MSc Economics

Contracting Under Asymmetric Information and Asymmetric Awareness

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Affidavit

I, James Glover

hereby declare

that I am the sole author of the present Master's Thesis,

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Abstract

I study a principal-agent contracting problem in which problems of asymmetric information and asymmetric awareness are present. I consider an insurance situation in which two types of agents know only a subset of the possible contingencies leading to damage over some good. The insurer is fully aware. However the insurer does not know which contingencies the insuree is originally aware of. The model is reduced to study the case of two types and three states or contingencies. A type who becomes aware of a contingency updates his awareness by assigning some residual probability to that event. The principal has the choice of which contingencies to cover, along with how to structure the terms of his contract to appeal to the various types. I hypothesize that the distribution of types of agents, updating parameters and levels of *suspicion* has an effect on the terms of the optimal contract. First a general solution method of the optimization problem of the principal is offered together with general characterizations of solutions. It is shown that certain distributions of types induce the principal to reveal information out of self interest, thereby altering the awareness of the different types of insurees and inducing more complete contracts.

1 Introduction

> *"There may be an interesting interaction between unforeseen contingencies and asymmetric information...there is a serious issue as to how parties form probability distributions over payoffs when they cannot even conceptualize the contingencies and actions that yield those payoffs, and as to how they end up having common beliefs ex ante...we should have some doubts about the common assumption that the parties to a contract have symmetric information when they sign the contract...Asymmetric information would therefore be the rule in such circumstances and would be unlikely to disappear through bargaining and communication."* - (Tirole, 1999)

Problems of asymmetric information have a long history in contract theory. In their most general form, hidden information problems model situations of a principal contracting with one more agents, or types of agents, whose valuation of the contract offered by the principal is private information. The standard analysis shows that the principal, despite not having sufficient information to distinguish one type from another, can, nevertheless, *screen* agents by structuring the terms of the contract to offer the "correct" incentives. Of late, there has been a growing interest in the concept of *unawareness* (see the literature review) - the idea that certain individuals are unable to conceptualize certain states of the world (See, for example, Schipper, 2011; Board & Chung, 2010; Heifetz et al., 2006). A number of studies (Auster, 2013; Filiz-Ozbay, 2008; Grant, Kline & Quiggin, 2014) have incorporated the framework of unawareness into contracting models as a means to more accurately reflect behaviour we see in the real world. In this thesis I study strategic interactions resulting from situations in which information and awareness interact.I look at the case of a fully aware insurer, and an insuree of limited awareness. The model is similar in construction to that of Filiz-Ozbay (2011) with two fundamental differences. First, where in Filiz-Ozbay the principal is *omniscient* that is not the case in my model. He is fully *aware* of all states of the world, but exactly which contingencies each of the other agents in the model are aware of is private information. Second, I include additional *types* of agents to which the principal can cater. Hence the decision problem is compounded not only by the awareness of individual actors within the model, but also by the addition of a distribution of types influencing the optimal strategy of the principal. To keep the analysis tractable in this first stage, I limit the study to a case of a three state world with *overlapping* awareness sets. I impose that each of the agents is aware of one and only one common contingency and each is aware of one and only contingency that the other is unaware of. I first define the problem, and the model. Next I analyze the optimal contract in the three-state, two-type case and offer a characterization of the equilibrium strategy. I ask the question of how beliefs of each type are formed, and how this influences the decisions of the principal over which contingencies to reveal and which types to cater to.

2 Literature Review

2.1 Unawareness

In its most basic formulation, unawareness is the inability of an agent to conceptualize certain states of the world. The states of which the agent is unaware are so far removed from her decision making process that they do not even enter her mind as a possibility. She is unaware of such a state, and unaware of her unawareness. Although there is an extensive body of knowledge of unawareness in the realm of philosophical logic and knowledge - for example see Konolidge (1986) for a more philosopical exposition of unawareness - modelling unawareness has proven to be a substantial task for decision theorists and microeconomists working in the area. Perhaps the earliest anticipations to the study of unaware agents comes from Geanakopolos (1989) and the most modern canonical models from Board & Chung (2011) or Heifetz et al. (2008). A variety of approaches have been used to model unaware agents. Early theoretical foundations were laid and questions posed by Geanakopolos (1989, 1992), Rustichini, Dekel & Lipman (1998) but it was Rustichini & Modica (1999) who estalbished the first general model of unawareness. Later studies such as Halpern (2001), Heifetz et al. (2005) and Board & Chung (2011) have consistently refined the theoretical constructs, or taken new approaches entirely. Often, the theoretical approach to modeling unaware agents been linguistic in that it has relied on the axioms of formal mathematical logic to construct *languages*. With such an approach, one can fully characterize the agent by the events in his language that he is able to articulate. Below I introduce the main approaches upon which my thesis, and many papers, are based.

The Subjective State Space Approach The subjective state-space approach is succinctly expounded in Heifetz et al. (2006). I follow the notation used in their model as closely as possible.

The authors first define a partial ordering over a complete lattice of disjoint spaces $\mathcal{S}_{\{\alpha\}_{\alpha \in A}}$ complete with a partial order $\preceq$ expressing the *richness* of vocabulary used in a given state-space. Recall that the pair $(\mathcal{S}, \succeq)$ is called a *lattice* if it holds that

$$\inf\{x, x'\}$$

and

$$\sup\{x, x'\}$$

exists for every $x, x' \in \mathcal{S}$. Further, a lattice is complete if both

$$\inf(S)$$

and

$$\sup(S)$$

exist for every $S \subseteq \mathcal{S}$. The ordering $S \succeq S'$ is read "S is less *expressive* that S'" A state $\omega \in S \in \mathcal{S}_\alpha$ for some α can be expressed with the vocabulary available in $\mathcal{S}_\alpha$.

Example 2.1. ***Adapted From Heifetz et al. (2006)*** *Consider a world which can be described by three basic propositions* (a, b, c) *which can be either true or false. This induces the complete lattice of disjoint state spaces:*

$$\mathcal{S} = \{S_{\{a,b,c\}}, S_{\{a,b\}}, S_{\{a,c\}}, S_{\{b,c\}}, S_{\{a\}}, S_{\{b\}}, S_{\{c\}}\}$$

For example, the state-space with the richest vocabulary is given as:

$$S_{\{a,b,c\}} =$$
$$\{(a, b, c), (a, \neg b, c), (a, b, \neg c), (\neg a, b, c), (a, \neg b, \neg c), (\neg a, \neg b, c), (\neg a, b, \neg c), (\neg a, \neg b, \neg c)\}.$$

A possible less expressive state is:

$$S_{\{a,b\}} = \{(a, b), (a, \neg b), (\neg a, b), (\neg a, \neg b)\}$$

Which the authors denote as $S_{\{a,b\}} \preceq S_{\{a,b,c\}}$.

The above example warrants further explaination. A state $\omega \in S$ also exists in some sense, as we will see, as a state $\omega_{S'} \in S'$. However each space allows agents to express ω differently. For example the state $\omega = (a, \neg b, c) \in S_{\{a,b,c\}}$ exists as the state $\omega_{S'} = \{a, \neg b\} \in S_{\{a,b\}}$. However this state is lacking an aspect when restricted to $S_{\{a,b\}}$, namely c is missing from the description of $\omega_{S'}$. It is this kind of characterization of state-spaces in terms of the richness of their descriptive capability which motivates our appellation of the state $S_{\{a,b,c\}}$ as "more expressive" than state $S_{\{a,b\}}$.

Returning to the paper at hand, projections between state-spaces are defined, $r_S^{S'}$ is the projection from space S to S' where $S \preceq S'$. Note that it must be the case that this projection is both surjective and commutative. Recall that a function $f : X \to Y$ is *surjective* if $f(X) = Y$. Projections commute means that $S'' \succeq S' \succeq S \implies r_S^{S''} = r_S^{S'} \circ r_{S'}^{S''}$. For a given state $\omega \in S \preceq S'$ define $\omega_S := r_S^{S'}(\omega)$ (read as "the restriction of state ω from the more expressive space S' to the less expressive space S"). For any collection of states $B \subset S$ define the *extension of states in B to at least as expressive vocabularies* as $B^{\uparrow} = \bigcup_{S \succeq S'} (r_S^{S'})^{-1}(B)$. Denote $\Sigma = \bigcup_{\alpha \in A} S_\alpha$ for some indexing set A. Then $E \subseteq \Sigma$ is an event if it is of the form of $B^{\uparrow}$. B is called the *basis* of event E and $S = S(E)$ is the base space.

Example 2.2. ***Continued from Example 2.1*** *The restriction of the state space $(a, b, \neg c), (a, \neg b, \neg c)$ in $S_{\{a,b,c\}}$ to the less expressive space $S_{\{a,b\}}$ are given by:*

$$r_{S_{\{a,b\}}}^{S_{\{a,b,c\}}}((a, b, \neg c)) = (a, b) \in S_{\{a,b\}}$$

$$r_{S_{\{a,b\}}}^{S_{\{a,b,c\}}}((a, \neg b, \neg c)) = (a, \neg b) \in S_{\{a,b\}}$$

Further, consider the event "a is true", this event has base-space $S_{\{a\}}$ and basis $\{(a)\}$ and

$$\{a\}^{\uparrow} = \{(a), (a, b), (a, b, c), (a, c), (a, \neg b), (a, \neg c), (a, \neg b, \neg c), (a, b, \neg c), (a, \neg b, c), (a, \neg b, \neg c)\}$$

The basis of this event is (a) and the base-space is $S_{\{a\}}$.

Finally $\Pi_i : \Sigma \to \mathcal{P}(\Sigma)$ is agent *i's possibility correspondence* and captures the states associated with event $E \in \Sigma$ he considers possible. These possibility correspondences must satisfy the following properties:

Properties On Possibility Correspondences

1. $\omega \in S \implies \Pi_i(\omega) \subseteq S'$ for some $S' \preceq S$
2. $\omega \in \Pi_i^{\uparrow}(\omega) \; \forall \, \omega \in \Sigma$
3. $\omega \in \Pi_i(\omega') \implies \Pi_i(\omega') = \Pi_i(\omega)$
4. $\omega \in S', \omega \in \Pi_i(\omega)$ and $S \preceq S' \implies \omega_S \in \Pi_i(\omega_S)$
5. $\omega \in S', S \preceq S' \implies \Pi_i^{\uparrow}(\omega) \subseteq \Pi_i^{\uparrow}(\omega_S)$
6. $S \preceq S' \preceq S'', \omega \in S''$ and $\Pi(\omega) \subseteq S' \implies (\Pi_i(\omega))_S = \Pi_i(\omega_S)$

The first of these properties is referred to as *confinedness* in the paper. It says that given the "true" state ω the states which I consider possible are all contained in some space S' and as such they can all be expressed with the same vocabulary. The second property, *generalized reflexivity*, guarantees that what an individual knows to be true obtains. The third *stationarity* says that if I consider a certain ω' possible at the true state ω then every state I consider possible at ω' I must also consider possible at ω and vice versa. The authors show that such a property implies introspection - an individual knows what she knows. The remaining properties state that after the restriction of ω to ω_S the agent does not learn anything new, does not forget anything she previously knew and does not become aware or unaware of anything. This has parallels to ideas such as perfect recall in game theory. This completes the formulation of the subjective-state-space awareness structure.

Definition 2.1. A **subjective-state-space unawareness structure** with n agents is a tuple $(\mathcal{S}, r_S^{S'}, \preceq, \{\Pi_i\}_{i=1}^n))$ such that:

1. $\mathcal{S}$ is a complete lattice of disjoint spaces.

2. $\preceq$ is a partial order over $\mathcal{S}$ relating the expressiveness of states to one another.

3. $r_S^{S'}$ represents a class of functions such that for $S \preceq S'$, $r_S^{S'} : S' \to S$ is the restriction of the more expressive space S' to the less expressive space S.

4. $\{\Pi_i\}_{i=1}^n$ is a sequence of *possibility correspondences* such that $\Pi_i : E \to \Sigma$ satisfies properties (1) to (6) of the properties on possibility correspondences.

Define the *awareness operator* as a function $A_i : \Sigma \to \mathcal{P}(\Sigma)$:

$$A_i(E) = \begin{cases} \{\omega \in \Sigma : \Pi(\omega) \subseteq S(E)^{\uparrow}\} & \exists\, \omega : \Pi(\omega) \subseteq S(E)^{\uparrow} \\ \emptyset^{S(E)} & \text{Else} \end{cases}$$

Where $\emptyset^S = \neg S^{\uparrow}$ is a "logical contradiction phrased with expressive power available in S". Intuitively, the $\emptyset^S$ event reflects that idea in my language I can express a logical fallacy, it is the "nothing" event in that it represents an event that can never happen but I can nevertheless express in my language.

A state in a given state space can be represented by a number of logical propositions. State spaces comprising a *richer vocabulary* in a sense describe more aspects of a state, the projections of such spaces to less descriptive spaces eliminate these aspects. In this way, a state can be represented, or described, in every state space in varying degrees of richness.

This *canonical* formulation of unawareness excels in its formality and precision. Such an approach permits clarity on exactly what unawareness is in a mathematically formal sense. However, often in contract theory a different approach involving a modelling of subjective probability spaces which are in a sense restricted has been used which I now turn to discuss.

2.2 Unawareness and Contract Theory

Within the last decade or so, a number of meaningful applications of unawareness to contract theory have been made. Brief discussions of a number of contributions which this paper references are offered below.

Incorporating into Contract Theory (Filiz-Ozbay (2011)) Filiz-Ozbay (2011) analyzes the effect of *evolving awareness* in an insurer-insuree setting. In this model, the insurer has full awareness over the states of the world Ω (hereafter referred to as contingencies) and hence assesses their likelihood with the objective probability measure μ. The insuree is aware of only $\Omega' \subset \Omega$ and assesses the contingencies in Ω with the measure $\mu(\cdot|\Omega)$. The author assumes a one-to-one damage mapping $c : \Omega \to \mathbb{R}_+$ and, for simplicity, reduces the entire model to damage levels where $c(\Omega) = S$ and $c(\Omega') = S' \subset S$. Hence a contract is written on *damage levels* each of which represents a given contingency, rather than contingencies themselves.

