TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

Institute of Fluid Mechanics & Heat Transfer

Flow of power-law polymer melts through thin gaps: rationale modelling of momentum and heat transfer

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Outlook

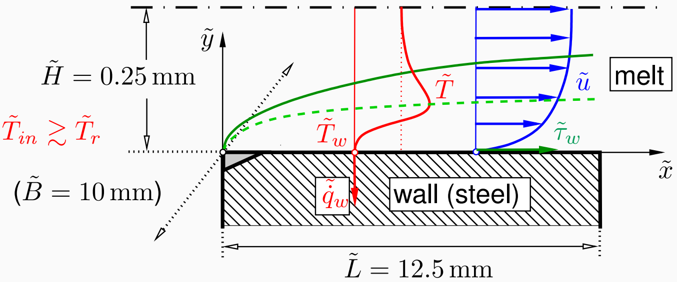
- inspection analysis
  - least-degenerate theoretical model
- validation again experiments ?
- main achievements
  - channel/boundary layer model
  - recovery/adiabatic wall temperature
  - explains predominantly thermal degradation
- further activities within FFG BRIDGE project *KUFO-Verschleiß*

Problem formulation and input data

- incompressible additivated single-phase Ultramid® melt (A3WG3/5/7/10)
- static constitutive law: pseudoplastic/structurally viscous
  Ostwald-de-Waele (Herschel–Bulkley) fluid, saturation/solidification ignored

$$\tilde{\tau} = \tilde{\mu}\,\partial\tilde{u}/\partial\tilde{y}, \quad \tilde{\mu} = \tilde{\mu}_0(\tilde{p},\tilde{T})\big(\tilde{\dot{\gamma}}/\tilde{\dot{\gamma}}_r\big)^{n-1}, \quad 1 > n > 0, \quad \tilde{\dot{\gamma}} \sim |\partial\tilde{u}/\partial\tilde{y}|$$

- planar channel flow, internal boundary layer (slip layer)



- $\tilde{H}/\tilde{L} = 0.02 \ll 1$
- controlled flow rate $\tilde{Q} = 20 \ldots 400\,\mathrm{cm}^3/\mathrm{s}$
- $\tilde{T}_r = 290\,^\circ\mathrm{C}$
- sought $\tilde{T}_w$, $\tilde{\dot{q}}_w$, $\tilde{\tau}_w$

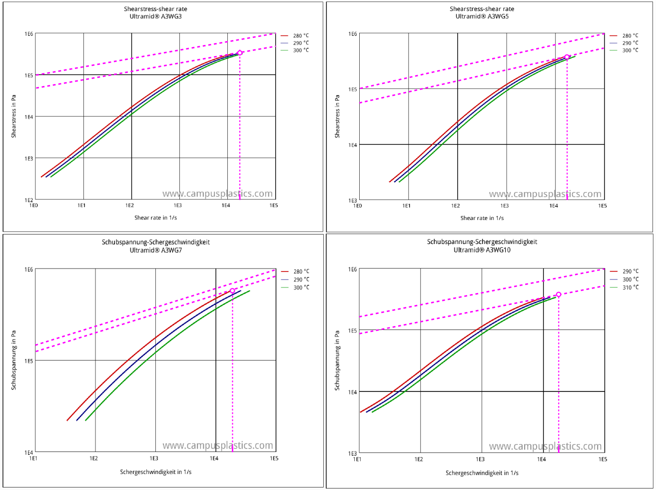
serious challenge

- rare thermo-physical data: $\tilde{\dot{\gamma}}_r,\ n,\ \big[\tilde{\mu}_r,\ \tilde{c}_{(p)},\ \tilde{\lambda}\big](\tilde{p},\tilde{T})$

Rheological model by extrapolation: BASF/ALBIS data sheets

$\tilde{\tau}$ vs. $\dot{\tilde{\gamma}}$, $\tilde{T}$

$n \doteq 0.20$



$$\tilde{U} = \tilde{Q}/(2\tilde{H}\tilde{B}) \doteq \quad 4 \ldots 80\,\mathrm{m/s}$$

