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Financial frictions and the interbank market

A Master's Thesis submitted for the degree of
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Affidavit

I, Fabian Greimel,

hereby declare

that I am the sole author of the present Master's Thesis,

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37 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

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List of Abbreviations

bp basis point

CRS constant returns to scale

DSGE dynamic stochastic general equilibrium

Fed Federal Reserve Bank

ICC incentive compatibility constraint

NK New Keynesian

OLG overlapping generations

QE quantitative easing

RBC real business cycle

WW2 World War II

Abstract

This thesis analyzes the role of the interbank market in the "canonical model of financial intermediation and business fluctutations" by Gertler and Kiyotaki (2011). In the model the interbank market arises because only a fraction of firms is allowed to invest in each period. While Gertler and Kiyotaki (2011) analyze only two special cases quantitatively, I provide a numerical solution of their model with general interbank friction. The solution is used to show that a change in the interbank friction affects the economy only marginally. I show that the interbank market is very small compared to total credit: an upper bound is determined by aggregate investment, the fraction of firms that may invest, and asset prices. The degree of financial frictions is the fraction of bank assets, that bankers can divert. This gives rise to an incentive compatibility constraint (ICC). If it binds, there will be interest spreads between deposits, interbank trade and returns on loans. The tighter it binds, to higher the spreads. Since the (inverse) degree of interbank friction is modelled as the fraction of interbank assets which cannot be diverted, a change in the friction parameter essentially will not translate to a change in prices or quantities.

1 Introduction

The financial crisis 2008–2009 has caused a severe recession in the US and elsewhere. Due to the nature of crisis—triggered by the financial sector—the Great Recession could not be explained by most of the standard macroeconomic models[1], simply because these models abstract from banks and financial intermediation. This modelling approach implies *frictionless* financial markets.

On the contrary, the financial crisis has shown that *financial frictions* matter. Adrian et al. (2012) show that, during the crisis, it became more expensive for firms to get credit. And in fact, firms were also less likely to take out loans. The mechanisms at work had already been analyzed long before the crisis. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) provided the tools to replicate these stylized facts of a financial crisis as an extension to otherwise standard macroeconomic models[2]. After the crisis, there has been a lot of work building upon this framework—part of which is analyzed in section 2. Gertler and Kiyotaki (2011) consolidate their earlier work in a "canonical model of financial intermediation and business fluctuations". They add two aspects to the literature: First, they add an interbank market to the model. The interbank market arises because liquidity requirements differ across banks. Anecdotal evidence suggests that a freezing interbank market has played an important role in the propagation of the crisis. So it might be important to model its economic impact. And second, they provide a detailed analysis of *unconventional* monetary policy. Gertler and Kiyotaki (2011) show how the Federal Reserve Bank's (Fed) credit policies during the Great Recession—like quantitative easing (QE) or equity injection into banks—can be analyzed in their framework with an interbank market. However, they do not provide a detailed discussion about the size of the interbank market in their model and how the interbank market's role in the real world is captured by the model.

Outline This master's thesis analyzes the *minor role* of the interbank market in the given framework. It extends the work by Gertler and Kiyotaki (2011) by providing (*i*) a numerical solution of the model with a *general interbank friction*[3] and (*ii*) an in-depth discussion of the role of the interbank market. Gertler and Kiyotaki (2011, in particular figure 2) analyze only two special cases quantitatively. The more general solution facilitates the analysis of how the degree of interbank friction affects the model outcomes.

I will argue that the role of the interbank market is very limited. This is due to the relatively small size of the interbank market, and thus the little weight that the interbank friction ω actually has in the model. The friction parameter ω will be interpreted as a measure of trust in the interbank market: low ω means lower trust, i.e. more friction.

This thesis is structured as follows. Section 3 describes a model of financial intermediation and business fluctuations with general interbank friction (as in Gertler and Kiyotaki, 2011, appendix 1). Sections 4.3 and 4.4 will show that the role of the interbank market is very limited in this model. In the last section I will discuss implications for policy makers and give suggestions how to increase the importance of the interbank

[1]These standard models are built on the basic real business cycle (RBC) modelling framework which does not include money or inflation. Many models are thus extended to the New Keynesian framework, including prices, inflation and monetary policy.

[2]Neither Bernanke and Gertler (1989) nor Kiyotaki and Moore (1997) explicitly include banks in there model. But *financial frictions* are still at work. See section 2.

[3]The relative degree of interbank market friction will be denoted by $\omega \in [0, 1]$. Gertler and Kiyotaki (2011) analyze only the special cases $\omega \in \{0, 1\}$.

market in macroeconomic models.

2 Financial frictions in the literature

Before the Great Recession, most standard macroeconomic models[4] did not include financial markets explicitly. This relied on the assumption that financial markets are complete[5] and *frictionless*.

These markets can be described as markets with perfect trust across agents (lying is not possible, due to perfect information). Moreover, wealth (debt level and asset holdings) does not affect whether an agent will be granted credit. That is because contracts are perfectly enforceable: lenders cannot default on their debt. That is why trust and creditworthiness play no role and "funds are liquid and can flow to the most profitable project or to the person who values the funds most" (Brunnermeier et al., 2012, p.1). This was in constrast to common knowledge: At least since the Great Depression in the 1930s it had been known that financial markets are *not* frictionless. They might propagate, amplify or even generate shocks to the real economy. In the decades prior to 2007 these frictions seemed small, so most models abstracted from them. However, macroeconomic models with financial frictions have been around at least since Bernanke and Gertler (1989).

This section provides a selective overview of the macroeconomic literature with financial frictions. It will discuss some empirical findings, the classical contributions and the most closely related articles of the recent literature.

2.1 Empirical evidence on financial frictions

This section should motivate the theoretical work below by pointing at related empirical work that has been conducted recently. Adrian et al. (2012) show that the Great Recession lead to a drop in loans *to the corporate sector* in the US—both on an aggregate and on a firm level. On the other hand, financing through corporate bonds increased. It almost made up for the decrease in credit. However, a large fraction of the economy is not captured by the data: Smaller—non-corporate—firms have no access to bond finance, so they might have suffered more from decreasing credit. In the firm-level data, Adrian et al. (2012) find that both the number and the volume of loans decreased during the crisis, while increasing for bonds. What is more, external finance has become more expensive. Adrian et al. (2012, section 3.2.1) show that the cost of new debt rose sharply during the crisis—from 99 basis points (bps) in Q2:2007 to 403bps in Q2:2009.

They show that changes in banks' loans are debt financed: changes in debt and assets are almost perfectly correlated. On the other hand, equity is "sticky"—the correlation to assets is almost zero. Thus, if banks change their leverage ratio

$$\frac{\text{assets (loans)}}{\text{net worth}},$$

they do this through a change in assets. This implies that bank leverage co-moves with the business cycle.

Schularick and Taylor (2012) analyze how credit and money aggregates behave in the years before and after financial crises. They use data of fourteen countries and 140 years. They find that lagged credit growth has some predictive power for predicting financial crises in logit regressions—it does better than money supply. The difference

[4]That is, dynamic stochastic general equilibrium (DSGE) models.

[5]Completeness of financial markets means there is perfect information and perfect enforcement of debt contracts; the assets are state-contingent (the pay-off depends on the state of the world) and assets can be traded in each possible state.

between money and credit growth is strong only after World War II (WW2), when money and credit growth started to decouple. Schularick and Taylor (2012) also find an increase in inflation and money growth rates after WW2. In particular, after financial crises hit, inflation went slightly up after 1945, while inflation had become negative before 1938. This suggests that the policy responses to financial crises have been more expansionary post WW2.

These findings suggest that it is important to include imperfect financial markets to macroeconomic models.

2.2 Financial frictions in macroeconomic models

In order to introduce financial frictions into macroeconomic models, agents can be given the opportunity to lie (misreport private information) or steal (break debt contracts). A different way is to set an exogenous debt limit for borrowers. These assumptions will have two implications: (*i*) external funds (debt) are more expensive than internal funds (equity), and (*ii*) wealth of a lender plays a role: the more net worth she has, the more debt she may obtain.

This thesis will analyze the model by Gertler and Kiyotaki (2011) who explicitly model banks as financial intermediaries. Their predecessors did not explicitly mention banks when they analyzed financial frictions, but they used similar mechanisms.

Costly state verification The models[6] by Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) and Bernanke et al. (1999) place the friction with entrepreneurs, who transform investment to capital goods through "projects". These are financed both internally (with their own net worth) and externally (loans from the lenders). They are given the opportunity to hide their true returns and pay out less than agreed. The principal (the household as lender) can monitor the entrepreneur at a cost, which is dead weight loss. Similarly to the newer models, the entrepreneurs' balance sheets play a key role.

The friction arises because there is asymmetric information: The entrepreneurs' projects have stochastic outcomes, which are private information of the entrepreneurs. The entrepreneurs can report a lower outcome (cutting down the lenders return) and consume the difference. In order to avoid misreporting, the lenders have the possibility to monitor the entrepreneurs at a costs ("costly state verification"). Expected agency costs (i.e. monitoring costs) are decreasing in the level of net worth. That is because with higher net worth the lender's stake in the project gets smaller, and she has less of an incentive to pay the monitoring costs.

Once net worth is sufficiently high (the project can be financed purely from net worth[7]) there is no more need to monitor, and thus no more agency costs at all. This can be ruled out in a quantitative analysis, for example by a finite expected lifetime of borrowers. In that case external finance is needed and there are agency costs. That is where the *financial accelerator* kicks in: in bad times the weaker balance sheets reduce investment demand—which amplifies the downturn.

Carlstrom and Fuerst (1997) show that financial frictions produce hump-shaped impulse responses to productivity shocks in an otherwise standard real business cycle (RBC) model. They argue that financial frictions can be used to endogenize capital

[6]Even though the financial frictions are modelled the same way, the contexts differ. While the analysis by Bernanke and Gertler (1989) is set in an overlapping generations (OLG) framework, Carlstrom and Fuerst (1997) do quantitative analysis in an RBC framework with an infinitely lived representative agent and Bernanke et al. (1999) add the New Keynesian structure with Inflation and monetary policy.

[7]Bernanke and Gertler (1989) talk about "full-collateralization"

adjustment costs. Bernanke et al. (1999) use both financial frictions and capital adjustment costs to get stronger feedback effects. After a negative productivity shock, net worth falls. Subsequently asset demand and, thus, asset prices fall. This further depresses the net worth (which is related to assets via the balance sheet relation, see figure 4 below).

Collateral constraints and exogenous borrowing limits The models by Kiyotaki and Moore (1997, 2012), Gertler and Karadi (2011) and Gertler and Kiyotaki (2011) share the idea that lenders cannot force borrowers to repay their debt—unless debt is secured. If the debt level of a borrower is high enough, she might have the incentive to break the debt contract and not to repay the loan. This gives rise to an *icc* which puts an upper bound on debt. This ensures that the debtor will always repay. Inspired by the events of the Great Recession, Gertler and Karadi (2011) and Gertler and Kiyotaki (2011) explicitly introduce banks as financial intermediaries in their models. As opposed to the earlier literature, the banks are considered as the *borrowers* who receive funds from depositors (or other banks). In this setup banks have the possibility to steal a fraction of these assets. They will only receive funds as long as the expected value of future profits (the value of the bank) exceeds the funds that can be diverted—that is, as long as they have no incentive to divert funds.

In order to achieve borrowing and lending in equilibrium, a model must feature heterogeneity of agents. One way to achieve this is to assume that some agents are less patient than others. However, there needs to be a bound on borrowing, to prevent the impatient agents to borrow infinite amounts. While Kiyotaki and Moore (1997) introduce an endogenous bound through the collateral constraint, Eggertsson and Krugman (2012) set an exogenous debt limit. They show how a drop in the debt limit leads to higher interest rates and a debt-deflation mechanism. Eggertsson and Mehrotra (2014) extend this idea in an overlapping generations (OLG) setup. They argue that decreasing debt limits might push economies into "secular stagnation"—that is, extended periods with zero interest rates (but a negative natural interest rate, so that the zero lower bound binds). The channels at work are similar in all models mentioned.

The literature on financial frictions is much broader. It is surveyed in Brunnermeier et al. (2012). Other approaches include the work by Fostel and Geanakoplos (2008, 2015) and Geanakoplos (2010), who analyze the financial crisis in a microeconomic framework.

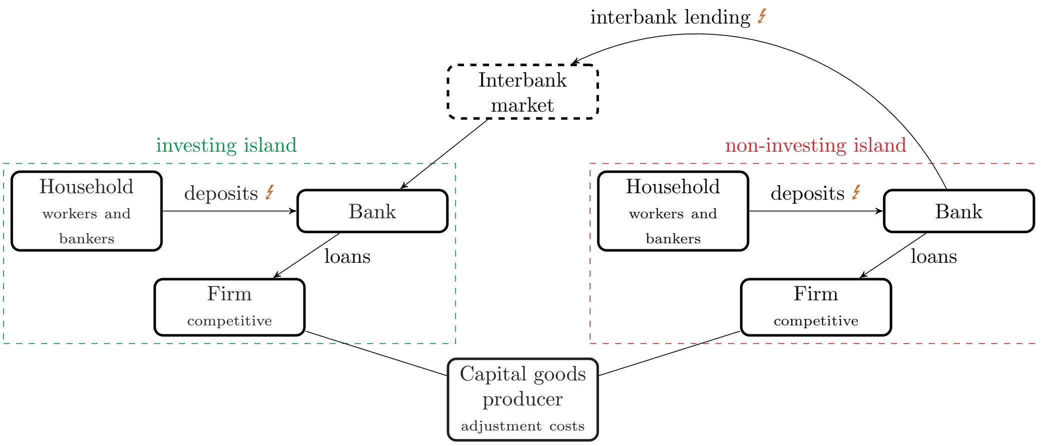


Figure 1: A flow chart of the model.

