INSTITUT FÜR HÖHERE STUDIEN
INSTITUTE FOR ADVANCED STUDIES
Vienna

TU WIEN
CONTINUING EDUCATION CENTER

**MSc Economics**

# Human Capital Depreciation and the Cyclical Volatility of Unemployment and Vacancies

A Master's Thesis submitted for the degree of
"Master of Science"

supervised by
Tamás Papp

Andreas Gulyás

0402144

Vienna, 14.6.2011

# Preface

I am indebted to Tamás Papp not only for his invaluable help, support and useful comments for this master thesis, but also for his advice during my studies at the Institute for Advanced Studies. I would also like to thank Michael Reiter and Christian Haefke for constructive comments. All remaining errors are solely my responsibility.

**MSc Economics**

# Affidavit

I, Andreas Gulyás

hereby declare

that I am the sole author of the present Master's Thesis,

Human Capital Depreciation and the Cyclical Volatility of Unemployment and Vacancies

33 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

Vienna, 14.6.2011

Signature

# Contents

# List of Figures

# List of Tables

**Abstract**

This paper extends a Diamond-Mortensen-Pissarides (DMP) model with human capital depreciation. Unemployed workers face the risk of losing human capital which must be compensated by additional on-the-job training after a successful match. Firms must bear the cost of training and thus also take into account the composition of the unemployment pool for vacancy creation. This leads to an amplification of the volatility of labor market tightness. However, the calibration and simulation of the model shows that the inclusion of human capital depreciation in a DMP model with fixed matching costs increases amplification by only a relatively small margin.

JEL Classification Numbers: E32 J32 J64

# 1 Introduction

The Diamond [1982], Mortensen [1982] and Pissarides [1985] (DMP henceforth) matching-model became the workhorse framework for modelling labor market frictions. Shimer [2005] questioned the model's ability to match the volatility of unemployment and vacancies observed in U.S. data. Shimer's parametrization implied a relatively high surplus of matches, which for a given labor productivity fluctuation leads to relatively low surplus volatility. Therefore the model can only account for about a twentieth of the U.S. labor market tightness volatility. It is now well understood that lowering the match-surplus amplifies the labor market tightness volatility. Hagedorn and Manovskii [2008] argued that Shimer's calibration of opportunity cost of unemployment considers only the unemployment benefit, whereas possible home-production and the value of leisure should also be considered. They recalibrate workers' outside options by matching cyclical responses of wages and the average profit rate to U.S. data. With the lower match surplus, their calibration is able to match the observed labor market tightness volatility. However, as Costain and Reiter [2008] noted, this approach yields counterfactual predictions in other dimensions, namely the unemployment rate becomes too sensitive to changes in unemployment benefits (UB).

Several ways have been proposed to resolve this empirical puzzle. Among others, Shimer [2004] and Hall [2005] argued that the model's inability to match the observed volatility hinges on a high degree of wage flexibility. Pissarides [2009] on the other hand showed that the quantitative predictions do not depend on the wage setting of incumbent workers, but only on the wage determination of new jobs which seems to be very flexible (see Haefke et al. [2009]). By introducing sunk fixed matching costs, Pissarides is able to increase the volatility of unemployment and vacancies, without creating unrealistic responses of the unemployment rate to changes in UB. In a recent paper, Silva and Toledo [2009b] show that Pissarides' result crucially depends on the fixed matching costs being sunk.

Pissarides makes the assumption that the matching costs are constant across workers. This assumption seems reasonable for flow vacancy costs, but not for fixed matching costs such as training costs. As reported by Barron et al. [1997] the amount of on-the-job training varies considerably. In this paper, I consider a DMP model where two types of unemployed coexist. While the first pool of unemployed maintains its human capital, the second type loses a significant part of its human capital. The main idea is similar to Pissarides [1992]. He models labor search frictions in an overlapping generations model and assumes that workers who were unable to find a job in the last period experience a decrease in productivity. With this setting the effects of a temporary labor productivity shock can persist for a long time.

The focus of this paper however is to evaluate the amplification mechanism of labor market volatilities of the DMP model if two types of unemployed coexist. I will refer to the first pool as short term unemployed, and to the second as long term unemployed. These are only labels, as both types face the same job-finding probability. If a firm that cannot observe the unemployed's state ends up with a long term unemployed, it has to pay an additional training cost, which is sunk, to make up for the loss of human capital.

By calibrating the model, I show that it is able to reconcile both a high enough volatility of labor market tightness and an empirically plausible unemployment rate response to a change in UB. However, comparative statics shows that in comparison to the standard DMP model with fixed matching costs, the incorporation of human capital depreciation does not add much to the implied volatilities. The business cycle statistics of the model reveal an even lower amplification mechanism as the long run effects computed by comparative statics. Overall, composition effects of the unemployment pool do not seem to be a driving force of labor market dynamics in this model.

The paper is organized as follows. The next section adds human capital depreciation into a DMP model. Section 3 discusses the parametrization and section 4 analyzes the elasticities obtained from comparative statics. Section 5 extends the model with labor productivity as an aggregate state and presents simulation results. Section 6 concludes.

# 2 The Model

Time is continuous. The labor market consists of a unit mass of workers and a large mass of firms. The job finding and vacancy filling process is time-consuming and costly, which is captured by a Cobb-Douglas matching function of the form $m(u,v) = Au^{\alpha}v^{1-\alpha}$. By denoting labor-market tightness as $\theta$ and exploiting the constant-returns-to-scale property, the job finding and filling rate can be written as $\lambda_w = \frac{m(u,v)}{u} = A\theta^{\alpha}$ and $\lambda_f = \frac{m(u,v)}{v} = A\theta^{1-\alpha}$, respectively.

A successful match produces a flow value $p$ with a linear production technology, until the match is exogenously separated with Poisson arrival probability $\sigma$. The firm ends up with a vacant position and the worker joins the unemployment pool $u = u_1 + u_2$, which consists of two types. The first pool $u_1$, where newly unemployed end up, has not experienced human capital depreciation. With poisson arrival rate $\tau$, they lose a significant fraction of their human capital and transit to the second pool $u_2$. This two state approach is an approximation for a continuous human capital depreciation. Independent of the state, the unemployed receive a flow compensation $b$, which can be interpreted as unemployment benefits, utility of leisure and home-production. The firm observes the unemployment state only after a successful match, which implies the same job finding probability $\lambda_w$ for both types of unemployed. The Hamilton-Jacobi-Bellman equations for the workers are given by

$$rW = w + \sigma(U_1 - W) \tag{1}$$

$$rU_1 = b + \lambda_w(W - U_1) + \tau(U_2 - U_1) \tag{2}$$

$$rU_2 = b + \lambda_w(W - U_2) \tag{3}$$

where $r$ represents the interest rate and $w$ the wage rate. Since the job-finding probabilities are the same for both types, $U_1$ and $U_2$ coincide.

