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Abstract

In this thesis, I analyze the effect of private assets on the retirement decision of a worker in a frictional labor market. Therefore I introduce a search and matching model, where asset accumulation of an individual depends on the wage bargained between firm and worker, and is interrupted by stochastic shocks to unemployment. I not only inspect the influence of assets on the retirement age, but also the employment status (employed and unemployed). The results show that both assets and employment status affect the optimal retirement decision as well as the distribution of retirement ages in the economy.

Contents

CHAPTER 1

Introduction

Retirement decision making is very complex. On the one hand there are factors related to the labor market like unemployment rate, productivity or wage. On the other hand, we have socio-economic factors like gender, martial status and health. Each of them and many more have an impact on retirement. An attempt to characterize some of these factors is made by Coile (2015). Retirement has the greatest financial impact on an individual in a lifetime. It indicates the time of lower income and the exhausting of savings generated in labor force. As a consequence, retirement decission is a balancing act between receiving higher income through working longer and having enough savings for retirement.

Labor market related incentives for retirement receive a growing interest and more and more scholars engage in the topic. First, the change of rigid state pensions to more flexible ones in addition to private pension, put a new focus on retirement decision. Besides, the raise of the statutory retirement age changes the behavior of an old worker in general. Second after the great recession in 2008 we saw a decline in retirement age. So the effect of recessions like reduction of productivity, higher unemployment rates or loss of wealth may influence the retirement decision as well. Most of the papers regarding retirement decision focus on social security and/or health conditions as a main determinant (Gustman and Steinmeier (2005),French (2005)) There are few papers, which address the influence of the employment status on the retirement age. One of the first papers dealing with the issue are Hairault et al. (2015) and Coile and Levine (2006). They show a direct link between labor market conditions and retirement age. They also prove that an unemployment worker will retire earlier than an employed one. Brown et al. (2010) show empirical evidence between private assets and retirement decision, but further investigations have not been done yet. The impact of private assets in combination of labor market status has not been dealt with by now. In this paper I investigate this combination by establishing a theoretical model and providing further analyses.

First I introduce a model of the retirement behavior of workers. In my paper workers can accumulate assets over time by choosing consumption and saving in each period. Hereby their incentive is to smooth consumption over their lifetime. In other words, workers save in periods with higher income (when they are employed) to finance consumption in periods with lower income (when they are unemployed or retired). This behavior is called risk averse and enables me to study the impact of private assets on retirement decisions. It is common in this field of research to choose risk neutral workers, who are indifferent about consumption over time. Therefore risk neutral workers have no incentive to save assets for the future and retirement. This attitude invalidates studying the question how private assets effects retirement decision. Ergo the risk averse worker has a crucial role in my model. The other crucial ingredient of the model are frictions on the labor market. Retirement decisions are not only the result of labor supply but also of labor demand, see Frimmel et al. (2018) or Hairault et al. (2015). This papers show, that retirement age drops, if there is a low labor demand. As seen in the recession of 2008, unemployed workers retire earlier under such conditions, since the possibility of getting a job is low. Therefore retirement age drops as a whole. This paper discusses the combined role of labor market status and private assets on retirement age. Section 2 summarizes papers with empirical evidences and section 3 gives an overview of the content of papers, which are strongly connected to this one. Section 4 introduces my model and first shows results at the individual level. Then a simulation for a population of workers is used to determine the distribution of retirement ages in the aggregate economy. Variations of my baseline model show the influence of some parameters on retirement. Section 5 concludes.

CHAPTER 2

Empirical evidence

In this section I gather empirical evidences as a foundation of my theoretical work. The first section provides prove for a correlation between labor market status and retirement age. The second section shows the effect of wealth on retirement age empirically.

2.1 The effect of labor market status on retirement

Hairault et al. (2015) is the first paper I refer to for empirical evidence. It uses the data of the Health and Retirement Survey (HRS), which is a survey on Americans over the age of 50, repeated every two years. 8 waves are used from 1992 to 2006. Due to the complexity of the data different groups within the labor market are characterized by Hairault et al. (2015) as follows. Full time workers have to work at least 35 h per week and for at least 36 weeks a year. A part-time worker works at least 25 hours a week or full time but less than 36 weeks. Unemployed workers are currently looking for a full time or part time job and are not retired. If a person mentions that he or she is retired, but working part time or looking for a part time job, the person is classified as partly retired. Retired persons, who are not partly retired, are classified as retired. Disabled and other individuals, who do not fit any of this criteria were excluded in this analysis.

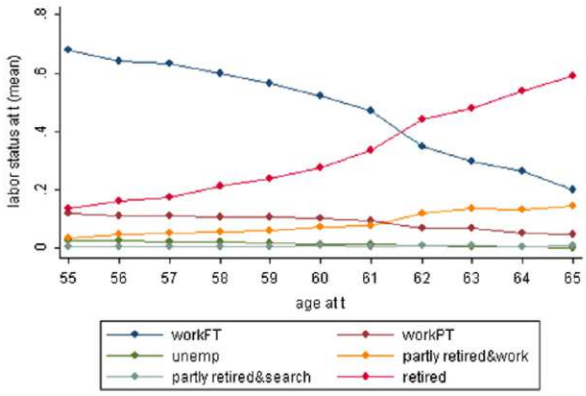

Fig. 1. Labor force distribution.

Figure 2.1: Hairault et al. (2015), Page 40; Figure 1

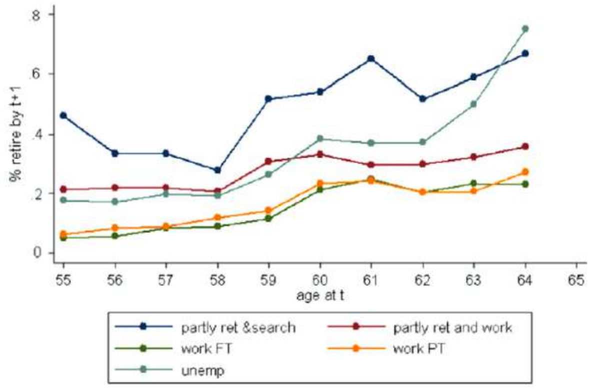

Fig. 2. Retirement propensity.

Figure 2.2: Hairault et al. (2015), Page 40; Figure 2

Figure 2.1 gives a good overview of the distribution of these groups on the labor market. Figure 2.2 on the other hand clearly shows a discrepancy between the retirement age of unemployed and employed. Full time workers and part time workers have nearly the same pattern by age. Unemployed have a higher rate of retiring in the next year than the two working classes. This gap even widens after the age of 60. The higher rate is a natural consequence, since retirement causes less financial loss for an unemployed worker than for an employed worker. This effect increases in age, because the possible benefit of getting hired reduces . Partly retired rates are difficult to explain. This group already receives pension benefits and therefore their ambitions to work again varies strongly.

This descriptive analysis points out that unemployed workers are more likely to retire by age than employed. To strengthen this conjecture the paper provides an econometric analysis. This way the selection effect, what describes the indirect influence of other characteristics on the result, can be reduced. Such characteristics are age, gender, education, household size, martial status, education, health, pension wealth of the household, job specific variables and the geographical component. The result of this analysis is shown in the following table.

Table 2
Probit regressions – dependent variable: retire by next wave.

Control variables	(1)	(2)	(3)	(4)
Unemployment	0.165***	0.176***	0.169***	0.169***
	(7.64)	(7.94)	(5.61)	(5.64)
Part-time		0.0280***	−0.000866	
		(3.76)	(−0.13)	
Proba continue work			−0.00149***	−0.00148***
			(−25.49)	(−25.73)
Observations	20,159	20,159	19,045	19,045

Omitted category: *works* in columns (1) and (4), *works full-time* in columns (2) and (3). Marginal effects; *t* statistics in parentheses.
*** $p < 0.001$.

Table 2.1: Hairault et al. (2015), Page 42; table 2

Table 2.1 shows that the probability to be retired in the next wave of the survey increases by 16.5 pp. when looking for a job compared to working. Columns (2) and (3) show that controlling for part time work does not significantly change this result.

2.2 The effect of wealth on retirement

It is very difficult to measure the causal effect of wealth on retirement. It follows naturally, that wealth lowers the ambition of working due to the financial stability. But wealth grows over a lifetime and therefore affects the retirement decision over a long period. This makes it more difficult to measure it. Other factors like the wealth of the family, expected income in future or even martial status influence wealth and therefore retirement age greatly. This makes it hard to investigate wealth separately. Therefore a shock in wealth is the most promising way to estimate a causal effect. Picchio et al. (2018) study the effect of winning a lottery on retirement age, while Brown et al. (2019) focus on inheritances. Their paper uses data from HRS from 1994 to 2002. The survey asks if the person has received an inheritance during the last wave and if so how much. In this regards, it also asks for the likelihood of inheriting money in the next 10 years. The persons are asked for a subjective probability of receiving any inheritance and the expected value of the inheritance awaited. If one cannot state the exact value they are expecting, they were asked to classify the amount in one of the following ranges: \$0-10.000, \$10.000-50.000, \$50.000-250.000, \$250.000-1.000.000 and over \$1.000.000. Due to the long time horizon, the authors can observe changes in the behavior of the survey participants. The paper splits the results into two groups. The first, in the paper called Person-Wave Sample, describes the short term effect of inheritance receipt over a period of 2 years. The other is a long term observation over 10 years. The first results are summed up in the following table.

Table 5: Effect of Inheritance Receipt on Retirement

Variable	Depend Var: Labor Force Exit					
	(1)	(2)	(3)	(4)	(5)	(6)
	Person-Wave Sample			Long-Difference Sample		
Inheritance Flag	0.0241 *			0.0360 **		
	(.0132)			(.0187)		
Inheritance Value		0.0210 *			0.0295 **	
		(.0108)			(.0074)	
Inh Value / HH Income			0.0111			0.0107
			(.0078)			(.0076)
# of Obs	17,801	17,801	17,733	4,508	4,508	4,485
Mean of Depend Var	0.192	0.192	0.192	0.541	0.541	0.541

Note:
1) The sample is limited to individuals who were working at the previous wave. The two-year change and long-difference samples are described in more detail in the text.
2) Inheritance value is measured in 100,000s of $2002.
3) All regressions include controls for age, gender, marital status, race, education, current and lifetime income, net worth, health status, pension type, industry, occupation, region, and wave; see text for details.
4) * indicates significance at the 10% level, ** indicates significance at the 5% level.

Table 2.2: Brown et al. (2010), Page 429; Table 2

The first column shows the effect of receiving inheritance in the short run. It concludes that when a person inherits, he or she is 2.4 percentage points more likely to retire over a a two-year period. This corresponds to an increase of13% over the baseline retirement rate. The next column shows that also the value of the inheritance has an effect on the probability to retire. Receiving an inheritance of $100,000 increases the rate of retirement by 2.1 points or 11 % from the baseline retirement rate. The last column states the effect of an inheritance at the size of an household income (≈$55.000). Although there is no statistical significance for this scenario, the coefficient is positive. The fourth to sixth columns refer to the long term observations and show similar results The only difference to the short-run estimates is that the percentage points compared to the base line retirement age are smaller.