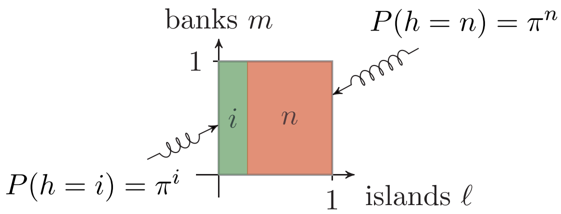


Figure 2: There is a continuum of island, with a continuum of banks on each island. Each period the islands are assigned a type (investing or non-investing) independently.

3 The model

This section describes the "canonical model of financial intermediation and business fluctuations" as in Gertler and Kiyotaki (2011). The model extends an RBC model with financial intermediaries (banks). As seen in figure 1 it consists of households (workers and bankers), financial intermediaries and firms. The final goods producers are competitive and the capital goods producers are subject to capital adjustment costs. Firms *cannot* borrow from households directly, they have to obtain funds from banks in the form of loans.

Household members, banks and final goods producers are located on a continuum of islands $\ell \in [0, 1]$. As illustrated in figure 2, there is a continuum of banks $m \in [0, 1]$ and final goods producers on each island. Loans flow only within an island. There are two types of islands $h \in \{i, n\}$. Firms on investing islands i may invest, firms on non-investing islands n have to roll over the loans for their existing capital stock. The assignment[8] of investment opportunities is independent each period. The fraction of investing types is π^i and the fraction of non-investing islands is $\pi^n = 1 - \pi^i$ each period.

These random investment opportunities create heterogenous liquidity needs on different island types. Banks can equilibrate their liquidity needs through an interbank market. Both the deposit and the interbank markets are subject to financial frictions because bankers get the possibility to steal a fraction θ of their assets and close the bank. They will only do so if the divertable funds exceed the value of expected future

[8]Technically, h is an independent random variable $[0, 1]^2 \to \{i, n\}$ such that $\Pr(h = i) = \pi^i$ and Pr is the Lebesgue measure on the unit square.

profits of the bank,

$$\text{divertable funds} > \text{value of continuing business.}$$

This puts an endogenous bound on the banks' balance sheets (as shown in figures 4 and 5). Household deposits and interbank lending will be treated differently. A fraction ω of interbank lending will be regarded as "safe".

In the following subsections I will describe each agent's maximization problem and the optimality conditions.

3.1 Households

The households are "big families" with a continuum of members. The household lives forever. A fraction f of members are workers, $1-f$ are bankers. Every period a fraction $1-\sigma$ of bankers become workers, and vice versa. When a banker exits, the bank's assets (terminal wealth) are transferred to her household. New bankers receive a lumpsum "start-up transfer" from the household. The households maximize their expected discounted lifetime utility[9] from consumption C_t and labor L_t subject to their budget contraint. The household earns a wage rate W_t on its labor supply, and a riskless gross return R_{t-1} on deposits (savings) D_{t-1} made in the previous period. There are lump-sum transfers and taxes which are summarized as a constant. The maximization problem is given by

$$\max_{(C_t,L_t,D_t)_t} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon} \right)$$
$$\text{s.t. } C_t = W_t L_t + R_{t-1} D_{h,t-1} - D_{ht} + \text{const}$$

β is the discount factor; χ is the relative utility weight of labor, ε is the inverse of the Frisch labor elasticity and γ is the habit formation coefficient. If γ is non-zero, the household will have a preference for "smooth" consumptions paths. After exogenous shocks, consumers will avoid "jumps" in their consumption.

The first order conditions give rise to a standard Euler equation

$$\mathrm{E}_t \underbrace{\beta \frac{u_{C,t+1}}{u_{C,t}}}_{\Lambda_{t,t+1}} R_t = 1, \tag{1}$$

where $\Lambda_{t,t+s}$ is the stochastic discount factor, and a labor supply equation

$$\mathrm{E}_t\, u_{C_t} W_t = \chi L_t^{\varepsilon}, \tag{2}$$

where u_{Ct} is the marginal utility of consumption at time t,

$$u_{C_t} = \frac{1}{C_t - \gamma C_{t-1}} - \beta\gamma\, \mathrm{E}_t \frac{1}{C_{t+1} - \gamma C_t}. \tag{3}$$

3.2 Final goods producers

On each island there is a continuum of final goods producers. They have an identical constant returns to scale (CRS) Cobb-Douglas production technology. Since capital is

[9] This formulation implies very strong assumptions. These include time-separable and time-invariant preferences, separability between consumption and labor supply, constant discount factor, the consumer being an expected-utility maximizer, the consumer having *very specific* preferences (log-utility with habit formation).

heterogenous for different islands, we have to assume that labor is perfectly mobile, so that the wages are equalized. That is why it is enough to look at the aggregate level. Given aggregate capital stock K_t and labor supply L_t the output is given by

$$Y_t = F(z_t, K_t, L_t) = \exp(z_t) K_t^\alpha L_t^{1-\alpha}$$

where z_t is an exogenous productivity process

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}.$$

Each period, only those firms on investing islands may invest. Since they are chosen independently, they hold a fraction π^i of the previous period's capital stock:

$$\begin{aligned} K_t &= \underbrace{\psi_t\big((1-\delta)\pi^i K_{t-1} + I_{t-1}\big)}_{\text{aggregate of type } i} + \underbrace{\psi_t(1-\delta)\pi^n K_{t-1}}_{\text{aggregate of type } n} \\ &= \psi_t\big((1-\delta)K_{t-1} + I_{t-1}\big), \end{aligned} \tag{4}$$

where δ is the depreciation rate and ψ_t is an exogenous process, determining the capital quality,

$$\ln \psi_t = \rho_\psi \ln \psi_{t-1} + \varepsilon_{\psi,t}.$$

Capital quality can be thought of as measuring economic obsolescence. It introduces exogenous variation in the value of capital. Firms pay the marginal product for labor and capital,

$$\begin{aligned} W_t &= F_L(z_t, K_t, L_t) \\ Z_t &= F_K(z_t, K_t, L_t). \end{aligned}$$

3.3 The capital goods producer

The capital goods producers produce new capital using the economy's final output as input. The capital goods are sold to firms on *investing* islands at the price of capital Q_t^i. Production of capital goods is subject to adjustment costs in the gross rate of change in investment[10].

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\eta_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{5}$$

The function satisfies $f(1) = 0$ and $f'(1) = 0$, so that there are no (marginal) adjustment costs in steady state. The adjustment costs are assumed to be convex: $f'' = \eta_I > 0$. η_I can be interpreted as the inverse elasticity of net investment to the price of capital. The capital goods producers choose a sequence of investments $(I_\tau)_{\tau=t}^\infty$ to maximize the discounted sum of profits:

$$\max_{(I_\tau)_\tau} \mathrm{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau}\Big(\underbrace{Q_\tau^i I_\tau}_{\text{revenue}} - \underbrace{\left(1 + f\left(\frac{I_\tau}{I_{\tau-1}}\right)\right) I_\tau}_{\text{costs}}\Big)$$

Capital goods producers are assumed to be owned by the households. That is why they share the same stochastic discount factor $\Lambda_{t,s}$. Profits (if there are any) are transferred to the households lump-sum. The first-order necessary condition (for I_t) of the maximization problem above determines the price of assets Q_t^i:

$$Q_t^i = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} \cdot f'\left(\frac{I_t}{I_{t-1}}\right) - \mathrm{E}_t \Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right). \tag{6}$$

[10]The adjustment costs assumed here are in line with those in Christiano et al. (2005, p. 15). They compare their specification to previously more common adjustment costs in levels: “In particular, in results not reported here, we found that the alternative adjustment cost model does not match the strong, hump-shaped response of investment [to a monetary policy shock] in the data.” (Christiano et al., 2005, p. 38). Their specification has since become quite common in the literature.

3.4 The banks and financial frictions

Banks collect deposits from households and supply loans to firms *on their island*. At the beginning of the period they choose loans s_t^h and interbank lending b_t^h contingently on the island type h, and deposits d_t. This can be thought of as collecting deposits *before*—and giving loans *after*—investment opportunities arrive (i.e. the island types are known). This is shown in figure 3. Once the investment opportunities have arrived, funds are scarce on the investing islands and abundant on the non-investing islands. That is why banks trade liquidity with banks of the other type (on a different island) on the interbank market. After the island type is revealed, banks execute their trades on the interbank and credit markets. It is assumed that banks borrow to the local firms frictionlessly. Funds circulate across islands only through the interbank market (see also figure 1).

Given the bank's *states*[11] $(s_{t-1}, b_{t-1}, d_{t-1})$, and bank's island type h, the end-of-period-t net worth n_t^h is given by the gross payoff of last period's assets,

$$n_t^h = (Z_t + (1-\delta)Q_t^h)\psi_t s_{t-1} - R_{bt} b_{t-1} - R_t d_{t-1}, \tag{7}$$

where Z_t are the divident payments by the non-financial firms. The firms repay the loans[12] each period, subject to depreciation and a capital value shock. Note that the net worth depends on the island type only via the asset prices Q_t^h. Given their island type h and the net worth n_t^h, the flow-of-funds (balance sheet) constraint for each bank is

$$Q_t^h s_t^h = n_t^h + b_t^h + d_t. \tag{8}$$

This relationship is shown in figures 4 and 5.

Banks pay dividends only when they exit—which happens with probability $1-\sigma$. In that case they transfer their total (end-of-period-t) net worth n_t^h to their household. Thus, the banks' objective function is the expected present discounted value of future dividends—the end-of-period-t value of the bank:

$$V_t = \mathrm{E}_{t,h} \sum_{i=0}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda_{t,t+i} n_{t+i}^h \tag{9}$$

Given the survival probability σ in each period, the probability of exiting in period t is given by $\sigma^{t-1}(1-\sigma)$. $\Lambda_{t,t+i}$ is the stochastic discount factor

$$\Lambda_{t,t+s} = \beta^s \frac{u_{C,t+s}}{u_{Ct}},$$

analogous to the discount factor of the households (as defined in section 3.1).

The model tries to capture the fact, that banks are constrained in obtaining funds from depositors and other banks. This is motivated by the possibility of bank runs and the fact that the interbank market dried out during the Great Recession. The friction is introducing through a moral hazard problem: At the end of each period bankers have the possibility to divert funds. That is, they may transfer a share θ of divertible assets

$$Q_t^h s_t^h - \omega b_t^h = n_t^h + d_t + (1-\omega) b_t^h$$

to their families. ω is the share of "safe" interbank assets. It measures the relative degree of friction in the interbank market. If it is zero, there is the same kind of

[11] That is, the bank's choices from the period before.

[12] In the real world, loans are debt. Here, loans are modelled as equity. The return on loans is not fixed a priori, but determined by the realizations of island type, capital quality and productivity. Unlike debt in the real world, it is thus a residual claim.