Firms open vacancies which are filled with rate $\lambda_f$. Similar to Pissarides [2009], when a worker arrives, the firm has to pay a sunk training cost. However, I assume that the cost of training depends on the worker's type. It amounts to $H$ for short term unemployed and to $H(1+\phi)$ for long term unemployed. Hence, the expected training costs can be written as $H\left(1 + \frac{u_2}{u_1+u_2}\phi\right)$. In order to simplify notation, the fraction $\frac{u_2}{u_1+u_2}$ will be henceforth denoted as $\eta$. Therefore, the asset pricing equations for the value of vacancies $V$ and jobs $J$ are represented by the following equations:

$$rJ = p - w + \sigma(V - J) \tag{4}$$

$$rV = \lambda_f \left(J - V - H\left(1 + \eta\phi\right)\right) \tag{5}$$

The DMP model without flow cost of vacancies, but with fixed matching costs is in fact a special case of this model. It can be recovered by either turning off the transitions to $u_2$ (i.e. setting $\tau = 0$) or assuming no additional training costs for long term unemployed (i.e. assuming $\phi = 0$).

As in the standard DMP framework, free entry on the firm side is assumed. Firms post vacancies until the value of a new vacancy is zero. Thus in equilibrium, the free entry condition

$$rV = 0 \tag{6}$$

is fulfilled.

To close the model, Nash bargaining for the wage determination is assumed. The total match surplus $S = J - V + W - U_1$ is shared among workers and firms. Wage is renegotiated every instance and hence the fix matching costs $H$ and $H\left(1 + \phi\right)$ are sunk. Formally,

$$\beta S = W - U_1 \tag{7}$$

$$\left(1 - \beta\right) S = J - V \tag{8}$$

where $\beta$ denotes the workers' bargaining power.

Combining the definition of the total match surplus $S$ and the asset pricing equations for firms and workers yields after some manipulations

$$S = \frac{p - b}{r + \sigma + \beta\lambda_w} \tag{9}$$

From the free entry condition (6), the following expression for $S$ can be derived

$$S = \frac{H\left(1 + \phi\eta\right)}{1 - \beta} \tag{10}$$

Combining equation (9) and (10) results in the following expression for the equilibrium job-finding probability

$$\lambda_w = \frac{(p-b)(1-\beta)}{\beta H(1+\phi\eta)} - \frac{r}{\beta} - \frac{\sigma}{\beta} \tag{11}$$

Using the equilibrium job-finding probability, the wage can can be expressed as:

$$w = p - (r+\sigma) H (1+\phi\eta) \tag{12}$$

For the full characterization of the model, the law of motion for both types of unemployed is given by the differential equations

$$\dot{u}_1 = (1-u)\sigma - \lambda_w u_1 - \tau u_1 \tag{13}$$

$$\dot{u}_2 = \tau u_1 - \lambda_w u_2 \tag{14}$$

It turns out that it is advantageous to use $u$ and $\eta$ as state variables instead of $u_1$ and $u_2$. The law of motion for $u$ follows immediately from equations (13) and (14) and is given by

$$\dot{u} = (1-u)\sigma - \lambda_w u \tag{15}$$

Taking the time derivative of $\eta = \frac{u_2}{u}$ yields

$$\dot{\eta} = \frac{\dot{u}_2}{u} - \frac{u_2}{u^2}\dot{u} \tag{16}$$

which can be rewritten as

$$\dot{\eta} = \tau(1-\eta) - \eta\left(\frac{1-u}{u}\right)\sigma \tag{17}$$

This model differs in two ways from the standard DMP model. Instead of a flow vacancy cost, it assumes fixed sunk matching costs similar to Pissarides [2009]. Second, it incorporates human capital depreciation by assuming two types of unemployed. Observe that the DMP model with fixed matching costs is a special case of the model presented. It can be either recovered by turning off transitions to long term unemployment, i.e. setting $\tau = 0$ or by making training cost of long term unemployed exactly as expensive as training short term unemployed ($\phi = 0$).

# 3 Parametrization

This section describes the parametrization of the model in detail, which is then subsequently used to assess the quantitative implications of the model. The calibration is summarized in table 1. It follows in its main aspects Shimer [2005]. One unit of time corresponds to a month. The interest rate $r$ is set to 0.004, which implies an annual real interest of about 5 percent. The separation rate $\sigma$ is chosen to match the average duration of a job to that of the U.S. of two and a half year. The flow unemployment compensation $b$ is set to 0.4, the value used by Shimer [2005]. This is on the lower end of what is used in the literature and implies a relatively high surplus value with a steady state wage of about 0.95. Labor productivity $p$ and the matching function scale parameter $A$ are normalized to one. The choice of the matching function elasticity follows again Shimer [2005] and is equal to 0.72.

The four parameters $\tau$, $H$, $\phi$ and $\beta$ are left to pin down. Given $\tau$, $H$ and $\phi$, the workers' bargaining share $\beta$ is chosen such that the job finding probability $\lambda_w$ matches the observed monthly U.S. rate of 0.45.

The loss of human capital is hard to measure and hence there is little direct evidence on it. The transition rate $\tau$ and the additional cost $\phi$ of training a long term unemployed are the two key parameters in the model determining the steady state fraction of unemployed workers with a human capital loss $\eta$ and how expensive it is to train a long term unemployed in relation to short term unemployed. These two "unobserved" parameters are calibrated in the following way. I fix the transition rate $\tau$ to an arbitrary but plausible value of 1/12. This parametrization implies that it takes on average 1 year until unemployed workers experience a significant loss of their human capital. This yields a steady state fraction $\eta$ of about 0.16. This share is in the same ballpark as the share of unemployed for over 27 weeks in the U.S, with its long run average of 14 per cent[1].

The training cost parameter $\phi$ is not set directly, but instead calibrated such that a certain fraction $\chi$ of total training costs is spent on long term unemployed. The fraction is defined as training resources consumed by long term unemployed $H(1+\phi)\eta$ divided by the total training costs $H(1+\phi\eta)$. A share of training expenditures for long term unemployed that exactly equals the share of long term

[1]This fraction is computed by dividing the number of unemployed persons according to the definition of the U.S. Department of Labor by the number of unemployed for over 27 weeks. The long run average refers to the time period between January 1951 and April 2011. Time series are retrieved from the St. Louis federal reserve economic data base.

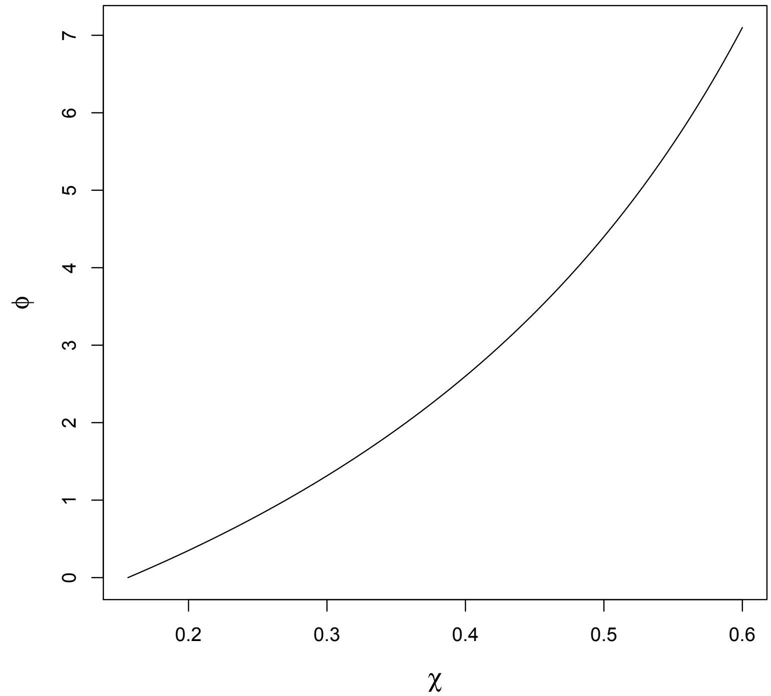


Figure 1: Training cost parameter $\phi$ in dependence of $\chi$ for $\tau = 1/12$. $\phi$ increases relatively fast with $\chi$. If the economy spends around 40 per cent of its total training costs on long term unemployed, then a long term unemployed is around 2.5 times more expensive to train than a short term. At $\chi = \eta = 0.156$, $\phi = 0$.