The process of evolving awareness enters the model in the following way. First the insurer specifies a contract $C = (A, t(\cdot), k)$. Here, A are the announced contingencies, $t : A \to \mathbb{R}$ are the transfers from insurer to insuree in the case that contingency $a \in A \setminus S'$ materializes and k is the premium payed from insuree to insurer upon signing the contract. Reading a contract, therefore, can have the effect of expanding the awareness of the insuree to $S' \cup A$ her *extended awareness set*.

This is the crux of the paper, and the insuree must now generate beliefs over $S \cup A$. This is a probability distribution $P_C \in \Delta(S' \cup A)$. The formation of beliefs is a part of the solution concept that the authors form. But, briefly, the article introduces to notions of belief formation in regards to a contract.

Filiz-Ozbay first imposes a *compatibility* condition on the belief of the insuree consisting of two main requirements. The first requirement of this condition on beliefs implies that, from the perspective of the insuree, the insurer makes a utility gain from offering contract C whenever it is accepted. The second requirement says that every contingency is considered possible, and "updated" beliefs when restricted to the initial awareness set S' must match. That is, the relative weights of contingencies in S' cannot be altered in distribution P_C - but no requirements are made on contingencies $a \in A$. Intuitively, a condition such as compatibility seems natural. If a potential insuree, believes that the insurer makes a loss from the contract I become wary of its terms. The second point intuitively suggests the idea that an insuree can judge the frequency of events of which he is aware. So, conditional on those events of which he is aware, his estimation of their likelihood with respect to his subjective beliefs, matches the principals when restricted to those events of which he is aware. Filiz-Ozbay requires beliefs to be compatible in equilibrium.

Definition 2.2. (Ozbay(2008)) An equilibrium is a triplet $(C^*, D^* : \mathbb{C} \to \{\text{buy}, reject\}$, $(P_C^*)_{C\in\mathbb{C}})$ such that:

1. $C^* \in \underset{C\in\mathbb{C}}{\arg\max} \mathbb{E}U_1(C, D^*(C))$
2. With respect to the equilibrium belief P_C^* the insuree's decision rule $D^*(C)$ is optimal. Further the insuree must reject the offer if $\exists\, a \in A : t(a) > a$.
3. P_C^* is compatible wherever possible.

The authors show that under the requirement of compatibility there is always an equilibrium which does not extend the awareness of the insuree (Theorem 3.1 in Filiz-Ozbay).

However, Filiz-Ozbay concedes that compatible beliefs are still not enough. There can exist situations in which, although the insuree believes the insurer makes a utility gain, according to the insurees equilibrium belief P_C^* the insurer is not a VNM utility maximizer. As such, an additional requirement, *consistency* on equilibrium beliefs is imposed.

Definition 2.3. (Ozbay (2008)) An equilibrium $(C^* = (t^*, A^*, k^*), D^*, P_C^*)$ is *consistent* if for every contract C the insuree assesses, with respect to belief P_C^*, the insurer to be a utility maximizer. ie:

$$\mathbb{E}U_1^0[C^*, D^*|P_C^*] \geq \mathbb{E}U_1^0[C, D^*|P_C^*]\,\forall\, C \in \mathbb{C}$$

Consistency, however, does not rule out the possibility of incomplete contracts - an incomplete contract in Filiz-Ozbay is one in which A is such that $S' \cup A \neq S$. In fact, the paper suggests that incompleteness in the contractual form arises from the insurers strategic incentive to exploit the unawareness of the insuree. Consistency of beliefs with respect to a contract is again a very natural notion. If I believe that the insurer can make a gain by offering some other contract then

I again become wary of his reasons for offering the contract he posts. So, the consistency requirement says that in equilibrium, my beliefs support the actions of the insurer.

Finally, it is shown that incorporating multiple insurers promotes awareness. Skipping some of the more technical details the following theorem is proofed:

Theorem 2.1. (Filiz-Ozbay(2008)) If the number of insurers is large enough, then in any symmetric equilibrium where the insuree buys the contract, the offer is a complete full insurance contract and insurers make zero profits.

Intuitively, the argument is as follows. As the number of firms gets larger, the insurer has two competitive options; 1) Announce contingencies which are hidden by other insurers 2) Offer a different contract on the same contingencies. The chance of attracting a consumer by offering the same contract decreases with number of insurers in the market. Therefore, an insurers expected gain from following the strategies of the others decreases with number of insurers in the market. As such, it is better to extend the awareness of the insurer. Essentially firms engage in a process of competition over awareness and prices such that each will cover all contingencies fully, and earn zero in expectation.

The paper by Filiz-Ozbay is not only clear and concise but provides a number of powerful and interesting results. Filiz-Ozbay succintly makes the case for unawareness as a driving force in the incompleteness of contracts, and goes on to show that competition among insurers promotes completeness in the contractual form. Its theoretical foundation is that of the earlier discussed paper by Heifetz et al., save for the fact that the mechanics of this model limits discussion to mutually exclusive events to which agents assign subjective probabilities. It is this idea that I incorporate into the following work, and allows for updating among and between unaware agents. A further interesting example of this sort of approach is found in Zhao (2013).

Framing Contingencies in Contracts (Zhao (2013)) Zhao in his paper *Framing Contingencies in Contracts* again builds on the model proposed by Filiz-Ozbay to examine how insurers *frame* the contingencies offered in a contract. By this he means the process by which terms in a contract can be used to promote or restrict awareness. The awareness structure is similar to that of Filiz-Ozbay: The insurer is aware of the entire set of contingencies leading to damage whereas the insuree knows only a subset of these. Consider the following example of how framing interacts with awareness. Suppose $\Omega = \{a, b, c\}$ is the set of possible damages to a good v. The insuree is aware only of a. Further, the insurer can offer two contracts to an insuree. The first contract the insurer can offer promises to transfer x units of value in the case contingency a. The second, promises x in the event that a "damage" occurs. Intuitively, we know that the agent will prefer the second contract: Although she is unaware of states (a, b) explicitly, she is aware of the possibility that other damages may occur to the good. This is where the papers by Zhao and Filiz-Ozbay differ: Zhao allows for a modeling of awareness of unawareness. I explain the basic method by which this is achieved.

First, the agent is aware of, and can *express*, some subset of a partition of Ω. Let K_0 denote the agents initial awareness and $\mathcal{L}(K_0)$ be the events which can be expressed by the agent. Let $X \subset \Omega$ represent a *general* event. For example, in Zhaos model, the *general* event X can be summarized by the sentence *"a damage occurs to good v"*. This event X captures a general concept, regardless of whether or not the individual contingencies $x \in X$ are available in the agents mind during the decision making process. This is what drives Zhao's model, this general event X captures the idea that there are contingencies in the world of which the insuree is unaware, but at the same time she is aware that she is unaware of these contigencies, and hence is aware of the general concept X (along with its complement X^C). Beliefs over these events are defined in the following way. For $Z \in \{X, X^C\}$ let $\alpha_Z(K_0)$ denote the *residual probability* of the unforeseen event Z. This function represents the degree to which the agent is aware of her unawareness of unforeseen contingencies in Z. Thus, whilst the objective probability space is given as $(\Omega, 2^\Omega, \mu)$ where μ is objective probability measure, the agent perceives a somewhat restricted version of this space $(K_0, \mathcal{L}(K_0), \mu^{K_0})$ where:

$$\mu^{K_0}(\omega) = \frac{\mu(\omega)}{\sum\limits_{\omega \in \Omega} \mu(\omega) + \sum\limits_{Z \in \{X, X^C\}} \alpha_Z(K_0)} \, \forall\, \omega \in K_0$$

$$\mu^{K_0}(Z \setminus K_0) \frac{\alpha_Z(K_0)}{\sum\limits_{\omega \in \Omega} \mu(\omega) + \sum\limits_{Z \in \{X, X^C\}} \alpha_Z(K_0)} \, \forall\, Z \in \{X, X^C\}$$

Using this method of belief formation, the utility of the insuree becomes "general-event dependent" in the sense that the choice the insuree makes, between accepting and rejecting the contract depends on whether or not the underlying concept of this general event "occurs". The utility function is given by $U : \Omega \times S \to \mathbb{R}$:

$$u(\omega, S) = \begin{cases} u^X(s) & \omega \in X \\ u^{X^C}(s) & \omega \in X^C \end{cases}$$

where S represents the choice set of the agent and the principal. That is, S is the set which denotes the contingencies on which the contract is signed along with the transfers, together with the choice of whether to accept or reject on the part of the agent.

After defining the maximization problem of the insurer, Zhao offers the key result of the paper (Proposition 1 in Zhao (2008)). Briefly this says that the insurer exploits the contract if and only if the contract leaves the insuree unaware of some payoff-relevant contingencies. It is easy to read this as a tautology - the insurer is exploitative if he exploits the unawareness of the insuree. But it is not trivial that this carries over to optimizing behavior. That is, it is not trivial that in an optima the insurer is exploitative if and only if the contract is vague on certain contingencies.

Using this Zhao turns to look at home insurance contracts, and models the choice of the insurer between offering the exploitative or non-exploitative contract as dependent on $\alpha_Z(K_0)$ - the agents degree of awareness of unawareness. There are a range of α values such that it is more profitable to announce than to not-announce extra contingencies (Proposition 2 in Zhao (2008)).

Zhao finally channels Filiz-Ozbay (2008) by analyzing the case of a *justifiability* constraint - the constraint that the insurer must appear to be a VNM utility maximizer according to the insuree - corresponding to the consistency requirement of equilibrium beliefs in Filiz-Ozbay. Under this constraint a full insurance result obtains, and again the insurer will announce the contigency as long as the insuree does not overestimate the existence of contingencies in Z.

Zhao's paper relies more explicitly on the modelling of agents with languages who view a certain partition of the restricted state-space. These are the events in the model. The language of the agents determine their ability to express such events and in turn their awareness of an event. Whilst the objective probability space would be given by $(\Omega, 2^\Omega, \mu)$ - the space viewed by the principal - the subjective space is a restriction of that to $(\Omega, \mathcal{L}(K), \mu^K)$. In this way, events can be modelled as non-mutually exclusive.

2.3 Executive Summary of the Literature

The literature on the subject of unawareness is relatively heterogenous. First, there are number of competing approaches and ideas on exactly how unaware agents should be modelled in terms of mathematical formalism. The subjective-state-space approach offered by Heifetz et al. (2006) is often the formal underpinning of most applications of unawareness to contract theory. Auster (2013), Zhao (2011) and Filiz-Ozbay (2011) are all examples of this. Moreover, there are various approaches on how to incorporate unawareness into principal-agent contracting models. The two main approaches to be contrasted are that of Zhao and Filiz-Ozbay. Where Zhao's model allows a discussion of non-mutually exclusive events which partition the state-space, Filiz-Ozbays perhaps simpler, or more intuitive, model restricts discussion to mutually exclusive events, which are modelled as contingencies in an insurance contract. Further, Filiz-Ozbay's formulation of beliefs is less restrictive. Whilst both impose a common initial measure - conditional on those contingencies of which the unaware agent is aware - Filiz-Ozbay's equilibrium concept allows for more flexibility in how beliefs are formed they merely have to be compatible with the contract. On the other hand, Zhao's formulation of unawareness and awareness of unawareness is interesting in its ability to capture how agents update their entire probability space after reading the contract. Although all the papers I have discussed have their merits, this thesis leans more in the direction of Filiz-Ozbay in terms of modelling agents aware of certain mutually exlusive events who's awareness evolves in the interaction of insurance contracting situation.

3 Model

The following is a proposed model of a situation in which strategic interactions depend on both asymmetric information and asymmetric awareness. Asymmetric awareness enters the model by supposing the agent is aware of only a subset of these causes complete with a restricted/subjective probability measure. The principal, on the other hand, is aware of the full set along with the objective probability measure over this set. Asymmetric information is captured via the idea that the principal, although aware of the agents limited awareness, does not know exactly which contingencies the agent considers. The agent can update her awareness when offered a contract. She reads the terms, evaluates the contingencies mentioned and assigns a subjective probability assessment to these new contingencies. However, I further suppose that reading terms of which she was previously unaware corresponds - in a sense - to becoming aware of her own unawareness as in Zhao (2011). In this setting, I interpret this as a proxy for suspicion. When new contingencies are announced to the agent, she becomes aware of the general event that there are other contingencies of which she is still unaware and can use this position when bargaining with the principal. The insurer must offer a contract to the insuree for protection against some damages.

3.1 Two Types, Three States

Awareness, Information and Types Let $\Omega = \{a, b, c\}$ be the entire set of causes leading to damages of a good v owned by an agent. The *insurer* has perfect knowledge of the contigencies $\omega \in \Omega$ along with their probabilities given by $\mu(\cdot)$, the objective probability measure. The *insuree* on the other hand is a aware only of a certain $K \subset \Omega$ set of contingencies and evaluates their probabilities with a restricted probability measure $\mu_K(\cdot)$. Although the insuree may not be able to accurately assess the probability of some event occurring it is unproblematic to assume that she assesses the relative weights of events correctly. Intuitively, the contingencies of which she is aware, she can observe and hence can judge the relative frequency of them in the same way as the insurer. That is, $\mu_K(\cdot) = \mu(\cdot|K)$. A type $K \in \Theta$ is associated with a given K that characterizes their awareness. There is a distribution $f(\cdot)$ over Θ of which the principal is aware. For this, the most basic version of the model, I assume that there are just two types of agents: A who are aware of set $A = \{a, b\}$ occur with probability $f(A) = \pi$, and B who are aware of $B = \{b, c\}$ and occur with probabiity $f(B) = 1 - \pi$.

Although an interesting question is how the insurer behaves given no information on these types I impose that the insurer knows the cardinality of each of the awareness sets along with the number of types. That is, he knows there are two types who are each aware of two contingencies. The asymmetric information arises from the fact that the principal does not know *which* $\omega \in K$ the agent is aware of, despite knowing the cardinality of K.

Further, without loss of generality I introduce the following assumption for clarity in the three state case.

Following Filiz-Ozbay I introduce a damage function $d : \Omega \to \mathbb{R}_+$ and reduce the model to damage levels $S = d(\Omega)$. As such, the words "event" and "damage" will be interchanged for fluidity where there is no room for confusion.

Abusing notation slightly let $d(a) = a$, $d(b) = b$ and $d(c) = c$. Then, I impose the following conditions on the relationship between the damage levels.

Assumption 3.1. $b < a$ and $b < c$

As such, the least salient state is state b of which both are aware initially and bargaining occurs over b plus the states of which each type becomes aware. To keep things general, we do not impose anything on the relationship between a and c.

