**MSc Economics**

# Endogenous Technical Progress and Business Cycles

A Master's Thesis submitted for the degree of
"Master of Science"

supervised by

Michael Reiter

Gregor Lüschen

1127071

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INSTITUT FÜR HÖHERE STUDIEN
INSTITUTE FOR ADVANCED STUDIES
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I, Gregor Lüschen

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Endogenous Technical Progress and Business Cycles

56 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

Vienna, June, 8, 2013

Signature

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# Abbreviations

| | |
|---|---|
| **BEA** | Bureau of Economic Analysis |
| **BGP** | Balanced Growth Path |
| **NSF** | National Science Foundation |
| **R&D** | Research and Development |
| **RBC** | Real Business Cycle |
| **TFP** | Total Factor Productivity |
| **USPTO** | United States Patent and Trademark Office |

# *Abstract*

In this paper, I discuss a real business cycle model featuring endogenous technical progress. In the model, the high-frequency fluctuations of the conventional business cycle are driven through a shock to wage markups. Through procyclical variations in research and development expenditures as well as in adoption expenditures they are propagated into a persistent medium-term cycle. I evaluate the model by comparing moments of artificial data generated by the model economy to the actual data moments. I put special interest on the behavior of variables at the heart of the endogenous technology mechanism. I find that the model performs well overall. The behavior of some technology variables implied by the model is somehow at odds with the data, though the discrepancies are not very robust. In particular, the model cannot account for the Granger causality of adoption on output, which might point to the presence of a supply shock directly affecting adoption.

# Chapter 1

# Introduction

Models of endogenous technical progress can be characterized as model economies that give rise to sustained economic growth in the presence of time-stationary exogenous productivity and preferences. In these models, long-run economic growth depends on individual decisions and the resources devoted to growth-enhancing activities. Prominent examples of endogenous technology models include Lucas (1988) and Romer (1990).

The traditional real business cycle literature originating from Kydland and Prescott (1982) uses a variant of the standard neoclassical growth model with endogenous labor supply. In this model, output per capita follows a random walk with drift where the drift is caused through exogenous technical progress. The fluctuations of the business cycle are driven by shocks to technology, that cause transient cyclical fluctuations in the level of economic activity about the trend. Thus, the model generally allows for the joint analysis of growth and cycles but denies an interdependence of the two phenomena. Models of endogenous technical progress allow for a persistent impact of transitory stochastic innovations on the level of economic activity.

An interesting applications of endogenous technology models in real business cycle analysis is for instance the analysis of lasting impacts of high-frequency fluctuations. Stadler (1990) uses a very simple framework of learning-by-doing to analyze the impact of cycles on long-run growth. In his model, workers accumulate knowledge during production. The state of knowledge and productivity hence depends on the accumulated level of past economic activity and transitory decreases in productivity generate lasting negative impacts on the economy. Comin and Gertler (2006) show that the US economy was characterized through medium-frequency oscillations of considerable size and length they refer to as the medium-term cycle. They use an endogenous growth model based on Romer (1990) that generates such fluctuations as a persistent response to the high-frequency fluctuations of the conventional business cycle.

The insights that endogenous technology models and other endogenous growth models can deliver on the dependence between cycles and long run behavior could also be of interest for policy advice. Based on the traditional real business cycle model, the government should abstain from stabilizing the economy, since its response is the optimal reaction to a change in the level of technology. Using a learning-by-doing model, Martin and Rogers (2000) show that if future benefits of learning-by-doing are not fully internalized, a stabilization policy is optimal even in the absence of nominal rigidities. Based on a stochastic AK-model with diminishing returns to investment, Barlevy (2004) revisits the question of the welfare cost of cycles. He finds that cycles might be associated with significant welfare costs, not originating from the variations in consumption per se, but from their impact on the long-run growth rate.

In the remainder of this paper, I will attempt a critical review of Comin and Gertler (2006). Chapter 2 gives an overview over their model. Chapter 3 discusses the calibration and solution method. In chapter 4, I evaluate the model by comparing moments of artificial data generated by the model economy with the moments of the actual data. In particular, I evaluate the model against data on technology variables that are at the heart of the model mechanism but are not considered by Comin and Gertler. Chapter 5 concludes.