unemployed, i.e. $\chi = \eta$, implies that the two types are equally expensive to train ($\phi = 0$). Figure 1 plots the training cost parameter $\phi$ in dependence of $\chi$ for a fixed value of $\tau = 1/12$. In my baseline calibration $\chi$ is set to 0.25 implying that long term unemployed are 80 per cent more expensive to train.

As mentioned above, the two parameters $\tau$ and $\chi$ are not directly observable and hence hard to pin down. Therefore these two parameters are subject to an extensive sensitivity analysis in section 4, where I discuss the business cycle implications for a whole range of plausible values of $\tau$ and $\chi$.

The fixed matching cost parameter $H$ is interpreted as on-the-job training cost. Using the 1992 Small Business Administration survey (SBA) and the 1982 Employment Opportunity Pilot Project survey (EOPP), the study of Barron et al. [1997] provides direct evidence for the amount of training received by newly hired workers. In both surveys, almost every new employee receives some form of on-the-job

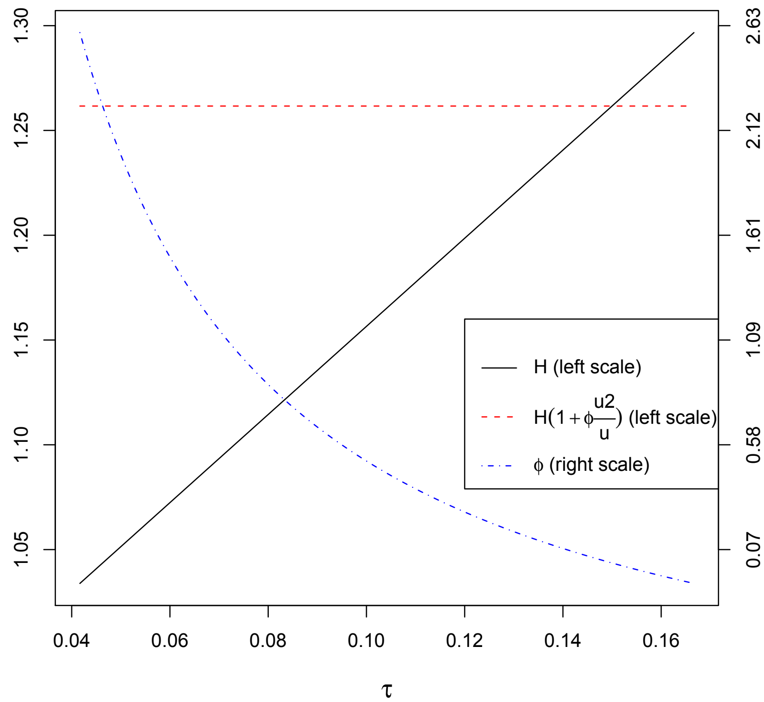


Figure 2: Calibration of training cost parameter with $\chi = 0.25$. $H$ and $\phi$ are always set such that the expected training cost $H\,(1 + \phi\eta)$ remains constant.

training (95 per cent for the EOPP data and 98 per cent for the SBA data). On average, this amounts to 142 (150) hours of training in the first three months for the EOPP (SBA) survey. The numbers of the two surveys are very similar and therefore I follow Silva and Toledo [2009a] and focus on the EOPP. With the additional 87.5 hours spent by other workers training a new hire, firms have to bear the cost of 229.5 hours of on-the-job training in the first three months. Assuming a 40 hour week, these costs amount to about 44 per cent of quarterly wage. Using this evidence, $H$ is calibrated such that average steady state training cost $H\,(1 + \phi\eta)$ is equal to 44 per cent of the steady state quarterly wage. Figure 2 illustrates the calibration of training costs. An increase of $\tau$ increases the share of long term unemployed and hence the fraction $\eta$. $H$ and $\phi$ are set such that the average training cost is constant for every $\tau$.

The next two chapters use this parametrization to assess the business cycle implications of this model.

| Parameters | | Value | Target Equation | Source |
|---|---|---|---|---|
| $\sigma$ | Separation rate | 0.033 | | Shimer [2005] |
| $b$ | Unemployment compensation | 0.4 | | Shimer [2005] |
| $r$ | Interest rate | 0.004 | | Shimer [2005] |
| $p$ | Labor productivity | 1 | | Normalization |
| $A$ | Matching function scale | 1 | | Normalization |
| $\alpha$ | Matching function elasticity | 0.72 | | Shimer [2005] |
| $\tau$ | Poisson rate of human capital loss | 0.083 | | Definition |
| | | | | |
| $H$ | Basic Training cost parameter | 1.121 | $(1+\phi\eta)\,H = \frac{229.5}{520} \times 3 \times \bar{w}$ | Barron et al. [1997] |
| $\phi$ | Additional training cost for $u_2$ | 0.800 | $\chi = \frac{H(1+\phi)\eta}{H(1+\phi\eta)} = 0.25$ | Definition |
| $\beta$ | Workers' bargaining share | 0.473 | $\lambda_w = 0.45$ | Shimer [2005] |
| | | | | |
| Implied values | | | | |
| $u_1$ | Short term unemployment | 0.058 | | |
| $u_2$ | Long term unemployment | 0.011 | | |
| $\varepsilon_{\theta p}$ | Elasticity of $\theta$ with respect to $p$ | 7.862 | | |
| | $(\tau = 0,\ \phi = 0)$ | (7.000) | | |
| $\frac{d\log(u)}{db}$ | Semi-elasticity of $u$ with respect to $b$ | 2.050 | | |
| | $(\tau = 0,\ \phi = 0)$ | (1.824) | | |

Table 1: Parametrization. In absence of direct evidence, the parameters $\tau$ and $\chi$ are hard to pin down. This table provides the baseline calibration which is used for the model simulation. Section 4 discusses in detail a sensitivity analysis for a whole range of plausible values for $\tau$ and $\chi$.

# 4 Comparative Statics

It is well understood that lowering the match surplus in the standard DMP model amplifies volatility of labor market tightness. On the other hand, as noted by Costain and Reiter [2008], this leads to an unrealistic sensitivity of the unemployment rate to changes in UB. The focus of this section will be to investigate to what extent the introduction of human capital depreciation to a DMP model with fixed matching costs affects the implications along these two dimensions. Using comparative statics, this chapter explores the long run behavior of the model to changes in $p$ and $b$ and section 5 discusses the simulation results in detail, which gives more insights into the business cycle dynamics of the model.