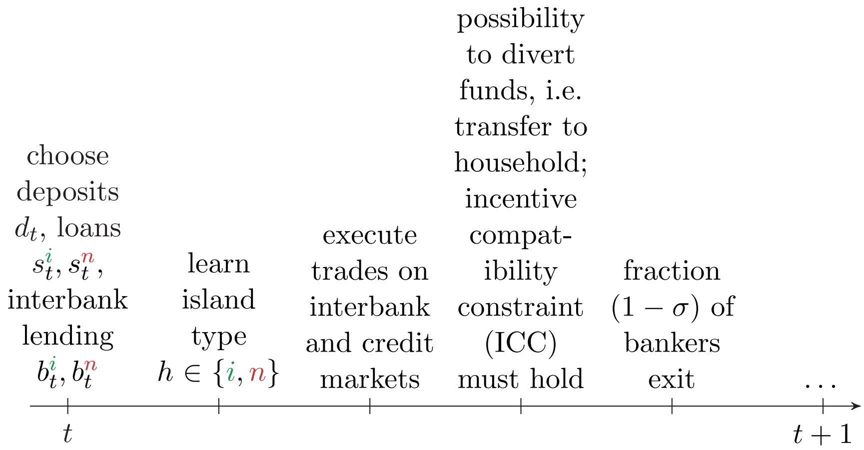


Figure 3: Timing of the banks' problem

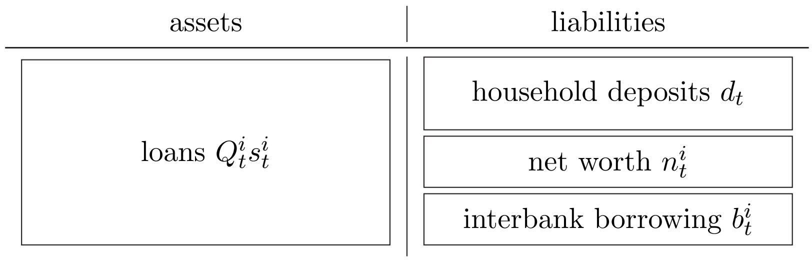


Figure 4: A bank's balance sheet on an investing island

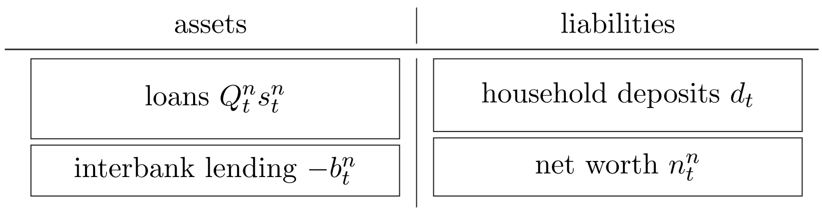


Figure 5: A bank's balance sheet on an non-investing island

friction for household deposits and interbank borrowing. If it is one, interbank lending is safe, so the interbank market is frictionless. The *end-of-period value* of the bank $V_t(s_t^h, b_t^h, d_t)$ must be higher than the outside option (diverting), so that the banker does not have an incentive to divert funds. This is captured by an ICC

$$V_t(s_t^h, b_t^h, d_t) \geq \theta(Q_t^h s_t^h - \omega b_t^h), \quad h \in \{i, n\}. \tag{10}$$

for each island type. The bank receives funds only as long as the ICC is satisfied, no matter which island is drawn. Otherwise the lenders will expect to lose a fraction $\geq \theta$ of their funds.

It will be shown that the banks' end-of-period value function is linear in the choices,

$$V_t(s_t^h, b_t^h, d_t) = \mathcal{V}_{st} s_t^h - \mathcal{V}_{bt} b_t^h - \mathcal{V}_{dt} d_t, \tag{11}$$

$\mathcal{V}_{st}$ is the marginal value of additional loans to firms, $\mathcal{V}_{bt}$ and $\mathcal{V}_{dt}$ are the marginal costs of deposits by households and interbank borrowing, respectively[13]. The bank's problem is formally summarized below. The details are found in appendix A.

3.4.1 The banks' problem

The banks maximize their expected terminal wealth (9), subject to the constraints (7), (8) and (10).

Let $V(s_t, b_t, d_t)$ be the banks' *end-of-period-t* value function, depending on the choices in period t. Given equation (9) it can be written in recursive form as

$$V(s_t, b_t, d_t) = \mathrm{E}_t \, \Lambda_{t,t+1} \sum_h \pi^h \Big((1-\sigma) n_{t+1}^h + \sigma \max_{s_{t+1}^h, b_{t+1}^h, d_{t+1}} V(s_{t+1}^h, b_{t+1}^h, d_{t+1}) \Big), \tag{12}$$

where the choice is subject to the ICC constraint (10). The end-of-period problem can be written as a Lagrangian, using the Lagrange multipliers λ_t^h for the ICC constraint,

$$\begin{aligned} \mathscr{L}_t &= V(s_t^h, b_t^h, d_t) + \lambda_t^h \Big(V(s_t^h, b_t^h, d_t) - \theta(Q_t^h s_t^h - \omega b_t^h) \Big) \\ &= (1 + \lambda_t^h) V(s_t^h, b_t^h, d_t) - \lambda_t^h \theta (Q_t^h s_t^h - \omega b_t^h) \end{aligned} \tag{13}$$

Using complementary slackness, one can summarize the banks problem as

$$\begin{aligned} V(s_{t-1}, b_{t-1}, d_{t-1}) = &\mathrm{E}_{t-1} \, \Lambda_{t-1,t} \sum_h \pi^h \Big((1-\sigma) n_t^h + \sigma \max_{s_t^h, b_t^h, d_t} (1 + \lambda_t^h) V(s_t^h, b_t^h, d_t) \\ &- \lambda_t^h \theta (Q_t^h s_t^h - \omega b_t^h) \Big) \\ \text{s.t. } n_{t+1}^h = &(Z_{t+1} + (1-\delta) Q_{t+1}^h) \psi_{t+1} s_t - R_{bt+1} b_t - R_{t+1} d_t, \end{aligned} \tag{14}$$

The first order necessary conditions, the verification of the linear value function and more details are found in appendix A.

[13] Here the notation differs from Gertler and Kiyotaki (2011). Their $\mathcal{V}_t$ is my $\mathcal{V}_{dt}$.

3.4.2 The incentive constraint and credit spreads

Building on the first order necessary conditions (derived in appendix A.1) the guessed value function is verified in appendix A.3. The undetermined coefficients are given by

$$\mathcal{V}_{st} = \mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega^h_{t+1} \big(Z_{t+1} + (1-\delta) Q^h_{t+1}\big) \psi_{t+1}, \tag{15}$$

$$\mathcal{V}_{bt} = R_{bt} \cdot \mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega^h_{t+1}, \tag{16}$$

$$\mathcal{V}_{dt} = R_t \cdot \mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega^h_{t+1} = \frac{R_t}{R_{bt}} \mathcal{V}_{bt}. \tag{17}$$

with $1-\sigma$ being the probability the banker exits and

$$\Omega^h_{t+1} = 1 - \sigma + \sigma\big(\mathcal{V}_{bt+1} + \lambda^h_{t+1}(\mathcal{V}_{bt+1} - \theta\omega)\big). \tag{18}$$

As shown in appendix A.3, the value function has a different representation (42) in terms of net worth. Then, Ω^h_{t+1} is the marginal value of net worth of bankers on island type h.

Combining these equations with equations (27) and (28)—which are combinations of the first order conditions—reveals how the ICCs (10) relate to interest rate spreads:

$$\frac{\theta\omega\bar{\lambda}_t}{1+\bar{\lambda}_t} = (R_{bt} - R_t) \cdot \mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega^h_{t+1}$$

$$\frac{\lambda^h_t \theta(1-\omega)}{1+\lambda^h_t} = \mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega^h_{t+1} (R^{\tilde{h}h}_{k,t+1} - R_{bt})$$

where

$$R^{\tilde{h}h}_{k,t+1} = \frac{Z_{t+1} + (1-\delta) Q^h_{t+1}}{Q^{\tilde{h}}_t} \psi_{t+1}$$

is the return to bank lending depending on the current island type $\tilde{h}$ and the island type next period h.

Note that the returns on interbank lending R_{bt} and household deposits R_t are determined in the period before they are paid. λ^h_t are Lagrange multipliers for the incentive compatibility constraint for each island type for $h \in \{i, n\}$, and thus non-negative. $\bar{\lambda}_t$ is their average:

$$\bar{\lambda}_t = \sum_h \pi^h \lambda^h_t. \tag{19}$$

By the complementary slackness conditions of a constrained optimum, the multiplier is non-zero only if the contraint binds. Since for a given degree of friction $\theta \neq 0$ and $\omega \neq 0, 1$ a higher spread is equivalent to a higher multiplier[14], a higher spread implies a more tightly binding ICC.

On the other hand, if the constraint is not binding on an island, the multiplier must be zero. So there will be no credit spread on this island. If the constraint binds on neither island, the bank lending spread will be zero, too: $R_{bt} = R$.

The above equations also show that if there is no general friction ($\theta = 0$, no funds can be diverted), there will be no spreads at all. If there is symmetric friction on the

[14] That is because $\frac{\partial}{\partial\lambda} \frac{\lambda}{1+\lambda} = \frac{1}{(1+\lambda)^2} > 0$.

interbank and deposit markets ($\omega = 0$, interbank lending is not safe), there will be no spread between deposits and interbank lending. If there is no friction on the interbank market ($\omega = 1$, interbank lending is safe) there will be no spread between the credit and interbank markets.

3.4.3 Evolution of aggregate net worth

There is a high degree of heterogeneity across islands. The islands might have different histories of investment opportunities. For example, two islands of type i are different if in the previous period one was of type i and one of type n. In this section I will show that one can aggregate over islands of one type, because only aggregate quantities of the previous period matter.

This is due to the island structure shown in figure 2, since the investment opportunities are drawn independently[15]. But we have to make an additional assumption.

Assumption 1 (Rates of return are equal across islands)**.** *Like Gertler and Kiyotaki (2011), I make an assumption about banks' rates of return in order to get tractability. In particular, banks are allowed to change islands at the beginning of each period so that ex ante rates of return are equal across islands (arbitrage). This happens the following way: A fraction of banks on islands with low return move to islands with high return. They sell their loans to other banks that remain on the island in exchange for interbank loans.*

Let aggregate loans S_{t-1}, deposits D_{t-1} and $B_{t-1} = 0$ be the integral over all banks for banks $j \in [0,1]^2$, e.g.

$$S_{t-1} = \int_{[0,1]^2} s_{j,t-1} \, \mathrm{d}\lambda(j)$$

aggregate net worth of islands of type h can then be written as

$$\tilde{N}_t^h = \int_{[0,1]^2} \mathbf{1}_h(h_j) n_{j,t} \, \mathrm{d}\lambda(j)$$

since h is independent,

$$= \underbrace{\int_{[0,1]^2} \mathbf{1}_h(h_j) \, \mathrm{d}\lambda(j)}_{P(h_j = h) = \pi^h} \int_{[0,1]^2} n_{j,t} \, \mathrm{d}\lambda(j) = \pi^h \int_{[0,1]^2} n_{j,t} \, \mathrm{d}\lambda(j)$$

plug in for net worth, and assume that returns equal across islands,

$$= \pi^h \int_{[0,1]^2} \big(Z + (1-\delta) Q_t^h\big) \psi_t s_{j,t-1} - R_{b,t-1} b_{j,t-1} - R_{t-1} d_{j,t-1} \, \mathrm{d}\lambda(j)$$

$$= \pi^h \Big(\big(Z + (1-\delta) Q_t^h\big) \psi_t S_{t-1} - R_{b,t-1} \underbrace{B_{t-1}}_{=0} - R_{t-1} D_{t-1} \Big). \tag{20}$$

the interbank market has to net out on aggregate. This is the net worth *before* bankers learn if they stay or exit. Only a fraction σ of bankers survives. They are replaced by new bankers, who receive a *start-up transfer* from their household. Gertler and Kiyotaki (2011) assume that the transfer is a fraction $\frac{\xi}{1-\sigma}$ of total assets of exiting bankers

$$\frac{\xi}{1-\sigma} (1-\sigma) \pi^h \big(Z_t + (1-\delta) Q_t^h\big) \psi_t S_{t-1}.$$

[15] That is, the random variable h described in footnote 8 is independent.

This gives a nice expression for aggregate net worth on each island:

$$
\begin{aligned}
N_t^h &= \underbrace{\sigma\pi^h\Big(\big(Z+(1-\delta)Q_t^h\big)\psi_t S_{t-1} - R_{t-1}D_{t-1}\Big)}_{\text{net worth of surviving bankers}} + \underbrace{\xi\pi^h\big(Z+(1-\delta)Q_t^h\big)\psi_t S_{t-1}}_{\text{start-up transfer}} \\
&= \pi^h\Big(\big(Z+(1-\delta)Q_t^h\big)\psi_t S_{t-1}(\sigma+\xi) - \sigma R_{t-1}D_{t-1}\Big). \qquad (21)
\end{aligned}
$$

Note that net worth depends on asset returns $(Z_t + (1-\delta)Q_t^h)\psi_t$. That impact will be greater, the higher the banks leverage $\frac{Q_t^h S_t^h}{N_t^h}$. In particular, a decline in capital quality ψ_t will directly reduce net worth.

3.5 Market clearing and equilibrium

Since interbank lending nets out in aggregate, bank loans S_t^h are financed by deposits D_t and total net worth N_t,

$$\sum_{h\in\{i,n\}} Q_t^h S_t^h = D_t + N_t$$

In equilibrium, the loan and labor markets clear. Firms have to roll over loans for their total capital stock each period. So total loans equals total capital:

$$\left.\begin{aligned} S_t^i &= I_t + (1-\delta)\pi^i K_t \\ S_t^n &= (1-\delta)\pi^n K_t \end{aligned}\right\} \implies S_t = I_t + (1-\delta)K_t = K_{t+1} \qquad (22)$$

Market clearing on the labor market implies that

$$\underbrace{F_L(z_t, K_t, L_t)}_{=W_t} u_{Ct} = \chi L_t^{\varepsilon}. \qquad (23)$$

Finally, the aggregate resource constraint is given by

$$Y_t = C_t + \left(1 + f\left(\frac{I_t}{I_{t-1}}\right)\right) I_t + G_t. \qquad (24)$$

where G_t is the government expenditure which is calibrated to be 20% of output in steady state, $G_t = 0.2 \cdot Y^*$.