# Chapter 2

# Model

Comin and Gertler (2006) point out, that the US economy has experienced strong medium-frequency oscillations between periods of robust growth and relative stagnation, that common filtering techniques used in identifying the business cycle assign to the trend. In view of the fact that periods of robust growth are marked by few if any conventional recessions, they postulate a causal relationship between the high-frequency variations of the conventional cycle and the medium-frequency oscillations and attempt to model them jointly. They refer to variations at frequencies of 50 years or less as the medium-term cycle and to those variations at frequencies of eight years or less as the conventional business cycle. In the data, they are able to document that the medium-term cycle is characterized by strong and cyclical variations in both, embodied and disembodied productivity, suggesting that productivity plays a crucial role. Further, they show that research and development (R&D) expenditures are procyclical over both, the conventional and the medium-term cycle. This motivates the use of an expanding variety model based on Romer (1990) in which the cyclical response of R&D and adoption propagates high-frequency variations through persistent effects on the state of technology and productivity.

The model features two final good sectors producing a capital and a consumption good. In each sector, innovators develop new technologies that are implemented by adopters and subsequently marketed by monopolistically competitive intermediate good producers. As the number of implemented intermediate input varieties

increases, final goods production becomes more efficient. A higher pace of innovation in the capital goods sector drives down the relative price of capital and allows the model to capture embodied technical change.
The model fully endogenizes the movements of productivity and technology over the cycle. The only source of uncertainty are stochastic innovations to the wage markup demanded by households. At the high frequency, an increase in wage markups causes firm exit, increasing markups and decreasing capital utilization rates in final good markets which is reflected in procyclical variations in embodied and disembodied productivity. It also causes R&D and adoption expenditures to decline and thereby has a persistent depressing effect on the state of technology, allowing the model to generate a persistent medium-term cycle.

## 2.1 Agents

### Households

There is a continuum of households of measure unity indexed by h. The households maximize the following lifetime utility function, which is seperable in consumption, $C_t$ and hours worked $L_t^h$

$$E_t \quad \sum_{i=0}^{\infty} \beta^i \left[ \log C_{t+i} - \frac{\left(L_{t+i}^h\right)^{1+\zeta}}{1+\zeta} \right] \tag{2.1}$$

where $\beta$ is the discount factor and $\zeta$ the inverse of the Frisch labor supply elasticity. They can save by investing into physical capital or making one-period loans to innovators and adopters. Let $B_t$ denote loans granted in period $t-1$ that are payable at t and $\Pi_t$ profits from monopolistic competitors. Household j faces the budget constraint

$$C_t + B_{t+1} + P_t^K K_{t+1} + T_t = W_t^h L_t^h + \Pi_t + \left[D_t + P_t^K\right] K_t + R_t B_t \tag{2.2}$$

where the consumption good is chosen as the numeraire. Each household supplies a differentiated type of the labor input and enjoys market power in the labor market. The labor demand faced by household h is given by

$$L_t^h = \left(\frac{W_t^h}{W_t}\right)^{-\frac{\mu_{w,t}}{1-\mu_{w,t}}} L_t \tag{2.3}$$

where $\mu_{w,t}$ denotes the symmetric equilibrium wage markup factor that is assumed to follow an exogenous stochastic process. The demand for the labor composite $L_t$ and the wage index $W_t$ are given by the Dixit-Stiglitz aggregators

$$L_t = \left[\int_0^1 \left(L_t^h\right)^{\frac{1}{\mu_{w,t}}} \; dh\right]^{\mu_{w,t}}$$
$$W_t = \left[\int_0^1 \left(W_t^h\right)^{\frac{1}{1-\mu_{w,t}}} \; dh\right]^{1-\mu_{w,t}}$$

The household's problem is to choose sequences of consumption, hours, loans and capital that maximize (2.1) subject to (2.2) and (2.3). Since the household has market power, he sets his real wage as a markup over the marginal rate of substitution between consumption and leisure.

