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Viscous layer dynamics of the Euler-Prandtl stage in marginal separation

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Transitional separation bubble (LSB)

Experimental observations

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Smoke flow visualisation of a LSB on an Eppler 387 airfoil at $\alpha = 2°$ and $\mathrm{Re} \approx 10^5$ [G. Cole, T. Mueller, 1990]

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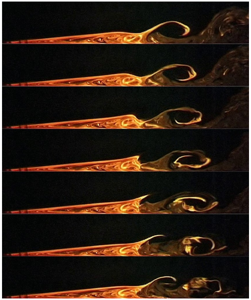

LSB in a laminar water tunnel: Λ-vortex generation cycle at $\mathrm{Re} \approx 10^5$
(provided by U. Rist, M. Lang)

LSB, high-Re asymptotic structure

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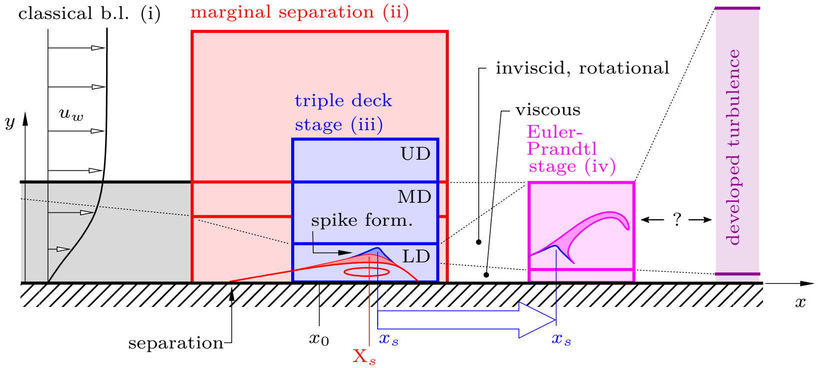

Early stages of the laminar-turbulent transition process
[S. Braun, S. Scheichl, D. Kuzdas, 2021]

Finite-time blow-up of triple deck stage

Self-similar blow-up structure as $x \to x_s, t \to t_s$ [J. Elliott, F. Smith, 1987]:

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LD: Sublayer leading order

$$\frac{2}{5}\frac{\partial \hat{\Psi}}{\partial \hat{\eta}} + \frac{3}{5}\hat{x}\frac{\partial^2 \hat{\Psi}}{\partial \hat{\eta}\partial \hat{x}} + \frac{1}{2}\hat{\eta}\frac{\partial^2 \hat{\Psi}}{\partial \hat{\eta}^2} + \frac{\partial \hat{\Psi}}{\partial \hat{\eta}}\frac{\partial^2 \hat{\Psi}}{\partial \hat{\eta}\partial \hat{x}} - \frac{\partial \hat{\Psi}}{\partial \hat{x}}\frac{\partial^2 \hat{\Psi}}{\partial \hat{\eta}^2} = -\frac{\partial \hat{\mathcal{P}}}{\partial \hat{x}} + \frac{\partial^3 \hat{\Psi}}{\partial \hat{\eta}^3}$$

$$x - x_s = (t_s - t)^{\frac{3}{5}}\hat{x}, \; y = (t_s - t)^{\frac{1}{2}}\hat{\eta}$$

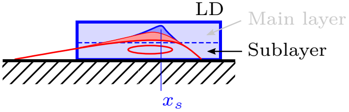
Boundary conditions:

$$\hat{\Psi} = \hat{\Psi}_{\hat{\eta}} = 0 \quad \text{for} \quad \hat{\eta} = 0,$$

$$\hat{\Psi} \sim \hat{\mathcal{B}}(\hat{x})\hat{\eta} + \ldots \quad \text{as} \quad \hat{\eta} \to \infty$$

Relation between pressure $\hat{\mathcal{P}}$ and slip velocity $\hat{\mathcal{B}}$ (Bernoulli-equ.):

$$-\hat{\mathcal{P}}' = 2\hat{\mathcal{B}}/5 + 3\hat{x}\hat{\mathcal{B}}'/5 + \hat{\mathcal{B}}\hat{\mathcal{B}}'$$