Table 1: Baseline parameter choices for the numerical analysis

parameter	value	description
β	0.99	consumers' discount factor
γ	0.5	habit formation parameter
χ	5.584	relative weight of labour in utility
ε	0.1	inverse Frisch labour elasticity
α	0.33	capital share
δ	0.025	depreciation rate of the capital stock
$\bar{\omega}$		steady state fraction of save interbank assets (interbank friction)
π^i	0.25	probability of new investment opportunities to arrive
$\bar{\theta}$	calibrated	steady state fraction of divertable asset (general degree of friction)
ξ	calibrated	transfer to entering bankers
σ	0.972	Survival rate of the bankers
η_I	1.5	Inverse elasticity of net investment to the price of capital
Gshare	0.2	Steady state proportion of government expenditures
$\bar{G}$	calibrated	steady state government expenditure
ρ_ψ	0.66	autoregressive parameter of log capital quality ψ_t
ρ_ω	0.66	autoregressive parameter of ω_t
ρ_θ	0.66	autoregressive parameter of θ_t
LR	4	steady state leverage ratio
SPREAD	0.0025	steady state average credit spread

All parameters are taken from Gertler and Kiyotaki (2011) except the autoregressive coefficients of the friction parameters. The steady state degree of interbank friction $\bar{\omega} \in [0,1]$ will be varied in the numerical analysis.

4 Numerical analysis and crisis simulation

In this section I will analyze the model quantitatively. After a description of the parameter calibration, I will discuss how to simulate a crisis in this model. The financial crisis is modelled as an exogenous shock to capital quality. The model features a *financial accelerator* which amplifies the shock. I fail to replicate the impulse responses by Gertler and Kiyotaki (2011, figure 2). However, I will provide arguments why their figures might not correspond to the parameterization given in their paper. I will show that the interbank market is very small relative to total credit. The interbank friction affects the solution of the model only through the ICC. Since the interbank market is very small, a change in the degree of interbank friction cannot have an important impact on the economy: *The interbank market plays only a minor role.*

4.1 Calibration

I follow Gertler and Kiyotaki (2011) with their parameter choices, which are given in table 1. In the quantitative analysis, however, I assume that the degrees of friction θ_t and ω_t are time dependent, following AR(1) processes,

$$\theta_t = (1-\rho_\theta)\bar{\theta} + \rho_\theta \theta_{t-1}$$
$$\omega_t = (1-\rho_\omega)\bar{\omega} + \rho_\omega \omega_{t-1}.$$

While the steady state degree of general friction $\bar{\theta}$ is calibrated, the steady state degree of interbank friction $\bar{\omega}$ is not. Instead, results are compared for different levels of friction. $\bar{\theta}$ and ξ are set to hit following targets in steady state:

1. An average credit spread of 100 bps per year. Given this period's island type h, the expected return is

$$\mathrm{E}_{h'} R^{hh'}_{k,t+1} = \sum_{h'} \pi^{h'} \frac{Z_{t+1} + (1-\delta) Q^{h'}_{t+1}}{Q^h_t} \psi_{t+1}$$

   where h' is next year's island type. So the average interest rate across all islands is

$$\mathrm{E}_t R_{k,t+1} := \sum_h \pi^h \, \mathrm{E}_{h'} R^{hh'}_{k,t+1}.$$

   Now we can target the average spread (risk premium) to be

$$\mathrm{E}_t (R_{k,t+1} - R_t) \overset{\text{cali}}{=} \frac{0.01}{4}$$

   since in the model one period corresponds to a quarter.

2. An economy-wide leverage ratio of 4. The leverage ratio is the ratio of the value of total bank loans to total banks' net worth:

$$\frac{Q^i_t S^i_t + Q^n_t S^n_t}{N^i_t + N^n_t} \overset{\text{cali}}{=} 4.$$

4.2 Crisis experiment: The financial accelerator

One way to trigger a financial crisis in the model is a negative shock to capital quality (ψ). This shock in capital quality depresses the value of the loans the banks have in their balance sheets. It is motivated by the idea that the Great Recession was triggered by a decline in housing prices. Mass default led to massive write-offs in banks' balance sheets. Note, however, that the model is very simplistic. It features neither a housing market nor mortgage-backed securities.

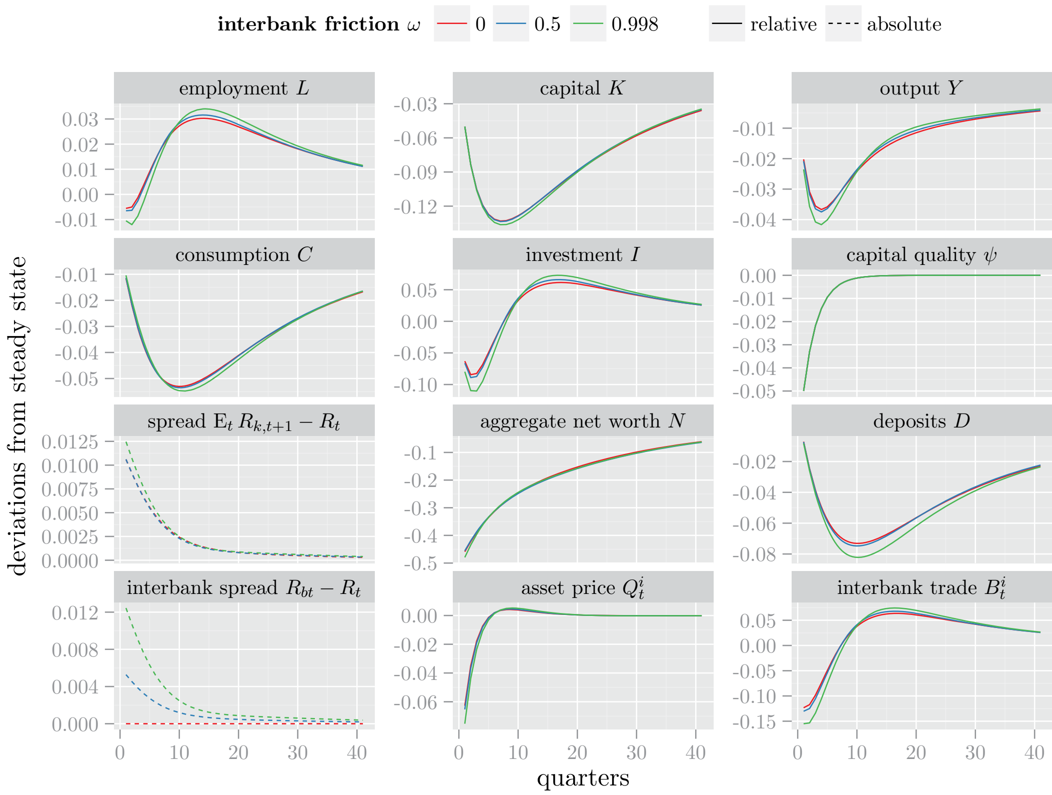
The financial accelerator Figure 6a shows the impulse responses to a 5% shock in capital quality. The shock translates one-for-one to capital (which equals total loans) in the first period, and is amplified to lead to a much larger drop in net worth (almost 50%).

The mechanism of the "financial accelerator" is revealed by the evolution of banks' net worth (21). It is determined by quantities and returns from the previous period ($S_{t-1}, D_{t-1}, R_{t-1}$), the exogenous capital quality ψ_t and asset prices Q^h_t.

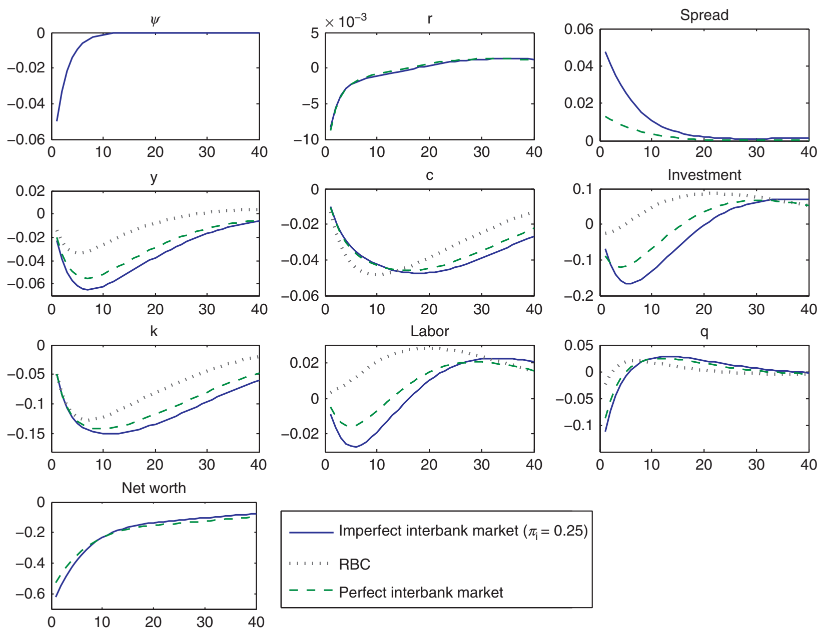
There is a *direct effect*: net worth decreases by 5% of the previous assets. The direct link of assets and the capital stock is due to market clearing on the credit market, see equation (22).

There is also *price effect.* The lower capital quality drives down the price of capital Q^h_t. This further shrinks the banks' net worth. Banks have to "fire-sell" bank assets in order to satisfy the balance sheet identity. This is reflected in a drop in investment. This drop in investment demand further depresses asset prices, and thus investment demand, and so on.

Comparison to Gertler and Kiyotaki (2011, figure 2) Gertler and Kiyotaki (2011) provide a general model of the interbank market in their appendix. They analyze only special cases quantitatively, though. Figure 6b shows their figure 2 which compares the cases of perfect ($\omega = 1$) and imperfect ($\omega = 0$) interbank markets.



(a) More friction (smaller ω) leads to less pronounced impulse responses. The response of the expected spread does not change a lot.



(b) Comparison a perfect ($\omega = 1$) and imperfect ($\omega = 0$) interbank markets (a reprint of Gertler and Kiyotaki, 2011, figure 2). If there are frictions, the impulse responses are stronger. The response to the expected spread changes a lot.

Figure 6: Comparing the results of the crisis experiment: Impulse responses to a 5% shock in capital quality for different degrees of interbank friction ω.

Table 2: Comparing the calibrated parameter θ

ω	θ	θ_{GK11}
0.00	0.42	0.13
1.00	0.38	0.38

The values from Gertler and Kiyotaki (2011, table 1) are denoted by θ_{GK11}, my calibrations by θ. I use my model with $\omega = 0.998$ for comparison with their model where $\omega = 1$.

The case of a perfect interbank market is matched pretty well. All impulse responses are essentially equivalent in figure 6a and Gertler and Kiyotaki (2011). However, if the interbank friction increases, my results show that the impulse responses become weaker in magnitude. This contradicts Gertler and Kiyotaki (2011). The difference is especially stark for the spread, which changes a lot in their figure, but only little in mine.

Table 2 shows that Gertler and Kiyotaki (2011) report large differences in the calibrated value of the general degree of friction θ. When going from the highest to the lowest interbank friction, the general friction almost triples. In my calibrations it even decreases slightly. As with the impulse responses, my results are very close for the case where $\omega = 1$. Figure 7 shows the U-shaped relationship of the calibrated value and ω.

Higher friction, lower levels It might seem counterintuitive that a lower friction leads to stronger impulse responses in a crisis. However, it is not only the impulse responses that matter, but also the steady state values. Figure 7 shows how the steady state values of the key variables change with different degrees of steady state interbank friction $\bar{\omega}$. Higher $\bar{\omega}$ (lower friction) leads to higher levels of output Y, consumption C, investment I, capital C and labor L in steady state. That is, the friction reduces the efficiency of the economy.

4.3 So small: Providing an upper bound for interbank lending

Figure 7 shows that the steady state level of interbank lending is very small relative to total credit. As it turns out, this is due to the construction of the model. This section formally derives an upper bound of interbank lending. Under most sensible parameter choices, interbank lending will be smaller than aggregate investment, which is δK^* in steady state.

That is because the interbank market only reflects the differential need for loans on the two types of islands. The need for an interbank market comes from heterogenous liquidity needs. A fraction π^i of firms may invest, the remaining firms must not. By contrast, all firms have to roll over the loans for their (depreciated) capital stock $(1 - \delta) \cdot K_t$ in every period. The additional liquidity needs on investing islands are comparatively small. In steady state, investment will just keep the aggregate capital stock constant,

$$I^* = \delta \cdot K^*.$$

The difference in liquidity needs is relatively small across island types. This can be seen figure 8.