$$W_t^h = \mu_{w,t} \left(L_t^h\right)^{\zeta} C_t \tag{2.4}$$

### Final Good Producers

As afford mentioned, there is a capital (k) and a consumption goods sector (c). In each sector x there is a final good composite, given by the following CES aggregate over the output of the $N_{x,t}$ monopolistic competitors active in sector x at time t

$$Y_{x,t} = \left[\int_0^{N_{x,t}} \left(Y_{x,t}^j\right)^{\frac{1}{\mu_{x,t}}}\right]^{\mu_{x,t}} \tag{2.5}$$

where $\mu_{x,t}$ denotes the symmetric equilibrium markup factor. Each period, all final good firms active in sector x incur an operating cost $b_x \psi_t$ where $\psi_t$ denotes the social value of the capital stock and $b_x$ is a sector specific scale parameter. Let $P_t^I$ denote the efficient price of capital, i.e. the price that would arise in a perfectly competitive environment and $K_t$ the aggregate capital stock. Then,

$$\psi_t = P_t^I K_t \tag{2.6}$$

These fixed costs have to be trending in order to allow for constant markup factors along the balanced growth path (BGP). Otherwise, the entry and exit mechanism discussed below would drive markups to zero over time. A possible interpretation is that operating costs increase with the sophistication of the economy, measured by the social value of the capital stock. Note that $\psi_t$ does not vary strongly over the cycle. Thus, the responses of the endogenous variables to markup shocks are not driven by variations in the operating costs.

It is assumed that a higher number of active firms drives down markups through increasing competitive pressure. Formally,

$$\mu_{x,t} = \mu(N_{x,t}); \quad \mu^{'}(\cdot) < 0 \tag{2.7}$$

Free entry into the final good sectors implies that firms make zero profits in equilibrium. Let $\Pi(\mu_{x,t}, P_{x,t}Y_{x,t}^j)$ denote firm profits. Then,

$$\Pi(\mu_{x,t}, P_{x,t}^j Y_{x,t}^j) = b_x \psi_t \tag{2.8}$$

hast to hold in equilibrium. By this mechanism, the model generates procyclical entry and countercyclical fluctuations in markups. In booms, firm profits increase which leads to entry of new competitors driving down the markup factor. The number of firms increases until decreasing markups drive profits down to zero.

All final good producers indexed by j have access to a technology represented by the production function

$$Y_{x,t}^j = \left[ \left( U_{x,t}^j K_{x,t}^j \right)^{\alpha} \left( L_{x,t}^j \right)^{1-\alpha} \right]^{1-\gamma} \left[ M_{x,t}^j \right]^{\gamma} \tag{2.9}$$

where $\alpha$ denotes the capital share and $\gamma$ the intermediate good share of value added. They can produce using capital services, $U_{x,t}^j K_{x,t}^j$, labor $L_{x,t}^j$ and an intermediate good composite $M_{x,t}^j$. $U_{x,t}^j$ denotes the capital utilization rate. The deprecation rate of capital rented by firm j is assumed to depend positively on the utilization rate. The intermediate good composite $M^j_{x,t}$ is a CES aggregate over the $A_{x,t}$ implemented varieties marketed in sector x

$$M^j_{x,t} = \left[\int_0^{A_{x,t}} \left(M^{j,k}_{x,t}\right)^{\frac{1}{\nu}} \, dk\right]^{\nu} \tag{2.10}$$

where $\nu$ is the symmetric equilibrium markup factor in the intermediate good market. Final good producers maximize profits subject to the production function and demand function, taking factor prices, markup factors and operating costs as given. The first order conditions are

$$\alpha(1-\gamma)\frac{P^j_{x,t}Y^j_{x,t}}{K^j_{x,t}} = \mu_{x,t}\left[D_t + \delta(U^j_{x,t})P^K_t\right] \tag{2.11}$$

$$(1-\alpha)(1-\gamma)\frac{P^j_{x,t}Y^j_{x,t}}{L^j_{x,t}} = \mu_{x,t}W_t \tag{2.12}$$

$$\alpha(1-\gamma)\frac{P^j_{x,t}Y^j_{x,t}}{U^j_{x,t}} = \mu_{x,t}\delta^{'}(U_t)P^K_t K^j_{x,t} \tag{2.13}$$

$$\gamma\frac{P^j_{x,t}Y^j_{x,t}}{M^j_{x,t}} = \mu_{x,t}P^M_{x,t} \tag{2.14}$$

### Intermediate Good Producers

As mentioned above, at each time t there is a continuum of measure $A_{x,t}$ of monopolistically competitive intermediate good producers in sector x. Each of them has obtained the right to market his variety through the adoption process discussed below. Intermediate good producers have access to a technology that allows them to convert one unit of the consumption composite into one unit of their variety.