Contracts

Definition 3.1. A **contract** in this model can be defined as a tuple $C = (Q, t(\cdot), k)$ where:

1. Q are the events covered by the contract (the announced contingencies).
2. $t : Q \to \mathbb{R}_+$ is a function specifying transfers to be paid in the event an $q \in Q$ materizalizes.
3. k is a premium paid to the insurer.

Then the **space of all contracts** $\mathbb{C}$ is the set of all such tuples.

The definition of a contract involves the use of a damage function to reduce the model to only damage levels.

Example 3.1. *Let damages be:*

$$d(\omega) = \begin{cases} 500 & \omega = a \\ 200 & \omega = b \\ 700 & \omega = c \end{cases}$$

A contract which promises transfers of 500 *in the event that a obtains and* 300 *in the event that c obtains, together with a premium of* 500 *is a tuple:*

$$C = (\{a, c\}, (t(a) = 500, t(b) = 0, t(c) = 300), 500)$$

There is an implicit assumption that offering such a contract to the A type (for example) alters his awareness, that is, agents are able to conceive of a notion once they are presented with it.

Updating Awareness and Suspicion After reading a contract which announces terms of which the insuree was previously unaware, she must form beliefs over these contingencies. Let B_K denote the the distribution of type K over the set of announced contingencies Q.

Definition 3.2. The updated probability distribution of type K B_K is the distribution defined by:

$$B_K(s) = \frac{(1-\alpha_K)\mu_P(s)}{\sum_{s\in K}\mu_P(s)} \,\forall\, s \in K$$

Where α_K is an exogenously given parameter reflecting the residual weight type K gives to the newly announced contingencies:

$$B_K(s) = \alpha_K$$

for $s \in Q \setminus K$
And if $K \subseteq Q \implies B_K(\cdot) = \mu(\cdot|K)$

α_K is a parameter that is given exogenously acting as a measure of updating for a given type. The second condition of the above definition says that if no new contingencies are revealed we get our original distribution back.

For example, A could be offered a contract based on $\{a, c\}$. After reading such a contract the A type resigns residual probability α_A to c and updates her beliefs over the contingencies $\{a, b\}$ in accordance with definition 3.2.

Note that with this kind of belief formation, the insurers profit is dependent not only on the terms of the contract, but also on the way in which beliefs are formed. That is, the terms of the optimal contract of following a given strategy (the choice of who to appeal to) depends on how suspicious each type becomes *ex ante*. For reasons that will become clear, for the rest of this paper I will refer to a type who, after becoming aware of a contingency x assigns $\alpha_K < \mu(x)$. Figure

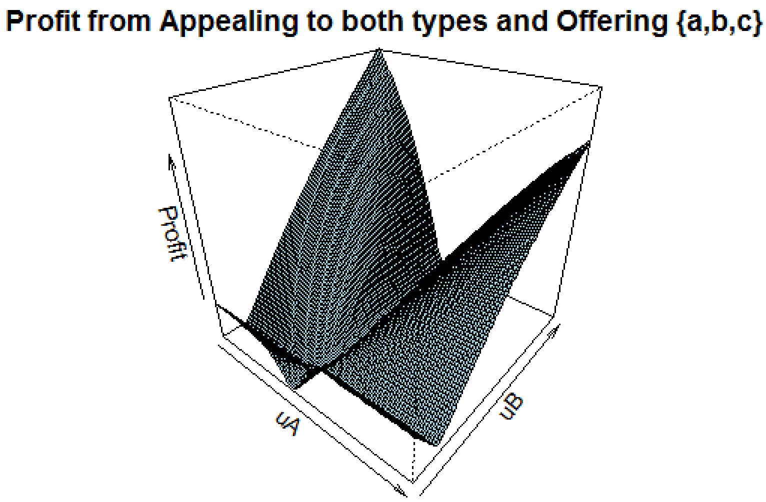

Figure 1: Profit of the Insurer As a function of the Updating Process

1 shows the profit of the insurer from following the strategy of announcing all contingencies and appealing to both types.

As I hope figure 1 demonstrates, the optimal profit is highly dependent on the way beliefs are formed. As such, the optimal strategy depends not only on the distribution of types, but also on this updating process.

The Insurers Problem For both insurer and insuree, utilities depend not only on the terms of the contract, but also on their strategies. For the insuree of either type the strategy space is simply $S_K = \{\text{Accept, Reject}\}$.

The insuree's strategy can be summarized via a decision rule $D_K : \mathbb{C} \to \{0, 1\}$ where 0 indicates rejection and 1 indicates acceptance for $K \in \{A, B\}$. The expected utility of type K from contract $C = (Q, t(\cdot), k)$ can be written as:

$$\begin{aligned}\mathbb{E}U_K(C|D_K(C)) = {} & [\sum_Q u(v - s + t(s) - k)B_K(s) + \sum_{K\setminus Q} u(v - s)B_K(s)]D_K(C) \\ & + [\sum_Q u(v - s)B_K(s) + \sum_{K\setminus Q} u(v - s)B_K(s)](1 - D_K(C))\end{aligned}$$

Example 3.2. ***Continued from Example 3.1*** *Suppose the insurer offers the contract identified in Example 3.1. Let* $u(x) = \sqrt{x}$.

The objective probabilities of the various contingencies are:

$$\mu_P(\omega) = \begin{cases} 0.25 & \omega = a \\ 0.5 & \omega = b \\ 0.25 & \omega = c \end{cases}$$

Let $v = 1000$, then the objective utility is given by:

$$\sqrt{1000 - 500 + 500 - 500} \times 0.25 + \sqrt{1000 - 200 + 0 - 500} \times 0.5$$
$$+\sqrt{1000 - 700 + 300 - 500} \times 0.25 \approx 14.88$$

After reading such a contract, A becomes aware of c and B becomes aware of a. Hence each must update. Suppose $\alpha_A = 0.1$ and $\alpha_B = 0.2$ then beliefs are given by:

$$B_A(s) = \begin{cases} 0.3 & s = a \\ 0.6 & s = b \\ 0.1 & s = c \end{cases}$$

and

$$B_B(s) = \begin{cases} 0.2 & s = a \\ 0.53 & s = b \\ 0.27 & s = c \end{cases}$$

Reservation utilities are given by:

$$\mathbb{E}(U(C)|A, 0)$$
$$=$$
$$\sqrt{1000 - 500} \times 0.3 + \sqrt{1000 - 200} \times 0.6 + \sqrt{1000 - 700} \times 0.1 \approx 25.4$$

$$\mathbb{E}(U(C)|B, 0)$$
$$=$$
$$\sqrt{1000 - 500} \times 0.2 + \sqrt{1000 - 200} \times 0.53 + \sqrt{1000 - 700} \times 0.26 \approx 24.175$$

And the value of purchasing the contract is:

$$\mathbb{E}(U(C)|A, 1)$$

$$=$$

$$\sqrt{1000-500+500-500}\times 0.3+\sqrt{1000-200+0-500}\times 0.6+$$
$$\sqrt{1000-700+300+500}\times 0.1\approx 18.1$$

$$\mathbb{E}(U(C)|B,1)$$
$$=$$
$$\sqrt{1000-500+500-500}\times 0.2+\sqrt{1000-200+0-500}\times 0.53$$
$$+\sqrt{1000-700+300-500}\times 0.37\approx 16.37$$

And as such both insurees will reject such a contract contract.

For an insurer the strategy space is simply terms of the contract. The insurer structures the terms of the contract such that he maximizes the expected payoff he gets from the A types and B types together. Given the distribution $f(A)=\pi$ the insurers utility function is given by:

$$\mathbb{E}U_P(C|D_A(C),D_B(C))=$$
$$\pi[k-\sum_Q t(s)\mu(s)]D_A(C)+(1-\pi)[k-\sum_Q t(s)\mu(s)D_B(C)$$

The insurers problem is formally given by the following definition.

Definition 3.3. The problem of the insurer is:

$$\max_{C\in\mathbb{C}}\mathbb{E}U_P(C|D_A(C),D_B(C))$$

The insurer must choose a contract which maximizes his expected payoff. In the interest of being explicit, with two types, this is equivalent to solving the following problem.

$$\max_{k,t(\cdot),Q}\pi[k-\sum_Q t(s)\mu(s)]D_A(C)$$
$$+(1-\pi)[k-\sum_Q t(s)\mu(s)D_B(C)$$

subject to, for $K \in \{A, B\}$ it holds:

$$\sum_{Q} u(v + t(s) - k)\mu(s|K) + \sum_{K \setminus Q} u(v - s - k)\mu(s|K)$$
$$\geq \sum_{Q \cup K} u(v - s)\mu(s|K) \tag{1}$$
$$\Longleftrightarrow$$
$$D_K(C) = 1$$

where the above is the participation constraint of the K type.

With this in hand we are ready for an equilibrium definition.

Definition 3.4. An **equilibrium** of the two-type model is a contract $C = (Q^*, t^*(\cdot), k^*)$ and a tuple of strategies $(D_A^*(C^*), D_B^*(C^*))$ such that:

1. The insurees of either type decides optimally on the contract given their beliefs ie. $(D_A^*(C^*), D_B^*(C^*)$ are such that:

$$D_K^*(C^*) = \begin{cases} 1 & \Longleftrightarrow \; \mathbb{E}[u_K(C^*|1)|B_K] \geq \mathbb{E}[u_K(C^*|0)|B_K] \;\&\; \mathbb{E}[U_P|B_K] \geq 0 \\ 0 & \Longleftrightarrow \; \mathbb{E}[u_K(C^*|0)|B_K] > \mathbb{E}[u_K(C^*|1)|B_K] \end{cases}$$

   $\forall\, K$.

2. Given the decisions of each type, the contract C^* is optimal for the insurer. ie:

$$\mathbb{E}[U_P(C^*|D_A^*(C^*), D_B^*(C^*))] \geq \mathbb{E}[U_P(C, D_A^*(C), D_B^*(C))]$$

   $\forall\, C \in \mathbb{C}$

3. $k^* \geq 0$ and $t^*(\omega) \leq d(\omega) \,\forall\, \omega \in \Omega$

An equilibrium is a contract, which in itself consists of a set of announced contingencies, a transfer rule and a premium. This contract must be optimal for the insurer given the decisions of each type, which in turn must be optimal for each insuree. Further, we require that premiums are non-negative - the insurer will never pay someone to take a contract - and transfers must not exceed damages since such a contract could serve to give the insurer infinite profit.

4 Solution Method

The problem the insurer faces is different from the standard insurance problem in that he faces uncertainty in terms of the distribution of types and as such of the awareness levels of the insuree he is dealing with. This problem is further compounded by the fact that announcing a contingency alters the participation constraint of a type. If an insuree reads a contract announcing a contingency of which she was previously unaware then her utility function updates to include a state which accounts for that contingency, and her beliefs update. Hence her participation constraint has the potential to be significantly different *ex ante*. Even further, it may be optimal to simply cater to one type, or it may be optimal to cater to both types. The solution method I propose to the problem is the following. The insurer faces a sequence of optimization problems subject to varying constraints. Each of these problems corresponds to a different choice or *strategy* the insurer plays. First, he must choose which contingencies to cover. Each $Q \subseteq \{a, b, c\}$ is a potential set of contingencies on which the insurer can offer cover. Further, he must optimize the terms of the contract depending on which type he chooses to cater to. That is, for each Q he find the optimal contract which the A type would accept, the optimal contract which the B type would accept and the optimal contract which both types. This corresponds to solving three optimization problems for each Q. For a given Q the insurer solves the relevant maximization problem once subject to the participation constraint of the A type, once subject to the participation constraint of the B type and once subject to the participation constraint of both types. In all cases, contracts must be compatible and transfers cannot exceed damages. This process eventuates by giving the insurer a menu of contracts such that for any π he simply picks the contract which maximizes his expected payoff.

4.1 Characterizing Solutions to the Insurers Problem

In this section I characterize the equilibrium contract as a function of the parameters of the model.

Proposition 4.1. Appealing to K over a single contingency $x \in K$ yields the zero contract ie. The contract characterized by the system of equations

$$
\begin{aligned}
&k^* : \text{Participation Constraint of K Type Binds} \\
&t^*(x) = x
\end{aligned}
$$

Proof. WLOG I show this for the decision to appeal to A over b. The insurer solves:

$$\max k - \mu(b)t(b)$$

subject to

$$u(v-a-k)\frac{\mu(a)}{\mu(a)+\mu(b)} + \frac{\mu(b)}{\mu(a)+\mu(b)}u(v-b+t(b)-k)$$
$$= u(v-a)\frac{\mu(a)}{\mu(a)+\mu(b)} + u(v-b)\frac{\mu(b)}{\mu(a)+\mu(b)}$$

$$t(b) \leq b \tag{2}$$

Yielding first order conditions

$$1 - \lambda\Big[\frac{u'(v-a-k))\mu(a)}{\mu(a)+\mu(b)} + \frac{u'(v-b+t(b)-k)\mu(b)}{\mu(a)+\mu(b)}\Big] = 0 \tag{3}$$

$$-\mu(b) + \lambda\big[u'(v-b+t(b)-k)\frac{\mu(b)}{\mu(a)+\mu(b)}\big] - \gamma_b = 0 \tag{4}$$

$$\lambda\big[u(v-a-k)\frac{\mu(a)}{\mu(a)+\mu(b)} + \frac{\mu(b)}{\mu(a)+\mu(b)}u(v-b+t(b)-k)\big] = 0 \tag{5}$$

$$\gamma_b[t(b)-b] = 0 \tag{6}$$

Clearly 2 implies the constraint must bind and therefore it must be the case that:

$$u(v-a-k)\mu(a) + u(v-b+t(b)-k)\mu(b)$$
$$= u(v-a)\mu(a) + u(v-b)\mu(b)$$

Note that $\lambda = 0$ implies the contradiction $1 = 0$ and hence the constraint must bind.

Solving for λ yields

$$\lambda = \frac{\gamma_b + \mu(b)}{u'(v - b + t(b) - k)\mu(b)}$$

Substituting this expression into 3 implies:

$$1 - [\mu(b) + \gamma_b + \frac{\mu(a)u'(v-a-k)}{u'(v-b+t(b)-k)} + \frac{\gamma_b{}'}{u}(v-a-k)\mu(a)u'(v-b+t(b)-k)] = 0$$

$$\iff$$

$$\mu(b) + \gamma_b + \frac{\mu(a)u'(v-a-k)}{u'(v-b+t(b)-k)} + \frac{\gamma_b u'(v-a-k)\mu(a)}{u'(v-b+t(b)-k)}$$
$$=$$
$$\mu(b) + \mu(a) + \mu(c)$$

Case 1: $\gamma_b = 0$ then

$$\frac{u'(v-a-k)}{u'(v-b+t(b)-k)} = \frac{\mu(a) + \mu(c)}{\mu(a)}$$

Then concavity implies that $t^*(b) > b - a$ and the constraint binds at k.