My model differs in two respects from the standard DMP model. First, instead of a flow vacancy cost, matching costs are assumed to be fixed and sunk. Flow vacancy costs dampen the effect of labor productivity shocks through the externality generated by a change in vacancies. The expected hiring cost for the firm $c/\lambda_f(\theta)$ is increasing in labor market tightness $\theta$. Hence, a rise in labor market tightness makes hiring more expensive and reduces the effect of labor productivity shocks on tightness. With matching costs being fixed, this feedback effect is turned off in my model. Pissarides [2009] and Mortensen and Nagypal [2007] have shown that fixed hiring costs in a DMP framework can generate sufficiently high labor market tightness volatility.

Second, human capital depreciation is introduced via the two different types of unemployed. The key question in this paper is the role of human capital depreciation on the volatility of labor market tightness. Notice that if either $\tau = 0$ and/or $\phi = 0$ (equivalent to $\chi = \eta$), we are back in the standard DMP model with fixed matching costs.

The calibration of the fixed matching cost makes sure that the steady state average matching costs are independent of $\tau$ (see figure 2). Therefore, the model features the same average steady state matching costs as the model without human capital depreciation ($\tau = 0$). The bottom of table 1 reports the elasticity of labor market tightness and the semi-elasticity of unemployment with respect to $b$ for the model with human capital depreciation (baseline calibration) and the model without ($\tau = 0$ and $\phi = 0$).

As already argued by Pissarides [2009] and Mortensen and Nagypal [2007], the standard DMP model with fixed matching costs ($\phi = 0$) generates an elasticity of about 7, which is near to the 7.56 observed in U.S. data. The baseline calibration with a share of training costs spent on long term unemployed $\chi = 0.25$ amplifies the labor market tightness volatility to 7.9. The intuition of the amplification mechanism is easy to see and is depicted in figure 3. The increase in hiring raises the job finding probability and hence decreases the average unemployment spell. Therefore, fewer unemployed lose human capital (i.e. less unemployed transit to long term unemployment), which improves the composition of the unemployment pool. Through lower average training cost because of a lower $\eta$, hiring becomes even more attractive. As the upper panel of figure 4 shows, this effect is even stronger for higher $\chi$ (and hence higher $\phi$) holding $\tau$ fixed. This is because a higher $\chi$ puts a higher weight on the training costs of long term unemployed. The figure highlights that a $\chi$ larger than approximately 0.22 is able to generate volatilities observed in U.S. data.

This improvement of the model in the dimension of the elasticity of labor market tightness comes at a cost though. Exactly the same mechanism as in the tightness case increases the semi-elasticity of the unemployment rate with respect to $b$. An increase in $b$ decreases the match surplus, which increases the average unemployment length and hence worsens the unemployment pool. The lower panel of figure 4 plots the positive relationship between the semi-elasticity and $\chi$, while holding $\tau$ at 1/12. Although the baseline calibration implies a plausible value for the response of unemployment to a change in UB, higher values of $\chi$ yield numbers well above the empirically realistic value of 2, estimated by Costain and Reiter [2008].

As discussed in section 3, because of missing evidence on human capital depreciation and training costs, the parameters $\tau$ and $\chi$ are hard to pin down. Figure 5 shows the $(\tau, \chi)$ combinations that are able to reconcile both, empirically plausible values for the elasticity of labor market tightness and the semi-elasticity of unemployment. It depicts the $\varepsilon_{\theta p} = 7.56$ and $\varepsilon_{ub} = 2$ isoclines in $(\tau, \chi)$ space. The model yields roughly similar elasticities if $\chi$ is a certain multiple of $\tau$ ($\chi$ must be approximately $3\tau$). It can be seen that the baseline parametrization is roughly in line with this finding.

The results show that for $\tau$ and $\chi$ values which imply empirically plausible implications, the extension of the standard DMP model with human capital depreciation does not add much to the model. The somewhat higher implied elasticities could also have been achieved by the DMP model with fixed matching costs with a slight modification of the parametrization. The key to understand why the incorporation of human capital depreciation fails to amplify the volatilities by much lies in the model's elasticity of the share of long term unemployed $\eta$ with respect to labor productivity $p$. In order to derive this elasticity, consider the dynamic equation (13) for $u_2$ with $u - u_2$ substituted for $u_1$:

$$\dot{u}_2 = (u - u_2)\tau - \lambda_w u_2 \tag{18}$$

The steady state relationship of this equation is given by

$$0 = \tau - (\lambda_w(\eta) + \tau)\,\eta \tag{19}$$

Substituting the equilibrium job finding probability from equation (11) into this expression and taking the total derivative yields after some manipulations

$$\hat{\eta} = \frac{(1-\beta)\,p}{H\left[(r+\sigma)\,\phi\eta - \tau\,(1+\phi\eta)\,\beta - \lambda_w\beta\right]}\hat{p} \tag{20}$$

where $\hat{\eta} = \frac{d\eta}{\eta}$ and $\hat{p} = \frac{dp}{p}$.

This elasticity is equal to -1.857 in the baseline calibration, which implies that the fraction $\eta$ does not change by much if $p$ changes. Consider a two standard deviation shock to labor productivity of 0.04[2]. This increase in $p$ decreases the share of long term unemployed only by 1.17 percentage points. That change lowers the expected training costs $H(1+\phi\eta)$ only by less than 1 per cent. Even this effect is dampened by a rise in the wage rate, which is increasing in $\eta$. The later effect though is negligible for small values of $r$ and $\sigma$. Figure 3 shows that for labor productivities in the range of $\pm 2$ standard deviations, $\eta$ and the expected training costs only vary between [0.145,0.169] and [1.252,1.273], respectively.

The next chapter analyzes whether the short run implications of this model differ from the long run effects found by comparative statics.