### Innovators

In each sector innovators develop new blueprints for intermediate goods. They then sell the rights to these goods to adopters, who convert them into usable form. They finance there activity through loans obtained from households. Innovators conduct R&D using the final consumption good composite as an input. Denote by $S^p_{x,t}$ the amount innovator p spends on R&D. Let $Z^p_{x,t}$ be the stock of blueprints innovator p developed. Then,

$$Z^p_{x,t+1} = \varphi_{x,t} S^p_{x,t} + \phi Z^p_{x,t} \tag{2.15}$$

where $\varphi_{x,t}$ is a productivity term that the innovator takes as given and $\phi$ is the product survival rate. Each period, a constant fraction $1 - \phi$ of intermediate goods becomes obsolete, be they implemented or not. R&D productivity on the individual level is given by

$$\varphi_{x,t} = \chi_x Z_{x,t} \left[ \psi_t^{\rho} S_{x,t}^{1-\rho} \right]^{-1} \tag{2.16}$$

where $\chi_x$ is a sector specific scale parameter and $0 < \rho < 1$. The framework gives rise to a positive spillover of the total number of blueprints in sector x, $Z_{x,t}$, on individual R&D productivity. Productivity further depends negatively on aggregate R&D expenditures $S_{x,t}$ and the social value of the capital stock. The latter is simply a balanced growth restriction that helps to render the growth rate of the number of varieties stationary. The former introduces a congestion externality of aggregate R&D expenditures on indivdual R&D productivity.
Assuming free entry into innovation implies that innovators break even in equilibrium. Given the linearity of the technology they have access to, this implies that the marginal cost of developing an additional variety has to equal the marginal revenue. Denote by $J_{x,t}$ the value of an unimplemented variety in sector x at time t. Then, the zero-profit condition takes the form:

$$\frac{1}{\varphi_{x,t}} = E_t \left[ \Lambda_{t+1} J_{x,t+1} \right] \tag{2.17}$$

In booms, the value of unadopted varieties increases. Given the stochastic discount factor and the social value of the capital stock, (2.17) and (2.16) imply that aggregate sectoral R&D expenditures have to increase for innovators to break even. Through this mechanism, the model generates procyclical variations in R&D.

### Adopters

Adopters buy blueprints from innovators. Once an adopter bought a blueprint from an innovator, he succeeds to adopt it with a certain probability. If he fails, he may try again in the following period. Adopter q's instantaneous probability of success is given by:

$$\lambda_{x,t}^{q} = \lambda\left(\frac{A_{x,t}}{\psi_t} H_{x,t}^{q}\right); \quad \lambda^{'}(\cdot) > 0; \quad \lambda^{''}(\cdot) < 0 \tag{2.18}$$

The adopter is able to increase his success probability on a blueprint by spending a higher amount $H_{x,t}^{q}$ of the consumption composite on adoption. His adoption expenditures are financed through loans from households. There is a positive spill-over effect from the total number of implemented varieties $A_{x,t}$. Further, $\lambda_{x,t}^{q}$ depends negatively on the social value of the capital stock. This will yield a stationary equilibrium growth rate of the number of implemented varieties. In order to characterize the optimal adoption decision, note that the value of an implemented variety $V_{x,t}$ is given by the discounted stream of profits it generates. Recursively,

$$V_{x,t} = \Pi_{x,t} + E_t\phi\left[\Lambda_{t+1}V_{x,t+1}\right] \tag{2.19}$$

where $\Pi_{x,t}$ denotes the profits made by intermediate good producers in sector x. The value of an unimplemented variety to an adopter is given by:

$$J_{x,t} = -H_{x,t}^{q} + \phi E_t\left\{\Lambda_{x,t+1}^{q}\left[\lambda_{x,t}^{q}V_{x,t+1} + (1-\lambda_{x,t}^{q})J_{x,t+1}\right]\right\} \tag{2.20}$$

The first order condition w.r.t. $H_{x,t}^{q}$ is:

$$1 = \lambda^{'}\left(\frac{A_{x,t}}{\psi_t} H_{x,t}^{q}\right)\frac{A_{x,t}}{\psi_t}\phi E_t\left\{\Lambda_{t+1}\left[V_{x,t+1} - J_{x,t+1}\right]\right\} \tag{2.21}$$

The adopter increases his adoption expenditures per variety until an infinitesimal increase in the probability of realizing the rise in value through adoption is worth the marginal costs. In booms, the increase in value associated with adoption increases, i.e. $V_{x,t}$ increases relative to $J_{x,t}$. The adopter reacts by increasing

$H^q_{x,t}$ until (2.21) holds with equality. Thus, adoption expenditures are procyclical.