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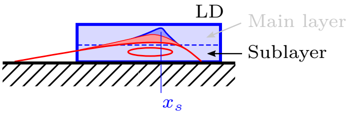

Numerical treatment

Spectral collocation method (Chebyshev polynomials)

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Ansatz to capture singular behaviour as $\hat{\eta} \to \infty$

$$\hat{\Psi}(\hat{x}, \hat{\eta}) = \hat{\mathcal{B}}(\xi)\frac{\eta_V}{d(\xi)} + f(\xi, \eta_V)$$

with the blow-up coordinates and scaling function
$\xi = \hat{x}$, $\eta_V = \hat{\eta} d(\xi)$, $d(\xi) = \left(1 + \xi^2\right)^{-5/12}$

Domain mapping

$x \in (-\infty; \infty)\,,\ y \in (0; \infty) \Rightarrow s, r \in [-1; 1]$ Gauss-Lobatto points

$x(s) = x^* + B\tan\left(\frac{\pi s}{2}\right)$ $s_j = -\cos\left(\frac{j\pi}{m}\right),\ j = (0, ..., m)$

$y(r) = C\tan\left(\frac{\pi(r+1)}{4}\right)$ $r_i = -\cos\left(\frac{i\pi}{n}\right),\ i = (0, ..., n)$

Differentiation $f_i^{(m)} = \Sigma_j D_{ij}^{(m)} f_j$
[J.P. Berrut and L.N. Trefethen, 2004] $D_{ij}^{(1,x)} = s_x D_{ij}^{(1,s)},\ D_{ij}^{(1,y)} = r_y D_{ij}^{(1,r)}$

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Finite-time blow-up

$\hat{\mathcal{A}}(0) = 0.6,\ \hat{\mathcal{A}}'(0) = 0.6,\ (m \times n) = (160 \times 80)$

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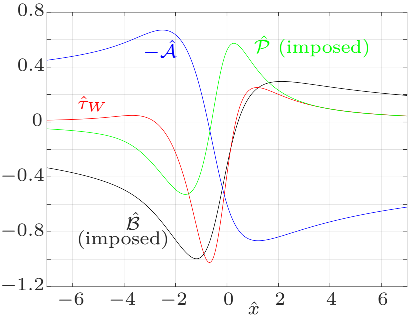
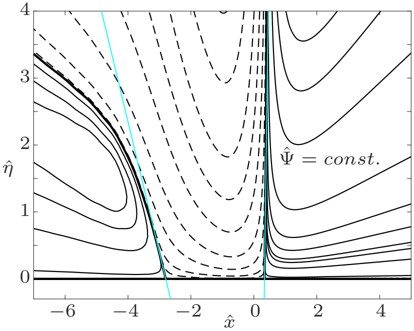
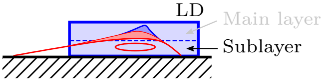
$$\frac{2}{5}\frac{\partial\hat{\Psi}}{\partial\hat{\eta}} + \frac{3}{5}\hat{x}\frac{\partial^2\hat{\Psi}}{\partial\hat{\eta}\partial\hat{x}} + \frac{1}{2}\hat{\eta}\frac{\partial^2\hat{\Psi}}{\partial\hat{\eta}^2} + \frac{\partial\hat{\Psi}}{\partial\hat{\eta}}\frac{\partial^2\hat{\Psi}}{\partial\hat{\eta}\partial\hat{x}} - \frac{\partial\hat{\Psi}}{\partial\hat{x}}\frac{\partial^2\hat{\Psi}}{\partial\hat{\eta}^2} = -\frac{\partial\hat{\mathcal{P}}}{\partial\hat{x}} + \frac{\partial^3\hat{\Psi}}{\partial\hat{\eta}^3}$$







Slope of separation/ reattachment streamline [K. Oswatitsch, 1958]:
$\tan\theta_0 = -3\tau_{wx}/p_x$