[2]see table 2 for a summary of U.S. labor market data

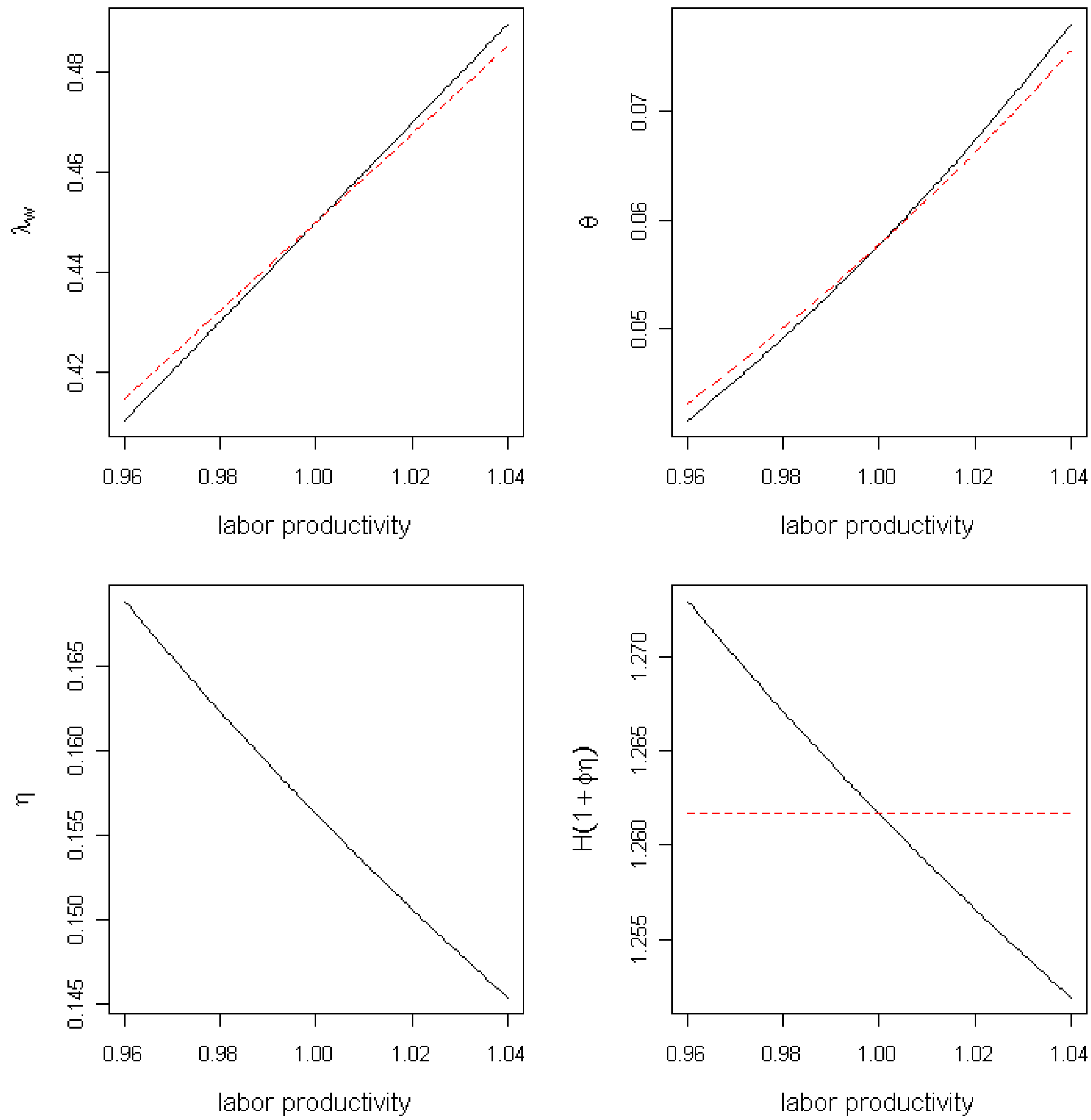


Figure 3: Comparative statics (in log deviations from $p = 1$). The black solid line depicts the DMP model with human capital depreciation and the red dashed line indicates the standard DMP model with fixed matching costs ($\tau = 0$ and $\phi = 0$).

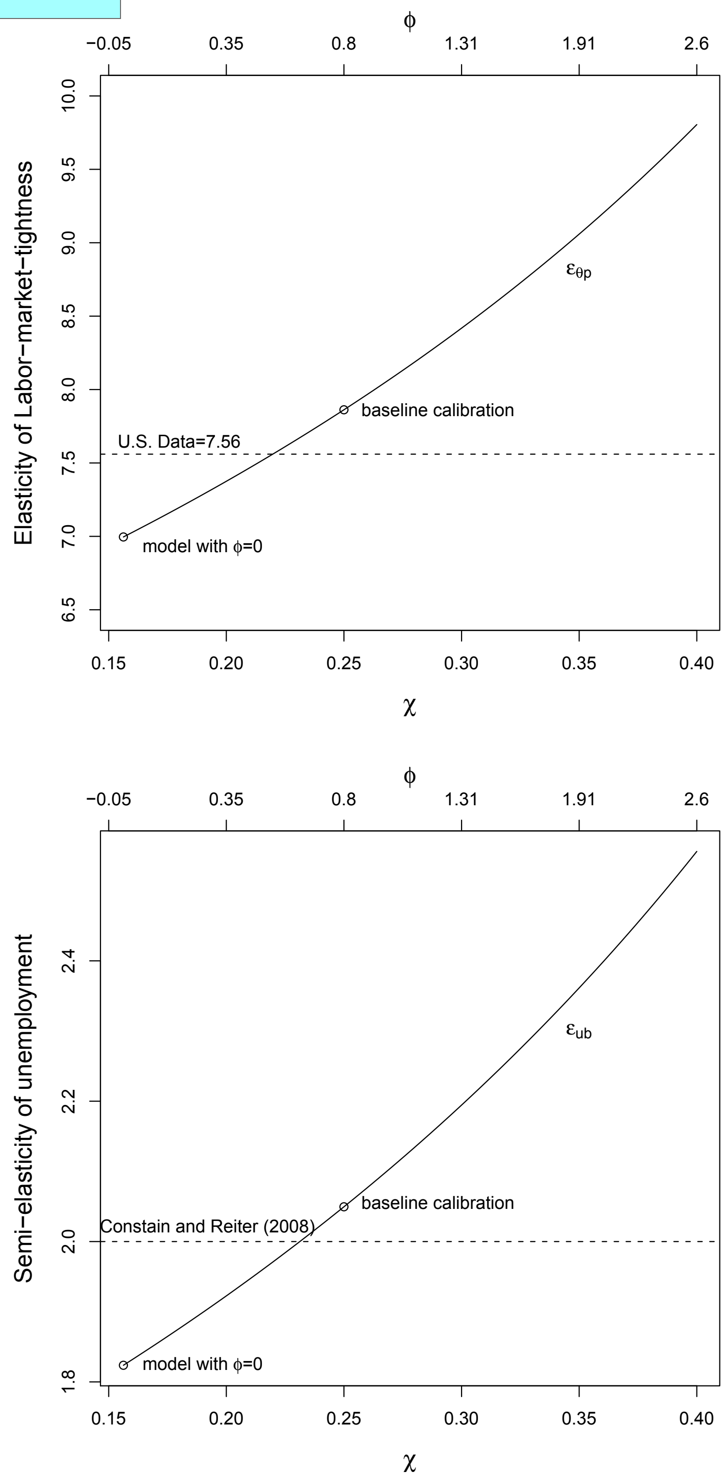


Figure 4: Elasticity of labor market tightness (upper panel) and semi-elasticity of unemployment with respect to $b$ ($\tau = 1/12$). The dashed lines indicate the observed elasticity observed in US Data (Pissarides [2009]) and the semi-elasticity of unemployment reported by Costain and Reiter [2008]. The baseline calibration of $\chi = 0.25$ implies elasticities near to the empirically plausible values.