Case 2: $\gamma_b > 0$ then $t(b) = b$ and the constraint binds at k^*.

I claim that case 2 is always the profit maximizing action.

Consider case 1 in which $t(b) < b$, note from the participation constraint that $k^* > k$.

Let $x = t(b) - b$ them since the participation constraint binds it must be the case that:

$$u(v - x - k) > u(v - b) \implies b - x > k \implies b > k$$

Further note that

$$u(v - b + t(b) - k) > u(v - b) \implies t(b) > k$$

Hence we have:

$$b > t(b) > k$$

We want to show that:

$$k - t(b)\mu(b) < k^* - b\mu(b)$$

Now since $k^* > k$ it remains to show that $0 < k - k^* < t(b) - b$ but this holds since $t(b) < b$. Hence $t(b) = b$ and k^* such that the constraint binds characterizes the optimal solution.

□

The above proposition yields a characterization of the insurers problem in the case that no compatibility constraint as in Filiz Ozbay is imposed. Note however that since the constraint binds we must have

$$u(v - k) > u(v - b)$$

$$\Longrightarrow$$

$$k < b = t(b)$$

Hence

But this implies:

$$k^* u'(v - a) < u'(v - b)[t^*(b) - k^*]$$

$$\Longrightarrow$$

$$t^*(b) > \frac{u'(v-a)\mu(a) + u'(v-b)\mu(b)}{u'(v-b)\mu(b)} k^*$$

$$\Longrightarrow$$

$$\frac{\mu(b)}{\mu(a) + \mu(b)} t^*(b) > \frac{u'(v-a)\mu(a) + u'(v-b)\mu(b)}{u'(v-b)} \frac{k^*}{\mu(a) + \mu(b)}$$

But note that

$$b < a \implies \frac{u'(v-a)}{u'(v-b)} > 1$$

and as such:

$$\frac{u'(v-a)\mu(a) + u'(v-b)\mu(b)}{u'(v-b)} \frac{1}{\mu(a) + \mu(b)} > \frac{\mu(a) + \mu(b)}{\mu(a) + \mu(b)} = 1$$

Hence:

$$\frac{\mu(b)}{\mu(a) + \mu(b)} t^*(b) > \frac{u'(v-a)\mu(a) + u'(v-b)\mu(b)}{u'(v-b)} \frac{k^*}{\mu(a) + \mu(b)} > k^*$$

So, according to the insuree, the insurer makes a loss on such a contract. In general this is something we would want to abstract from, as in Filiz-Ozbay, such a requirement was not implemented in this paper but is an interesting avenue for future research.

Proposition 4.2. The decision to appeal to only one insuree over the contingencies of which they are originally aware, K, yields the contract characterized by the following system of equations:

$$t(s) = s \, \forall \, s \in K$$
$$u(v - k^*) = \sum_K u(v - s)\mu(s|K)$$

Proof. I show this, WLOG, for the decision to appeal to A on $\{a, b\}$.

$$\max k - t(a)\mu(a) - t(b)\mu(b) \tag{7}$$

subject to

$$u(v - a + t(a) - k)\frac{\mu(a)}{\mu(a) + \mu(b)} + u(v - b + t(b) - k)\frac{\mu(b)}{\mu(a) + \mu(b)} \geq$$
$$u(v - a)\frac{\mu(a)}{\mu(a) + \mu(b)} + u(v - b)\frac{\mu(b)}{\mu(a) + \mu(b)} \tag{8}$$

$$t(a) \leq a \tag{9}$$
$$t(b) \leq b \tag{10}$$

Consider the first order conditions:

$$1 - \lambda[u'(v - a + t(a) - k)\frac{\mu(a)}{\mu(a) + \mu(b)} + u'(v - b + t(b) - k)\frac{\mu(b)}{\mu(a) + \mu(b)}] = 0 \tag{11}$$

$$-\mu(a) + \lambda u'(v - a + t(a) - k)\frac{\mu(a)}{\mu(a) + \mu(b)} - \gamma_a = 0 \tag{12}$$

$$-\mu(b) + \lambda u'(v - b + t(b) - k)\frac{\mu(b)}{\mu(a) + \mu(b)} - \gamma_b = 0 \tag{13}$$

First note that $\lambda = \gamma_a = \gamma_b = 0$ yields the contradiction $\mu(a) = \mu(b) = 0$.

Next note that $\lambda > 0$ but $\gamma_a = \gamma_b = 0$ yields the contradiction $1 - \mu(a) - \mu(b) = 0$ after solving 12 and 13 for λ and substituting into 11.

$\lambda = 0, \gamma_a > 0, \gamma_b > 0 \implies t^*(a) = a, t^*(b) = b$. If the constraint does not bind, then this cannot be optimal since increasing k^* until it does gives the insurer more profit. Note that it cannot be the case that $\lambda = \gamma_i = 0$ since this would imply that $\mu(i) = 0$.

Hence the only possible solution is characterized by $\lambda > 0, \gamma_a > 0, \gamma_b > 0$.

This implies that $t^*(a) = a$ and $t^*(b) = b$ and the constraint binds. Hence the system:

$$t^*(a) = a$$

$$t^*(b) = b$$

$$u(v - k^*) = u(v - a)\frac{\mu(a)}{\mu(a) + \mu(b)} + u(v - b)\frac{\mu(b)}{\mu(a) + \mu(b)}$$

Characterizes the solution.

□

Proposition 4.3. (Characterization of a solution to appealing to the alternative type on K)

If $\alpha_B \leq \mu(a)$ then the optimal contract resulting from the decision to appeal to the B type over $\{a, b\}$ is characterized by the equation system:

$$t^*(a) = a$$

$$t^*(b) = b$$

$$\begin{aligned} & u(v - a + t^*(a) - k^*)\frac{\alpha_B}{(1 - \alpha_B)}\big[\mu(b) + \mu(c)\big] + u(v - b + t^*(b) - k^*)\mu(b) \\ +& u(v - c - k^*)\mu(c) = \\ & u(v - a)\frac{\alpha_B}{(1 - \alpha_B)}\big[\mu(b) + \mu(c)\big] + u(v - b)\mu(b) + u(v - c)\mu(c) \end{aligned}$$

Proof. The insurer solves:

$$\max k - t(a)\mu(a) - t(b)\mu(b)$$

Subject to

$$
\begin{aligned}
&u(v-a+t(a)-k)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]+u(v-b+t(b)-k)\mu(b)\\
&+u(v-c-k)\mu(c)\geq\\
&u(v-a)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]+u(v-b)\mu(b)+u(v-c)\mu(c) \qquad (14)
\end{aligned}
$$

$$t(a)\leq a \qquad (15)$$

$$t(b)\leq b \qquad (16)$$

Yielding FOCS

$$
\begin{aligned}
&1-\lambda\Big[\frac{\alpha_B}{1-\alpha_B}[\mu(b)+\mu(c)]u'(v+t(a)-a-k)\\
&+u'(v+t(b)-b-k)\mu(b)-u'(v-c-k)\mu(c)\Big]=0 \qquad (17)
\end{aligned}
$$

$$-\mu(a)+\lambda[\frac{\alpha_B}{1-\alpha_B}u'(v+t(a)-a-k)[\mu(b)+\mu(c)]]-\gamma_a=0 \qquad (18)$$

$$-\mu(b)+\lambda[u'(v+t(b)-b-k)\mu(b)]-\gamma_b=0 \qquad (19)$$

Note first that the constraint must bind. Then, solving for λ we have:

$$
\begin{aligned}
\lambda &= \frac{\mu(a)+\gamma_a}{u'(v+t(a)-a_k)}\frac{1-\alpha_B}{\alpha_B}\frac{1}{\mu(b)+\mu(c)}\\
&= \frac{\gamma_b+\mu(b)}{u'(v-b+t(b)-k)\mu(b)}
\end{aligned}
$$

Substituting these expressions into 17 it must be that:

$$1-[\mu(a)+\gamma_a+\mu(b)+\gamma_b+\lambda u'(v-c-k)\mu(c)]=0$$

And therefore:

$$\gamma_a+\gamma_b+\lambda u'(v-c-k)\mu(c)=\mu(c)$$

Case 1: $\gamma_a=\gamma_b=0$

From here we get that

$$\lambda u'(v-c-k)=1$$

$$\iff$$

$$1 = \frac{u'(v-b+t(b)-k)}{u'(v-c-k)}$$

$$\Longrightarrow$$

$$t(b) = b - c < 0$$

Yielding a contradiction.

Case 2: $\gamma_a = 0,\ \gamma_b > 0$

In this case we have that $t^*(b) = b$

And it must be the case that:

$$\gamma_b + \lambda u'(v-c-k)\mu(c) = \mu(c)$$

Substituting in and solving for γ_b yields:

$$\gamma_b = \frac{\mu(c)[u'(v-k) - u'(v-c-k)]}{1 + \frac{\mu(b)}{\mu(c)}\frac{u'(v-c-k)}{u'(v-k)}}$$

But note that by concavity

$$u'(v-k) < u'(v-c-k)$$

since premiums must be positive as long as a contract is offered.

$$\Longrightarrow$$

$$\gamma_b < 0$$

which violates the KKT conditions.

Case 3: $\gamma_b = 0, \gamma_a > 0$

$$\gamma_a + \frac{\mu(a) + \gamma_a}{u'(v-a+t(a)-k)[\mu(b)+\mu(c)]}\frac{1-\alpha_B}{\alpha_B}u'(v-c-k)\mu(c) = \mu(c)$$

And note that $t(a) = a$.

Then solving for γ_a we have:

$$\gamma_a = \frac{\mu(c)\left[1 - \frac{\mu(a)}{\mu(b)+\mu(c)}\frac{1-\alpha_B}{\alpha_B}\frac{u'(v-c-k)}{u'(v-k)}\right]}{1 + \frac{\mu(a)}{\mu(b)+\mu(c)}\frac{1-\alpha_B}{\alpha_B}\frac{u'(v-c-k)}{u'(v-k)}}$$

Consider the numerator of this fraction which simplifies to:

$$u'(v-k) - \frac{\mu(a)}{\mu(b)+\mu(c)}\frac{1-\alpha_B}{\alpha_B}u'(v-c-k)$$

Note that for any $k > 0$ by the concavity of $u(\cdot)$

$$u'(v-c-k) > u'(v-k)$$

Also note that

$$\alpha_B \leq \mu(a) \implies \frac{\mu(a)}{\mu(b)+\mu(c)}\frac{1-\alpha_B}{\alpha_B} \geq 1$$

Then, since the denominator of the above expression for γ_a is always positive. This would imply that the γ_a multiplier is negative. A contradiction.

Case 4: $\gamma_a > 0$ and $\gamma_b > 0$

In this case transfers and damages are exactly equal and the premium is such the constraint binds.

□

Consider the previous proposition. If $\alpha_B \leq \mu(a)$ we find that $t(a) = a$ and $t(b) = b$. For the constraint to bind it must therefore be the case that the premium k_B^{*ab} must be less than the premium k_A^{*ab}. Therefore, it becomes easy to see that offering the $\{a, b\}$ contract to the B type when he underestimates the contingency a is dominated by simply offering it to the A type. I prove this result formally in the next section. Further, it is important to note than for certain α_A this contract will appear to be loss-inducing according to the insuree, to avoid this I assume alpha is parametrized in a way such that this does not occur.

Proposition 4.4. (Characterization of a solution to the problem of appealing to both types on $\{a, b\}$.)

When $\alpha_K \leq \mu(x)$ for the relevant x, the optimal contract is characterized by by the system of equations:

$$t^*(a) = a$$

$$t^*(b) = b$$

$$
\begin{aligned}
&u(v-a+t^*(a)-k^*)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]+u(v-b+t^*(b)-k^*)\mu(b)\\
&+u(v-c-k^*)\mu(c)=\\
&u(v-a)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]+u(v-b)\mu(b)+u(v-c)\mu(c)
\end{aligned}
$$

Further the premium under this solution must be strictly less than the premium under the decision to appeal to solely the A type.

Proof. Consider the problem.

$$\max k - t(a)\mu(a) - t(b)\mu(b) \tag{20}$$

subject to

$$
\begin{aligned}
&u(v-a+t(a)-k)\mu(a)+u(v-b+t(b)-k)\mu(b)\\
&\geq u(v-a)\mu(a)+u(v-b)\mu(b)
\end{aligned}\tag{21}
$$

$$
\begin{aligned}
&u(v-a+t(a)-k)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]+u(v-b+t(b)-k)\mu(b)\\
&+u(v-c-k)\mu(c)\geq\\
&u(v-a)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]+u(v-b)\mu(b)+u(v-c)\mu(c)
\end{aligned}\tag{22}
$$

$$
\begin{aligned}
t(a)&\leq a\\
t(b)&\leq b
\end{aligned}
$$

The resulting first order conditions are:

$$
\begin{aligned}
&1-\lambda\big[u'(v-a+t(a)-k))\mu(a)+u'(v-b+t(b)-k)\mu(b)\big]\\
&-\gamma\big[u'(v-a+t(a)-k)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]+\\
&u'(v-b+t(b)-k)\mu(b)+u'(v-c-k)\mu(c)\big]\\
&=0
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&-\mu(a)+\lambda\big[u'(v-a+t(a)-k)\mu(a)\big]+\\
&\gamma\big[u'(v-a+t(a)-k)\frac{\alpha_B\big[\mu(b)+\mu(c)\big]}{(1-\alpha_B)}\big]-\gamma_a\\
&=0
\end{aligned}
\tag{24}
$$

$$
-\mu(b)+(\lambda+\gamma)\big[u'(v-b+t(b)-k)\mu(b)\big]=0 \tag{25}
$$

$$
\gamma_a(t(a)-a)=0 \tag{26}
$$

$$
\gamma_b(t(b)-b)=0 \tag{27}
$$

$$
\begin{aligned}
&\lambda\big[u(v-a+t(a)-k)\mu(a)+u(v-b+t(b)-k)\mu(b)\\
&-u(v-a)\mu(a)-u(v-b)\mu(b)\big]=0
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
&\gamma\big[u(v-a+t(a)-k)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]+u(v-b+t(b)-k)\mu(b)\\
&+u(v-c-k)\mu(c)-u(v-a)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]\\
&-u(v-b)\mu(b)-u(v-c)\mu(c)\big]=0
\end{aligned}
\tag{29}
$$

There are a number of cases leading to different potential solutions of this problem. I shall go through each.