The estimates shown in Table 2.2 may be biased. Inheritance receipt is often not random and expected by a household. For example, if a person from a rich family is more likely to receive inheritance and is also more likely to retire earlier without any inheritance, this would introduce a spurious correlation between inheritance and retirement. Further, the estimates may not correctly reflect the effects of change of wealth on retirement. When inheritance payments are expected, persons will adjust their behavior early on. For these reasons the paper takes the survey information on the expectation of inheritance receipt into account. In this second analysis, the dependent variable becomes an indicator whether one retires earlier than expected. With these new modifications the following results are obtained.

THE EFFECT OF INHERITANCE RECEIPT ON RETIREMENT

TABLE 3.—EFFECT OF EXPECTED VERSUS UNEXPECTED INHERITANCE RECEIPT ON RETIREMENT LONG-DIFFERENCE SAMPLE

	Dependent Variable: Retire Earlier Than Expected					
Variable	(1)	(2)	(3)	(4)	(5)	(6)
Inheritance flag	0.1280* (0.688) [.0491]					
Inheritance value		0.1333** (.0414) [.0506]				
Inheritance value/Household income			0.1047** (.0258) [.0398]			
Inheritance flag—expected				0.1141 (.0862) [.0438]		
Inheritance flag—unexpected				0.1477 (.0989) [.0570]		
Inheritance value—expected					0.1126* (.0413) [.0427]	
Inheritance value—unexpected					0.2720** (.0943) [.1033]	
Inheritance flag—amount expected						0.1970 (.1243) [0.765]
Inheritance flag—amount more						0.1546* (.0818) [.0596]
Inheritance flag—amount less						−0.1692 (.1828) [−.0623]
Number of observations	2,499	2,499	2,485	2,499	2,499	2,499
Mean of dependent Variable	0.382	0.382	0.382	0.382	0.382	0.382
F-test: Expected = Unexpected (or Amount more = amount less)				0.785	0.116	0.096

Note: Coefficient estimates from a probit model are reported with standard errors shown in parentheses. Marginal effects (evaluated at the sample means) are shown in brackets. The sample is limited to individuals who were working at wave 2 and provided an expected retirement date at wave 1. Long-difference samples are described in more detail in the text. Inheritance value is measured in 100,000s of 2002 dollars. All regressions include controls for the death of a parent, age, gender, marital status, race, education, current and lifetime income, net worth, health status, pension type, industry, occupation, and region. See text for details. * Significance at the 10% level. ** Significance at the 5% level.

Table 2.3: Brown et al. (2010), Page 431; Table 3

This time the paper only considers the long time difference data sample. The first 3 columns show the results with the new variables. They are similar to the results of table 2.2. For the last 3 columns more explanatory variables are added. These variables display if an inheritance is expected, if the amount a person received is what he or she had expected and whether the amount is more, equal or less than expected. The results indicate that inheritances, which are unexpected or larger than expected have a significant impact on early retirement. For example, if we raise an unexpected inheritance by the value of $100000, a person is 10 % more likely to retire. In conclusion, wealth has a causal influence on the retirement and it is substantial for retirement decision making.

CHAPTER 3

Supporting literature

In this section I summarize the two papers *Equilibrium unemployment and retirement by Jean-Olivier Hairault, François Langot, André Zylberberg* (2015) and *Labor-market frictions, incomplete insurance and severance payments by Etienne Lalé* (2019) . The models described therein serve as building blocks of the model I develop in Section 4. I give an overview of these papers, including goal, environment, mechanisms and conclusions.

3.1 Hairault et al. (2015)

The main goal of this paper is to understand the effect of the labor market status (being employed or unemployed) on the retirement decision. It is a continuation of Coile and Levine (2006), which proves the effect of labor market status on the retirement age empirically. This is the first paper, which proposes and investigates an underlying theoretical mechanism. First the paper extends the empirical evidence of the correlation between unemployment rate and retirement age as described in section 2.1. In the next step the paper introduces a model where retirement decisions depend on being unemployed or employed . The model highlights the important role of labor market frictions and how they affect the retirement gap between employed and unemployed workers. Once the model is introduced it is extended by social security policies.

The paper assumes endogenous retirement age and labor market frictions to generate a gap in retirement age. Without any calculations this result comes quite naturally. An individual values unemployment less than employment because of lower income. The drop in income experienced by retiring is therefore lower for an unemployed worker. As a consequence, the optimal transition to retirement comes earlier for an unemployed. The size of this retirement gap depends on the labor market tightness. Labor market tightness describes the ratio between supply of vacancies and the demand of them by unemployed workers. Low labor market tightness decreases the probability for unemployed workers to find a job. Therefore unemployed workers may be discouraged and take the rational choice to retire earlier. Whereas employees put more effort in keeping a job, due to the labor market situation. As a result labor market tightness has also a crucial role in retirement considerations.

The paper also introduces a social security system for workers. I will not describe this part of the paper here, since it has no relevance for my model.

3.1.1 Environment

In this model time is continuous. The life cycle of a worker starts with age $\tau = 0$ and ends with $\tau = T$. When a worker reaches $\tau = T$, the worker dies and is replaced by a worker with age $\tau = 0$. Every period in time, a worker can be in employment, unemployment or retirement. Workers are risk neutral and consume their income in every period. They optimally choose their retirement age in respect to his or her current employment status. Workers, either unemployed or employed, face age-dependent disutility $z(\tau)$. For employees $z(\tau)$ represents the disutility of work. Whereas for unemployed workers $z(\tau)$ displays the disutility of searching for a job. The function is normalized to $z(0) = 0$ and increases with age, such that at some point retirement will be chosen optimally. The retirement age for employed and unemployed workers is denoted by A_e and A_u, respectively.

3.1.2 Labor market

The labor market is filled with a continuum of firms. A firm produces from one unit of labor y units of output. Each employee supplies one unit of labor. If a worker is unemployed or retired he or she gains a value b by home production. The paper assumes $y > b$, which is standard in literature. The labor market is perfectly segmented by age. The paper uses a CRS matching function $M(v(\tau), u(\tau))$, where $v(\tau)$ represents the number of vacancies and $u(\tau)$ the number of unemployed of age τ. This function also states that $M(v(a), u(b)) = 0$ for a $a \neq b$, which implies that a job directed to a worker at age a can only be productive with a worker at age a. It also states that there is no on-the-job Search. The matching function M has the following properties, $M_1 > 0$, $M_2 > 0$, $M_{1,1} < 0$ and $M_{2,2} < 0$, where the indices represents the derivative in respect to the first or second variable. With the matching function we can define Poisson rates at which a vacancy meets an unemployed and vice versa.

$\frac{M(v(\tau),u(\tau))}{v(\tau)} = q(\theta(\tau))$ with $q'(\theta(\tau)) < 0$

$\frac{M(v(\tau),u(\tau))}{u(\tau)} = p(\theta(\tau))$ with $p'(\theta(\tau)) > 0$

where $\theta(\tau) \equiv \frac{v(\tau)}{u(\tau)}$ represents the market tightness for a worker at age τ. Matches and therefore jobs are dissolved with an exogenous rate s.

3.1.3 Bellman equation

Workers

The paper defines the value function for retired $R(\tau)$, unemployed $U(\tau)$ and employed $W(\tau)$ workers as follows.

$$rR(\tau) = b + \bar{U}'(\tau) \tag{3.1}$$
$$rU(\tau) = b - z(\tau) + p(\theta(\tau))[W(\tau) - \bar{U}(\tau)] + \bar{U}'(\tau) \tag{3.2}$$
$$rW(\tau) = w(\tau) - z(\tau) + s[\bar{U}(\tau) - W(\tau)] + W'(\tau) \tag{3.3}$$

where $\bar{U} = \max(U(\tau), R(\tau))$

For employed workers their value function (3.3) is determined by the wage $w(\tau)$ minus the disutility of work. In addition at the rate s jobs will be dissolved. In this case workers choose between unemployment or retirement and loose the value of employment.
Unemployed workers receive b by the home production minus the disutility $z(\tau)$ for the job search. This time there is a rate for finding a job $p(\theta(\tau))$. At this rate a former unemployed person finds a job, thus receives $W(\tau)$ and looses $\bar{U}(\tau)$. The value for retired is straightforward with the income b from home production.

By construction an unemployed worker never accepts a job at a time $\tau > A_u$. At the same time if an employed worker looses a job after A_u, he or she will retire immediately. Therefore $A_u \leq A_e$ and this indicates the retirement gap for the first time.

Firms

For a firm the value functions for a job filled with a worker of age τ $J(\tau)$ and for a vacancy $V(\tau)$ targeted to a worker of age τ are defined as follows.

$$rJ(\tau) = y - w(\tau) + s[V(\tau) - J(\tau)] + J'(\tau) \text{ for } \tau \leq A_e \tag{3.4}$$
$$rV(\tau) = -\iota + q(\theta(\tau))[J(\tau) - V(\tau)] \text{ for } \tau \leq A_u \tag{3.5}$$

where ι are the costs for an open vacancy. Vacancies will be filled until age $\tau = A_u$, because after this no unemployed worker is in the labor force. Similar to this jobs only will be open until $\tau = A_e$, because after this age all workers move to retirement. The paper postulates a free entry condition for firms. This means that as long as $V(\tau) > 0$ new firms enter the market until all rents are exhausted. Therefore a free entry condition leads directly to $V(\tau) = 0$

Consequently (3.5) can be rewritten as

$$\frac{\iota}{q(\theta(\tau))} = J(\tau) \tag{3.6}$$

Job surplus and wage

When a match is formed a joint surplus $S(\tau)$ is generated, since hiring and searching are costly. This surplus is defined as follows.

$$S(\tau) = J(\tau) + W(\tau) - \bar{U}(\tau)$$

Wages are defined in a Nash bargain. A firm has the inside option to hire a person or, if already hired, to keep the employee. Its outside option is to have a vacancy. Due to the free entry condition, the value for a vacancy is 0. On the other hand, the inside option of a worker is to work. Whereas the outside option is to be unemployed or retired. The Nash bargain implies that workers receive a share ζ of the surplus

$$J(\tau) = (1-\zeta)S(\tau)$$
$$W(\tau) - \bar{U}(\tau) = \zeta S(\tau)$$

With (3.3) and (3.4) one can calculate:

$$(r+s)[S(\tau) + \bar{U}(\tau)] = y - z(\tau) + s\bar{U}(\tau) + J'(\tau, \epsilon) + W'(\tau)$$

and in respect to that

$$(r+s)S(\tau) = y - z(\tau) - r\bar{U}(\tau) + \bar{U}'(\tau) + S'(\tau)$$

From (3.1) and (3.2) we get

$$r\bar{U}(\tau) + \bar{U}'(\tau) = \begin{cases} b - z(\tau) + \zeta p(\theta(\tau))S(\tau) & \text{for } \tau \leq A_u \\ b & \text{for } A_u < \tau \leq A_e \end{cases}$$