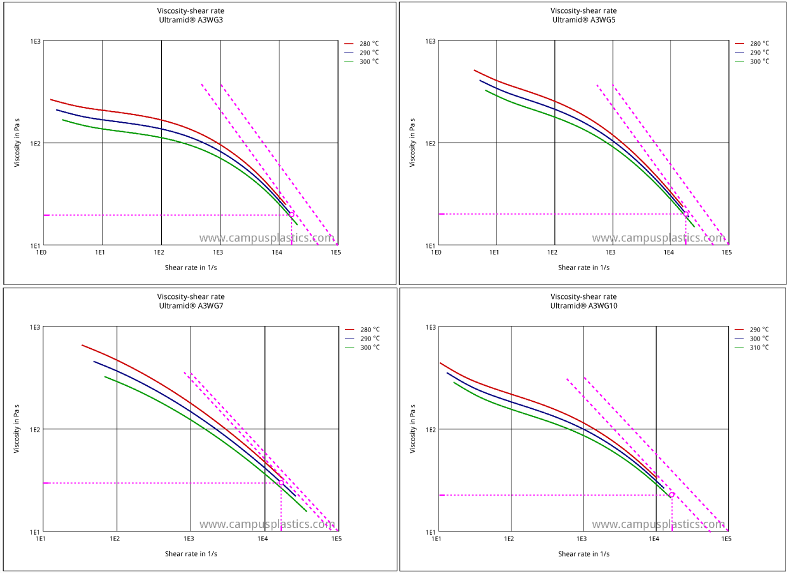
$$\dot{\tilde{\gamma}}_r = \tilde{U}/\tilde{H} \doteq \quad 1.6 \times 10^4 \ldots 3.2 \times 10^5\,\mathrm{s}^{-1}$$

Rheological model by extrapolation: BASF/ALBIS data sheets

$\tilde{\mu}$ vs. $\tilde{\dot{\gamma}}, \tilde{T}$

$$\tilde{\dot{\gamma}}_r = 1.6 \times 10^4\,\mathrm{s}^{-1}$$

$$n - 1 \doteq -0.80$$



log–log regression

$$\tilde{\tau} = \tilde{\mu}\frac{\partial\tilde{u}}{\partial\tilde{y}}, \quad \tilde{\mu} = \tilde{\mu}_r\left(\frac{|\partial\tilde{u}/\partial\tilde{y}|}{\tilde{\dot{\gamma}}_r}\right)^{n-1}\exp\left(2.30\,\epsilon\,\frac{\tilde{T}_r - \tilde{T}}{\tilde{T}_r}\right)$$

$$\tilde{\mu}_r \doteq 21.5\,\mathrm{Pa\,s}, \quad \tilde{\dot{\gamma}}_r \doteq 1.6\times 10^4\,\mathrm{s}^{-1}, \quad n \doteq 0.20, \quad \tilde{T}_r \doteq 563\,\mathrm{K}, \quad \epsilon \doteq 0.2\ldots 1.0$$

$$\tilde{\rho} \doteq 1070\,\mathrm{kg/m^3}, \quad \tilde{c} \doteq 2570\,\mathrm{J/(kg\,K)}, \quad \tilde{\lambda} \doteq 0.205\,\mathrm{W/(m\,K)} \quad \Rightarrow \quad Pr = \tilde{\mu}_r\tilde{c}/\tilde{\lambda} \doteq 2.695\times 10^5$$

$\tilde{Q}\ \ (\tilde{U})\ =$	20 (4)	400 (80)	$\mathrm{cm^3/s}\ \ (\mathrm{m/s})$	$\propto \tilde{U}$	
$\tilde{\tau}_r = \tilde{\mu}_r(\tilde{U}/\tilde{H})^n\,\tilde{\dot{\gamma}}_r^{1-n} \doteq$	3.44 ✓	6.26 ✓	bar	$\propto \tilde{U}^n$	$\propto \tilde{\dot{\gamma}}_r^{1-n}\tilde{\mu}_r$
$\tilde{p}_r = \tilde{\tau}_r\tilde{L}/\tilde{H} \doteq$	172	313	bar	$\propto \tilde{U}^n$	$\propto \tilde{\dot{\gamma}}_r^{1-n}\tilde{\mu}_r$
$Re = \tilde{\rho}\,\tilde{U}^2/\tilde{p}_r \doteq$	10^{-3}	0.22 !	$\ll 1$	$\propto \tilde{U}^{2-n}$	$\propto \tilde{\dot{\gamma}}_r^{n-1}\tilde{\mu}_r^{-1}$
$Pe = Re\,Pr \doteq$	270	5.93×10^4	$\gg 1$	$\propto \tilde{U}^{2-n}$	$\propto \tilde{\dot{\gamma}}_r^{n-1}$
$Br\ (Na) = \tilde{\tau}_r\tilde{U}\tilde{H}/(\tilde{\lambda}\tilde{T}_r) \doteq$	3	109	$O(1)$	$\propto \tilde{U}^{1+n}$	$\propto \tilde{\dot{\gamma}}_r^{1-n}\tilde{\mu}_r$