Case 1: $\lambda=\gamma=\gamma_a=\gamma_b=0$

This yields the contradiction $\mu(a) = \mu(b) = 0$

Case 2: $\lambda > 0$, $\gamma > 0$

Suppose both are positive. Then both constraints must bind at an optimum. Hence it must be the case that.

$$\begin{aligned}
&u(v-a+t(a)-k)\mu(a) + u(v-b+t(b)-k)\mu(b) - u(v-a)\mu(a) - u(v-b)\mu(b) = \\
&u(v-a+t(a)-k)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big] + \\
&u(v-b+t(b)-k)\mu(b) + u(v-c-k)\mu(c) \\
&-u(v-a)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big] - u(v-b)\mu(b) - u(v-c)\mu(c)
\end{aligned} \tag{30}$$

$$\iff$$

$$\begin{aligned}
&u(v-a+t(a)-k)\mu(a) - u(v-a)\mu(a) = \\
&u(v-a+t(a)-k)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big] + u(v-c-k)\mu(c) \\
&-u(v-a)\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big] - u(v-c)\mu(c)
\end{aligned} \tag{31}$$

But note that since $u(v-c-k) - u(v-c) < 0 \, \forall \, k > 0$ and since this must be the case when a contract is offered then we have:

$$\begin{aligned}
&u(v-a+t(a)-k)\mu(a) - u(v-a)\mu(a) = \\
&[u(v-c-k) - u(v-c)]\mu(c) + [u(v-a) - u(v-a+t(a)-k)]\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big] < \\
&[u(v-a+t(a)-k) - u(v-a)]\frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big]
\end{aligned} \tag{32}$$

But

$$\alpha_B \leq \mu(a) \implies 1-\alpha_B \geq 1-\mu(a) = \mu(b)+\mu(c) \implies \frac{\alpha_B}{(1-\alpha_B)}\big[\mu(b)+\mu(c)\big] \leq \mu(a)$$

Which gives the contradiction

$$[u(v-a) - u(v-a+t(a)-k)]\mu(a) =$$

$$<$$

$$[u(v-a)-u(v-a+t(a)-k)]x$$

Where $x \leq \mu(a)$.

Case 3: $\lambda = 0$, $\gamma = 0$

This yields the contradiction $1 = 0$ from equation 23.

Case 4: $\lambda > 0$, $\gamma = 0$

Then it must be the case that:

$$\lambda = \frac{\mu(a)+\gamma_a}{u'(v-a+t(a)-k)\mu(a)}$$

$$=$$

$$\frac{\mu(b)+\gamma_b}{u'(v-b+t(b)-k)\mu(b)}$$

Substituting these expressions into 23 we have:

$$1-(\mu(a)+\gamma_a+\mu(b)+\gamma_b)=0$$

$$\Longleftrightarrow$$

$$\gamma_a+\gamma_b=\mu(c)$$

But, consider $\gamma_a = 0$

Then we must have must have $\gamma_b = \mu(c)$ and hence $t(b) = b$ and from our previous expressions then it must be that:

$$\frac{\mu(b)+\mu(c)}{\mu(b)} = \frac{u'(v-k)}{u'(v-a+t(a)-k)} > 1$$

Hence by concavity we have

$$v-k < v-a+t(a)-k$$

therefore

$$a < t(a)$$

yields a contradiction. A symmetric idea follows when $\gamma_b = 0$. So it must be the case that $\gamma_a, \gamma_b > 0$ and hence $t^*(a) = a$, $t^*(b) = b$ and the constraint for the A type binds. Note however, that proposition 4.7 in section 4.2 implies that this contract will never be accepted by the B type when $\alpha_B < \mu(a)$ and hence this is *no solution to this problem.*

So we are left with:

Case 5: $\gamma > 0, \lambda = 0$

From the first order conditions we get:

$$\gamma = \frac{\gamma_a + \mu(a)}{u'(v - a + t(a) - k)[\mu(b) + \mu(c)]}$$
$$=$$
$$\frac{\mu(b) + \gamma_b}{u'(v - b + t(b) - k)}$$

And hence from 23

$$1 - [\gamma_a + \mu(a) + \mu(b) + \gamma_b + \gamma u'(v - c - k)\mu(c)]$$
$$\Longleftrightarrow$$
$$\gamma_a + \mu(a) + \mu(b) + \gamma_b + \gamma u'(v - c - k)\mu(c)]$$
$$= \mu(a) + \mu(b) + \mu(c)$$

There are a number of subcases here, and the proof follows exactly the same reasoning as the previous proposition. The result therefore follows. Note that the premium must be less than the premium when appealing directly to A. Hence it must be the case that the A type's participation constraint is satisfied and non-binding.

□

Hence proposition 4.4 shows there is something of a tradeoff between catering to a type and making them aware of a certain contingency. If they become suspicious such that they underestimate a salient state then in order to attract the A type a lower premium must be paid.

Figure 2 shows the insurers utility from following such a strategy as a function of the updating process of the A type. Note that it should not be troubling that the plane does not fluctuate in accordance with α_B since $\alpha_B = 0$ here. In the uppermost plane of the graph there is point where the function flattens out. This is exactly the point where the B constraint binds.

Proposition 4.5. (Characterization of a solution to the problem of appealing to either type across the whole space $\Omega = \{a, b, c\}$)

A solution to the problem of appealing to a single A here to be explicity type by offering cover on all contingencies $\{a, b, c\}$ is characterized by the system of equations:

$$t^*(a) = a$$
$$t^*(b) = b$$

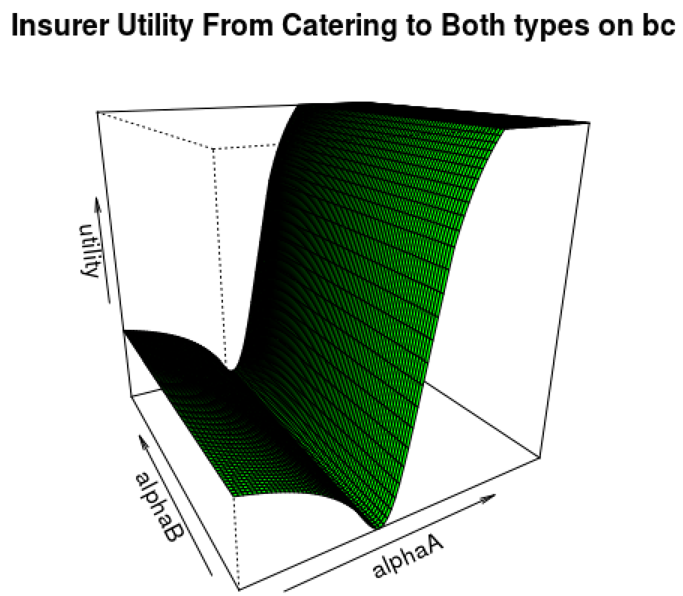

Figure 2: Insurer Utility From Catering to both types over $\{b, c\}$

$$\frac{u'(v-c+t(c)-k)}{u'(v-k)}\frac{\alpha_A}{1-\alpha_A}\frac{1-\mu(c)}{\mu(c)}=1$$

$$\begin{aligned}&u(v-c+t^*(c)-k^*)\alpha_A+u(v-k^*)\frac{(1-\alpha_A)\mu(b)}{\mu(a)+\mu(b)}+u(v-k)\frac{(1-\alpha_A)\mu(a)}{\mu(a)+\mu(b)}\\&=u(v-c)\alpha_A+u(v-b)\frac{(1-\alpha_A)\mu(b)}{\mu(a)+\mu(b)}+u(v-a)\frac{(1-\alpha_A)\mu(a)}{\mu(a)+\mu(b)}\end{aligned}$$

for the A type.

Proof. WLOG I show for the A type. The proof and reasoning for the B type are symmetric.

The insurer solves:

$$\max_{k,t(\cdot)} k-t(a)\mu(a)-t(b)\mu(b)-t(c)\mu(c) \tag{33}$$

subject to

$$
\begin{aligned}
&u(v-c+t(c)-k)\frac{\alpha_A}{1-\alpha_A}(1-\mu(c))+u(v-b+t(b)-k^*)\mu(b)\\
&+u(v-a+t(a)-k)\mu(a)\\
&=u(v-c)\frac{\alpha_A}{1-\alpha_A}(1-\mu(c))+u(v-b)\mu(b)+u(v-a)\mu(a) \qquad (34)
\end{aligned}
$$

$$t(a) \leq a \qquad (35)$$

$$t(b) \leq b \qquad (36)$$

$$t(c) \leq c \qquad (37)$$

Yielding first order conditions:

$$
\begin{aligned}
&1-\lambda\big[u'(v-c+t(c)-k)\frac{\alpha_A}{1-\alpha_A}(1-\mu(c))\\
&+u'(v-b+t(b)-k)\mu(b)+u'(v-a+t(a)-k)\mu(a)]=0 \qquad (38)
\end{aligned}
$$

$$-\mu(a)+\lambda[u'(v-t(a)+a-k)\mu(a)]-\gamma_a=0 \qquad (39)$$

$$-\mu(b)+\lambda\big[u'(v-t(b)+b-k)\mu(b)\big]-\gamma_b=0 \qquad (40)$$

$$-\mu(c)+\lambda\big[\frac{u'(v-t(c)+c-k)\alpha_A}{1-\alpha_A}(1-\mu(c))\big]-\gamma_c=0 \qquad (41)$$

$$\gamma_a(t(a)-a)=0 \qquad (42)$$

$$\gamma_b(t(b)-b)=0 \qquad (43)$$

$$\gamma_c(t(c)-c)=0 \qquad (44)$$

$$\qquad (45)$$

$\lambda = 0$ yields the contradicition $0 = 1$

Solving for λ we find:

$$\lambda = \frac{\mu(a) + \gamma_a}{u'(v - a + t(a) - k)\mu(a)} = \frac{\mu(b) + \gamma_b}{u'(v - b + t(b) - k)\mu(b)}$$
$$= \frac{\mu(c) + \gamma_c}{u'(v - c + t(c) - k)} \frac{1 - \alpha_A}{\alpha_A[1 - \mu(c)]}$$

substituting these expressions into 38 we find.

$$1 - [\mu(a) + \gamma_a + \mu(b) + \gamma_b) + \mu(c) + \gamma_c] = 0$$

$$\iff$$

$$\gamma_a = \gamma_b = \gamma_c = 0$$

From here dividing the expressions for λ by one another we find $t(a) = a$, $t(b) = b$ and $t(c)$ is characterized by:

$$\frac{u'(v - c + t(c) - k)}{u'(v - k)} \frac{\alpha_A}{1 - \alpha_A} \frac{1 - \mu(c)}{\mu(c)} = 1$$

Note that $\alpha_A = \mu(c) \implies t(c) = c$ as we would expect.

Further, $\alpha_A < \mu(c) \implies u'(v - c + t(c) - k) > u'(v - k)$ and by concavity $v - c + t(c) - k < v - k \implies t(c) < c$ and vice versa. □

The above figure shows the insurers utility from following the strategy of offering B a complete contract as a function of how he updates. Note that when B correctly estimates the contingency he becomes aware of the utility of the insurer from following such a strategy takes a minimum. Intuitively this is because, whilst B is less willing to take insurance covering him for the a state when he underestimates he overestimates the contingencies of which he is originally aware, ie. $\{b, c\}$ which leaves the insurer some room to exploit this. Note that as $\alpha_B \to \mu(a)$ the utility of the insurer is decreasing. When $\alpha_B < \mu(a)$ then the B type underestimates the likelihood of state a and hence he overestimates the alternate states b and c. Hence the insurer can exploit this fact by promising these transfers at a higher premium. The insurer knows the objective probability of these transfers and hence in expectation his utility must be higher.

A full characterization of the solution to the problem of offering both types $\{a, b, c\}$ is unfortunately rather complex. As such it is omitted here.

Figure 4 shows how the utility of the insurer fluctuates as beliefs are updated. It takes a minimum at the point where each of the types updating parameter

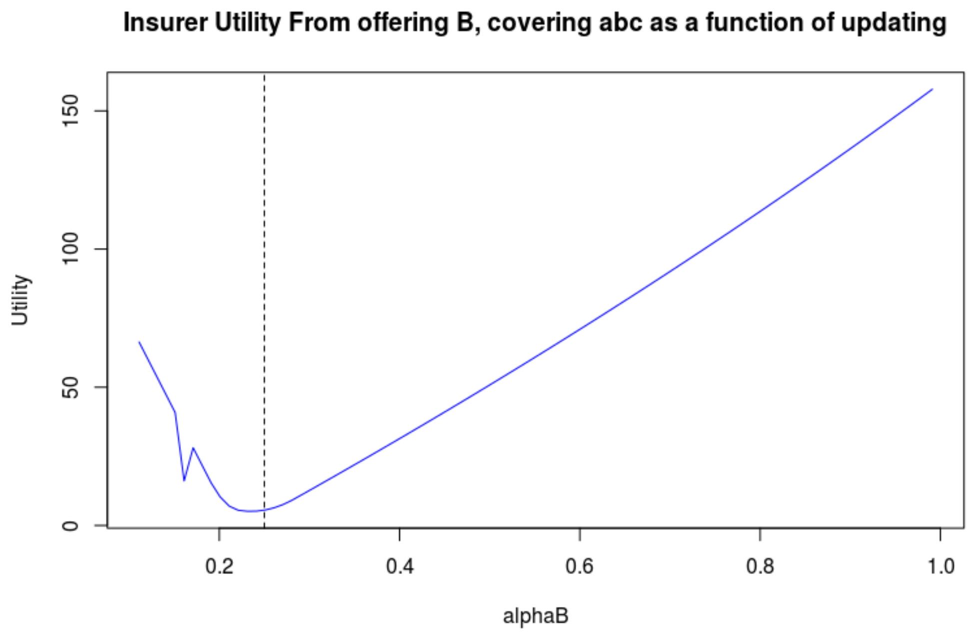

Figure 3: Insurer utility from covering B across the entire set. Vertical line at $\mu(a) = 0.25$

matches the objective probability. In this case there is no room for exploitation in either direction across either of the types.