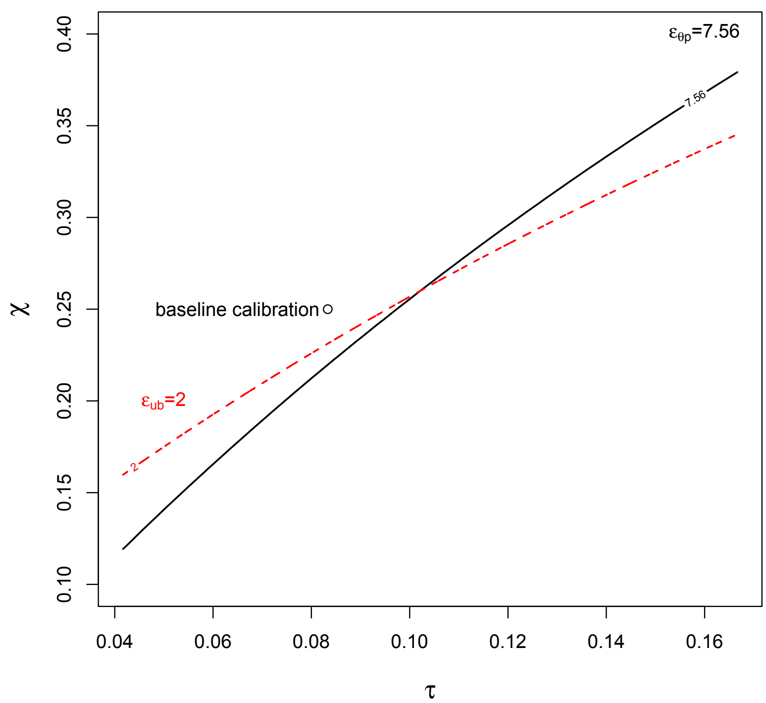


Figure 5: Tightness and unemployment elasticity for different $(\tau, \chi)$ combinations. The figure depicts the $\varepsilon_{\theta p} = 7.56$ (black solid line) and $\varepsilon_{ub} = 2$ (red dashed line) isocline. The model implies roughly similar volatilities for all combinations $(\tau, \chi)$ where $\tau$ is a certain fraction of $\chi$.

# 5 Simulation

In order to simulate the model and obtain business cycle statistics, the model is extended with $p$ as an aggregate state. The derivation is straight forward and is shown in the appendix. I assume that $\log p$ follows an Ornstein-Uhlenbeck process of the form

$$d\log(p) = \rho\log(p)\,dt + s dz \tag{21}$$

with mean reversion parameter $\rho$ and innovation standard deviation $s$ and where $z$ denotes the Wiener process.

The parameters $\rho$ and $s$ are calibrated to match the cyclical volatility (0.02) and the autocorrelation (0.878) of average U.S. labor productivity, reported by Shimer [2005], which yields $\rho = 0.034$ and $s = 0.007$.

As shown in the appendix the model can be written in the form

$$\begin{bmatrix} \dot{u}(t) \\ \dot{\eta}(t) \end{bmatrix} = F\left(\begin{bmatrix} u(t) \\ \eta(t) \\ \log(p(t)) \end{bmatrix}\right) \tag{22}$$

The model is simulated by a discrete approximation of equation (21) and (22). Formally

$$\begin{bmatrix} u(t+\Delta) \\ \eta(t+\Delta) \end{bmatrix} = \begin{bmatrix} u(t) \\ \eta(t) \end{bmatrix} + F\left(\begin{bmatrix} u(t) \\ \eta(t) \\ \log(p(t)) \end{bmatrix}\right)\cdot\Delta \tag{23}$$

and

$$\log p(t+\Delta) = \log p(t) + \rho\log p(t)\,\Delta + s\sqrt{\Delta}\epsilon_t \tag{24}$$

where the step size is one week ($\Delta = 4$) and $\epsilon_t \sim N(0,1)$ is assumed to be iid.

The computation of the business cycle statistics follows Shimer [2005]. The model is simulated 10,000 times for 1,212 quarters, where the first 1,000 data points are burned in. The logarithm is taken of the resulting time series and the standard deviations of the cyclical components from a HP-trend with smoothing parameter $10^5$ are computed. The model is simulated for the baseline calibration of table 1 and for the standard DMP model with fixed matching costs ($\tau = 0$ and $\phi = 0$) which are reported in table 3 and table 4, respectively. As a comparison, table 2 summarizes U.S. labor market statistics calculated by Shimer [2005].

The simulation results confirm the results from the comparative statics calcula-

| | $u$ | $v$ | $\theta$ | $\lambda_w$ | $p$ |
|---|---|---|---|---|---|
| Standard deviation | 0.190 | 0.202 | 0.382 | 0.118 | 0.020 |
| Quarterly autocorrelation | 0.936 | 0.940 | 0.941 | 0.908 | 0.878 |
| | Correlation matrix | | | | |
| $u$ | 1.000 | -0.894 | -0.971 | -0.949 | -0.408 |
| $v$ | | 1.000 | 0.975 | 0.897 | 0.364 |
| $\theta$ | | | 1.000 | 0.948 | 0.396 |
| $\lambda_w$ | | | | 1.000 | 0.396 |
| $p$ | | | | | 1.000 |

Table 2: Summary Statistics, Quarterly U.S. Data 1951-2003 (Source: table 1 Shimer [2005]).

| | $u$ | $v$ | $\theta$ | $\lambda_w$ | $p$ | $\eta$ |
|---|---|---|---|---|---|---|
| Standard | 0.039 | 0.117 | 0.154 | 0.043 | 0.020 | 0.035 |
| deviation | (0.005) | (0.015) | (0.021) | (0.006) | (0.003) | (0.005) |
| Quarterly | 0.933 | 0.878 | 0.900 | 0.900 | 0.878 | 0.946 |
| autocorrelation | (0.016) | (0.029) | (0.024) | (0.024) | (0.029) | (0.013) |
| | Correlation matrix | | | | | |
| $u$ | 1.000 | -0.952 | -0.973 | -0.973 | -0.951 | 0.983 |
| | | (0.011) | (0.006) | (0.006) | (0.011) | (0.004) |
| $v$ | | 1.000 | 0.997 | 0.997 | 0.999 | -0.886 |
| | | | (0.001) | (0.001) | (0.000) | (0.026) |
| $\theta$ | | | 1.000 | 1.000 | 0.996 | -0.919 |
| | | | | (0.000) | (0.001) | (0.019) |
| $\lambda_w$ | | | | 1.000 | 0.996 | -0.919 |
| | | | | | (0.001) | (0.019) |
| $p$ | | | | | 1.000 | -0.883 |
| | | | | | | (0.026) |
| $\eta$ | | | | | | 1.000 |

Table 3: Simulation results with baseline calibration (see table 1). The numbers report the deviations of the log-series from a HP-trend with smoothing parameter $10^5$. Bootstrapped standard errors across the 10,000 model simulations are shown in parentheses.