Constitutive law: more advantageous representation

classical Arrhenius–Andrade (or the like) relation

$$\frac{\tilde{\tau}}{\tilde{\tau}_r} = \left(\frac{|\partial \tilde{u} / \partial \tilde{y}|}{\dot{\tilde{\gamma}}_r} \right)^n \exp\left[b \left(\frac{\tilde{T}_r}{\tilde{T}} - 1 \right) \right]$$

$$\tilde{\tau}_r, \dot{\tilde{\gamma}}_r = const, \quad n \doteq 0.20, \quad \tilde{T}_r \doteq 563\,\mathrm{K}, \quad b = ?$$

long-chain aliphatic compounds

$$b\,\tilde{T}_r \gtrsim 2500\,\mathrm{K} \quad \Rightarrow \quad b \gtrsim 4.44$$

Governing eqs.: non-dimensional slender-layer approximation

$$x = \frac{\tilde{x}}{\tilde{L}}, \quad y = \frac{\tilde{y}}{\tilde{H}}, \quad t = \frac{\tilde{t}\,\tilde{U}}{\tilde{L}}, \quad u = \frac{\tilde{u}}{\tilde{U}}, \quad v = \frac{\tilde{v}\tilde{H}}{\tilde{L}\tilde{U}}, \quad \tau = \frac{\tilde{\tau}}{\tilde{\tau}_r}, \quad p = \frac{\Delta\tilde{p}}{\tilde{p}_r}, \quad T = \frac{\tilde{T}}{\tilde{T}_r}$$

continuity

$$\partial_x u + \partial_y v = 0, \quad \int_0^1 u \, \mathrm{d}y = 1$$

$$u = \partial_y \psi, \quad v = -\partial_x \psi$$

momentum (local equilibrium)

$$\underbrace{Re}_{\ll 1} (\partial_t + u\partial_x + v\partial_y)u \sim -p'(x) + \partial_y \tau, \quad \tau = (\partial_y u)^n \exp[\beta(1 - T)], \quad \beta = 2.30\,\epsilon$$

thermal energy

$$\underbrace{Pe\,(\partial_t + u\partial_x + v\partial_y)T}_{\text{convection}} \sim \underbrace{Br\,\tau\,\partial_y u}_{\text{dissipation}} + \underbrace{\partial_{yy} T}_{\text{heat conduction}}$$

equilibrium with thermal correction

$$p'(x) \sim \tau_y, \quad \tau \sim (\partial_y u)^n [1 + 2.30\,\epsilon(T-1)] \quad (n \doteq 0.20, \quad \epsilon \ll 1)$$

$$y = 0: \quad u = 0, \quad y = 1: \quad \partial_y u = \tau = 0, \quad \int_0^1 u \, \mathrm{d}y = 1$$

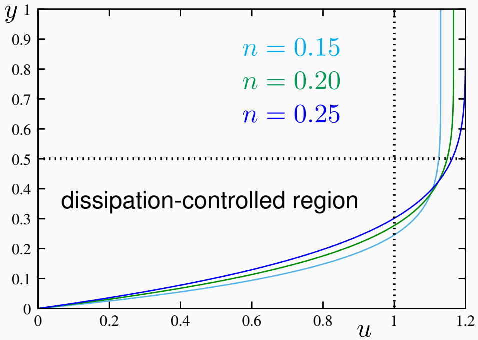
leading order

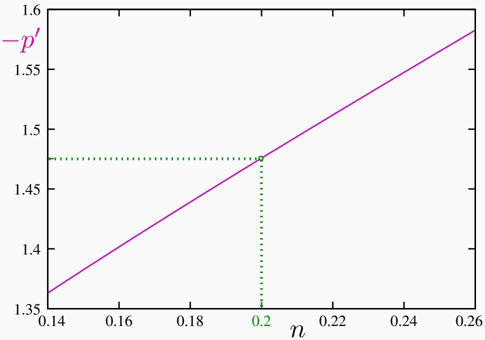
$$\tau \sim -p'(x)(1-y)$$

$$u(y) \sim \frac{1+2n}{1+n}\left[1-(1-y)^{1+1/n}\right], \quad \psi(y) \sim \frac{(1+2n)y - n[1-(1-y)^{2+1/n}]}{1+n}, \quad v \sim 0$$