For the job surplus one can derive

$$(r+s)S(\tau) - S'(\tau) = \begin{cases} y - b - \zeta p(\theta(\tau))S(\tau) & \text{for } \tau \leq A_u \\ y - b - z(\tau) & \text{for } A_u < \tau \leq A_e \end{cases}$$

Let ω be the minimum of the two options, then one can integrate the function to receive

$$S(\tau) = Ce^{(r+s)\tau} - \int_0^\tau \omega(t)e^{-(r+s)(t-\tau)}\,dt$$

Since the joint surplus is zero when every worker has retired, the paper assumes that $S(A_e) = 0$. With this condition one can calculate $C = \int_0^{A_e} \omega(t)e^{-(r+s)(t-\tau)}\,dt$. In combination with (3.6) it follows that

$$S(\tau) = \begin{cases} \int_0^{A_u}[y - b - \frac{\zeta}{1-\zeta}c\theta(t)]e^{-(r+s)(t-\tau)}\,dt + \int_{A_u}^{A_e}[y - b - z(t)]e^{-(r+s)(t-\tau)}\,dt & \text{for } \tau \leq A_u \\ \int_\tau^{A_e}[y - b - z(t)]e^{-(r+s)(t-\tau)}\,dt & A_u < \tau \leq A_e \end{cases}$$

The surplus describes two different stages in the life cycle of a worker. In the second stage, after all unemployed workers have retired, disutility of work is the only force that pushes an employee to retire. Whereas in the first stage, the bargaining power and the job tightness have a direct impact on the surplus. If the job tightness is high, it is easier to form a match to generate a surplus. A high bargaining power of the workers leads to a reduction of vacancies, since the share received by the firms reduces. $z(\tau)$ affects both, unemployed and employed workers, at the same rate so it has no effect on the surplus in the first stage.

3.1.4 Results

Retirement age

With $W(\tau) - \bar{U}(\tau) = \zeta S(\tau)$ and $S(A_e) = 0$, (3.3) can be reduced to

$$rW(\tau) = w(\tau) - z(\tau) + s[\zeta S(\tau)] + W'(\tau)$$

and be integrated to

$$W(\tau) = \max_{A_e} \int_{\tau}^{A_e} e^{-r(t-\tau)}[w(t) - z(t) - s\zeta S(t)\, dt] + e^{-r(A_e-\tau)} R(A_e)$$

With the first order condition (FOC) with respect to A_e

$$b = w(A_e) - z(A_e)$$

and the wage equation $w(A_e) = \zeta y + (1-\zeta)(b + z(A_e))$ a condition for the equilibrium retirement age of employed workers is given by

$$y - z(A_e) = b$$

Since all unemployed workers are retired at age $\tau > A_u$ the condition depends only on the disutility of work. When the output minus the disutility hits the level of home production, an employee retires. In the same way (3.2) can be reduced to

$$rU(\tau) = b - z(\tau) + p(\theta(\tau))\zeta S(\tau) + \bar{U}'(\tau)$$

and again integrated to

$$U(\tau) = \max_{A_u} \int_{\tau}^{A_u} e^{-r(t-\tau)}[b - z(t) + p(\theta(t))\zeta S(t)\, dt] + e^{-r(A_u-\tau)} R(A_u)$$

The FOC for retirement age A_u in this case is

$$b - z(A_u) + p(\theta(A_u))\zeta S(A_u) = b$$

As one can see, A_u is determined by the age at which disutiliy of searching catches up with the expected value of getting a job.
This equation can be rewritten in respect to (3.6) and the sharing rules as follows

$$\frac{\zeta}{1-\zeta}\iota\theta(A_u) = z(A_u)$$

This highlights the important role of the labor market tightness for the retirement age of unemployed. If the opportunities of finding a job at a age τ get better, retirement will be delayed. The probability of getting a job is represented by $\theta(\tau)$. Focusing on (3.6), one can see that labor market tightness depends directly on $J(\tau)$. But $J(\tau)$ is determined by A_e. This implies that $\theta(A_u)$ is a function of A_e. Therefore A_e has an impact on A_u.

Summary

With the function for the surplus $S(\tau)$ and the conditions for optimal retirement, labor market equilibrium is finally defined and values for the surplus can be calculated recursively. With these values one can calculate the age of retirement and all the other endogenous variables. In the following paragraph I sum up the main propositions and conclusions.

The paper proves that if $z(T) > y - b$, there exists a unique pair (A_e, A_u) with $A_e > A_u$. This is the formal proof of the retirement gap.

The authors then conduct a range of comparative static analyses. First, they investigate the impact of the bargaining power ζ and the cost ι on A_u . Since higher vacancy posting cost or higher bargaining power leads to an increase on the value of searching for a new job, a higher retirement age A_u is resulting. But with higher costs, firms provide fewer vacancies and therefore the matching probability reduces. So the impact of these effects is ambiguous. Another point is that a reduction of the matching efficiency of $M(.,.)$ will reduce A_u, but A_e will stay the same. Increasing searcht frictions therefore increase the retirement gap.

The last result of the basic model is that if the productivity y increases, the effect on the two retirement ages differs. The paper proves that an increase in productivity raises A_e less than it raises A_u, which leads to a reduction of the retirement gap. With an increase of y, the expected surplus rises as well. Therefore companies open more vacancies due to the free entry condition. This leads to a better chance of finding a job, because of the higher labor market tightness. In addition an increase in y leads to a higher expected surplus and therefore to a higher retirement age for both groups. If a recession occurs, the retirement gap should therefore be larger due to the reduction in productivity. This theoretical prediction is in line with Coile and Levine (2011), who show the reduction of the average retirement age by the high number of unemployed in the Great Recession.

In the rest of the paper the authors introduce a social security system for workers. I will not describe this part of the paper here, since it has no relevance for my thesis.

3.2 Lalé (2019)

The paper´s main goal is to investigate severance payments and their role in the process of wage bargain. The paper considers risk averse persons to conclude the influence of severance payments on welfare. Risk averse individuals that smooth consumption over their lifetime, as well as discrete time are the main parallels to my model.

3.2.1 Environment

In this model time is discrete and runs forever. A worker has two separated life parts. The first is from age 0 to N_w and describes the working period. N_r describes the duration for the retirement period. N_w and N_r are exogenous in contrast to Hairault et al. (2015),which indicates that retirement age is not the focus of the investigation in this paper. If a worker dies after $N_w + N_r$ he or she will be replaced by a new entrant in the job market. This means that population size is constant. The utility function $u(.)$ for workers satisfies $u'(.) > 0$ and $u''(.) < 0$. Workers are therefore risk averse. The firms act risk neutral and there is a continuum of them at the labor market. Firms want to maximize their profit and use the real interest rate r to discount the future. This means, that they assess the future value of a today's investment with the real interest rate. Whereas workers use β as an subjective discount factor.

3.2.2 Labor market

There are three different possible statuses for a person in a life cycle: retired, unemployed, or employed. As mentioned the labor market is filled with a continuum of firms. The output is produced by a matched firm-worker-pair. It is given by $\lambda f(y_t, \tau)$ where τ is the age of the worker and λ is a parameter for the use and cost of capital. y_t is idiosyncratic to the matched pair and stochastically evolving according to a recurrent Markov process. This Markov process is defined by its transition function $G(. \mid y)$. Also there is an exogenous separation rate s_τ, which depends on the age of a worker. The matching function $M(u_t, v_t)$ is a constant-returns-to-scale function, where u_t are the numbers of unemployed and v_t is a measure for the vacancies for a time t. $\theta_t = \frac{v_t}{u_t}$ is again the labor market tightness and $q(\theta_t) = M(\theta_t^{-1}, 1)$ is the probability of a meeting of a perspective firm and $\theta_t q(\theta_t)$ for a prospective worker. Notice that in contrast to Hairault et al. (2015) the labor market is not segmented by age τ. If a match is formed, it has a potential output y_t, which is determined by the distribution $G_0(.)$. Based on the realisation, either the match dissolves immediately, or a matching pair is formed. If the pair is formed, it starts to produce at a level y_t. Otherwise the worker goes back into unemployment and the firm is left with a vacancy. A vacant position has ι costs for a firm.

3.2.3 Governmental policies

The paper investigates the effects of different social security policies. Therefore it has to include these policies in the basic model, which are described as follows.

- The government provides pension benefits to retired workers. These are a fixed amount ρ and compensated by a flat rate tax κ_ρ on labor income.
- Unemployed workers get unemployment insurance benefits (UI). When a worker becomes newly unemployed he or she receives an amount ν_1, and if these benefits have expired, they receive social assistance ν_0. The UI benefits are financed by a flat rate tax on wages κ_ν.
- Employed workers, with a long tenure within one firm, receive severance payments (SP) in case the match dissolves. These payments are a transaction between a firm and a worker and are denoted as T_τ, depending on the age of a worker.

The variable $i_u \in \{0,1\}$ indicates whether a worker cannot or can receive UI. Respectively i_e does the same with the possibility for SP. Therefore we get two transition matrices.

$$P_{i_u,j_u} = \begin{bmatrix} 1 & 0 \\ p_u & 1-p_u \end{bmatrix} \text{ and } P_{i_e,j_e} = \begin{bmatrix} 1-p_e & p_e \\ 0 & 1 \end{bmatrix}$$

where p_u is the probability of loosing UI and p_e describes the probability of becoming eligible to SP. At first for a certain time in a new job, a worker is not eligible for SP. When a worker is entitled to SP, the workers behavior changes. The greatest change is in the wage bargaining process. The bargaining power of a worker rises once he or she becomes eligible, since dissolving of a match forces a firm to pay SP.

3.2.4 Bellman equations

Workers

The workers and retirees face an inter-temporal budget constraint

$$a_{t+1} \leq (1+r)a_t + \xi_t^d - c_t,$$

where c_t is consumption and ξ_t^d is the disposable income. a_t describes risk free assets, which can be saved and earn the market interest rate r. It is also possible for workers to borrow money up to an exogenous limit of $\underline{a} \geq 0$. When they retire, borrowing money is not possible anymore and consequently $\underline{a} = 0$. To ensure that all the SP can be paid, all profits of firms are collected in a fund owned by all firms.

Since it is a discrete time system, we will denote variables and the age of workers in a one step ahead period with a ($'$), like $\tau' = \tau + 1$.

The value function of retirees is described as follows:

$$R(a,\tau) = \max_{c,a'}\{u(c) + \beta R(a',\tau')\} \tag{3.7}$$

subject to

$$a' \leq (1+r)a + \rho - c$$
$$a' \geq 0$$

for every $N_w + 1 \leq \tau \leq N_w + N_r$ In addition, $R(a, N_w + N_r + 1) = 0$ for every a, since at this age a worker has already passed away. For retirees utility by consumption and the expected future value of their assets are the only value generating objects. The budget constraint for retired workers reflects that retired workers live from pension benefits ρ and savings in form of assets.