On the liability side, note that interbank lending and borrowing have to net out on aggregate. That is, total loans have to match deposits D_t and net worth N_t:

$$Q_t^i S_t^i + Q_t^n S_t^n = N_t + D_t.$$

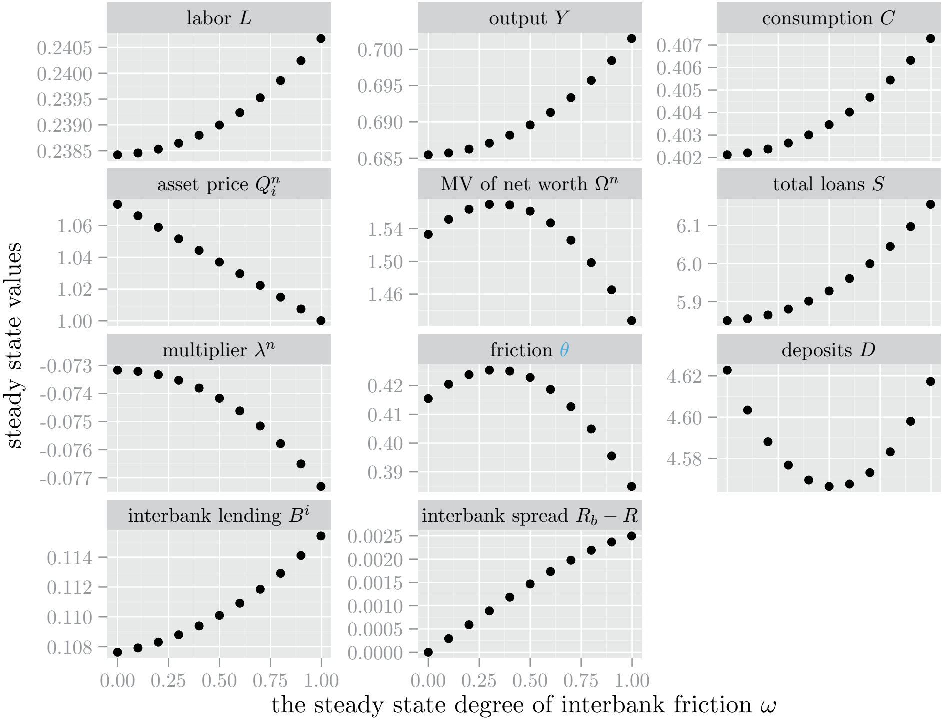


Figure 7: Steady state levels of selected variables for different parameterization of ω. All other variables are unchanged. Lower ω means *more* friction.

This gives rise to the following proposition.

Proposition. *Given the model framework of Gertler and Kiyotaki (2011), the aggregate interbank borrowing* B_t^i *across investing islands is given by*

$$B_t^i = \pi^n Q_t^n I_t + \Phi$$

where

$$\Phi = \pi^n \underbrace{(Q_t^n - Q_t^i)}_{\geq 0}\big((S_t^i - I_t)\psi_t - S_t^i\big).$$

With a small enough capital quality ψ_t*, there is an upper bound on interbank borrowing* B_t^i*,*

$$\psi_t \leq \frac{S_t^i}{S_t^i - I_t} \implies 0 \leq B_t^i \leq \pi^n Q_t^n I_t. \tag{25}$$

With the calibration used in their paper, $\pi^n Q_t^n \leq 1$ *in steady state. Thus,* $B^{i*} \leq I^*$*.*

Idea of the proof. The detailed proof is given in appendix B. The idea can be seen in figure 8. As pointed out before, interbank assets are 0 in aggregate (they are in zero net supply). Banks will trade on the interbank market only if their net worth and deposits do not suffice to satisfy the demand for credit. The interbank lending is given by

$$B_t^i = Q_t^i S_t^i - N_t^i - \pi^i D_t.$$

Total assets and total liabilities are equal on both islands. Since investment I_t is small, the distribution of assets over islands types is roughly the same as $\pi^i : \pi^n$. On

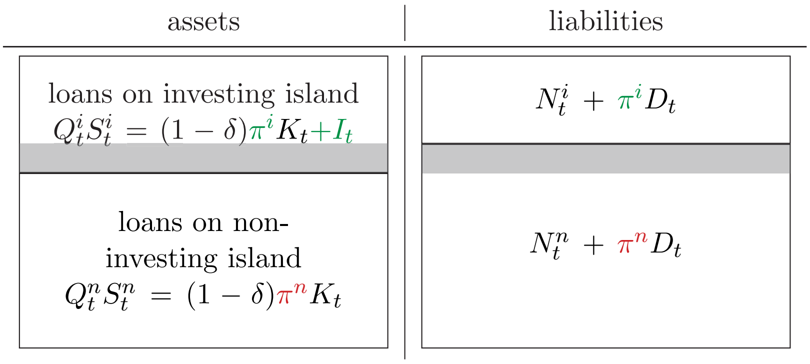


Figure 8: The aggregate balance sheet by island types. Identification of the interbank market as difference between assets and liabilities of each island.

the liability side, recall that net worth is given by

$$N_t^h = \pi^h\Big(\big(Z_t + (1-\delta)Q_t^h\big)\psi_t S_{t-1} - R_{t-1}D_{t-1}\Big)$$

The asset prices Q_t^i, Q_t^n are of the same magnitude on the two island types. That is why the distribution of net worth is also close to $\pi^i : \pi^n$.

Thus, assets and liabilities roughly have the same distribution across island types, so the need for interbank lending will be small. □

In the real world On the first glance, the data support a very small interbank market. Figure 9 shows the *balance sheet level* of interbank credit and total credit for US commercial banks. It can be seen that the interbank loans are between 0.7% and 8.6% of total credit. The steady state values of our model show that this ratio is of a similar magnitude: 1.8%.

However, it can be seen in the lowest panel of figure 9 that the interbank market was much bigger before the crisis than it is now. A macroeconomic model that wants to explain the Great Recession should probably aim at matching the levels before the crisis.

But there is an important aspect missing. These data reflect *long-term* interbank lending, that show up in the banks balance sheets. A big chunk of interbank lending is overnight lending. Banks use this instrument to satisfy their reserve requirements. These credits are usually paid back within a day, so they do not show up in banks balance sheets (which only show the *level* of outstanding credit as parts of assets and borrowing as parts of liabilities). That is why data on overnight trade is hard to get.

Demiralp et al. (2004, table 1) estimate a *daily volume* of $145 billion of overnight lending for the first quarter of 1998. That is, withing two days the flows of interbank lending exceed the balance sheet stocks: on average, around 58% of the stock was traded *per day*.

A model of the interbank market, that cannot replicate the large flows, might thus miss important aspects the effects that the interbank market can have on the economy. If these large daily flow play an important role, it is not adequate to use a quarterly model frequency. However, a reduced frequency will, other things equal, also lead to smaller flows. This is because the interbank flows are bounded by a multiple of investment. In steady state, investment is given by δK^*. The depreciation rate δ will be smaller, the shorter the time periods.

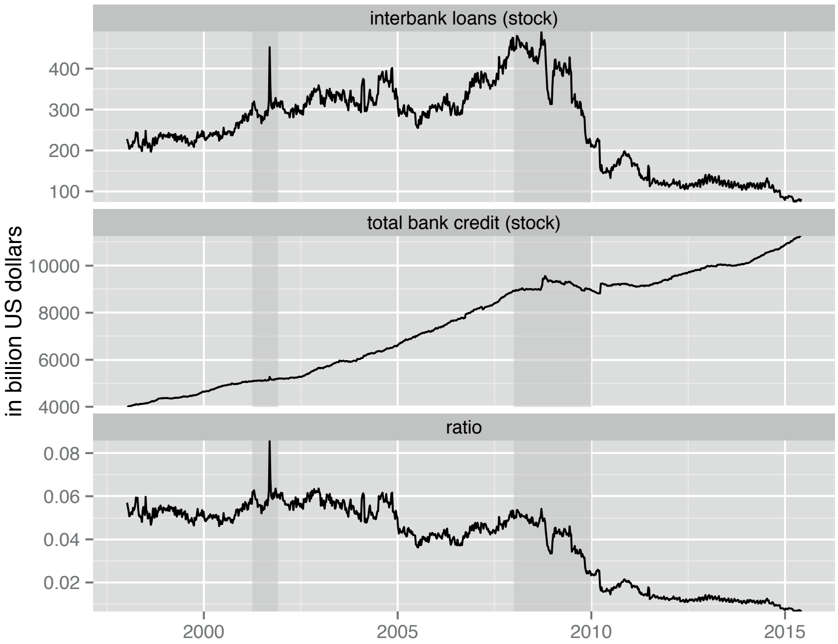


Figure 9: Total bank credit and interbank lending by US commerical banks. The shaded areas denote recessions. Source: FRED

4.4 Why the interbank market does not matter in Gertler and Kiyotaki (2011)

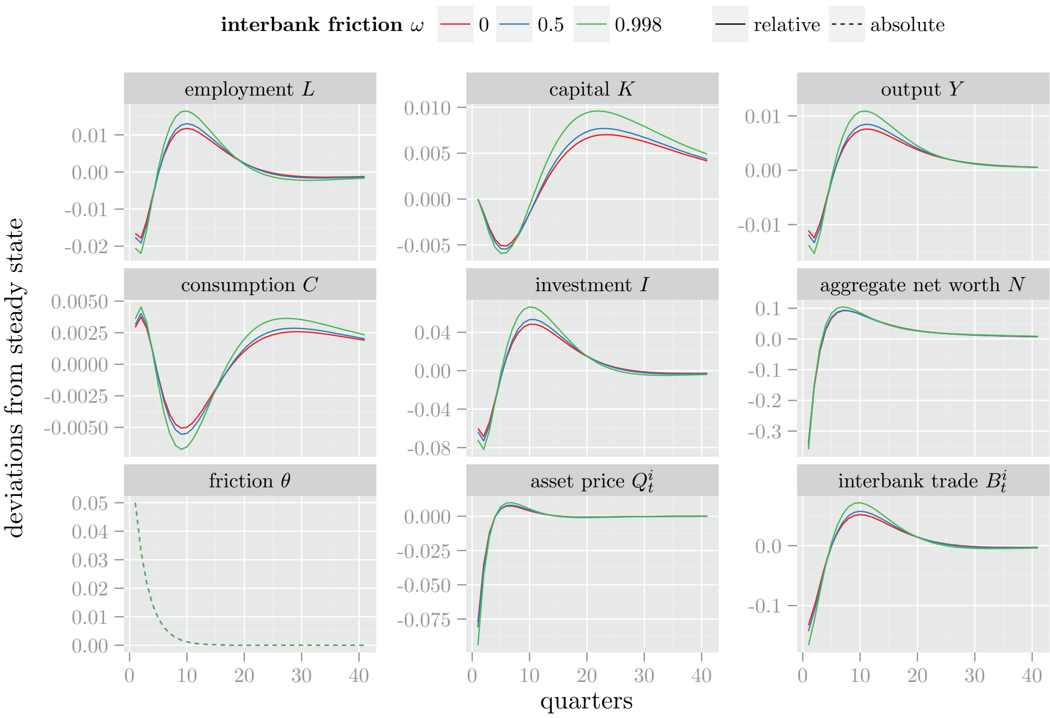
We have just seen that the size of the interbank market is very small. I will now show that that this implies that the friction on the interbank market is quantitatively irrelevant in the given model by Gertler and Kiyotaki (2011).

Let us first analyze how the economy reacts to a change in the degrees of friction. As opposed to Gertler and Kiyotaki (2011) I assume that the degrees of friction are time-varying. They follow AR(1) processes with means at the steady state levels $(\bar{\theta}, \bar{\omega})$. A negative shock to ω (higher friction) can be interpreted as sudden drop in trust among banks. This is motivated by the freezing of the interbank market after the crash of Lehman brothers.

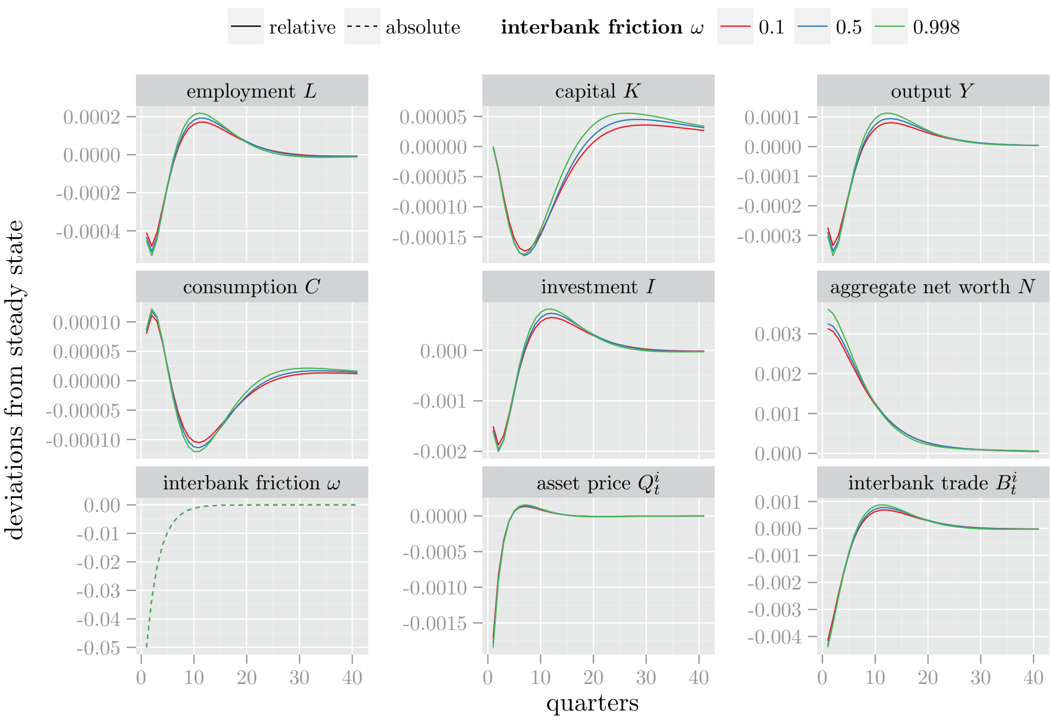
Figure 10 shows selected impulse responses to shocks to the degrees of friction (θ_t, ω_t). While the shape of the responses are very similar in panels 10(a) (θ_t) and 10(b) (ω_t) the magnitudes of the shocks vary a lot. While a 5% to the general friction leads to significant responses of output and total net worth, the responses to a shock to ω_t are negligible. The mechanism behind is very similar to the financial accelerator of section 4.2. There is no direct effect on the capital stock, though. The banks are forced to sell assets (disinvest) because more friction makes the ICC bind more tightly. This reduces the asset prices, banks sell even more assets, and so on.