$$0 > \tau_w \sim -p'(x) \sim (2+1/n)^n$$

cf. Weissenberg–Rabinowitsch–Mooney and Bagley-corrections





$$n = 0.2 \quad \Rightarrow \quad -p' = 7^{0.2} \doteq 1.476$$

$$\Delta\tilde{p} \doteq 254 \ldots 462\,\mathrm{bar}\,?$$

Stationary heat transfer on adiabatic wall (contact problem)

time scales

sensing $> 0.8\,\mathrm{s}$

$\tilde{Q}\ \ (\tilde{U})$	20 (4)	400 (80)	$\mathrm{cm}^3/\mathrm{s}\ \ (\mathrm{m/s})$
$\tilde{L}/\tilde{U}$	3.125	0.156	ms

$$\Rightarrow \quad \partial_t T \sim 0$$

heat transfer melt–*st*eel

$$\tilde{y}=0\colon \quad \tilde{\dot{q}}_w = \tilde{\lambda}\frac{\partial \tilde{T}}{\partial \tilde{y}}\bigg|_{\tilde{y}=0+} = \tilde{\lambda}_{st}\frac{\partial \tilde{T}}{\partial \tilde{y}}\bigg|_{\tilde{y}=0-} \quad \Rightarrow \quad \tilde{\lambda}\,\frac{\tilde{T}_r}{\tilde{H}}\frac{\partial T}{\partial y}\bigg|_{y=0+} \sim \tilde{\lambda}_{st}\,\frac{\tilde{T}_w-\tilde{T}_r}{\tilde{L}_{st}}$$

$$\frac{\partial T}{\partial y}\bigg|_{y=0+} \sim \underbrace{\frac{\tilde{\lambda}_{st}}{\tilde{\lambda}}}_{\doteq\, 83}\ \underbrace{\frac{\tilde{H}}{\tilde{L}}}_{0.02}\ \underbrace{\frac{\tilde{L}}{\tilde{L}_{st}}}_{\ll 1}\,(T_w-1), \qquad (\tilde{\lambda},\, \tilde{\lambda}_{St}) \doteq (0.205,\ 17)\,\mathrm{W/(m\,K)}$$

$$\Rightarrow \quad \partial_y T(x,0) \sim 0$$

Fully coupled steady-state problem: $[\psi, T](x,y)$, $p'(x)$

parameters

$$n, \quad b, \quad Pe, \quad Br(Pe, n) > 0$$

dissipation-controlled marching problem, Arrhenius factor

$$-p'(x)(1-y) = \tau = (\psi_{yy})^n \exp(b/T - b)$$

$$Pe\,(\psi_y \partial_x - \psi_x \partial_y)T = Br\,\tau\psi_{yy} + T_{yy}$$

$$x = 0\colon \quad T = 1, \qquad y = 0\colon \quad \psi = \psi_y = T_y = 0, \qquad y = 1\colon \quad \psi = 1, \quad \psi_{yy} = T_y = 0$$

$0 \le x, y \le 1\colon$ recovery temperature $\tilde{T}_w = \tilde{T}_r\, T(x,0)$, $\Delta\tilde{p} = \tilde{p}_r \int_1^x p'(s)\,\mathrm{d}s$

parabolic problem invariant against change of $\tilde{L}$

$$\xi = x/Pe, \qquad \pi(\xi) = p(x)/Pe, \qquad \pi'(\xi) = p'(x)$$

Canonical marching problem

$$-\pi'(\xi)(1-y) = \tau = (\psi_{yy})^n \exp(b/T - b)$$

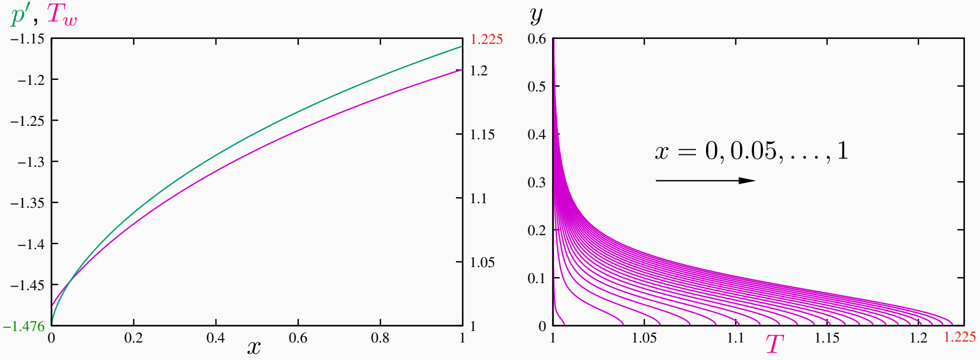
$$(\psi_y\partial_\xi - \psi_\xi\partial_y)T = Br\,\tau\psi_{yy} + T_{yy}$$

$$\xi = 0\colon\quad T = 1, \qquad y = 0\colon\quad \psi = \psi_y = T_y = 0, \qquad y = 1\colon\quad \psi = 1,\ \ T_y = 0$$