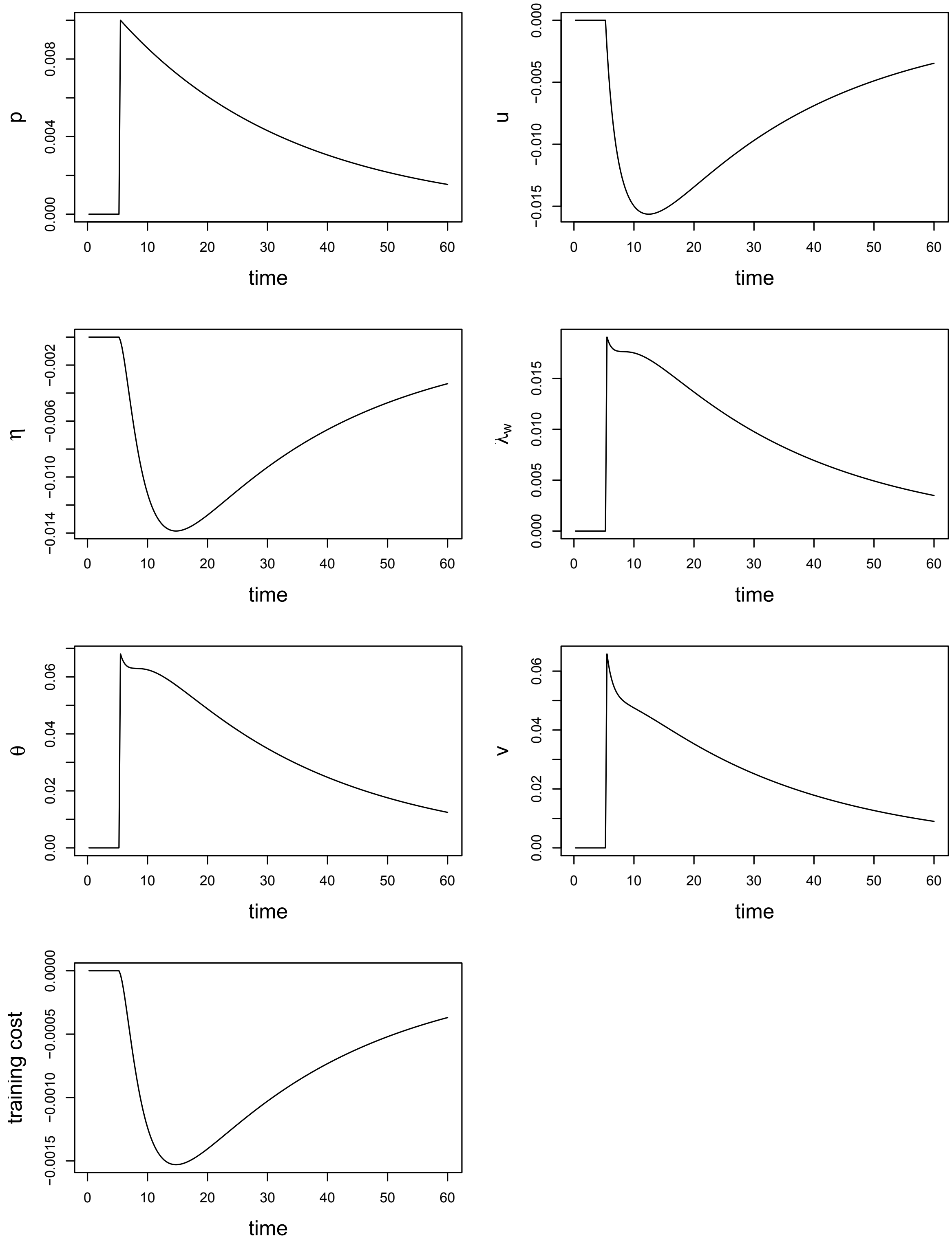


Figure 6: Impulse response to a one standard deviation labor productivity shock for the DMP model with human capital depreciation (Time period is one month)

| | $u$ | $v$ | $\theta$ | $\lambda_w$ | $p$ |
|---|---|---|---|---|---|
| Standard | 0.036 | 0.108 | 0.142 | 0.040 | 0.020 |
| deviation | (0.005) | (0.013) | (0.018) | (0.005) | (0.003) |
| Quarterly | 0.919 | 0.846 | 0.875 | 0.875 | 0.878 |
| autocorrelation | (0.020) | (0.035) | (0.029) | (0.029) | (0.029) |
| | | | Correlation matrix | | |
| $u$ | 1.000 | -0.942 | -0.967 | -0.967 | -0.967 |
| | | (0.013) | (0.008) | (0.008) | (0.008) |
| $v$ | | 1.000 | 0.996 | 0.996 | 0.996 |
| | | | (0.001) | (0.001) | (0.001) |
| $\theta$ | | | 1.000 | 1.000 | 1.000 |
| | | | | (0.000) | (0.000) |
| $\lambda_w$ | | | | 1.000 | 1.000 |
| | | | | | (0.000) |
| $p$ | | | | | 1.000 |

Table 4: Simulation Results for model with $\tau = 0$ and $\phi = 0$. The numbers report the deviations of the log-series from a HP-trend with smoothing parameter $10^5$. Bootstrapped standard errors across the 10,000 model simulations are shown in parentheses.

tions. The model with human capital depreciation is able to amplify the volatility of labor market tightness to a multiple of about 7.56 of the productivity's standard deviation[3]. But the comparison to the business cycle statistics of the standard DMP model with fixed matching costs shows that the effects of an inclusion of human capital depreciation are minor. The volatility changes from 0.142 to 0.154, an increase of about 8 per cent. The amplification is even smaller as found with comparative statics. The reason behind this is the propagation of a labor productivity shock. Figure 6 depicts the impulse response of the model's variables to a labor productivity shock. The job finding probability immediately reacts to a change in productivity and thus the unemployment rate and the average unemployment spell start to fall. Since the transition to the second state of unemployment (loss of human capital) takes in my calibration on average one year, the full effect on the fraction $\eta$ needs some time to materialize. As seen in the impulse response, $\eta$ lags the unemployment rate by about one quarter. Thus, the short run effect is smaller than the long run

[3]According to Shimer [2005], the ratio of the standard deviations should be 19.1. I follow Pissarides [2009], who argues that this is only true if there were no measurement errors, and no other shocks to labor market tightness. By accounting for this, he sets the target 7.56, which would be the regression coefficients of regressing productivity on tightness.

effect computed in the comparative statics section.

The other relationships are not much affected by the incorporation of human capital depreciation. It is not able to break the strong link between the job finding probability and productivity. The correlation between these two variables remains about three times as strong as observed in U.S. data. With the assumption of exogenous separations, the good resemblence of the Beveridge curve is not surprising.

# 6 Conclusion

In this paper I relax the assumption of Pissarides [2009] that the fixed matching costs are constant across all workers. In my model, two types of workers coexist, which need different amounts of on-the-job training. In contrast to the first type of workers without loss of human capital, workers who experience human capital depreciation are assumed to require additional on-the-job training. The unemployed's type is only revealed after a successful match. Therefore upon hiring, firms do not only take into account the current productivity level, but also expected training costs and thus the composition of the unemployment pool.

As shown in this study, the incorporation of human capital depreciation into a DMP model helps to increase the volatility of labor market tightness. The intuition for the amplification mechanism is easy to see. A positive labor productivity shock makes hiring more attractive and hence increases the job finding probability. This does not only lower the unemployment rate, but also fewer unemployed lose human capital and thus the share of long term unemployed decreases. The lower expected training costs induce firms to increase their hiring even more and thus amplifies the volatility of labor market tightness.

On the other hand, my model increases also the response of unemployment to a change of UB, although the calibration yields an empirically plausible response near to the one found by Costain and Reiter [2008].

The comparison to a standard DMP model with fixed matching costs shows that the inclusion of human capital depreciation does not amplify the volatility by much. The elasticity of labor market tightness, a broadly used measure for its volatility, is only about 12 per cent higher than in the model without human capital depreciation. The short run effects are even smaller. The simulation of the model shows that the ratio of the standard deviations of tightness and productivity gets only amplified by a mere 8 per cent. This difference between short and long run is due to the relatively slower response of the share of long term unemployed.