For the value functions for unemployed, two options are possible. Either a worker receives UI or not. The two options are indexed and determined by

$$U_{i_u}(a,\tau) = \max_{c,a'} \Big\{ u(c) + \beta \sum_{j_u=0,1} p_{i_u,j_u} \Big((1-\theta q(\theta)) U_{j_u}(a',\tau') + \theta q(\theta) \int \max\{W_0(y',a',\tau'), U_{j_u}(a',\tau')\}\, dG_0(y') \Big) \Big\} \tag{3.8}$$

subject to

$$a' \leq (1+r)a + (1-\kappa_\rho)\nu_{i_u} - c$$
$$a' \geq -\underline{a}$$

for every $1 \leq \tau \leq N_w$ with $U_{i_u}(a, N_w+1) = R(a, N_w+1)$ for every i_u and a.

Respectively the value of employment is also indexed with respect to the possibility for receiving SP.

$$W_{i_e}(y,a,\tau) = \max_{c,a'} \Big\{ u(c) + \beta \Big(s_{\tau'} U_1(a',\tau') + (1-s_{\tau'}) \sum_{j_e=0,1} p_{i_e,j_e} \int \max\{W_{j_e}(y',a',\tau'), U_1(a'+T_{\tau,j_e},\tau')\}\, dG(y' \mid y) \Big) \Big\} \tag{3.9}$$

subject to

$$a' \leq (1+r)a + (1-\kappa_\rho-\kappa_\nu) w_{i_e}(y,a,\tau) - c$$
$$a' \geq -\underline{a}$$

for every $1 \leq \tau \leq N_w$ with $W_{i_e}(y, a, N_w + 1) = R(a, N_w + 1)$ for every i_e, y and a. T_{τ, j_e} represents the SP payments depending on the age and $w_{i_e}(y, a, \tau)$ the wage bargained between firms and workers.

Firms

SP influence the wage bargain and therefore the value functions of firms. Likewise to the case for unemployed and employed workers, we have two scenarios

$$J_{i_e}(y, a, \tau) = \lambda f(y, \tau) - w_{i_e}(y, a, \tau) + \frac{1 - s_{\tau'}}{1 + r} \sum_{j_e = 0,1} p_{i_e, j_e} \int \max\{J_{j_e}(y', a', \tau'), -T_{\tau, j_e}, \} \, dG(y' \mid y)\Big)\} \quad (3.10)$$

for every $1 \leq \tau \leq N_w$ with $J_{i_e}(y, a, N_w + 1) = 0)$ for every i_e, y and a.

3.2.5 Wage

The wage is determined through a Nash bargain. The SP also separates the Nash product into two scenarios. The first one occurs if a worker is not eligible for SP. The two factors of the Nash product describe the options of the two bargaining parties. The value of a possible agreement is subtracted by the value calculated in case the bargain fails. Respectively the two options are called inside and outside options.

$$w_0(y, a, \tau) = \arg\max\{W_0(y, a, \tau) - U_1(a, \tau))^{\gamma} J_0(y, a, \tau)^{1-\gamma}\}$$

where $\gamma \in (0, 1)$ is the bargaining power of workers. Since SP is not included workers earn W_0 for working. The outside option is to get UI. The firm has J_0 as the inside option and the outside option is to hold a vacancy. Like in Hairault et al. (2015) the paper assumes a free entry condition for firms. Therefore the outside option is zero, since there is no profit to generate. The option of an unemployed worker, who receives U_0 and starts to work, is neglected in the paper.

The second case is that the worker receives SP

$$w_1(y, a, \tau) = \arg\max\{W_1(y, a, \tau) - U_1(a + T_\tau, \tau))^{\gamma} (J_1(y, a, \tau) + T_\tau)^{1-\gamma}\}$$

In this case T_τ represents the SP a firm has to pay if the employee leaves. This increases his or her asset stock for the outside option.

3.2.6 Additional conditions

Firstly, the free entry condition for firms implies that firms exhaust the present discounted value of job creation net of the cost of a vacancy. An unemployed and a vacancy meet at the end of a period. Therefore the free entry condition yields:

$$\frac{\iota}{q(\theta)} = \frac{1}{1+r} \sum_{\tau=1}^{N_w-1} \sum_{j_e=0,1} \int_{\mathcal{Y},\mathcal{A}} \max\{J_0(y', \bar{a}_{i_e}^U(a), \tau'), 0\}\, dG_0(y') \frac{\mu_{i_e}^U(a,\tau)}{u_{N_w-1}}\, da \tag{3.11}$$

$\mathcal{Y}$ and $\mathcal{A}$ denote the support for match productivity and asset holdings, respectively. $\mu_{i_e}^U(a,\tau)$ represents the distribution of unemployed workers and is scaled by the amount of job seekers u_{N_w-1}. The subscript refers to workers of age less than N_w. This implies that at N_w there are no job seekers anymore. $\bar{a}_{i_e}^U(a)$ represents a asset holding rule of an unemployed worker in equilibrium.

The next two conditions serve to balance the budget of the pension and UI schemes by the respective taxes.

$$\kappa_\rho \sum_{\tau=1}^{N_w} \Big(\sum_{j_e=0,1} \int_{\mathcal{Y},\mathcal{A}} w_{i_e}(y,a,\tau)\, d\mu_{i_e}^W(y,a,\tau) + \sum_{i_u=0,1} \nu_{i_u} \int_{\mathcal{A}} d\mu_{i_u}^U(a,\tau) \Big)$$
$$= \sum_{\tau=N_w+1}^{N_w+N_r} \int_{\mathcal{A}} \rho d\mu^R(a,\tau) \tag{3.12}$$

$$\kappa_\nu \sum_{\tau=1}^{N_w} \sum_{i_e=0,1} \int_{\mathcal{Y},\mathcal{A}} w_{i_e}(y,a,\tau)\, d\mu_{i_e}^W(y,a,\tau) = \sum_{\tau=1}^{N_w} (\sum_{i_u=0,1} \nu_{i_u} \int_{\mathcal{A}} d\mu_{i_u}^U(a,\tau) \tag{3.13}$$

The two equations and a fixed κ_ρ defines ρ. (2.13) gives the tax rate κ_ν.

With these conditions and equations, an equilibrium can be recursively defined and numerically solved.

External model settings

In this subsection I present a list of the functions and parameters set externally.

y' is an AR(1) process with $y' = (1-\delta)\phi + \delta y + \epsilon'$, where ϕ is the unconditional mean of the process, $\delta \in (0,1)$ the persistence and ϵ' white noise with $\epsilon' \sim \mathcal{N}(0,\sigma^2)$. In this case $\delta = 0.965$ borrowed by Chang and Kim (2006) and σ is calibrated as ≈ 0.19.

Due to the intuition of the Current Population Survey (CPS) for a hump shape productivity profile, $f(y,\tau) = y \times (\varepsilon_0 + \varepsilon_1\tau + \varepsilon_2\tau^2)$.

The matching function is standard Cobb-Douglas with $M(u,v) = mu^\chi v^{1-\chi}$, m=0.2747 and $\chi = 0.5$.

Like productivity the CPS also suggests that the separation rate is convex and decreasing over time. Therefore it is set

$$s_\tau = \bar{s} \times exp(\varsigma_0 + \varsigma_1\tau + \varsigma_2\tau^2)$$

where $\bar{s}$ is a scale parameter and the ς_i are computed from the data.

At last we have the utility function, which differentiates this paper from most in the labor market literature. Since the paper wants to generate risk averse workers, the utility function is CRRA,

$$u(c) = \frac{c^{1-\eta} - 1}{1 - \eta}$$

where η is set 2.

The rest of the parameters are computed with data, simplified or taken from official data of the US government and can be found in Lalé (2019), Table 1, Page 9.

Outcomes

The paper uses numeric interpolation to obtain optimal decision making recursively. Additional sensitivity analysis helps to understand the behavior of the model in general. With these tools some strong results can be formulated. The paper focuses on SP and its effects. Therefore the model suggests, that SP have a direct effect on the wage of a worker. Since the bargaining power of workers is strengthened by SP workers who are eligible to SP, will receive higher wages. On the contrary, since firms are aware of that, entry wages for newly employed workers drop. The effect of lower entry wages dominate the advantage of higher wages later on. As a result average wage declines. SP also has an indirect effect on firms and welfare. For a firm the value of a filled job decreases with SP. Consequently less vacancies will be opened. As a result the job finding rate drops, but the separation rate stays constant. It follows that unemployment periods prolong for workers. Therefore lower wages decrease pension benefits in equilibrium and longer unemployment periods increase UI taxes to balance the budget. These wage-shifting effects are so remarkable, that the paper suggest, that SP produces welfare losses.

Although the model suggested a strong wage effect, empirically it can not be ascertained. But the analysis supports the conclusion that there may be a connection.

CHAPTER 4

The model

In this section I describe my own model by its environment, value functions, first results and a sensitivity analysis for some parameters. The model borrows structural ideas from Lalé (2019) and combines it with the retirement gap of Hairault et al. (2015). It allows me to investigate the effects of assets and employment status on the optimal retirement age.

4.1 Environment

In my model time is discrete and runs forever. A worker enters the job market at the age of 21 with $\tau = 1$ and dies at age 80 ($T = 60$). In his or her lifetime there are three different possible employment statuses: employed, unemployed and retired. Whereas employed workers earn money by wages, retired and unemployed worker produce goods by home production b. Individuals derive utility from consumption.The utility function $u(.)$ is strictly concave and increasing. Workers are thus risk averse and have an incentive to accumulate assets over their lifetime. We denote τ as the age of a worker. β is the subjective discount factor, which is determent by the real interest rate r. Any worker before retirement experiences disutility z(τ), which displays the discomfort of working or looking for a job. Only if a worker is retired, disutilty vanishes. $z(.)$ is normalized with $z(0) = 0$ and convex increasing in age as in Hairault et al. (2015). This discomfort pushes a worker from the workforce into retirement. Once a worker chooses retirement he or she stays retired.

4.2 Labor market

The model has a continuum of firms, which transform one unit of labor into y units of output. Every worker provides 1 unit of labor. Again home production has a lower productivity i.e. $y > b$. There is no segregation of the labor market regarding age or any other characteristics of a worker. So every worker is capable of doing any job at any age as in Lalé (2019). A simplification from Lalé (2019) is that $p(\theta)$ is not determined in equations through a equation similar to (3.11), but regarded exogenous and henceforth simply denoted p.. Additionally I assume that the rate s, at which a job will be dissolved, is independent of age.

4.3 Bellman equations

Here we consider each value function for unemployed, employed, retired and firms. I define $\tau' = \tau + 1$ and respectively $a' = a + 1$ for the assets in the next period.

