Hedge Fund Index relative to FTSE 1oo returns (red line), and p-values of $\hat{J}_{\alpha,1}$ test (blue line) based on Ferenc-Fama 5 factors regressions over the period September 2006 to December 2011

References

Anderson, T. (2003). *An Introduction to Multivariate Statistical Analysis*. Wiley Series in Probability and Statistics. Wiley.

Chudik, A., M. H. Pesaran, and E. Tosetti (2011). Weak and strong cross-section dependence and estimation of large panels. *The Econometrics Journal 14*(1), C45–C90.

Cochrane, J. (2009). *Asset Pricing: (Revised Edition)*. Princeton University Press.

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance 25*(2), 383–417.

Gibbons, M. R., S. A. Ross, and J. Shanken (1989). A test of the efficiency of a given portfolio. *Econometrica: Journal of the Econometric Society*, 1121–1152.

Jarque, C. M. and A. K. Bera (1980). Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics letters 6*(3), 255–259.

Jensen, M. C. (1968). The performance of mutual funds in the period 1945–1964. *The Journal of finance 23*(2), 389–416.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The Review of Economics and Statistics 47*(1), 13–37.

Pesaran, M. (2010). Predictability of asset returns and the efficient market hypothesis. iza discussion papers. *Institute for the Study of Labor*.

Pesaran, M. H. and T. Yamagata (2012). Testing capm with a large number of assets.

Samuelson, P. A. (1965). *Proof that properly anticipated prices fluctuate randomly*. Number 6.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance 19*(3), 425–442.