Let us now analyze the effect of the friction analytically. The degrees of friction enter the model through the banks' ICC. The aggregate ICCs for each island type $h \in \{i, n\}$ at the end of period t are given by

$$
\begin{aligned}
V_t(S_t^i, B_t^i, \pi^i D_t) &\geq \theta(\underbrace{Q_t^i S_t^i}_{\approx 1.5} - \omega \underbrace{B_t^h}_{\approx 0.1}), \\
V_t(S_t^n, B_t^n, \pi^n D_t) &\geq \theta(\underbrace{Q_t^n S_t^n}_{\approx 4.5} + \omega \underbrace{B_t^n}_{\approx 0.1}).
\end{aligned}
\tag{26}
$$

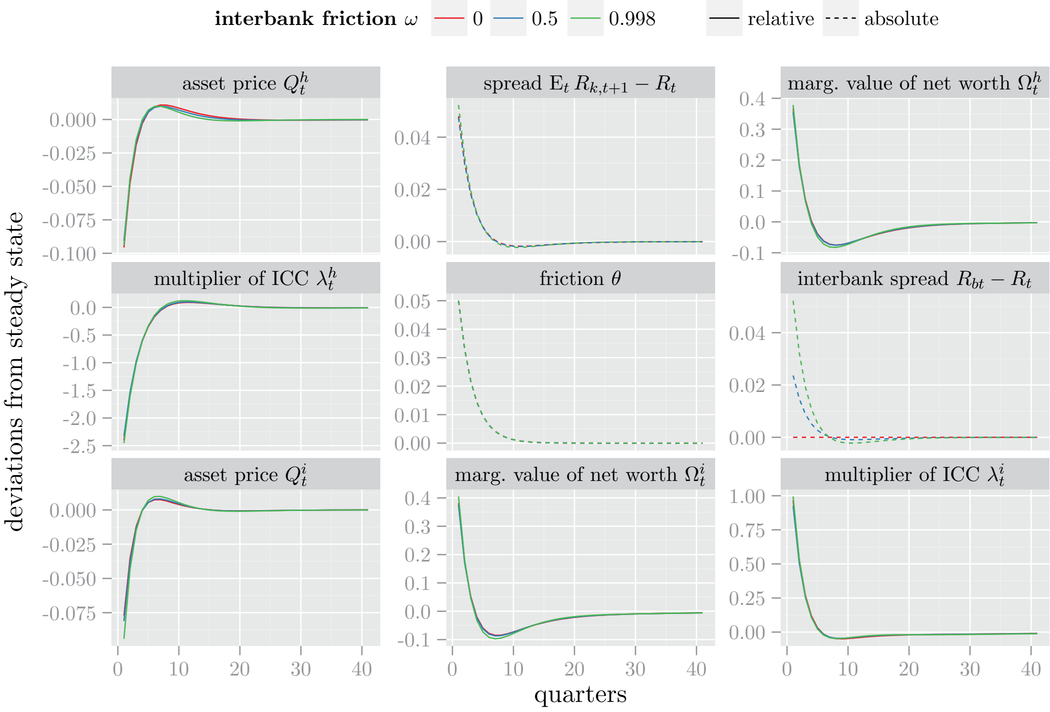


(a) 5% shock to general degree of friction θ_t (higher θ_t means more friction)

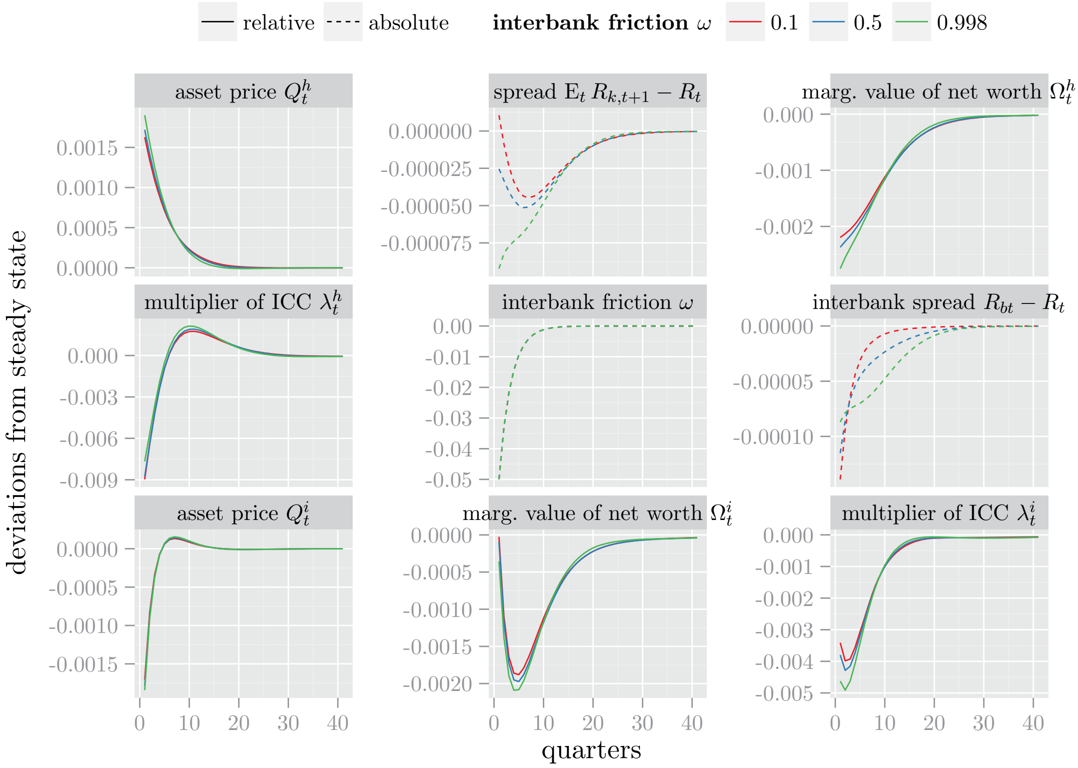


(b) -5% shock to degree of interbank friction ω_t (lower ω_t means more friction)

Figure 10: Impulse responses to shocks to the degrees of friction I.



(a) 5% shock to general degree of friction θ_t (higher θ_t means more friction)



(b) -5% shock to degree of interbank friction ω_t (lower ω_t means more friction)

Figure 11: Impulse responses to shocks to the degrees of friction: Multipliers, spreads, the asset price. While there is a significant response to a shock in θ_t, the response to ω_t is very small.

The magnitudes depend on the parameterization. They are taken from figure 7. The degrees of friction affect the tightness of the ICCs, thus the interest spreads in the economy (see section 3.4.2). The equations above reveal that changes in the interbank friction cannot have a big impact on spreads. Even going from one extreme to the other ($\omega = 0$ to $\omega = 1$) reduces the right hand side of (26) by only about 2% on non-investing islands and 6% on investing islands.

Figure 11 shows impulse responses to shocks to the friction parameters for a different set of variables. Among these variables are the Lagrange multipliers λ_t^h corresponding to the above constraints (26). The multipliers indicate how strong the contraint is binding. The plots confirm that there is only a very small reaction to a shock in the degree of interbank friction ω_t. By contrast, the reaction to a change in the general friction θ_t yields a much stronger response.

The plots also show that the impulse responses of prices (that is, asset prices and interest spreads) share the same pattern: they are almost unaffected by a change in the interbank friction.

5 Conclusions and outlook

I have shown that the interbank market plays only a minor role in the given model framework. This has implications for policy makers and points to promising research topics.

5.1 Implication for monetary policy

An important contribution of Gertler and Kiyotaki (2011) is to provide a framework to analyze unconventional monetary policy. The setup is similar to Gertler and Karadi (2011, 2013)—who have an even stronger focus on monetary policy—but it provides additional insight due to the existence of an interbank market.

In general, unconventional monetary policies (as opposed to interest rate policy) are measures that are taken to extend the liquidity in the economy. A famous instance are the QE programs by the Fed. The given framework provides a new way to think about the mechanism of these policies: Unconventional monetary policy aims at relaxing the lending constraints of the banks. There are different ways how the central bank can intervene. From a conceptual point of view, the simplest measures are lending facilities (direct lending), where the central bank lends to firms directly. This mechanism is not only used by Gertler and Kiyotaki (2011), but also by Gertler and Karadi (2011, 2013), and the findings of this Master thesis do not affect its relevance.

Gertler and Kiyotaki (2011) additionally discuss two more credit policies: (*i*) discount facilities, where the banks can borrow from the central bank, with a lower degree of friction. This policy strengthens the balance sheet of the banks through external fundings. And (*ii*) equity injections, where the central bank acquires bank net worth. Unfortunately, Gertler and Kiyotaki (2011) do not provide a quantitative analysis of the policies. I conjecture that the discount facilities will hardly have an effect, because they are modelled in the same way as interbank lending. Discount facilities can be thought of as an alternative to interbank trade.

The role of the interbank market is certainly more important than this model suggests. So it is certainly important how monetary policy can intervene, if the interbank market dries out. It is, thus, essential to change the model structure in way that gives more weight to the interbank market.

5.2 Giving the interbank market a larger role

Anecdotal evidence suggest that the interbank market has played an important role in the propagation of the financial crisis 2007–2009. In the given model the role of the interbank market is very limited since it is too small. In this section I point at possible paths to go.

Is the quarterly frequency adequate? As discussed earlier, the daily flows on the interbank market are very large compared to stocks. For 1998 it was estimated that the aggregate flows of two days exceed the stocks. This suggests that a quarterly frequency is inadequate, since it cannot mirror the comparatively large flows. However—without any other changes—increasing the frequency of the model will not help. I have shown that interbank lending is (under reasonable parameters) bounded by investment, and investment is δK^* in steady state. When going to monthly or daily frequency, the depreciation rate will fall accordingly. This puts an ever lower bound on interbank lending. Thus, the tight link of δ and interbank lending has to be broken. This is discussed below.

Increasing relative weight In the current model the share of interbank lending to total credit is very small. That is because banks have to roll over loans worth their total capital stock each period. This implies that

$$S_t = (1-\delta)K_t + I_t \text{ and } B_t \le \pi^n Q_t^n I_t.$$

(see the proposition on page 22). This is due to (*i*) the fact that a large proportion of loans is the same for all banks. This creates only a small degree of heterogeneity in the model. (*ii*) The proportion of net worth and deposits across islands is very similar to the proportion of credit demand across island types. There are two ways to deal with that. There needs to be more heterogeneity across firms. If firms hold *long-term assets* (or they have net worth) they do not have to repay their total capital stock each period. This will narrow the gap between total assets and idiosynchratic asset on investing islands. But additionally, there should be *greater heterogeneity across banks*, so there is a greater need for interbank lending. Bigio (2015) use firms-specific capital quality as a means to introduce greater heterogeneity.

Which friction matters more? The given model features two friction parameters. There is a general friction θ_t and an interbank friction ω_t. Let us interpret these parameters as measuring *trust*. Given that deposits are usually "safe" due to deposit insurance, I assume that the friction on the deposit market plays a lesser role than the friction on the interbank market.

However, taking the model seriously, it is not *trust* that matters, but the *possibility to steal*, which is not affected by deposit insurance. So, one would have to make an ad-hoc assumption that bankers cannot steal depositors' money as easily as they can steal other banks' money. The incentive constraints are equilibrium conditions, ensuring that the bank does not steal assets. So even though depositors do not fear a loss (because of deposit insurance), there will be a friction—as long as banks are allowed to steal.

One might still assume that the friction on the deposit market plays a lesser role than the friction on the interbank market. A friction on the deposit market would imply that the decrease in loans was due to a lack of deposits during the crisis. As it turns out, though, deposits did not change a lot in the crisis. This gives rise to an alternative ICC,

$$\begin{aligned} V_t(s_t^h, b_t^h, d_t) &\ge \theta_t(Q_t^h s_t^h - \omega_t b_t^h - \kappa_t d_t) \\ &= \theta_t(n_t^h + (1-\omega_t)b_t^h + (1-\kappa_t)d_t) \end{aligned}$$

where κ_t and ω_t denotes the share of "safe" deposits and interbank lending, and the friction on the interbank market is larger than on the deposit market, $\kappa_t \gg \omega_t$.