4.3.1 Workers

The value function for retired is:

$$R(a, \tau) = \max_{a',c}\{u(c) + \beta R(a', \tau')\} \tag{4.1}$$

s.t.

$$a' \leq (1 + r)a + b - c$$
$$a' \geq 0$$

where r is the interest rate. Since retirement ends with death, $R(a, T + 1) = 0$ for every a. The value function for retired is the sum of utility arising from consumption and the discounted future value.

The next value function describes the value for unemployed workers.

$$U(a, \tau) = \max_{a',c}\{u(c) - z(\tau) + \beta p\bar{W}(a', \tau') + \beta(1 - p)\bar{U}(a', \tau')\} \tag{4.2}$$

s.t.

$$a' \leq (1 + r)a + b - c$$

where

$$\bar{W}(a, \tau) = \max(W(a, \tau), R(a, \tau))$$

and

$$\bar{U}(a, \tau) = \max(U(a, \tau), R(a, \tau))$$

$\bar{W}(a, \tau)$ and $\bar{U}(a, \tau)$ reflect the optimal transition for a worker of the workforce to retirement. If $R(a, \tau)$ exceeds $U(a, \tau)$ ($W(a, \tau)$) an unemployed (employed) worker will retire. The probability p in (4.2) represents the possibility of getting hired or respectively staying unemployed with the alternative of retirement.

Next I introduce the value function for employed workers:

$$W(a,\tau) = \max_{a',c}\{u(c) - z(\tau) + \beta s \bar{U}(a',\tau') + \beta(1-s)\bar{W}(a',\tau')\} \tag{4.3}$$

s.t.

$$a' \leq (1+r)a + w(a,\tau) - c$$

The main difference between employed and unemployed is the transition rate s or p. For the budget constraints, wage replaces home production.

4.3.2 Firm

Since output minus wage determines profit of a firm, the value function is stated:

$$J(a,\tau) = y - w(a,\tau) + \beta s V(a,\tau) + \beta(1-s)\bar{J}(a',\tau') \tag{4.4}$$

with $\bar{J}(a,\tau) = \begin{cases} J(a,\tau) & \text{for } W(a,\tau) > R(a,\tau) \\ V(a,\tau) & \text{for } W(a,\tau) \leq R(a,\tau) \end{cases}$

where $V(a,\tau)$ describes the value of a vacancy in this period. As long as $W(a,\tau) > R(a,\tau)$ a worker chooses to work. But in the moment retirement provides the same value for a worker, he or she chooses to retire. Therefore the job becomes vacant. Since we assume a free entry condition for firms, $V(a,\tau) = 0$, (4.4) becomes

$$J(a,\tau) = y - w(a,\tau) + \beta(1-s)\bar{J}(a',\tau') \tag{4.5}$$

4.4 Wage

The wage of a worker is the result of a Nash bargain. A workers inside option is either to stay on the job or, if unemployed, to start at a job. The outside option is either unemployment or retirement. On the other side, a firm can either keep (hire) the worker in order to gain value or enforce (keep) a vacancy. Since I proclaim a free entry condition the value of a vacancy drops to zero. Therefore the Nash product has the following form:

$$w(a,\tau) = \arg\max(W(a,\tau) - \bar{U}(a,\tau))^{\gamma} J(a,\tau)^{1-\gamma} \tag{4.6}$$

where $\gamma \in (0,1)$ describes the worker´s bargaining power.
The first order condition (FOC) w.r.t $w(a,\tau)$ reads:

$$\gamma \frac{u'((1+r)a + w(a,\tau) - a')}{W(a,\tau) - \bar{U}(a,\tau)} = (1-\gamma)\frac{1}{J(a,\tau)}, \tag{4.7}$$

where a' and $W(a,\tau)$ are resulting from (4.3), $\bar{U}(a,\tau)$ is calculated by the optimal $U(a,\tau)$ in (4.2) and $R(a,\tau)$ in (4.1). $J(a,\tau)$ is computed from (4.5) and thus also influenced by the optimal wage.

4.5 Functions and Parameters

Every worker can set ones retirement age optimally according to ones current age and asset level. As mentioned before I assume risk averse workers to see the impact of private assets on retirement age. Therefore I need an appropriate utility function. In this model I use a CRRA-utility function:

$$u(c) = \frac{c^{1-\eta} - 1}{1 - \eta}, \tag{4.8}$$

where η is the coefficient of the relative risk aversion. It is fixed at 2, which has been used in Lalé (2019) and is common in literature.

Since I choose to set the discount factor for firms and individuals to be equal, I fix β as $\beta = \frac{1}{1+r} \approx 0.9615$ for $r = 4\%$.

I use data from the German labor market provided by Gartner et al. (2012)[1] to set the separation rate s at 0.058 and the employment rate θ at 0.701. I normalize the output to $y = 1$ and since set b to 0.4, drawing on Shimer (2005) . The bargain power of a worker is set to $\gamma = 0.5$ and borrowed from Lalé (2019).
As described before $z(\tau)$ rises over time and pushes worker out of labor force. So $z(\tau)$ is a convex function with high disutility at the end of a lifetime,

$$z(\tau) = z_{max} \frac{e^{\lambda * (\tau - 1) - 1}}{e^{\lambda * (59) - 1}}, \tag{4.9}$$

to ensure that $z(60) = z_{max}$ and $z(0) = 0$. For my benchmark results, I use $z_{max} = 10$ and $\lambda = 0.2$. The resulting age profile of disutility is shown in Figure 4.1.

[1] The quarterly rates given on page 106 forms a transition matrix A. Consequently A^4 represents the transition matrix for a year. Then the rates needed are the one representing a change of employment status in the Matrix A^4. Although it is common for labor market analysis to use at most quarterly rates, I uses yearly rates, since the results do not change drastically.

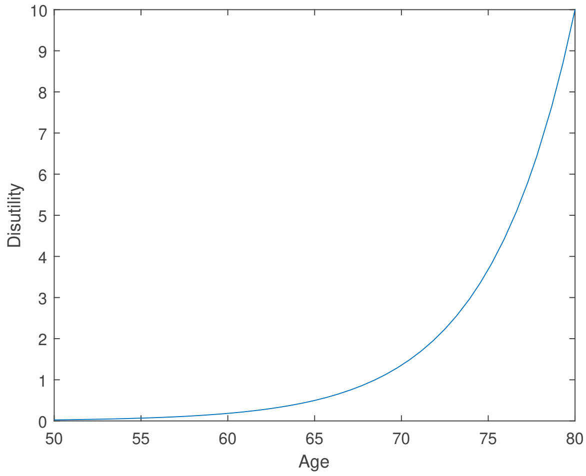

Figure 4.1: Disutility of work by age

The following table contains an overview of all the parameters.

Parameter		
Characterization	Parameter	Value
Length of a life cycle	T	60
Relative risk aversion coefficient	η	2
Real interest rate	r	0.04
Discount factor	β	0.9615
Job separation rate	s	0.058
Job finding rate	p	0.706
Output	y	1
Home production	b	0.4
Worker bargain power	γ	0.5
Maximal disutility	z_{max}	10
Growth rate of disutility	λ	0.2

4.6 Numerical Algorithm

To compute an equilibrium via numerical methods, I start with discretizing the asset space from [0,40] into a grid with 300 points. The grid is more dense in the beginning and is getting more loose towards the end. 60 points are located in 5 different sectors between 0 and 0.4, 0.4 and 1, 1 and 4, 4 to 10 and 10 to 40. The algorithm is described as follows:

For each age τ from 60 to 1:

- For each a on the asset grid:

  - Solve (4.1) to obtain $R(a,\tau)$.
  - Guess $w(a,\tau)$
  - Solve (4.2), (4.3) and (4.4) using golden section search. To evaluate the value functions off the asset grid, use cubic spline interpolation.
  - Calculate $\bar{W}$, $\bar{U}$ and $\bar{J}$
  - Obtain the residual of the Nash bargaining equation (4.7)
  - Update $w(a,\tau)$ with a new guess and continue until (4.7) is satisfied.

- Find the assets a on the grid , at which (4.2) becomes smaller than (4.1) and respectively (4.3) smaller than (4.1).This determines the asset threshold beyond which unemployed and respectively employed of age τ retire.

- Smooth the wage function $a \mapsto w(a,\tau)$ and recalculate all value functions with the smoothed wage.. This is necessary due to jumps in the wage function. These jumps are created by workers, who know that they are working in this period, but will retire in the next. This fact increases the wage for this period and forces a jump relative to the wage of workers who participate in both periods, which can lead to numerical problems. Therefore I approximate the wage function by a rational function. For a better fit, separate functions are fit on the regions separated by the asset thresholds determined above.

4.7 Outcomes

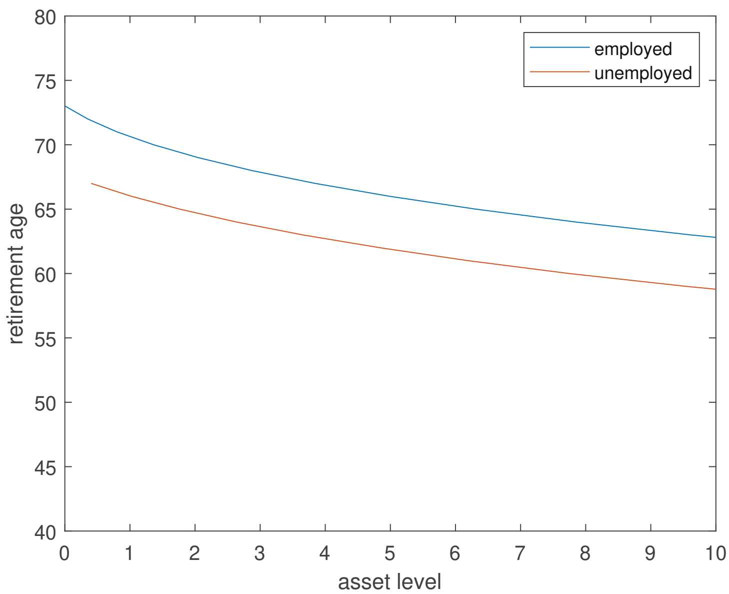

Figure 4.2: Age of retirement depending on the asset level

Figure 4.2 shows the first result. It displays how the retirement age depends on the asset level. Also it differentiates between the two possible work statuses. To set the asset levels in perspective, the mean of a yearly salary of a worker is 0.9195. The fact that the employment status has an huge impact on the timing of retirement is evident. It confirms the result of Hairault et al. (2015). As mentioned in Section 2.1 unemployed workers only have retirement as an outside option, whereas employees have unemployment and retirement to choose. At some point unemployed workers choose to retire, but employees are still in the labor market. From now on employees have only the outside option of retirement. Loosing the job at this point leads immediately to retirement.
For a specific asset level, the gap in retirement age between unemployed and employed workers stays nearly the same with 4, in some cases 5 years. Although assets have a big influence on the retirement age, they hardly affects the retirement gap . If one has an asset level of 10, which are 11 annual average salaries, one leaves the job market as an unemployed person 9 years and as an employed person 10 years earlier than having zero assets. The descending slopes of the curves indicate that a higher asset level leads to a lower retirement age for both cases .