- $\xi \ll 1\colon$ insulation boundary layer (BL) for $Br > 0 \quad \Rightarrow \quad T \to 1+$
- $b \ll 1\colon$ weak coupling, $\psi_\xi \sim 0$, problem linear
- $b = O(1)\colon$ full coupling, iterative numerical scheme

1. $i = 0, \quad T_i = 1$
2. $(\psi^*_{yy})^n = (1-y)\exp(b - b/T_i), \quad y = 0\colon\quad \psi^* = \psi^*_y = 0$
3. equilibrium: $-\pi'(\xi) = [\psi^*(\xi,1)]^{-n}, \quad \psi = \psi^*/\psi^*(\xi,1)$
4. marching in X, Chebychev collocation in $y \quad \Rightarrow \quad T$
5. $T_{i+1} = \omega_i T_i + (1-\omega_i)T$ (SUR, $0 \le \omega_i < 1$)
6. $i + 1 \mapsto i$, GO TO 2.

Canonical marching problem: $n = 0.2\,,\ b = 1.92\,,\ Pe = 270\,,\ Br = 3$



$$\tilde{T}_w \lesssim 1.22\,\tilde{T}_{in} \doteq 413\,^\circ\mathrm{C}$$

but $T_w \downarrow$ as

$Pe \uparrow$

$(n \downarrow,\ b \uparrow)$

Canonical insulation BL: $Pe \gg 1$

outer developed flow

$$T \sim 1, \quad y \ll 1: \;\; \psi \sim \sigma y^2/2, \;\; \sigma = \tau_w^{1/n} = 2 + 1/n \doteq 7, \quad \tau \to \tau_w$$

BL scaling b-independent, excess temperature

$$X = \frac{Br^{3/2}}{Pe}\,\sigma^{3n/2+1/2}\,x, \quad Y = Br^{1/2}\,\sigma^{n/2+1/2}\,y, \quad \psi \sim \frac{\Psi(X,Y)}{Br\,\tau_w}, \quad T \sim 1 + \theta(X,Y)$$

BL problem governs $T_w = 1 + \theta(X,0) > 1$

cf. Schlichting & Gersten, *Boundary-Layer Theory*, 9th ed. (2017), § 9.6

$$\Psi_{YY} = \exp[B\,\theta/(1+\theta)], \quad B = b/n \gtrsim 22.2$$

$$(\Psi_Y \partial_X - \Psi_X \partial_Y)\theta = \Psi_{YY} + \theta_{YY}$$

$$X = 0: \;\; \theta = 0, \quad Y = 0: \;\; \Psi = \Psi_Y = \theta_Y = 0, \quad Y \to \infty: \;\; \theta \to 0$$

Strength of dissipation: distinguished limit

$$D = \frac{Br}{Pe^{2/3}} \tau_w^{1+1/(3n)} \propto \tilde{U}^{(5n-1)/3} \begin{cases} \text{decreases} & \ldots \quad n < 0.2 \\ \text{increases} & \ldots \quad n > 0.2 \end{cases}$$

$$n \doteq 0.20 \quad \Rightarrow \quad D \doteq 0.203$$

$$X = D^{3/2} x \quad \Rightarrow \quad \text{BL for } \; 0 < x < 1 \quad \checkmark$$

BL problem: weak coupling, $B\theta \ll 1$

excess temperature

$$\theta \sim X^{2/3}\,\vartheta(X,\eta), \quad \eta = Y/X^{1/3}, \quad \Psi = Y^2/2$$