Introduce a distinction of savings and investment banks An alternative route is splitting up the banks into savings and investment banks (as in Hilberg and Hollmayr, 2011) Since investment banks have no access to external funds, most of their lending will be funded through lending from the savings banks. A friction on this market would have a similar impact to the general friction θ in the model by Gertler and Kiyotaki (2011). However, the interbank market does not emerge endogenously anymore, if the island structure of the model is given up. Banks of the same type will not trade, which is in contrast to main function of the interbank market in the real world.

References

Adrian, T., Colla, P. and Shin, H. S. (2012), Which financial frictions? Parsing the evidence from the financial crisis of 2007 to 2009, *in* D. Acemoglu, J. Parker and M. Woodford, eds, 'NBER Macroeconomics Annual 2012', Vol. 27, University of Chicago Press, pp. 159–214.

Bernanke, B. and Gertler, M. (1989), 'Agency costs, net worth, and business fluctuations', *The American Economic Review* **79**(1), pp. 14–31.
URL: *http://www.jstor.org/stable/1804770*

Bernanke, B. S., Gertler, M. and Gilchrist, S. (1999), The financial accelerator in a quantitative business cycle framework, *in* J. Taylor and M. Woodford, eds, 'Handbook of Macroeconomics', Vol. 1, Elsevier, pp. 1341–1393.

Bigio, S. (2015), 'Endogenous liquidity and the business cycle', *American Economic Review* **105**(6), 1883–1927.
URL: *http://www.aeaweb.org/articles.php?doi=10.1257/aer.20110035*

Brunnermeier, M. K., Eisenbach, T. M. and Sannikov, Y. (2012), Macroeconomics with financial frictions: A survey, Working Paper 18102, National Bureau of Economic Research.

Carlstrom, C. T. and Fuerst, T. S. (1997), 'Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis', *The American Economic Review* pp. 893–910.

Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005), 'Nominal rigidities and the dynamic effects of a shock to monetary policy', *Journal of political Economy* **113**(1), 1–45.

Demiralp, S., Preslopsky, B. and Whitesell, W. (2004), Overnight interbank loan markets, Research Paper Series 2004-29, Board of Governors of the Federal Reserve System.

Eggertsson, G. B. and Krugman, P. (2012), 'Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach', *The Quarterly Journal of Economics* **127**(3), 1469–1513.
URL: *http://qje.oxfordjournals.org/content/127/3/1469.abstract*

Eggertsson, G. B. and Mehrotra, N. R. (2014), A model of secular stagnation, Working Paper 20574, National Bureau of Economic Research.

Fostel, A. and Geanakoplos, J. (2008), 'Leverage cycles and the anxious economy', *American Economic Review* **98**(4), 1211–44.
URL: *http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.4.1211*

Fostel, A. and Geanakoplos, J. (2015), Financial innovation, collateral and investment, Discussion Paper 1903R, Cowles Foundation.

Geanakoplos, J. (2010), The leverage cycle, *in* 'NBER Macroeconomics Annual 2009, Volume 24', University of Chicago Press, pp. 1–65.

Gertler, M. and Karadi, P. (2011), 'A model of unconventional monetary policy', *Journal of monetary Economics* **58**(1), 17–34.

Gertler, M. and Karadi, P. (2013), 'QE 1 vs. 2 vs. 3. . . : A framework for analyzing large-scale asset purchases as a monetary policy tool', *International Journal of Central Banking* **9**(1), 5–53.

Gertler, M. and Kiyotaki, N. (2011), Financial intermediation and credit policy in business cycle analysis, *in* 'Handbook of monetary economics', Vol. 3A, Elsevier, pp. 547–599.

Hilberg, B. and Hollmayr, J. (2011), Asset prices, collateral and unconventional monetary policy in a dsge model, Working paper, European Central Bank.

Kiyotaki, N. and Moore, J. (1997), 'Credit cycles', *Journal of Political Economy* **105**(2), 211–248.
URL: *http://www.jstor.org/stable/10.1086/262072*

Kiyotaki, N. and Moore, J. (2012), Liquidity, business cycles, and monetary policy, Working Paper 17934, National Bureau of Economic Research.
URL: *http://www.nber.org/papers/w17934*

Schularick, M. and Taylor, A. M. (2012), 'Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008', *American Economic Review* **102**(2), 1029–61.
URL: *http://www.aeaweb.org/articles.php?doi=10.1257/aer.102.2.1029*

A Solving the banks' problem in Gertler and Kiyotaki (2011)

Gertler and Kiyotaki (2011) solve the banks' problem in different special cases. The calculations are not given in the article, so they shall be given in this appendix. The first special case—perfect interbank market ($\omega = 0$)—is easiest, so it will be omitted.

A.1 Derivations of the first order conditions

The banks' optimal decision The first order necessary conditions for an optimal solution are

$$(1+\lambda_t^h)\mathcal{V}_{st} = \lambda_t^h \theta Q_t^h \qquad (s_t^h)$$
$$(1+\lambda_t^h)\mathcal{V}_{bt} = \lambda_t^h \theta \omega, \qquad (b_t^h)$$
$$(1+\lambda_t^h)\mathcal{V}_{dt} = 0 \qquad (d_t)$$
$$\mathcal{V}_{st} s_t^h - \mathcal{V}_{bt} b_t^h - \mathcal{V}_{dt} d_t \geq \theta(Q_t^h s_t^h - \omega b_t^h) \qquad (\lambda_t^h)$$

Substracting (d_t) from (b_t^h) yields

$$(1+\bar{\lambda}_t)(\mathcal{V}_{bt} - \mathcal{V}_{dt}) = \theta\omega\bar{\lambda}_t. \tag{27}$$

where $\bar{\lambda}_t = \sum_h \pi^h \lambda_t^h$ is the expected Lagrangian multiplier. Substracting (d_t) from (s_t^h) yields

$$(1+\lambda_t^h)\left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}\right) = \lambda_t^h \theta(1-\omega). \tag{28}$$

The incentive constraint (λ_t^h) is equivalent to

$$\left(\theta - \frac{\mathcal{V}_{st}}{Q_t^h}\right) Q_t^h s_t^h - (\theta\omega - \mathcal{V}_{bt}) b_t^h \leq -\mathcal{V}_{dt} d_t$$

In order to get the formulation from the paper we need to complete to right hand side to $\mathcal{V}_t n_t^h = \mathcal{V}_t(Q_t^h s_t^h - b_t^h - d_t)$,

$$\left(\theta - \left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_t\right)\right) Q_t^h s_t^h - (\theta\omega - (\mathcal{V}_{bt} - \mathcal{V}_t))\, b_t^h \leq \mathcal{V}_t n_t^h. \tag{29}$$

A.2 The other special case: symmetric frictions $\omega = 1$

From $\omega = 1$ it immediately follows that $\mathcal{V}_{bt} = \mathcal{V}_{dt}$. Also note[16] that $Q_t^i < Q_t^n$. It will be hepful to define the excess value of assets (over deposits) on island h,

$$\mu^h = \frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{dt} \implies \mathcal{V}_{dt} = \mathcal{V}_{bt}$$

Additionally we define the leverage ratio[17] on each island,

$$\phi_t^h = \frac{Q_t^h s_t^h}{n_t^h}. \tag{30}$$

[16] An argument is given in Gertler and Kiyotaki (2011, section 2.3.2): "Because asset supply per unit of bank net worth is larger on investing islands than on non-investing islands, the asset price is lower, i.e., $Q_i^t < Q_t^n$. Intuitively, given that the leverage ratio constraint limits banks' ability to acquire assets, prices will clear at lower values on investing islands where supplies per unit of bank net worth are greater."

[17] This defintion is different from the paper, where the relationship is $\frac{Q_t^h s_t^h}{n_t^h} \leq \frac{\mathcal{V}_{dt}}{\theta - \mu_t^h} =: \phi_t^h$

It will also turn out handy to rewrite the value function in several ways using the above relations and the definition of net worth.

$$\begin{aligned}
V_t^h &= \mathcal{V}_{st} s_t^h - \mathcal{V}_{bt} b_t^h - \mathcal{V}_{dt} d_t \\
&= (\mu_t^h + \mathcal{V}_{dt}) Q_t^h s_t^h - \mathcal{V}_{dt}(b_t^h + d_t) \qquad (31)\\
&= (\mu_t^h + \mathcal{V}_{dt}) Q_t^h s_t^h - \mathcal{V}_{dt}(Q_t^h s_t^h - n_t^h) \\
&= \mu_t^h Q_t^h s_t^h + \mathcal{V}_{dt} n_t^h \\
&= \mu_t^h n_t^h \phi_t^h + \mathcal{V}_{dt} n_t^h \qquad (32)
\end{aligned}$$

We use the method of undetermined coefficients to verify the linear guess of the value function. We write the guessed value function and its recursive form:

$$\begin{aligned}
V_t^h &= \mathcal{V}_{st} s_t^h - \mathcal{V}_{bt} b_t^h - \mathcal{V}_{dt} d_t \\
&= \mathrm{E}_t \, \Lambda_{t,t+1} \sum_{h'} \pi^{h'} \Big((1-\sigma) n_{t+1}^{h'} + \sigma V_{t+1}(n_{t+1}^{h'}) \Big).
\end{aligned}$$

Note that h denotes the island in the current period and h' denotes the island in the following period, $h, h' \in \{i, n\}$. Take the derivative with respect to d_t to get

$$\begin{aligned}
\mathcal{V}_{dt} &= \mathrm{E}_t \, \Lambda_{t,t+1} \sum_{h'} \pi^{h'} \Big((1-\sigma) \frac{\partial n_{t+1}^{h'}}{\partial d_t} + \sigma \frac{\partial V_{t+1}(n_{t+1}^{h'})}{\partial n_{t+1}^{h'}} \underbrace{\frac{\partial n_{t+1}^{h'}}{\partial d_t}}_{R_{t+1}} \Big) \\
&= \mathrm{E}_t \, \Lambda_{t,t+1} \sum_{h'} \pi^{h'} R_{t+1} \underbrace{\Big((1-\sigma) + \sigma \frac{\partial V_{t+1}(n_{t+1}^{h'})}{\partial n_{t+1}^{h'}} \Big)}_{\Omega_{t+1}^{h'}} \qquad (33)
\end{aligned}$$

In order to get $\mathcal{V}_s$ we must compute μ_t^h. Using (31) take the derviatve with respect to s_t^h,

$$(\mu_t^h + \mathcal{V}_{dt}) Q_t^h = \mathrm{E}_t \, \Lambda_{t,t+1} \sum_{h'} \pi^{h'} \Big((1-\sigma) \frac{\partial n_{t+1}^{h'}}{\partial s_t^h} + \sigma \frac{\partial V_{t+1}(n_{t+1}^{h'})}{\partial n_{t+1}^{h'}} \underbrace{\frac{\partial n_{t+1}^{h'}}{\partial s_t^h}}_{Z_{t+1} + (1-\delta) Q_{t+1}^{h'}} \Big)$$

Dividing by the asset price Q_t^h and using that $R_{k,t+1}^{hh'} = \frac{Z_{t+1} + (1-\delta) Q_{t+1}^{h'}}{Q_t^h} \psi_{t+1}$ gives that

$$\mu_t^h + \mathcal{V}_{dt} = \mathrm{E}_t \, \Lambda_{t,t+1} \sum_{h'} \pi^{h'} R_{t+1}^{hh'} \underbrace{\Big((1-\sigma) + \sigma \frac{\partial V_{t+1}(n_{t+1}^{h'})}{\partial n_{t+1}^{h'}} \Big)}_{\Omega_{t+1}^{h'}}$$

Substracting (33) from this expression yields that

$$\mu_{t+1}^h = \mathrm{E}_t \, \Lambda_{t,t+1} \sum_{h'} \Omega_{t+1}^{h'} (R_{k,t+1}^{hh'} - R_{t+1}). \qquad (34)$$

Remains to calculate Ω_{t+1}^h. It helps to calculate the marginal value of net worth before. Use the representation of the bank's value as in (32):

$$\frac{\partial V_{t+1}(n_{t+1}^{h'})}{\partial n_{t+1}^{h'}} = \mu_t^h \phi_t^h + \mathcal{V}_{dt}.$$

From there it follows that

$$\Omega_{t+1}^{h'} = 1 - \sigma + \sigma (\mu_{t+1}^h \phi_{t+1}^h + \mathcal{V}_{dt+1}) \qquad (35)$$

A.3 The general case $\omega \in (0,1)$

The solution here is a bit more involved. First we rewrite equations (27), (28) and (29). (27) can be rewritten as

$$\mathcal{V}_{bt} - \mathcal{V}_{dt} = \frac{\theta\omega\bar{\lambda}_t}{1+\bar{\lambda}_t} \tag{36}$$

(28) can be rewritten as

$$\begin{aligned}
\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt} &= \lambda_t^h \theta(1-\omega) - \lambda_t^h \left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}\right) \\
\iff \lambda_t^h &= \frac{\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}}{\theta(1-\omega) - \left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}\right)} =: \frac{\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}}{\nu}
\end{aligned} \tag{37}$$