4.2 Characterization of Optimal Strategies

Note that in the previous sections I have been implictly treating the series of optimization problems as relatively unconnected. But, clearly the choice of which contract is an optimal contract depends on the distribution of types. To see this, compare figure 4 with figure 5, a graph of the equilibrium profit as a function of the updating process for $\pi = 0.2$.

Note that the lower most plane of this figure is at a value of approximately 150. Jumps in the graph indicate that at different regions of beliefs, different strategies dominate. For example a jump could indicate that given $\pi = 0.2$ as the A type overestimates it becomes more lucrative to cater exclusively to him and hence the strategy of the insurer switches at a given α_A.

Now that I have characterized the solutions to the sequence of problems that the insurer faces. I turn to analyze the optimal strategies resulting from these solutions as functions of the distribution of types.

Proposition 4.6. A type who becomes suspicious after updating will always reject the contract designed for the other.

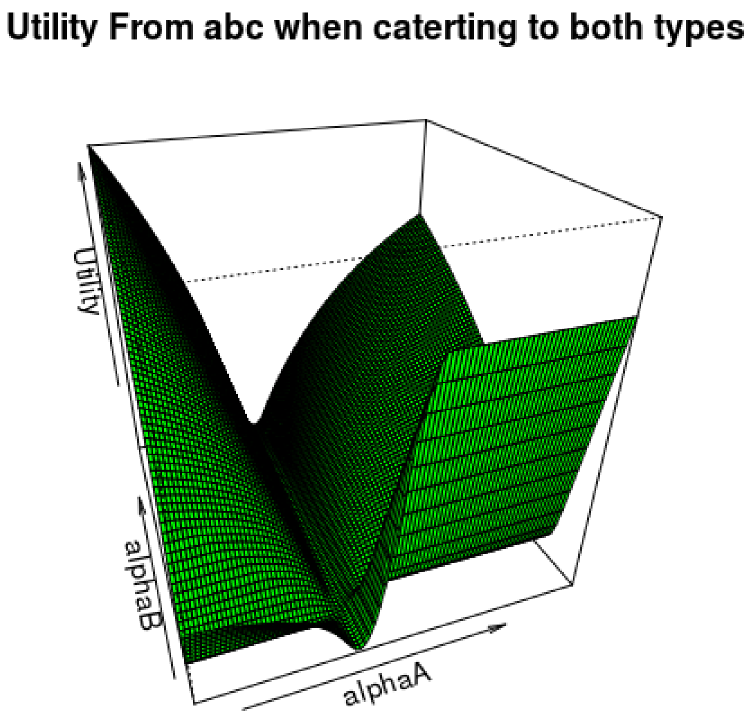

Figure 4: Insurer utility from covering both A and B across the entire set as a function of the updating process

Proof. To see this, consider the contract designed for the A type on $\{a, b\}$. This contract consists of transfers $t^*(a) = a$, $t^*(b) = b$ and a premium such that the participation constraint of the A type binds. Suppose the B type did accept the A type's contract. Note that the inequality must be strict by lemma 4.1. Then it must be the case that:

$$u(v - k_A^*)\frac{\alpha_B}{1-\alpha_B}(\mu(b) + \mu(c)) + u(v - k_A^*)\mu(b) + u(v - c - k_A^*)\mu(c)$$

$$>$$

$$u(v - a)\frac{\alpha_B}{1-\alpha_B}(\mu(b) + \mu(c)) + u(v - b)\mu(b) + u(v - c)\mu(c)$$

Suppose $\alpha_B = \mu(a)$ this implies that:

$$u(v - k_A^*)\mu(a) + u(v - k_A^*)\mu(b) + u(v - c - k_A^*)\mu(c)$$

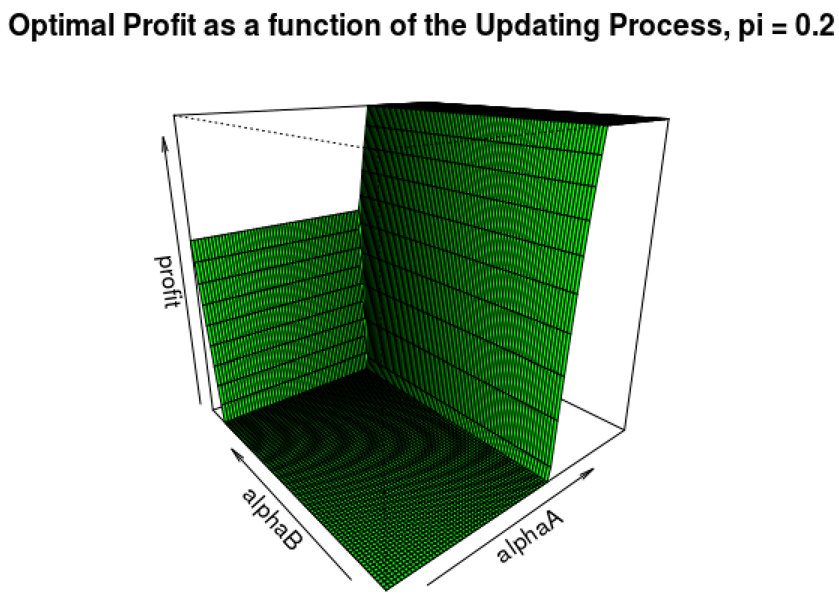

Figure 5: Optimal Profit as a Function of Updating

$$>$$

$$u(v-a)\mu(a)+u(v-b)\mu(b)+u(v-c)\mu(c)$$

And since the A type's constraint binds we have:

$$u(v-a)\mu(a)+u(v-b)\mu(b)+u(v-c-k_A^*)\mu(c)$$

$$>$$

$$u(v-a)\mu(a)+u(v-b)\mu(b)+u(v-c)\mu(c)$$

But then $u(v-c-k_A^*)\mu(c)<u(v-c)\mu(c)$ yields a contradiction.

For the case when $\alpha_B<\mu(a)$ consider the following:

$$u(v-k_A^*)\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+u(v-k_A^*)\mu(b)+u(v-c-k_A^*)\mu(c)$$

$$=$$

$$u(v-a)[\mu(a)[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+\mu(b)]]$$
$$+u(v-b)[\mu(b)[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+\mu(b)]]$$
$$+u(v-c-k_A^*)\mu(c)$$

I first claim that

$$\mu(a)[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+\mu(b)]$$
$$<$$
$$\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))$$

but this is true

$$\iff$$

$$\mu(a)\mu(b) < [\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))](1-\mu(a))$$
$$=$$
$$[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))^2]$$
$$<$$
$$[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))]$$
$$<$$
$$\alpha_B$$
$$<\mu(a)$$

Where the second to last inequality holds since $\alpha_B < \mu(a) \implies 1-\alpha_B > \mu(b)+\mu(c)$.

Which holds true since $\mu(b) \in (0,1)$.

Further note that since $\mu(c) > 0$

$$\alpha_B < \mu(a) \implies \frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+\mu(b) < \mu(a)+\mu(b) < 1$$

Hence we have:

$$u(v-k_A^*)\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+u(v-k_A^*)\mu(b)+u(v-c-k_A^*)\mu(c)$$

$$=$$

$$\begin{aligned}
&u(v-a)[\mu(a)[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+\mu(b)]]\\
&+u(v-b)[\mu(b)[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+\mu(b)]]\\
&+u(v-c-k_A^*)\mu(c)
\end{aligned}$$

$$<$$

$$\begin{aligned}
&u(v-a)[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+u(v-b)\mu(b)[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+\mu(b)]\\
&+u(v-c-k_A^*)\mu(c)
\end{aligned}$$

$$<$$

$$u(v-a)[\frac{\alpha_B}{1-\alpha_B}(\mu(b)+\mu(c))+u(v-b)\mu(b)+u(v-c)\mu(c)$$

So the B type's participation constraint is violated, hence he will never accept A's contract.

□

Proposition 4.7 gives a motivation for our labelling of a type who underestimates a contingency as a *suspicious*. Essentially after reading the A type's contract, which gives full insurance over A, B updates in such a way that he would never accept the contract at such a premium. This means that the insurer will never offer the A type's contract to B.

Proposition 4.7. The value of the objective to the solution of the problem offering A contingencies $\{a, b\}$ is always higher than the value of the objective to the solution of the problem of offering B contingencies $\{a, b\}$.

Proof. First note that from proposition 4.3 we have that $t^*(a) = a$ and $t^*(b) = b$. Hence it simply remains to show that $k_B^{*ab} < k_A^{*ab}$. But note that this follows freely from proposition 4.6 since if we had $k_B^{*ab} > k_B^{*ab}$ then the premium the B type accepts is greater than the premium the A type accepts, and transfers are the same in both cases, hence it must be the case that he accepts A's contract which contradicts the previous proposition 4.6.

□

Despite only specifying a result disregarding the distribution of types, proposition 4.8 is useful in the following way. The insurer, with probability π knows he is dealing with an A type. Then, given that he is in this state, if the B type will become suspicious of him (in that he underestimates the contingency which is revealed) the insurer will always choose to offer $\{a, b\}$ to the A type rather than the B type. Conversely, with probability $1 - \pi$ he is dealing with a B type, and hence will offer $\{b, c\}$ to him over offering it to the A type. Effectively, this proposition allows us to ignore certain nodes in the underlying game tree. For a numerical example, see section 5.

Proposition 4.8. The strategy of revealing a contingency to and catering to both types dominates the strategy of catering to the single type in and only if

$$\pi < \frac{k^*_{AB} - a\mu(a) - b\mu(b))}{k^*_A - a\mu(a) - b\mu(b)}$$

Proof. We know that $k^{*ab}_{AB} < k^{*ab}_A$ and in both cases $t^*(a) = a$, $t^*(b) = b$. The result follows from simple algebra. □

The previous proposition implies that certain distributions of types encourage the insurer to reveal contingencies out of self interest. Hence the fact that each type's level of awareness is private information can be seen as one potential driver of completeness in the contractual form.

Proposition 4.9. For all distributions, if the insuree becomes suspicious after updating then the strategy of concealing the additional contingency and appealing to that insuree over the contingencies of which he is originally aware dominates the strategy of revealing the contingencies.

Proof. WLOG I show this for the A type.

We want to show:

$$\pi(k^{*ab}_A - b\mu(b) - a\mu(a)) \geq \pi(k^{*abc}_A - t^*(c)\mu(c) - b\mu(b) - a\mu(a)$$

It therefore remains to show that:

$$k^{*ab}_A \geq k^{*abc}_A - t^*(c)\mu(c)$$

.

Suppose not:

$$k^{*ab}_A < k^{*abc}_A - t^*(c) < k^{*abc}_A - t^*(c)\mu(c) < k^{*abc}$$

.

Then, since the A type constraint binds under $k^{*ab} \implies$

$$u(v - k^{*abc})\mu(a) + u(v - k^{*abc})\mu(b) < u(v - a)\mu(a) + u(v - b)\mu(b)$$

But then note that by proposition 4.4 the constraint of the A type when made aware of c must also bind.

Then this means that

$$u(v - c + t^*(c) - k^{*abc}) > u(v - c)$$

And hence it must be the case that $t^*(c) > k^{*abc}$.

But then note that our assumption would imply $k^{*ab} < 0$ which is a contradiction.

Note that this line of reasoning holds independently of the distribution π and hence the statement of the proposition is fulfilled. □

To this point I have shown a number of things. Suppose the B type becomes suspicious after updating. Then, first, he will reject the A type's contract. Second, it will be better for the insurer to appeal to A over appealing to both types. Third, if the A type becomes suspicious after revealing c it will be better to simply offer the A type $\{b, c\}$.

This is summarized in the following theorem:

Theorem 4.1. $\alpha_A < \mu(c)$ and $\alpha_B < \mu(a) \implies$ the optimal strategy is to appeal to either A or B directly over the initial set K.

Proof. See propositions 4.6-4.9. Note further that the actual distribution π determines exactly which strategy is optimal for the insurer between appealing to A over $\{a, b\}$ and B over $\{b, c\}$. See figure 7 for further intuition. □

The intuition is the following. Suppose the B type underestimates the probability of the contingency of which he becomes aware then, after reading the A type's contract he becomes suspicious that the premium is essentially too high to warrant such coverage. Hence, he rejects the contract for the A type, and it becomes too costly for the insurer to insure him over $\{a, b\}$. Similarly, if revealing a contingency to the A type makes it costly to insure him on that contingency the insurer will never reveal such a contingency.

Conjecture 4.1. $\exists\, \alpha_A, \alpha_B, \pi$ such that it is optimal to appeal to both types on $\{a, b, c\}$ under a compatibility requirement even when both types underestimate the contingency of which they are made aware.

The above conjecture is motivated by the fact that there is a trade off between making an agent aware of a contingency-in that it is costly to insure them-and being able to capture a larger share of the market. Under compatibility the insurer is constrained to offering contracts which appear utility maximizing. As such, perhaps it is possible that contracts can be offered which reveal all contingencies to both types and still do better for the principal than all other contracts. This is an area for future research.

5 A Numerical Example

Now that I have introduced the general solution concept I turn to the following numerical example. Again there are three states $\Omega = \{a, b, c\}$ with two types of agents: A who are aware of $\{a, b\}$ and B who are aware of $\{b, c\}$. Each agent wants to insure a good of value $v = 2000$. Objective probabilities of a certain contingency bringing damage to the good the beliefs held by the principal and are given by. The objective probability measure of the principal is given by

$$\mu_P(\omega) = \begin{cases} 0.25 & \omega = a \\ 0.5 & \omega = b \\ 0.25 & \omega = c \end{cases}$$

Therefore, the initial belief system of each type is

$$\mu_A(\omega) = \begin{cases} 0.33 & \omega = a \\ 0.67 & \omega = b \end{cases}$$

$$\mu_B(\omega) = \begin{cases} 0.67 & \omega = b \\ 0.33 & \omega = c \end{cases}$$

Suppose each underestimates the likelihood of the contingency they become aware of. Specifically, suppose $\alpha_A = 0.2$ and $\alpha_B = 0.1$ then the updated beliefs are given by

The good has value $v = 2000$ to each type of consumers and the damages are:

$$d(\omega) = \begin{cases} 500 & \omega = a \\ 400 & \omega = b \\ 600 & \omega = c \end{cases}$$

For each agent $u(x) = \sqrt{x}$ and I will characterize the solution as a function of π.