This figure gives us a very good impression of the individual's optimal retirement decision. To understand its implications on the aggregated economy, I simulate the behavior of a large population. This program has the following structure:

1. Choose $N = 100000$ as size of the population.
2. Assume that every new worker with $\tau = 1$ has an asset stock of 0 and is unemployed.
3. Generate a matrix of random numbers between (0,1)
4. Use this matrix, the separation probability s and the job-finding probability p to simulate shocks in employment status.

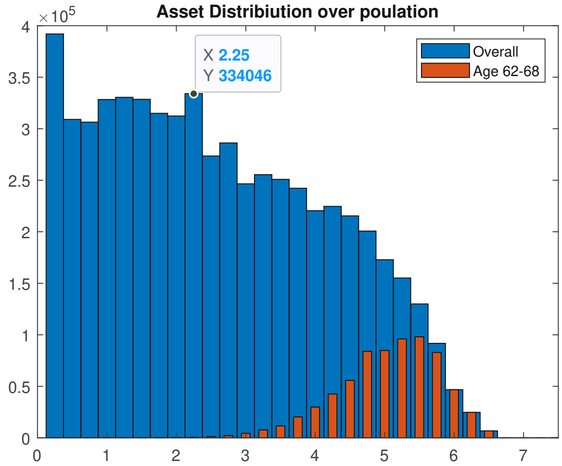

Figure 4.3: Distribution of asset levels in the simulated population

Figure 4.3 shows the range of the reached asset levels. They are between [0,6.5], which implies a maximum of ≈ 7.1 average yearly salaries. It also shows the distribution of assets for a worker at age 62 to 68. In this time retirement decisions will take place.

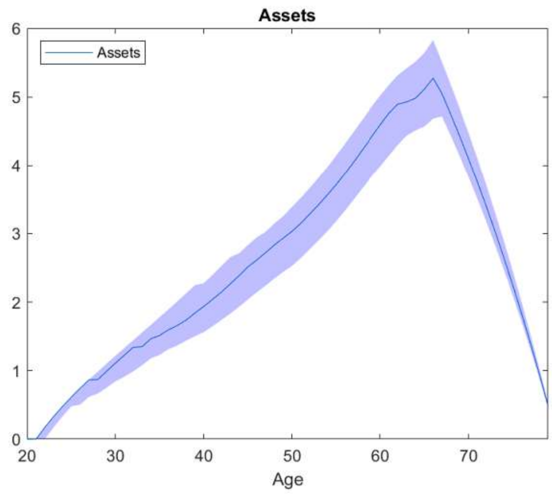

Figure 4.4: Simulated asset level of one worker

Figure 4.4 gives us a good sense on how a worker accumulates wealth over a lifetime and consumes the savings in retirement. The blue area indicates the data between the first and third quantile of the asset distribution at every given age, while the solid line indicates the mean.

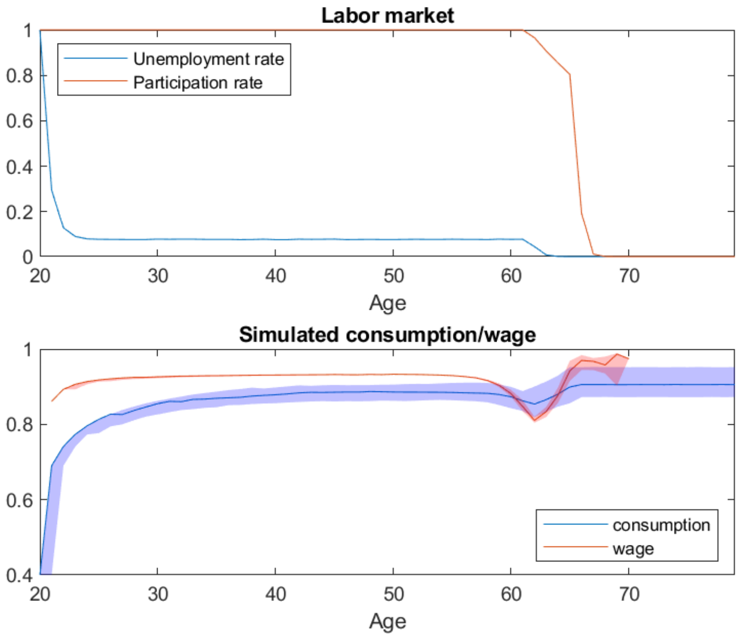

Figure 4.5: Simulated Wage, consumption and labor market rates

The upper part of Figure 4.5 shows that the unemployment rate drops from the initial value of 1 (all workers starts unemployed) to a stable rate of 0.1. Also it is apparent that there is a different retirement age between employed and unemployed. This gap is indicated by the drop of the curves to 0 and therefore the exit of the labor market. Since the unemployment rate drops to 0 before the participation rate, at some point, only employed workers are acting on the labor market anymore. The distribution of retirement ages is shown in Figure 4.6 below. The wage and consumption plot in the lower part of Figure 4.5 displays an overall stable consumption behavior. The wage curve is nearly constant until it reaches the point where the first retirement decisions are made. The average wage drops in this period only to rise to a higher level. The temporary reduction of wage explains the crinkle on the consumption curve. To explain the drop I start with the end of the working phase of an employed worker. As we know, employed workers stay longer in the labor market. Since disutility of work increases in age, they require a higher wage towards the end of working life to stay active. In fact we observe that retirement occurs when the wage required to continue working exceeds the highest wage that the firm is able to offer, which in the last period before retirement is y. To make up for these high wages close to retirement, firms optimally lower wages in the preceding years.

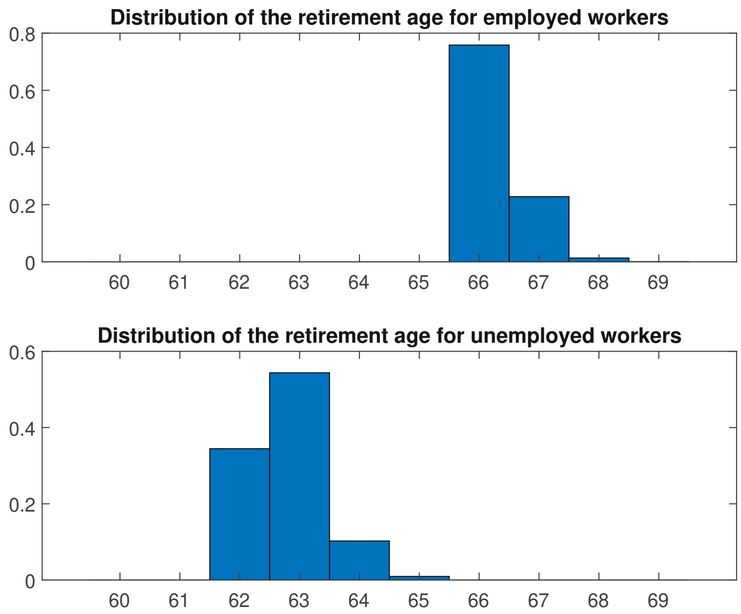

Figure 4.6: Comparison between the two labor market statuses

Figure 4.6 shows how the distribution of retirement age differs between employed and unemployed workers. The shift to the left indicates the important role of employment status on retirement age. But also the asset level plays an indirect role in Figure 4.6. Within each of these two groups, richer individuals retire earlier. The fact that unemployed worker are more likely on a lower asset level, additionally contributes to these workers retiring earlier.

The simulation results thus show that both employment status and wealth affect retirement decisions, and that they interact with one another.

4.8 Sensitivity analysis

In this section I analyse the sensitivity of the main results with respect to four key parameters, that affects retirement incentives .

4.8.1 Home production

Home production b influences the equilibrium directly by providing income in retirement but also in unemployment. These lead to opposite effect on retirement incentives. The first effect is that higher home production leads to higher pensions in retirement. With a higher $R(a, \tau)$ worker retire earlier. The second effect is that a higher b rises the unemployment income and therefore $U(a, \tau)$. increases the outside option of workers in the wage bargain. With higher wages $W(a, \tau)$ goes up too. As a result workers retire later. To see which of the effects dominates I change the parameter paribus ceteris to $b = 0.35$ and $b = 0.45$.

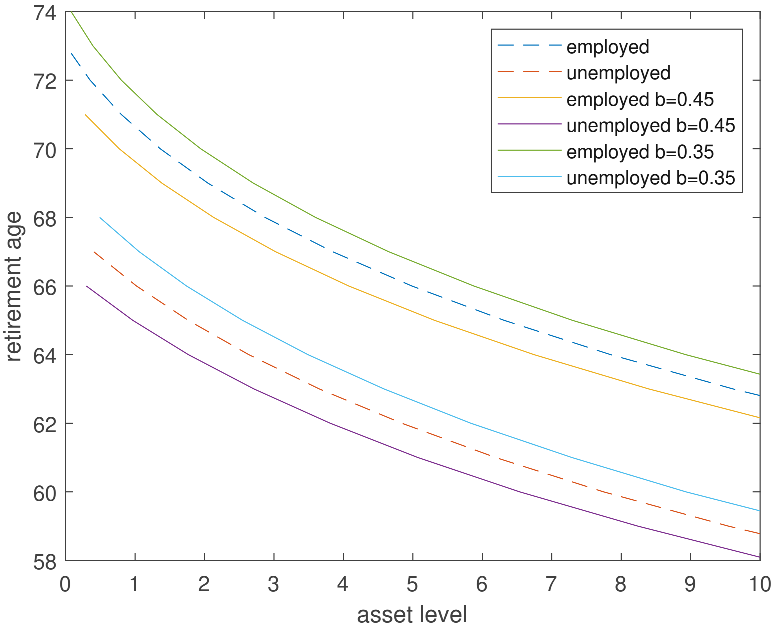

Figure 4.7: Comparison between the different parameter settings

Figure 4.7 shows that the first effect dominates and that with higher home production retirement age drops. The qualitative pattern of the curves does not change in any setting, which indicates that home production effects all asset levels equally. The second effect is confirmed by the average wage. The average wage rise to 0.9268 for the higher b and drops at 0.9088 for the lower b value. This leads to faster accumulation of assets, which strengthens the reduction in retirement age at the aggregate level.