The incentive compatibility constraint (29) can be rewritten as

$$\begin{aligned}
&\left(\theta - \left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_t\right)\right) Q_t^h s_t^h - (\theta\omega - (\mathcal{V}_{bt} - \mathcal{V}_t)) \underbrace{(Q_t^h s_t^h - n_t^h - d_t^h)}_{b_t^h} \leq \mathcal{V}_t n_t^h. \\
\iff &\underbrace{\left(\theta(1-\omega) - \left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}\right)\right)}_{=:\nu} Q_t^h s_t^h + (\theta\omega - (\mathcal{V}_{bt} - \mathcal{V}_{dt}))d_t \leq (\mathcal{V}_{bt} - \theta\omega) n_t^h
\end{aligned} \tag{38}$$

Plugging in (36) for $\mathcal{V}_{bt} - \mathcal{V}_{dt}$ gives

$$\begin{aligned}
\nu \cdot Q_t^h s_t^h &\leq (\mathcal{V}_{bt} - \theta\omega) n_t^h - \left(\theta\omega - \frac{\theta\omega\bar{\lambda}_t}{1+\bar{\lambda}_t}\right) d_t \\
&= (\mathcal{V}_{bt} - \theta\omega) n_t^h - \theta\omega \left(1 - \frac{\bar{\lambda}_t}{1+\bar{\lambda}_t}\right) d_t \\
&= (\mathcal{V}_{bt} - \theta\omega) n_t^h - \frac{\theta\omega}{1+\bar{\lambda}_t} d_t.
\end{aligned}$$

This gives the following expression

$$Q_t^h s_t^h \leq \frac{1}{\nu}\left((\mathcal{V}_{bt} - \theta\omega) n_t^h - \frac{\theta\omega}{1+\bar{\lambda}_t} d_t\right) \tag{39}$$

Combining this expression with (37) gives

$$\left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}\right) Q_t^h s_t^h \leq \lambda_t^h \left((\mathcal{V}_{bt} - \theta\omega) n_t^h - \frac{\theta\omega}{1+\bar{\lambda}_t} d_t\right) \tag{40}$$

Next, we rewrite the guessed *end-of-period* value function, so that it only contains $\mathcal{V}_{bt}$. First complete the last term to n_t.

$$\begin{aligned}
V_t^h(\cdot) &= \frac{\mathcal{V}_{st}}{Q_t^h} Q_t^h s_t^h - \mathcal{V}_{bt} b_t^h - \mathcal{V}_{dt} d_t \\
&= \left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}\right) Q_t^h s_t^h + (\mathcal{V}_{bt} - \mathcal{V}_{dt}) d_t + \underbrace{(Q_t^h s_t^h - d_t - b_t^h)}_{n_t^h} \mathcal{V}_{bt}
\end{aligned}$$

Now we use the rewritten first order conditions (36) and (37). We assume that the incentive compatibility constraints (40) bind use it, too.

$$
\begin{aligned}
&= \lambda_t^h \left((\mathcal{V}_{bt} - \theta\omega) n_t^h - \frac{\theta\omega}{1+\bar{\lambda}_t} d_t \right) + \frac{\theta\omega\bar{\lambda}_t}{1+\bar{\lambda}_t} d_t + n_t^h \mathcal{V}_{bt} \\
&= \lambda_t^h (\mathcal{V}_{bt} - \theta\omega) n_t^h - \frac{\lambda_t^h \theta\omega}{1+\bar{\lambda}_t} d_t + \frac{\theta\omega\bar{\lambda}_t}{1+\bar{\lambda}_t} d_t + n_t^h \mathcal{V}_{bt} \\
&= \big(\mathcal{V}_{bt} + \lambda_t^h (\mathcal{V}_{bt} - \theta\omega)\big) n_t^h + \frac{\theta\omega(\bar{\lambda}_t - \lambda_t^h)}{1+\bar{\lambda}_t} d_t \qquad (41)
\end{aligned}
$$

From this expression we obtain the marginal value of net worth

$$
\frac{\partial V_t(\cdot)}{\partial n_t^h} = \mathcal{V}_{bt} + \lambda_t^h (\mathcal{V}_{bt} - \theta\omega)
$$

Now we plug that information into the Bellman equation (the *end-of-period-t* value, where h is the type in period $t+1$)

$$
\begin{aligned}
V(s_t, b_t, d_t) = &\,\mathrm{E}_t\, \Lambda_{t,t+1} \Big(\sum_h \pi^h \big((1-\sigma) n_{t+1}^h + \sigma \big(\mathcal{V}_{bt+1} + \lambda_{t+1}^h (\mathcal{V}_{bt+1} - \theta\omega) \big) n_{t+1}^h \big) \\
&+ \sum_h \pi^h \frac{\theta\omega(\bar{\lambda}_{t+1} - \lambda_{t+1}^h)}{1+\bar{\lambda}_{t+1}} d_t \Big), \\
= &\,\mathrm{E}_t\, \Lambda_{t,t+1} \Big(\sum_h \pi^h \big[\underbrace{1 - \sigma + \sigma \big(\mathcal{V}_{bt+1} + \lambda_{t+1}^h (\mathcal{V}_{bt+1} - \theta\omega) \big)}_{\Omega_{t+1}^h} \big] n_{t+1}^h \\
&+ \frac{\theta\omega}{1+\bar{\lambda}_{t+1}} d_t \underbrace{\sum_h \pi^h (\bar{\lambda}_{t+1} - \lambda_{t+1}^h)}_{=0} \Big) \\
= &\,\mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega_{t+1}^h n_{t+1}^h. \qquad (42)
\end{aligned}
$$

Now we can finally use the method of undetermined coefficients. We use the previous equation, the linear guess (11) and the law of motion of banks' net worth (7).

$$
\mathcal{V}_{st} s_t^h - \mathcal{V}_{bt} b_t^h - \mathcal{V}_{dt} d_t = \mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega_{t+1}^h \left(\big(Z_{t+1} + (1-\delta) Q_{t+1}^h \big) \psi_{t+1} s_t - R_{bt} b_t - R_t d_t \right),
$$

Comparing coefficients on the left and right-hand-side yield

$$
\begin{aligned}
\mathcal{V}_{st} &= \mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega_{t+1}^h \big(Z_{t+1} + (1-\delta) Q_{t+1}^h \big) \psi_{t+1}, \qquad (43) \\
\mathcal{V}_{bt} &= R_{bt} \cdot \mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega_{t+1}^h, \\
\mathcal{V}_{dt} &= R_t \cdot \mathrm{E}_t\, \Lambda_{t,t+1} \sum_h \pi^h \Omega_{t+1}^h = \frac{R_t}{R_{bt}} \mathcal{V}_{bt}.
\end{aligned}
$$

For convenience, these equations are also given in the main text as equations (15), (16) and (17).

B Proof of the proposition

We want to find out the difference between loans $Q_t^h S_t^h$ and bank liabilities (net of interbank lending) $N_t^h + \pi^h D_t$ on each island. The strategy is the following. First we rewrite total assets in terms of asset of investing islands $h = i$, second we write total liabilities in terms of liabilities on investing islands. Then we use the balance sheet equality to put a bound on the interbank flows B_t^h.

(i), by market clearing of the asset markets we know that

$$Q_t^i S_t^i = Q_t^i\big(\pi^i(1-\delta)K_t + I_t\big)$$
$$Q_t^n S_t^n = Q_t^n \pi^n (1-\delta)K_t$$

We want to write total assets in terms of assets on investing islands. Observe that

$$Q_t^n \pi^n (1-\delta)K_t = \frac{\pi^n}{\pi^i}\frac{Q_t^n}{Q_t^i} Q_t^i \pi^i (1-\delta)K_t$$

Now we complete the right-hand side to the assets on the non-investing islands

$$\underbrace{Q_t^n \pi^n (1-\delta)K_t}_{Q_t^n S_t^n} = \frac{\pi^n}{\pi^i}\frac{Q_t^n}{Q_t^i} \underbrace{Q_t^i\big(\pi^i(1-\delta)K_t + I_t\big)}_{Q_t^i S_t^i} - \frac{\pi^n}{\pi^i} Q_t^n I_t$$

So total assets can be written as

$$Q_t^i S_t^i + Q_t^n S_t^n = \left(1 + \frac{\pi^n}{\pi^i}\frac{Q_t^n}{Q_t^i}\right) Q_t^i S_t^i - \frac{\pi^n}{\pi^i} Q_t^n I_t. \tag{44}$$

(ii), we want to rewrite total liabilities in terms of liabilities of investing islands i. By equation (21), average net worth of each island can be written as

$$\frac{N_t^h}{\pi^h} = \big(Z_t + (1-\delta)Q_t^h\big)\psi_t S_{t-1} - R_{t-1}D_{t-1}$$

So we can write the difference of the two islands types as

$$\frac{N_t^n}{\pi^n} - \frac{N_t^i}{\pi^i} = (1-\delta)\psi_t S_{t-1}(Q_t^n - Q_t^i)$$

Multiplying by π^n gives

$$N_t^n - \frac{\pi^n}{\pi^i} N_t^i = \underbrace{\pi^n (1-\delta)\psi_t S_{t-1}(Q_t^n - Q_t^i)}_{=:\Delta \geq 0}$$

Note that since $Q_t^n \geq Q_t^i$ (see footnote 16) this expression will be non-negative since capital quality $\psi_t > 0$. Moreover the expression will be "small". Let us expand the left-hand side so as to get liabilities $N_t^h + \pi^h D_t$ of each type.

$$N_t^n + D_t\pi^n - \frac{\pi^n}{\pi^i} N_t^i - D_t\pi^n = \Delta$$

$$\implies N_t^n + D_t\pi^n - \frac{\pi^n}{\pi^i}(N_t^i + D_t\pi^i) = \Delta$$

$$\implies N_t^n + D_t\pi^n = \frac{\pi^n}{\pi^i}(N_t^i + D_t\pi^i) + \Delta$$

$$\implies N_t^n + N_t^i + (\pi^n + \pi^i)D_t = \left(1 + \frac{\pi^n}{\pi^i}\right)(N_t^i + D_t\pi^i) + \Delta \tag{45}$$

(iii) Now we can compare the assets and liabilities on the investing island. By the balance sheet equality we know that (44) equals (45),

$$\left(1+\frac{\pi^n}{\pi^i}\right)(N_t^i + D_t\pi^i) + \Delta = \Big(1+\frac{\pi^n}{\pi^i}\underbrace{\frac{Q_t^n}{Q_t^i}}_{\geq 1}\Big)Q_t^iS_t^i - \frac{\pi^n}{\pi^i}Q_t^nI_t$$
$$= \left(1+\frac{\pi^n}{\pi^i}\right)Q_t^iS_t^i - \frac{\pi^n}{\pi^i}Q_t^nI_t + \underbrace{\left(\frac{\pi^n}{\pi^i}\frac{Q_t^n}{Q_t^i} - \frac{\pi^n}{\pi^i}\right)Q_t^iS_t^i}_{=:\Gamma}$$
$$\iff \frac{\pi^n}{\pi^i}Q_t^nI_t + \Delta - \Gamma = \underbrace{\left(1+\frac{\pi^n}{\pi^i}\right)}_{=1/\pi^i}\big(\underbrace{Q_t^iS_t^i - (N_t^i + D_t\pi^i)}_{B_t^i}\big)$$
$$\iff \pi^nQ_t^nI_t + \pi^i(\Delta - \Gamma) = B_t^i.$$

Let us now look at Γ

$$\Gamma = \frac{\pi^n}{\pi^i}\left(\frac{Q_t^n}{Q_t^i} - 1\right)Q_t^iS_t^i = \frac{\pi^n}{\pi^i}\left(\frac{Q_t^n - Q_t^i}{Q_t^i}\right)Q_t^iS_t^i$$
$$= \frac{\pi^n}{\pi^i}(Q_t^n - Q_t^i)S_t^i$$

Now let us look at Δ

$$\Delta = \underbrace{\pi^n(1-\delta)K_t}_{S_t^n}\psi_t(Q_t^n - Q_t^i)$$
$$= \frac{\pi^n}{\pi^i}(S_t^i - I_t)\psi_t(Q_t^n - Q_t^i)$$

I used equation (22). Remains to check the sign of $\Delta - \Gamma$

$$\Delta - \Gamma = \frac{\pi^n}{\pi^i}\underbrace{(Q_t^n - Q_t^i)}_{\geq 0}\big((S_t^i - I_t)\psi_t - S_t^i\big)$$

This is non-positive if the capital quality is not too large.

$$\Delta - \Gamma \leq 0 \impliedby (S_t^i - I_t)\psi_t - S_t^i \leq 0 \iff \psi_t \leq \frac{S_t^i}{S_t^i - I_t}$$

The other implication holds with strict equalities. As long as investment is non-negative, the upper bound on ψ_t is greater than or equal to one. So, if the capital quality is at steady state level or smaller,

$$\psi_t \leq \psi^* = 1 \implies \Delta - \Gamma \leq 0 \implies \pi^nQ_t^nI_t \geq B_t^i.$$