From our characterization of the optimal strategy we know the B type will not accept the contract on $\{a, b\}$ that is designed for A since he becomes suspicious of the insurers motives. Conversely, the A type will not accept the contract designed for B over $\{b, c\}$. Finally, note that offering $\{a, c\}$ is equivalent to offering set $\{a, b, c\}$. For example, we can see this by simply normalizing $b = 0$. Therefore, the corresponding nodes of the following game tree are ignored since they can never be played at an optima.

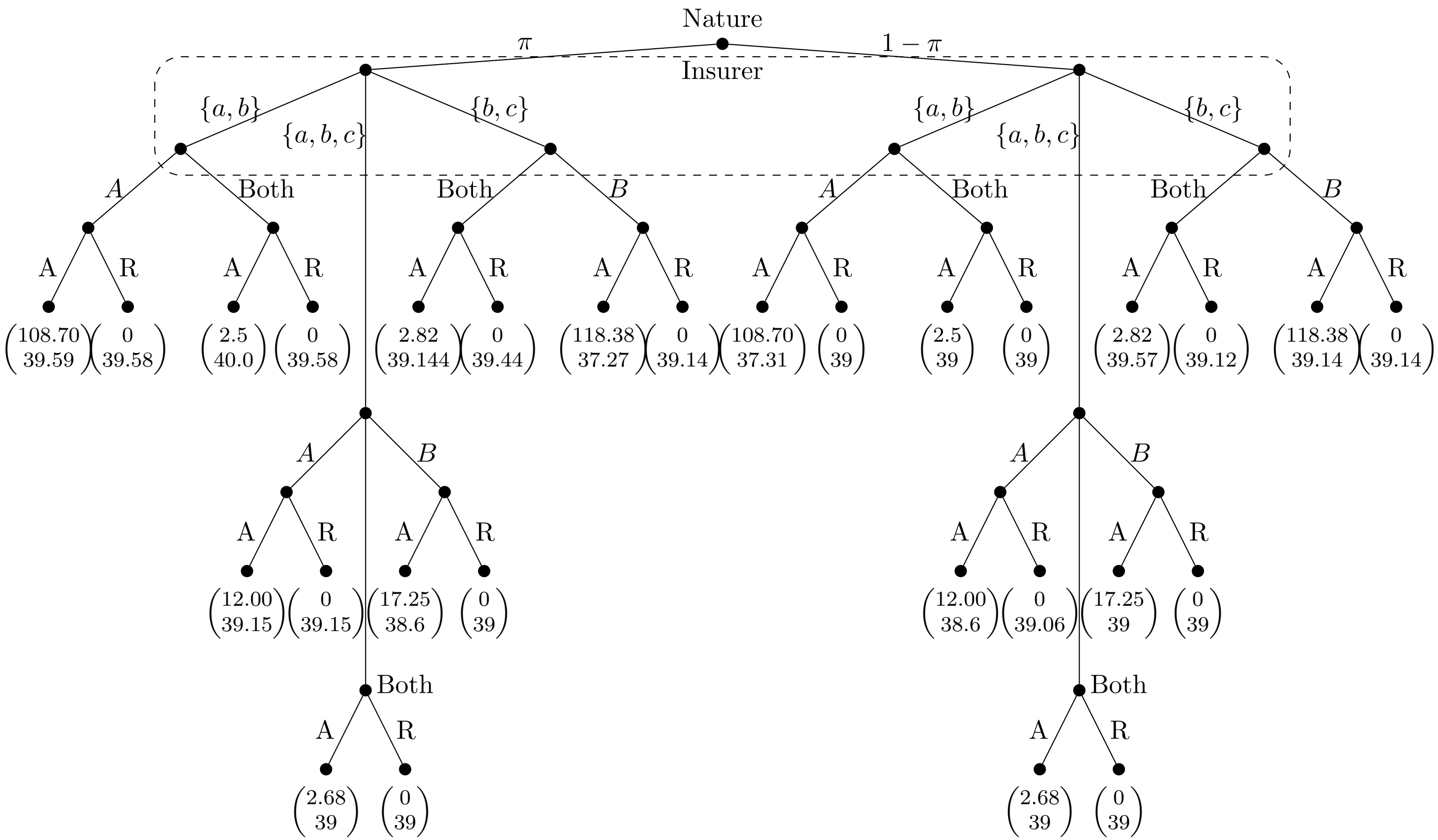

Figure 6: The Asymmetric Information/Asymmetric Awareness Game

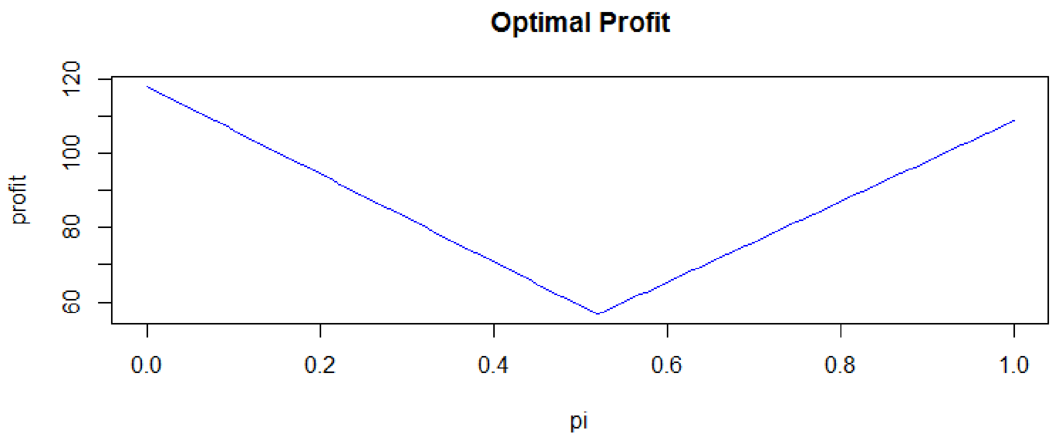

Figure 7: Optimal Profit When Both Types Underestimate

Solving the above is now a simple function of the distribution of types. Consider the following figure showing the optimal profit of the insurer as a function of π. Note that the graph is decreasing up to a point and then increasing. This is due to the fact that since both types underestimate the existence of the respective contingencies of which they are made aware it is never profitable to cater to both types simultaneously. Since doing so would require lower transfers and lower premiums on the part of the insurer. The kink in the curve is the exact point where it becomes profitable to cater only to the A type in this case.

The solution to the above game for $\pi = 0.2$ is for the insurer to cater to B on $\{b, c\}$ yielding an expected profit of

$$118.38 \times 0.8 + 0 * 0.2 = 94.519$$

Note that in all cases B rejects the contract of A and vice versa. This is to be contrasted with the case when both overestimate the existence of their respective contingencies. The optimal profit as a function of π in the case with the same damages when both overestimate at $\alpha_A = \alpha_B = 0.5$ is given below.

In the range of $\pi \in (0.17, 66)$ the strategy of appealing to both types on $\{b, c\}$ dominates all other strategies.

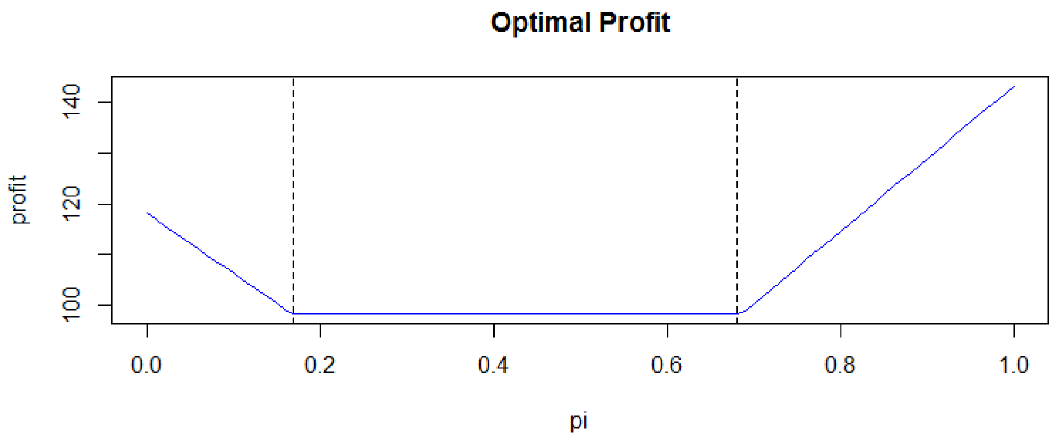

Figure 8: Optimal Profit When Both types Overestimate

6 Conclusion

This paper has presented a basic analysis of a three contingency case of insurance contracting when awareness and information interact. Two insurees each aware of a subset of cardinality two of a simple three state state-space. Types of insurees differ in terms of the contingencies of which they are originally aware. These contingencies are private information, hence the insurer must structure the terms of his contract to achieve a maximal payoff given this constrant. The main contribution of this paper is the presentation of a general solution method in a basic model of insurance contracting. Following the approach of Filiz-Ozbay I first presented the model itself before discussing characterizations of solutions for the various sequence of optimization problems the insurer faces when presented with such a problem. When presented with a contingency that was previous beyond the conception of a given type, that type must form some belief over its likelihood. It is shown that insuree's who become suspicious of the insurer in the sense that when they are presented with a new contingency demand a higher coverage are those who underestimate more salient states in the model. The insurer faces a general trade-off between making an agent aware of a contingency and catering to more than one type of insuree. It is shown that agents who become aware of a certain contingency and underestimate its likelihood will demand a lower transfer. But, in order to cater to both types the insurer must lower his premium. Hence in this case there exists a trade-off between information and awareness. Though only a first tentative step in this domain, the possibility for future work on this question is wide. Generalizing the model to arbitrary numbers of types and arbitrary levels of awareness would be the first task of any such endeavour. Further, characterizing α as a function of the characteristics of each type would be an interesting possibility. Beyond this applying such a framework to problems of moral hazard, signalling and screening would undoubtedly yield interesting results. Finally, this paper abstract, very informally, away from compatibility and consistency requirements. A proper implementation of such requirements might yield significantly different results.