Euler-Prandtl stage

Prandtl layer

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Unsteady Boundary Layer equation

$$\frac{\partial^2 \Psi}{\partial \bar{\eta} \partial \bar{t}} + \frac{\partial \Psi}{\partial \bar{\eta}} \frac{\partial^2 \Psi}{\partial \bar{x} \partial \bar{\eta}} - \frac{\partial \Psi}{\partial \bar{x}} \frac{\partial^2 \Psi}{\partial \bar{\eta}^2} = -\frac{\partial P}{\partial \bar{x}} + \frac{\partial^3 \Psi}{\partial \bar{\eta}^3}$$

with the scalings: $\bar{x} \sim \mathrm{Re}^{1/2} \tilde{x}/\tilde{L}$, $\bar{\eta} \sim \mathrm{Re}^{3/4} \tilde{y}/\tilde{L}$, $\bar{t} \sim \mathrm{Re}^{1/2} \tilde{t} \tilde{u}_\infty/\tilde{L}$

Boundary conditions:

$$\begin{aligned} \Psi = \Psi_{\bar{\eta}} = 0 \qquad & \text{for} \qquad \bar{\eta} = 0, \\ \Psi \sim B(\bar{x}, \bar{t})\bar{\eta} + \ldots \qquad & \text{as} \qquad \bar{\eta} \to \infty \end{aligned}$$

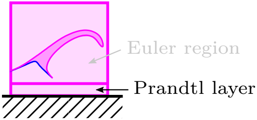
Initial condition: $(\bar{t} \to -\infty)$

$$\Psi\left(\bar{x}, \bar{\eta}, \bar{t}\right) \sim |\bar{t}|^{1/10} \hat{\Psi}(\hat{x}, \hat{\eta}) + |\bar{t}|^{-1/10} \hat{\Psi}_2(\hat{x}, \hat{\eta}) + |\bar{t}|^{-3/10} \hat{\Psi}_3(\hat{x}, \hat{\eta}) + \ldots$$

$$\text{with} \quad \bar{x} = |\bar{t}|^{3/5} \hat{x}, \quad \bar{\eta} = |\bar{t}|^{1/2} \hat{\eta}$$

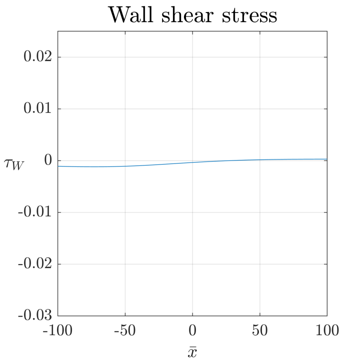
Relation between P and B (Bernoulli-equ.):

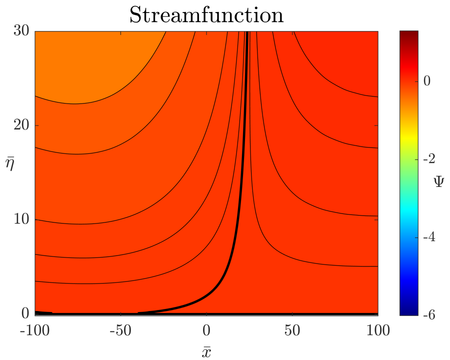
$$-P_{\bar{x}} = B_{\bar{t}} + B B_{\bar{x}}$$

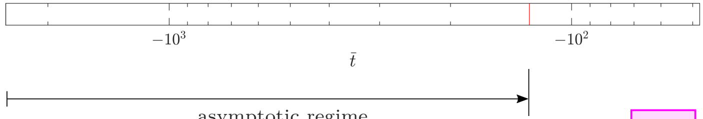


Euler-Prandtl stage

Prandtl layer dynamics, $(m \times n) = (120 \times 100)$

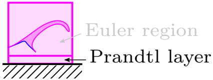






asymptotic regime

$(-\bar{t} \gg 1)$



Conclusions and future work

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- Early stages of laminar-turbulent transition
- Viscous layer dynamics of marginally separated boundary layer flow (TD and EP stages)
- Successfully calculated the finite-time blow-up profiles from the TD-stage

Future Work

- Inclusion of 3rd order TD blow-up profile as I.C.
- Refine spatial resolution of computational domain
- Prandtl layer: Improve transition from asymptotic regime to evolution at $\bar{t} \sim \mathcal{O}(1)$

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