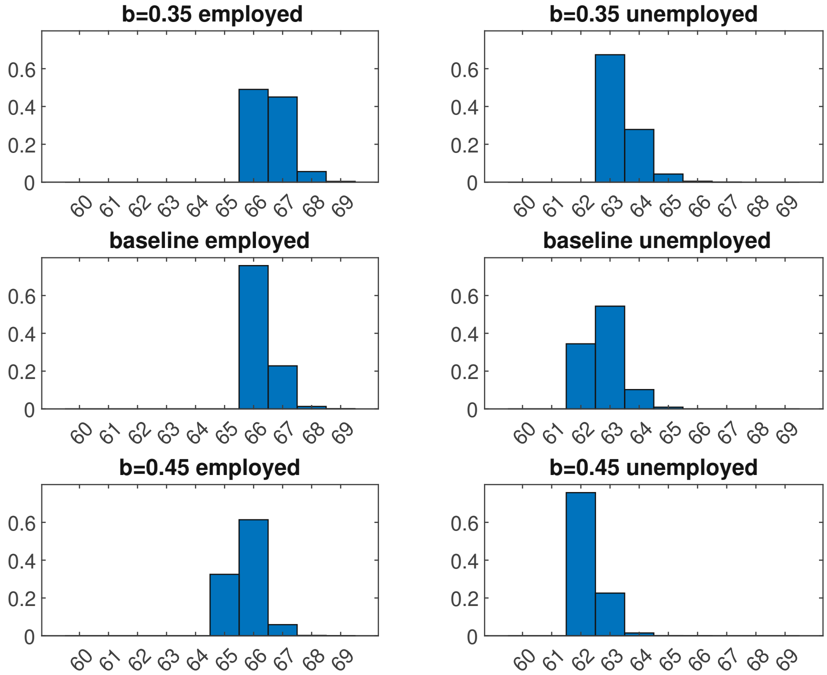

Figure 4.8: Change in distribution for b

Descriptive Statistics-Retirement Age		
Labor market status	Mean	Standard Deviation
Unemployed workers with $b = 0.35$	63.3798	0.5966
Unemployed workers baseline model	62.7765	0.6593
Unemployed workers with $b = 0.45$	62.2617	0.4871
Employed workers with $b = 0.35$	66.5732	0.6164
Employed workers baseline model	66.2561	0.4696
Employed workers with $b = 0.45$	65.7381	0.5691

The descriptive statistic and Figure 4.8 shows, that the changes in average retirement age for both groups are indeed nearly equal at 0.5 years. The results do not suggest a systematical relation between b and the dispersion of retirement ages as measured by their standard deviation. This is likely due to the assumption that retirement is only possible at the end of a year.

4.8.2 Maximum disutility

The second parameter I change paribus ceteris is the maximum of disutility. This has an direct equilibrium effect. I set z_{max} to 8 and then to 12.

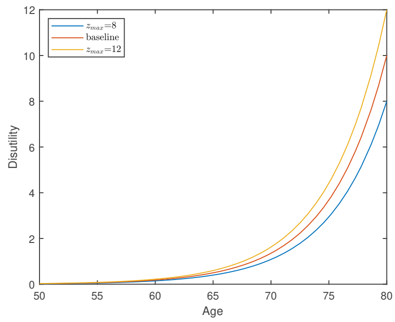

Figure 4.9: 3 different disutility function over age

With this change the function $z(\tau)$ differs clearly in the period of retirement decisions, as one can see in Figure 4.9.

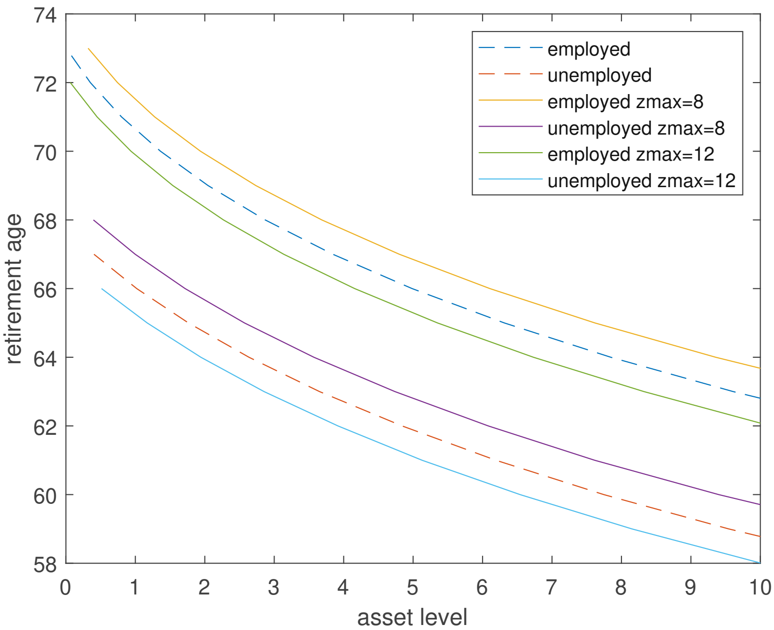

Figure 4.10: Comparison between the different parameter settings

Figure 4.10 indicates that a higher (lower) disutility forces a worker earlier (later) into retirement. Disutility has an equal impact on employed and unemployed workers. Therefore a change in z_{max} influences both groups in the same way.

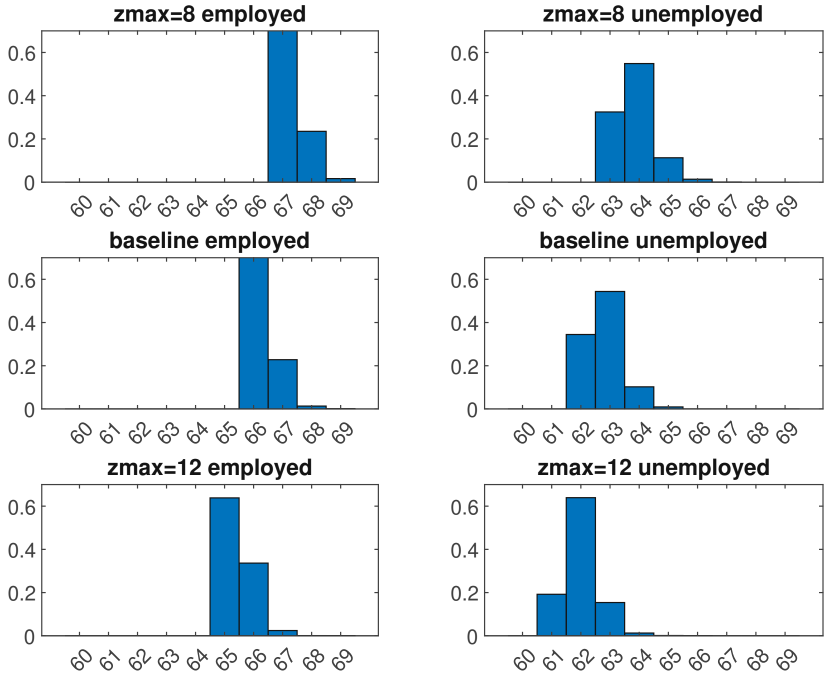

Figure 4.11: Change in distribution for b

Descriptive Statistics-Retirement Age		
Labor market status	Mean	Standard Deviation
Unemployed workers with $z_{max} = 8$	63.8167	0.6806
Unemployed workers baseline model	62.7765	0.6593
Unemployed workers with $z_{max} = 12$	61.9936	0.6471
Employed workers with $z_{max} = 8$	67.2679	0.4809
Employed workers baseline model	66.2561	0.4696
Employed workers with $z_{max} = 12$	65.3879	0.5403

The descriptive statistic supports Figure 4.11. The changes of retirement age are consistently equal with 1 year. The length of the period for retirement transition roughly stays the same in all scenarios.

4.8.3 Slope of disutility

A change in λ, which influences the slope of the exponential function, also changes the level of $z(\tau)$ at every age. To investigate the differential effect of the slope, I manipulate the disutility function in a way that $z(46)$, representative for age 66, is the same like in the baseline model for all cases. I choose 66, because at this the time, most of the employed workers retires in the baseline model. To do so I adjust z_{max} in a way that this can be achieved. This means in the case of $\lambda = 0.15$ $z_{max} \approx 5.225$ and for $\lambda = 0.25$ $z_{max} \approx 19.15$

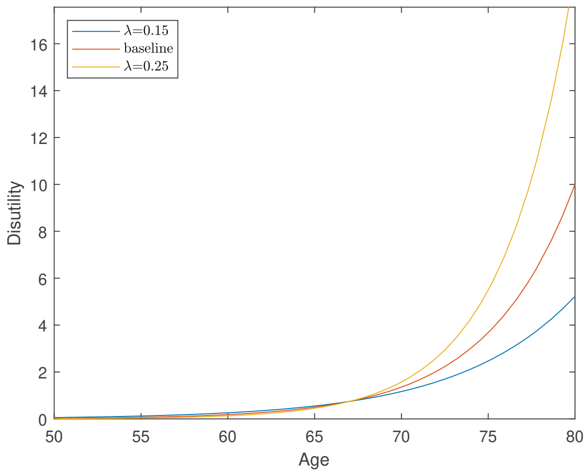

Figure 4.12: Disutility of work by age

Figure 4.12 visualizes the disutility function in the three different settings.By construction, they all give the same value at age 66, yet have a different slope. This changes retirement incentives of individuals as displayed in Figure 4.14.

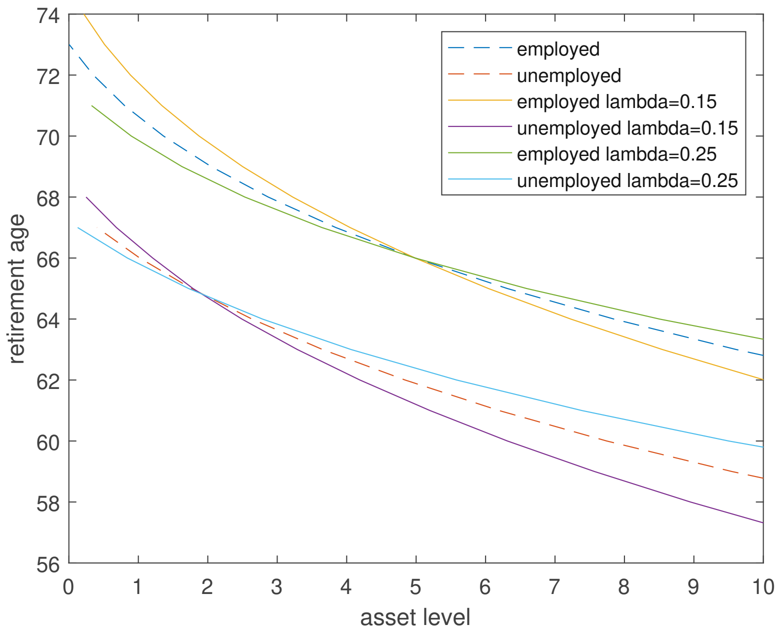

Figure 4.13: Comparison between the different parameter settings

For the first time the difference between the curves for different $z(\tau)$ does not stay the same over asset levels. By construction, the three curves for employed workers are intersect at age 66, whereas for unemployed workers the intersection is at the age of 65. The slope of the retirement schedule with $\lambda = 0.15$ is steeper than for $\lambda = 0.25$. Also for the first time, one can see different effect on the two employment groups. The impact of the change in $z(\tau)$ on lower asset levels for employed worker is higher than for unemployed, but the effect reverses when it comes to higher asset levels.