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7 Appendix

```
rm(list=ls())

#####VARIABLES FOR EXAMPLES################

Q <- c(0.25,0.5,0.25)
D <- c(500,200,700)
V = 2000
X <- c(1,1,1,1)

##########BELIEFS####################

init_beliefs <- function(b) {
A <- numeric(2)
A[1] = b[1]/(b[1]+b[2])
A[2] = b[2]/(b[1]+b[2])

B <- numeric(2)
B[1] = b[2]/(b[2]+b[3])
B[2] = b[3]/(b[2]+b[3])

return(list("A"=A, "B"=B))
}

init_beliefs(Q)


up_beliefs <- function(b, uA=0, uB=0){
A <- numeric(3)
A[1] = ((1-uA)*b[1])/(b[1]+b[2])
A[2] = ((1-uA)*b[2])/(b[1]+b[2])
A[3]= uA

B <- numeric(3)
B[1] = uB
B[2] = ((1-uB)*b[2])/(b[2]+b[3])
B[3] = ((1-uB)*b[3])/(b[2]+b[3])
return(list("A"=A, "B"=B))
}

################OBJECTIVE####################

objective <- function(b, cover) {
if (cover=="b") {
obj <- function(x) {
return(-x[1]+b[2]*x[3])
}
}
if (cover=="ab") {
obj <- function(x) {
return(-x[1]+b[1]*x[2]+b[2]*x[3])
}
}
if (cover=="ac") {
obj <- function(x) {
return(-x[1]+b[1]*x[2]+b[3]*x[4])
```

```
}
}
if (cover=="bc") {
obj <- function(x) {
return(-x[1]+b[2]*x[3]+b[3]*x[4])
}
}
if (cover=="abc" ){
obj <- function(x) {
return(-x[1]+b[1]*x[2]+b[2]*x[3] + b[3]*x[4])
}
}
return(obj)
}

objective(Q, cover="abc")(X)


############UTILITY FUNCTIONS############

utility <- function(b,v,d,offer,cover,uA=0,uB=0) {
if (offer=="A") {
if (cover=="b") {
z <- init_beliefs(b)$A
u <- function(x) {
return(sqrt(v-d[1]-x[1])*z[1]+sqrt(v-d[2]-x[1]+x[3])*z[2])
}
}
if (cover=="ab") {
z <- init_beliefs(b)$A
u <- function(x) {
return(sqrt(v-d[1]-x[1]+x[2])*z[1]+sqrt(v-d[2]-x[1]+x[3])*z[2])
}
}
if (cover=="ac") {
z <- up_beliefs(b, uA=uA, uB=uB)$A
u <- function(x) {
return(sqrt(v-d[1]-x[1]+x[2])*z[1]+sqrt(v-d[2]-x[1])*z[2]+sqrt(v-d[3]-x[1]+x[4])
    *z[3])
}
}
if (cover=="bc") {
z <- up_beliefs(b,uA=uA, uB=uB)$A
u <- function(x) {
return(sqrt(v-d[1]-x[1])*z[1]+sqrt(v-d[2]+x[3]-x[1])*z[2]+sqrt(v-d[3]-x[1]+x[4])
    *z[3])
}
}
if (cover=="abc") {
z <- up_beliefs(b,uA=uA, uB=uB)$A
u <- function(x) {
return(sqrt(v-d[1]-x[1]+x[2])*z[1]+sqrt(v-d[2]+x[3]-x[1])*z[2]+sqrt(v-d[3]-x[1]+
    x[4])*z[3])
}
}
}
if (offer=="B") {
if (cover=="b") {
```

```
z <- init_beliefs(b)$B
u <- function(x) {
return(sqrt(v-d[2]-x[1]+x[3])*z[1]+sqrt(v-d[3]-x[1])*z[2])
}
}
if (cover=="ab") {
z <- up_beliefs(b,uB=uB, uA=uA)$B
u <- function(x) {
return(sqrt(v-d[1]-x[1]+x[2])*z[1]+sqrt(v-d[2]-x[1]+x[3])*z[2] + sqrt(v-d[3]-x
    [1])*z[3])
}
}
if (cover=="ac") {
z <- up_beliefs(b, uB=uB,uA=uA)$B
u <- function(x) {
return(sqrt(v-d[1]-x[1]+x[2])*z[1]+sqrt(v-d[2]-x[1])*z[2]+sqrt(v-d[3]-x[1]+x[4])
    *z[3])
}
}
if (cover=="bc") {
z <- init_beliefs(b)$B
u <- function(x) {
return(sqrt(v-d[2]+x[3]-x[1])*z[1]+sqrt(v-d[3]+x[4]-x[1])*z[2])
}
}
if (cover=="abc") {
z <- up_beliefs(b,uB=uB, uA=uA)$B
u <- function(x) {
return(sqrt(v-d[1]-x[1]+x[2])*z[1]+sqrt(v-d[2]+x[3]-x[1])*z[2]+sqrt(v-d[3]-x[1]+
    x[4])*z[3])
}
}
}
if (offer=="both") {
if (cover=="b") {
ba <- init_beliefs(b)$A
bb <- init_beliefs(b)$B

u <- function(x) {
za <- function(x) {
return(sqrt(v-d[1]-x[1])*ba[1]+sqrt(v-d[2]-x[1]+x[3])*ba[2])
}
zb <- function(x) {
return(sqrt(v-d[2]-x[1]+x[3])*bb[1]+sqrt(v-d[3]-x[1])*bb[2])
}
return(c(za(x),zb(x)))
}
}

if (cover=="ab") {
ba <- init_beliefs(b)$A
bb <- up_beliefs(b,uB=uB)$B

u <- function(x) {
za <- function(x) {
return(sqrt(v-d[1]-x[1]+x[2])*ba[1]+sqrt(v-d[2]-x[1]+x[3])*ba[2])
}
zb <- function(x) {
```

```
return(sqrt(v-d[1]-x[1]+x[2])*bb[1]+sqrt(v-d[2]-x[1]+x[3])*bb[2] + sqrt(v-d[3]-x
    [1])*bb[3])
}
return(c(za(x),zb(x)))
}
}

if (cover=="ac") {
ba <- up_beliefs(b,uA=uA)$A
bb <- up_beliefs(b,uB=uB)$B

u <- function(x) {
za <- function(x) {
return(sqrt(v-d[1]-x[1]+x[2])*ba[1]+sqrt(v-d[2]-x[1])*ba[2]+sqrt(v-d[3]-x[1]+x
    [4])*ba[3])
}
zb <- function(x) {
return(sqrt(v-d[1]-x[1]+x[2])*bb[1]+sqrt(v-d[2]-x[1])*bb[2]+sqrt(v-d[3]-x[1]+x
    [4])*bb[3])
}
return(c(za(x),zb(x)))
}
}

if (cover=="bc") {
ba <- up_beliefs(b,uA=uA)$A
bb <- init_beliefs(b)$B

u <- function(x) {
za <- function(x) {
return(sqrt(v-d[1]-x[1])*ba[1]+sqrt(v-d[2]+x[3]-x[1])*ba[2]+sqrt(v-d[3]-x[1]+x
    [4])*ba[3])
}
zb <- function(x) {
return(sqrt(v-d[2]+x[3]-x[1])*bb[1]+sqrt(v-d[3]+x[4]-x[1])*bb[2])
}
return(c(za(x),zb(x)))
}
}

if (cover=="abc") {
ba <- up_beliefs(b,uA=uA,uB=uB)$A
bb <- up_beliefs(b,uB=uB,uA=uA)$B

u <- function(x) {
za <- function(x) {
return(sqrt(v-d[1]+x[2]-x[1])*ba[1]+sqrt(v-d[2]+x[3]-x[1])*ba[2]+sqrt(v-d[3]+x
    [4]-x[1])*ba[3])
}
zb <- function(x) {
return(sqrt(v-d[1]+x[2]-x[1])*bb[1]+sqrt(v-d[2]+x[3]-x[1])*bb[2]+sqrt(v-d[3]+x
    [4]-x[1])*bb[3])
}
return(c(za(x),zb(x)))
}
}
}
return(u)
```

```
}

##################CONSTRAINTS$#####################

res_util <- function(b,v,d,offer,cover,uA,uB) {
return(utility(b,v,d,offer=offer,cover=cover,uA=uA,uB=uB)(c(0,0,0,0)))
}

res_util(Q,V,D,offer="both", cover="abc", uA=0.2, uB=0.2)

part_constraint <- function(b,v,d,offer,cover,uA,uB) {
M = function(x) {
return(res_util(b,v,d,offer=offer,cover=cover,uA=uA,uB=uB) - utility(b,v,d,offer
    =offer,cover=cover,uA=uA,uB=uB)(x))
}
return(M)
}

defined_constraint <- function(v,d,offer,cover) {

if (offer=="A"){
if (cover=="b") {
z = function(x) {
return(c(x[1]+d[1]-v,
x[1]+d[2]-v-x[3]))
}
}
if (cover=="ab") {
z = function(x) {
return(c(x[1]+d[1]-v-x[2],
x[1]+d[2]-v-x[3]))
}
}
if (cover=="ac") {
z = function(x) {
return(c(x[1]+d[1]-v-x[2],
x[1]+d[2]-v,
x[1]+d[3]-v-x[4]))
}
}
if (cover=="bc") {
z = function(x) {
return(c(x[1]+d[1]-v,
x[1]+d[2]-v-x[3],
x[1]+d[3]-v-x[4]))
}
}
if (cover=="abc") {
z = function(x) {
return(c(x[1]+d[1]-x[2]-v,
x[1]+d[2]-v-x[3],
x[1]+d[3]-v-x[4]))
}
}
}

if (offer=="B"){
if (cover=="b") {
```

```
z = function(x) {
return(c(x[1]+d[2]-v-x[3],
x[1]+d[3]-v))
}
}
if (cover=="ab") {
z = function(x) {
return(c(x[1]+d[1]-v-x[2],
x[1]+d[2]-v-x[3],
x[1]+d[3]-v))
}
}
if (cover=="ac") {
z = function(x) {
return(c(x[1]+d[1]-v-x[2],
x[1]+d[2]-v,
x[1]+d[3]-v-x[4]))
}
}
if (cover=="bc") {
z = function(x) {
return(c(x[1]+d[2]-v-x[3],
x[1]+d[3]-v-x[4]))
}
}
if (cover=="abc") {
z = function(x) {
return(c(x[1]+d[1]-x[2]-v,
x[1]+d[2]-v-x[3],
x[1]+d[3]-v-x[4]))
}
}
}

if (offer=="both") {
if (cover=="b") {
z = function(x) {
return(c(x[1]+d[1]-v,
x[1]+d[2]-v-x[3],
x[1]+d[3]-v))
}
}
if (cover=="ab") {
z = function(x) {
return(c(x[1]+d[1]-v-x[2],
x[1]+d[2]-v-x[3],
x[1]+d[3]-v))
}
}
if (cover=="ac") {
z = function(x) {
return(c(x[1]+d[1]-v-x[2],
x[1]+d[2]-v,
x[1]+d[3]-v-x[4]))
}
}
if (cover=="bc") {
z = function(x) {
```

```
return(c(x[1]+d[1]-v,
x[1]+d[2]-v,
x[1]+d[2]-v-x[3],
x[1]+d[3]-v-x[4]))
}
}
if (cover=="abc") {
z = function(x) {
return(c(x[1]+d[1]-x[2]-v,
x[1]+d[2]-v-x[3],
x[1]+d[3]-v-x[4]))
}
}
}
return(z)
}

defined_constraint(V,D,offer="A", cover="b")(X)

constraint <- function(b,v,d,offer,cover,uA,uB) {
C = function(x) {
P = part_constraint(b,v,d,offer = offer,cover = cover,uA=uA,uB=uB)
D = defined_constraint(v,d,offer=offer,cover=cover)

return(c(P(x),D(x)))
}
return(C)
}

constraint(Q,V,D,offer="A", cover="ac",uA=0.9,uB=0.1)

####################Optimization############

library(nloptr)

profit <- function(UA,UB,RA,RB,P,O) {
if (UA >= RA) {
if (UB >= RB) {
return(list("P"=-O, "Accept" = "both"))
}
else {
return(list("P"=-O*P, "Accept"="A"))
}
}
else {
if (UB >= RB) {
return(list("P"=-(1-P)*O, "Accept"="B"))
}
else {
return(list("P"=0, "Accept"="neither"))
}
}
}

opt <- function(b,v,d,offer,cover,uA=0,uB=0,pi=0.5) {
obj <- objective(b,cover=cover)
constr <- part_constraint(b,v,d,offer=offer,cover=cover,uA=uA,uB=uB)
```

```
lb <- numeric(4)
lb[1:4]=0

ub <- numeric(4)
ub= c(Inf, d[1:3])

x0= c(1,1,1,1)



local_opts <- list( "algorithm" = "NLOPT_LD_MMA", "xtol_rel"  = 1.0e-7 )
opts <- list("algorithm" = "NLOPT_LN_COBYLA", "xtol_rel"  = 1.0e-7, "maxeval"
    = 10000, "local_opts" = local_opts)
res <- nloptr( x0=x0, eval_f=obj, lb=lb, ub=ub, eval_g_ineq=constr, opts=opts)

UA = round(utility(b,v,d,offer="A", cover=cover,uA=uA,uB=uB)(res$solution),2)
UB = round(utility(b,v,d,offer="B", cover=cover,uA=uA,uB=uB)(res$solution),2)

RA = round(res_util(b,v,d,offer="A", cover=cover,uA=uA,uB=uB),2)
RB = round(res_util(b,v,d,offer="B", cover=cover,uA=uA,uB=uB),2)

P = profit(UA,UB,RA,RB, pi, res$objective)$P
A = profit(UA,UB,RA,RB, pi, res$objective)$Accept

#return(utility(b,v,d,cover=cover,offer=offer,uA=uA,uB=uB)(res$solution))
return(list("insU" = -res$objective, "contract"=res$solution, "profit"=P, "
    Accept"=A))
}

opt(Q,V,D,offer="both", cover="abc", uA=0.1, uB=0.2)$Accept

utility(Q,V,D,offer="A", cover="ab")(opt(Q,V,D,offer="both", cover="ab", uA=0.1,
     uB=0.2)$contract)

###########Equilibrium############

equilibrium <- function(b,v,d,uA,uB,pi) {
A = matrix(nrow = 5, ncol=7)
A[1,] = c(opt(b,v,d,offer = "A", cover="b", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "A", cover="b", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "A", cover="b", uA=uA, uB=uB, pi=pi)$contract, "Ab")

A[2,] = c(opt(b,v,d,offer = "A", cover="ab", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "A", cover="ab", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "A", cover="ab", uA=uA, uB=uB, pi=pi)$contract, "Aab")

A[3,] = c(opt(b,v,d,offer = "A", cover="bc", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "A", cover="bc", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "A", cover="bc", uA=uA, uB=uB, pi=pi)$contract, "Abc")

A[4,] = c(opt(b,v,d,offer = "A", cover="ac", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "A", cover="ac", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "A", cover="ac", uA=uA, uB=uB, pi=pi)$contract, "Aac")

A[5,] = c(opt(b,v,d,offer = "A", cover="abc", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "A", cover="abc", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "A", cover="abc", uA=uA, uB=uB, pi=pi)$contract, "Aabc")
OA <- A[which.max(A[,1]),1]
```

```
stratA <- A[which.max(A[,1]),7]
AcceptA <- A[which.max(A[,1]),2]
contractA <- A[which.max(A[,1]),3:6]

B = matrix(nrow = 5, ncol=7)
B[1,] = c(opt(b,v,d,offer = "B", cover="b", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "B", cover="b", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "B", cover="b", uA=uA, uB=uB, pi=pi)$contract, "Bb")

B[2,] = c(opt(b,v,d,offer = "B", cover="ab", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "B", cover="ab", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "B", cover="ab", uA=uA, uB=uB, pi=pi)$contract, "Bab")

B[3,] = c(opt(b,v,d,offer = "B", cover="bc", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "B", cover="bc", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "B", cover="bc", uA=uA, uB=uB, pi=pi)$contract, "Bbc")

B[4,] = c(opt(b,v,d,offer = "B", cover="ac", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "B", cover="ac", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "B", cover="ac", uA=uA, uB=uB, pi=pi)$contract, "Bac")

B[5,] = c(opt(b,v,d,offer = "B", cover="abc", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "B", cover="abc", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "B", cover="abc", uA=uA, uB=uB, pi=pi)$contract, "Babc")
OB <- B[which.max(B[,1]),1]
stratB <- B[which.max(B[,1]),7]
AcceptB <- B[which.max(B[,1]),2]
contractB <- B[which.max(B[,1]),3:6]

both = matrix(nrow = 5, ncol=7)
both[1,] = c(opt(b,v,d,offer = "both", cover="b", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "both", cover="b", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "both", cover="b", uA=uA, uB=uB, pi=pi)$contract, "bothb")

both[2,] = c(opt(b,v,d,offer = "both", cover="ab", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "both", cover="ab", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "both", cover="ab", uA=uA, uB=uB, pi=pi)$contract, "bothab")

both[3,] = c(opt(b,v,d,offer = "both", cover="bc", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "both", cover="bc", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "both", cover="bc", uA=uA, uB=uB, pi=pi)$contract, "bothbc")

both[4,] = c(opt(b,v,d,offer = "both", cover="ac", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "both", cover="ac", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "both", cover="ac", uA=uA, uB=uB, pi=pi)$contract, "bothac")

both[5,] = c(opt(b,v,d,offer = "both", cover="ac", uA=uA, uB=uB, pi=pi)$profit,
opt(b,v,d,offer = "both", cover="ac", uA=uA, uB=uB, pi=pi)$Accept,
opt(b,v,d,offer = "both", cover="ac", uA=uA, uB=uB, pi=pi)$contract, "bothabc")
Oboth <- both[which.max(both[,1]),1]
stratboth <- both[which.max(both[,1]),7]
Acceptboth <- both[which.max(both[,1]),2]
contractboth <- both[which.max(both[,1]),3:6]

Z = matrix(nrow=3,ncol=7)
Z[1,] = c(OA,stratA, AcceptA, contractA)
Z[2,] = c(OB, stratB, AcceptB,contractB)
Z[3,] = c(Oboth, stratboth, Acceptboth,contractboth)
```

```
prof <- Z[which.max(Z[,1]),1]
contract <- Z[which.max(Z[,1]),4:7]
strat <- Z[which.max(Z[,1]),2]
accept <- Z[which.max(Z[,1]),3]

return(list("eprof"=prof, "econtract"=contract, "estrat"=strat, "eaccept"=accept
    ))
}
```