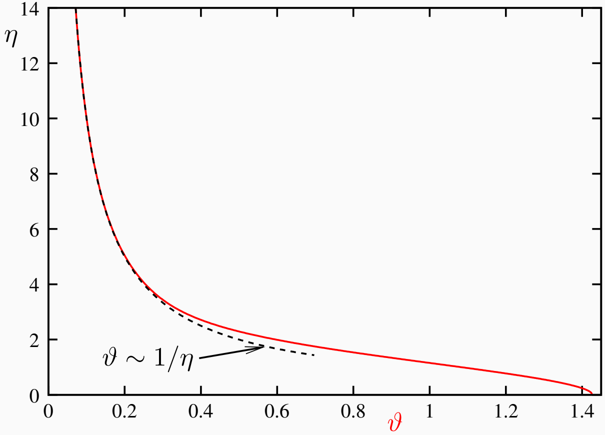
$$[\eta X \vartheta_X +]\; 2\eta\,\vartheta/3 - \eta^2\vartheta_\eta/3 = 1 + \vartheta_{\eta\eta}, \quad \vartheta_\eta(x,0) = \vartheta(x,\infty) = 0$$

closed-form similarity solution $\Rightarrow$ $T_w \sim 1 + \vartheta(0) X^{2/3}$

$$\vartheta(\eta) = \frac{3^{1/6}}{8\,\Gamma(5/6)\sqrt{\pi}}\Big\{2^{7/3}\,3^{1/6}\pi\, M\Big(-\frac{2}{3},\frac{2}{3},-\frac{\eta^3}{9}\Big)\Big[1+\frac{\sqrt{\pi}}{3}\,\eta^{3/2}\,\mathrm{e}^{\eta^3/18} I_{1/6}\Big(\frac{\eta^3}{18}\Big)\Big]$$
$$-\,9\,\Gamma(5/6)\,\Gamma(5/3)^2\,\eta^{5/2}\,\mathrm{e}^{\eta^3/18} I_{-1/6}\Big(\frac{\eta^3}{18}\Big) M\Big(-\frac{1}{3},\frac{4}{3},-\frac{\eta^3}{9}\Big)\Big\}$$

$$\vartheta(0) = 3^{1/3}\sqrt{\pi}/\big[2^{2/3}\Gamma(5/6)\big] \doteq 1.42665$$

Universal leading-order maximum excess temperature, no overshooting

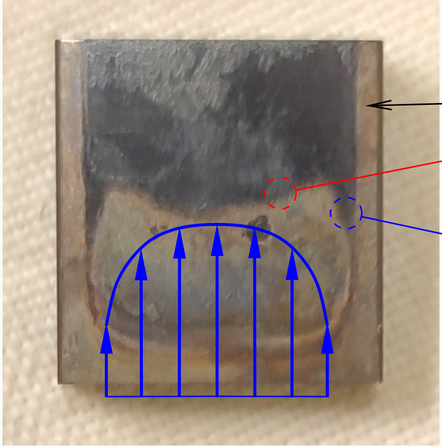


$$D \doteq 0.203 \quad \Rightarrow \quad (T_w - 1)/x^{2/3} \sim D\,\vartheta(0) \doteq 0.289 \quad \Rightarrow \quad \tilde{T}_w < 1.29\,\tilde{T}_{in} \doteq 453°\mathrm{C}$$

Maybe of interest: long channel, $Pe \ll 1$

temperature uniform across channel, reduced pressure drop

$$T \sim c(n, b, Br)\, x, \quad -p' \sim -p'(0)\exp(-b)$$



repeated tempering of platelet: coarsening, secondary precipitation

$\approx 450–525\,^\circ\mathrm{C}$

lateral BL

PM X190CrVMo20-4-1, Ultramid® A3W (PA66, no fibres), 210 blasts (300 cm^3/s)

no gravimetric wear!

Discussion & activities

discussion of results

- $\tilde{T}_w \lesssim 1.22\,\tilde{T}_{in} \doteq 413\,^\circ\mathrm{C} \quad \Rightarrow$ degradation, 2nd-order phase transition ?
- $\tilde{T}_w \downarrow$ by (i) heat transfer (iteratively), (ii) $\tilde{Q} \uparrow$!
- measurements $\quad \Rightarrow \quad$ slightly higher $\tilde{T}_w$ (glass fibres uninfluential) $\checkmark$

ongoing

- full marching problem including
  - relaxation & saturation (slip layer, limiting shear stress, $n \ll 1$)
  - streamwise & lateral BLs ($\tilde{B}/\tilde{L} \sim 1$)
  - publication: *J. Non-Newton. Fluids*

upcoming

- non-equilibrium flow $\quad \Rightarrow \quad$ micro-mechanical abrasive wear

The best is yet to come – thank you for your attention!

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