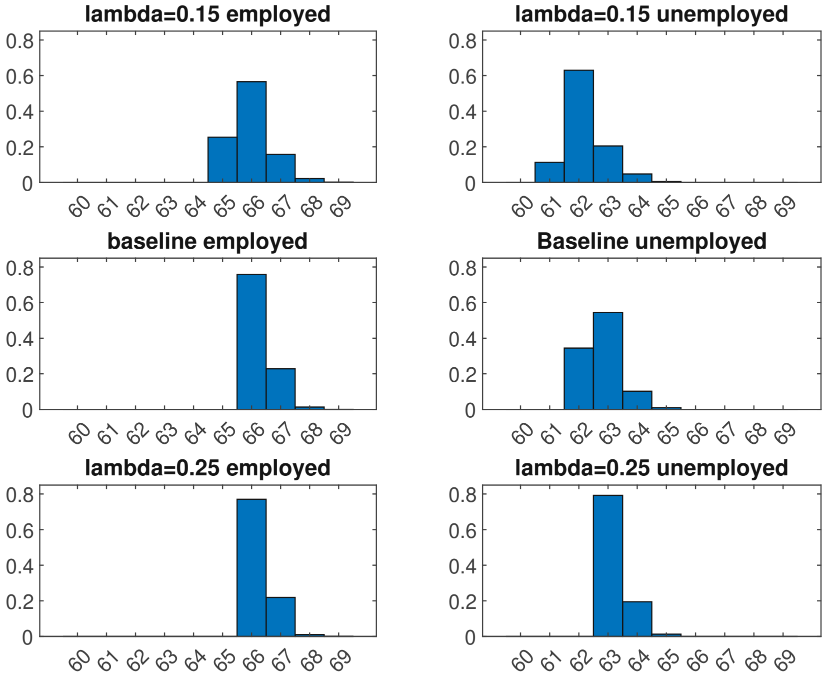

Figure 4.14: Change in distribution

In Figure 4.14 I see for the case $\lambda = 0.15$ a shift to the left and a widening of the distribution. This is explained by the fact that a reduced growth rate of disutility is not pushing workers out of the labor market that strongly. For $\lambda = 0.25$ the opposite effect should occur. For unemployed workers we see a shift to the right and a more compact distribution. For the employed workers the higher disutility seems to have nearly no impact. This can be explained by the fact that older employees have already a high assets level. Therefore Figure (4.14) indicates that the retirement age for employees with a high asset level and higher disutility does not vary in respect to the baseline model.

Descriptive Statistics-Retirement Age		
Labor market status	Mean	Standard Deviation
Unemployed workers with $\lambda = 0.15$	62.2041	0.7204
Unemployed workers baseline model	62.7765	0.6593
Unemployed workers with $\lambda = 0.25$	63.2204	0.4445
Employed workers with $\lambda = 0.15$	65.9516	0.7161
Employed workers baseline model	66.2561	0.4696
Employed workers with $\lambda = 0.25$	66.2405	0.4527

For unemployed workers, the descriptive statistics confirms the results of Figure 4.14. A higher λ raises the retirement age and shorten the period of retirement decisions.
For employed workers, the retirement age does not change significantly, due to the construction of the disutility function. But again we see a shorter period of retirement transitions.

4.8.4 Job-finding probability

The last parameter I analyse is the job finding rate p and I change it to 0.8 and 0.6. p influences the value of unemployment (4.2) directly. A higher p leads to an increase in the likelihood of moving to employment and a decrease in the likelihood of staying unemployment. The value of employment is higher than the value for unemployment and therefore $\bar{W} \geq \bar{U}$. As a result, a higher p increases the value of unemployment. A higher p does not influence the value of employment directly, but it strengthens the bargaining position of workers. This leads to higher wages and lower retirement age.
A higher $U(a, \tau)$ and therefore a higher $\bar{U}(a, \tau)$ also generates a higher $W(a, \tau)$ via(4.3). But since $\bar{U}(a, \tau)$ is multiplied by the separation rate, this effect is comparatively small.

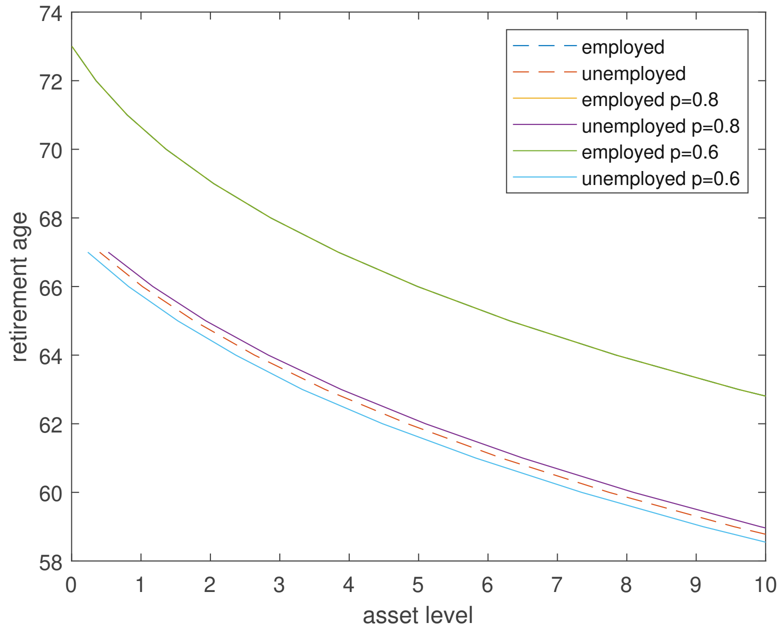

Figure 4.15: Comparison between the different parameter settings

Figure 4.15 shows that a a change in p only influence the value for unemployed. A raise (decline) of p leads to higher (lower) values for unemployed workers. Consequently retirement age for unemployed workers rises (drops). Again the gap between the different curves for unemployed workers stays nearly the same.

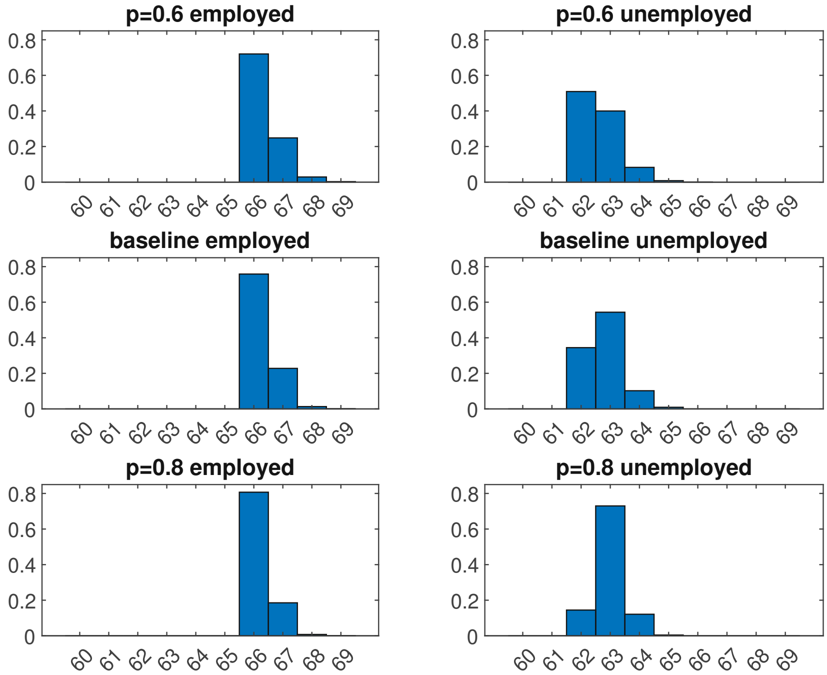

Figure 4.16: Change in distribution

Descriptive Statistics-Retirement Age		
Labor market status	Mean	Standard Deviation
Unemployed workers with $p = 0.6$	62.5917	0.6807
Unemployed workers basic model	62.7765	0.6593
Unemployed workers with $p = 0.8$	62.9850	0.5320
Employed workers with $p = 0.6$	66.3140	0.5382
Employed workers basic model	66.2561	0.4696
Employed workers with $p = 0.8$	66.2008	0.4200

As seen in Figure 4.16, a higher p raises the average retirement age of unemployed workers.The average retirement age for employed workers slightly decreases since higher wages lead to faster asset accumulation. p has also an influence on the period of retirement transitions. A higher p shortens this period as high asset levels are reached earlier.

CHAPTER 5

Conclusion

The empirical evidence has shown that the labor market status and assets have a crucial impact on retirement age. My model studies both effects as well as their interaction. Building on Lalé (2019), I have extended the work of Hairault et al. (2015) for the influence of assets on retirement behaviour. For the baseline parameterization, I find that the retirement gap stays almost constant at any given asset level. On aggregate, however, employed workers accumulate assets faster, allowing them to retire even a little earlier than in a model without asset accumulation. These results are relatively robust as evidenced by the sensitivity analysis. This robustness shows me the consistent and constant influence of assets on the retirement age. It also shows that the solutions are valid for a great number of economies with different parameter settings.

The simplicity of the model enables further investigations and useful extensions. I believe the model is very helpful for modelling working incentives for older or long-term unemployed workers in relation to their asset level. Since work becomes more painful with age and individuals who have experienced more phases of unemployment during their career choose to participate longer, they experience a double cost. This may be ameliorated by a suitably designed pension scheme.

Bibliography

Brown, J. R., Coile, C. C., and Weisbenner, S. J. (2010). The Effect of Inheritance Receipt on Retirement. *The Review of Economics and Statistics*, 92(2):425–434.

Coile, C. C. (2015). Economic determinants of workers' retirement decisions. *Journal of Economic Surveys*, 29(4):830–853.

Coile, C. C. and Levine, P. B. (2006). Bulls, bears, and retirement behavior. *ILR Review*, 59(3):408–429.

Coile, C. C. and Levine, P. B. (2011). The market crash and mass layoffs: How the current economic crisis may affect retirement. *The B.E. Journal of Economic Analysis Policy*, 11(1).

French, E. (2005). The Effects of Health, Wealth, and Wages on Labour Supply and Retirement Behaviour. *The Review of Economic Studies*, 72(2):395–427.

Frimmel, W., Horvath, T., Schnalzenberger, M., and Winter-Ebmer, R. (2018). Seniority wages and the role of firms in retirement. *Journal of Public Economics*, 164:19–32.

Gartner, H., Merkl, C., and Rothe, T. (2012). Sclerosis and large volatilities: Two sides of the same coin. *Economics Letters*, 117(1):106–109.

Gustman, A. L. and Steinmeier, T. L. (2005). The social security early entitlement age in a structural model of retirement and wealth. *Journal of Public Economics*, 89(2):441–463.

Hairault, J.-O., Langot, F., and Zylberberg, A. (2015). Equilibrium unemployment and retirement. *European Economic Review*, 79:37–58.

Lalé, E. (2019). Labor-market frictions, incomplete insurance and severance payments. *Review of Economic Dynamics*, 31:411–435.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.