In this given setting, composition effects of the unemployment pool do not seem to be a driving force of labor market dynamics. However, I made several crucial and counter-factual assumptions for my analysis. First, I assumed that firms do not observe any information from the worker, which implies that both types of unemployed face the same job finding probability. In data we observe that the job finding probability decreases with unemployment spell[4]. Therefore it seems likely that firms discriminate on the basis of the length of unemployment [5].

Second, in a recent paper Silva and Toledo [2009b] show that if a part of the fixed matching cost can be passed on to workers by firms, i.e. a share of the fixed costs becomes non-sunk, the amplification of labor market tightness volatility is significantly dampened. The results of my model are probably also prone to this critic. If firms are able to shift some of the higher training cost for long term unemployed to workers, the composition of the unemployment pool becomes less important.

The relaxation of these assumptions remains for future research.

[4]see Machin and Manning [1999] for a discussion about long term unemployment.

[5]Blanchard and Diamond [1994] studied a model in which firms fill the vacancy with the applicant with the shortest unemployment spell.

# References

J.M. Barron, M.C. Berger, and D.A. Black. On-the-job training. *Books from Upjohn Press*, 1997.

O.J. Blanchard and P.A. Diamond. Ranking, unemployment duration, and wages. *Review of Economic Studies*, 61(3):417–34, 1994.

J.S. Costain and M. Reiter. Business cycles, unemployment insurance, and the calibration of matching models. *Journal of Economic Dynamics and Control*, 32 (4):1120–1155, 2008.

P.A. Diamond. Wage determination and efficiency in search equilibrium. *The Review of Economic Studies*, 49(2):217, 1982.

C. Haefke, M. Sonntag, T. Van Rens, and Universitaet Kiel. Institut fuer Weltwirtschaft. *Wage rigidity and job creation.* Institute for the World Economy, 2009.

M. Hagedorn and I. Manovskii. The cyclical behavior of equilibrium unemployment and vacancies revisited. *The American Economic Review*, 98(4):1692–1706, 2008.

R.E. Hall. Employment fluctuations with equilibrium wage stickiness. *The American Economic Review*, 95(1):50–65, 2005.

S. Machin and A. Manning. The causes and consequences of longterm unemployment in europe. *Handbook of labor economics*, 3:3085–3139, 1999.

D.T. Mortensen. The matching process as a noncooperative bargaining game. *NBER Chapters*, pages 233–258, 1982.

D.T. Mortensen and E. Nagypal. More on unemployment and vacancy fluctuations. *Review of Economic Dynamics*, 10(3):327–347, 2007.

C.A. Pissarides. Short-run equilibrium dynamics of unemployment, vacancies, and real wages. *The American Economic Review*, 75(4):676–690, 1985.

C.A. Pissarides. Loss of skill during unemployment and the persistence of employment shocks. *The Quarterly Journal of Economics*, 107(4):1371, 1992.

C.A. Pissarides. The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica*, 77(5):1339–1369, 2009.

R. Shimer. The consequences of rigid wages in search models. *Journal of the European Economic Association*, 2(2-3):469–479, 2004.

R. Shimer. The cyclical behavior of equilibrium unemployment and vacancies. *The American Economic Review*, 95(1):25–49, 2005.

J.I. Silva and M. Toledo. Labor turnover costs and the behavior of vacancies and unemployment. *Macroeconomic Dynamics*, 13(supplement S1):76–96, 2009a.

J.I. Silva and M.E. Toledo. The unemployment volatility puzzle: The role of matching costs revisited. *MPRA Paper*, 2009b.

# A Appendix

This appendix shows the derivation of the model extension with $p$ as an aggregate state. As stated previously, $\log p$ is assumed to follow an Ornstein-Uhlenbeck process of the form

$$d\log(p) = \rho\log(p)\,dt + sdz \tag{25}$$

where $z$ is the Wiener process.

The Hamilton-Jacobi-Bellman (HJB) equations for the workers are now

$$\begin{aligned} rW(p,u,\eta) &= w(p,u,\eta) + \sigma(U_1(p,u,\eta) - W(p,u,\eta)) \\ &\quad + \mathcal{A}W(p,u,\eta) && (26) \\ rU_1(p,u,\eta) &= b + \lambda_w(p,u,\eta)(W(p,u,\eta) - U_1(p,u,\eta)) \\ &\quad + \tau(U_2(p,u,\eta) - U_1(p,u,\eta)) + \mathcal{A}U_1(p,u,\eta) && (27) \\ rU_2(p,u,\eta) &= b + \lambda_w(p,u,\eta)(W(p,u,\eta) - U_1(p,u,\eta)) \\ &\quad + \mathcal{A}U_1(p,u,\eta) && (28) \end{aligned}$$

where $\mathcal{A}$ is the generator operator, e.g. $\mathcal{A}W = W_p\rho\log(p) + \frac{1}{2}W_{pp}s^2 + W_u\dot{u} + W_\eta\dot{\eta}$.

The HJB equations for the firm side are given by:

$$\begin{aligned} rJ(p,u,\eta) &= p - w(p,u,\eta) + \sigma(V(p,u,\eta) - J(p,u,\eta)) \\ &\quad + \mathcal{A}J(p,u,\eta) && (29) \\ rV(p,u,\eta) &= \lambda_f(J(p,u,\eta) - V(p,u,\eta) - H(1+\phi\eta)) \\ &\quad + \mathcal{A}V(p,u,\eta) && (30) \end{aligned}$$

where $u$ and $\eta$ evolve according to the following differential equations:

$$\begin{aligned} \dot{u} &= (1-u)\sigma - \lambda_w u && (31) \\ \dot{\eta} &= (1-\eta)\tau - \eta\left(\frac{1-u}{u}\right)\sigma && (32) \end{aligned}$$

For notational simplicity I drop the states. The surplus is again given by

$$S = W - U + J - V \tag{33}$$

As in the standard model, Nash bargaining and the free entry condition are assumed to hold. Thus

$$\beta S = W - U_1 \tag{34}$$

$$(1-\beta) S = J - V \tag{35}$$

$$rV = 0 \tag{36}$$

Using the workers' and firm's HJB equations (26), (27), (28), (29), (30) together with the Nash bargaining equations (34), (35) and the free entry condition (36) one can derive the following equation for $S$

$$S\left(r + \beta\lambda_w + \sigma\right) = p - b + \mathcal{A}S \tag{37}$$

where $\mathcal{A}S = S_p \rho \log(p) + \frac{1}{2} S_{pp} s^2 + S_u \dot{u} + S_\eta \dot{\eta}$. The free entry condition (36) yields

$$S = \frac{H(1+\phi\eta)}{1-\beta} \tag{38}$$

From equation (38) follows immediately that

$$S_p = S_{pp} = S_u = 0 \tag{39}$$

and

$$S_\eta = \frac{H\phi}{1-\beta} \tag{40}$$

Substituting equations (38)-(40) into (38), and solving for the equilibrium job finding rate $\lambda_w$ yields

$$\lambda_w = \frac{(p-b)(1-\beta)}{\beta H(1+\phi\eta)} - \frac{r+\sigma}{\beta} + \frac{\phi}{\beta(1+\phi\eta)} \underbrace{\left[(1-\eta)\tau - \eta\left(\frac{1-u}{u}\right)\sigma\right]}_{=\dot{\eta}} \tag{41}$$