## 2.2 Equilibrium

The economy has a symmetric sequence of equilibrium allocations. The endogenous states are the aggregate capital stock $K_t$, the number of blueprints developed in each sector $Z_{x,t}$ and the number of implemented intermediate goods $A_{x,t}$.

**Market Clearing**

Denote by $Y_t$ aggregate net value added. It is equal to the sum of gross output over the sectors net of expenditures on intermediate goods and operating costs incured by firms:

$$Y_t = \sum_{x=c,k} \left[P_{x,t} Y_{x,t} - (A_{x,t})^{1-\nu} M_{x,t} - N_{x,t} b_x \psi_t\right] \tag{2.22}$$

The model features a trivial government sector, which finances it's expenditures through lump-sum taxes.

$$G_t = T_t \tag{2.23}$$

An expression for aggregate demand is given by:

$$Y_t = C_t + P_{k,t} Y_{k,t} + G_t + \sum_{x=c,k} \left[S_{x,t} + (Z_{x,t} - A_{x,t}) H_{x,t}\right] \tag{2.24}$$

Net value added is used for consumption, investment $P_{k,t}Y_{k,t}$, government expenditures and R&D and adoption expenditures. The capital stock evolves according to

$$K_{t+1} = (1 - \delta(U_t)) K_t + Y_{k,t} \tag{2.25}$$

where $U_t$ is the equilibrium utilization rate chosen by all final good producers. Aggregating over all firms, the final output composite in sector x, $Y_{x,t}$, takes the form

$$Y_{x,t} = (N_{x,t})^{\mu_{x,t}-1} \left[ (U_t K_{x,t})^{\alpha} L_{x,t}^{1-\alpha} \right]^{1-\gamma} [M_{x,t}]^{\gamma} \tag{2.26}$$

where $K_{x,t}$, $L_{x,t}$ and $M_{x,t}$ are the aggregate inputs used by all firms active in sector x. $(N_{x,t})^{\mu_{x,t}-1}$ reflects the returns to variety that arise with the CES aggregator. Through this magnitude, entry and exit of firms directly affects productivity.

Profit maximization by final good firms implies that

$$\alpha(1-\gamma)\frac{P_{x,t}^j Y_{x,t}^j}{K_{x,t}^j} = \mu_{x,t} \left[ D_t + \delta(U_{x,t}^j) P_t^K \right] \tag{2.27}$$

$$(1-\alpha)(1-\gamma)\frac{P_{x,t}^j Y_{x,t}^j}{L_{x,t}^j} = \mu_{x,t} W_t \tag{2.28}$$

$$\alpha(1-\gamma)\frac{P_{x,t}^j Y_{x,t}^j}{U_{x,t}^j} = \mu_{x,t} \delta^{'}(U_t) P_t^K K_{x,t}^j \tag{2.29}$$

$$\gamma\frac{P_{x,t}^j Y_{x,t}^j}{M_{x,t}^j} = \mu_{x,t} P_{x,t}^M \tag{2.30}$$

have to hold in equilibrium. The labor supply equation arising from the household problem is given by:

$$W_t = \mu_{w,t} (L_t)^{\zeta} C_t \tag{2.31}$$

Due to symmetry, $\mu_{w,t}$ is also the aggregate wage markup factor. Factor market clearing for capital and labor further requires:

$$L_t = L_{c,t} + L_{k,t} \tag{2.32}$$

$$K_t = K_{c,t} + K_{k,t} \tag{2.33}$$

The intertemporal Euler equation is given by

$$1 = E_t [\Lambda_{t+1} R_{t+1}] \tag{2.34}$$

$$\tag{2.35}$$

where $\Lambda_{t+1} = \beta \frac{C_t}{C_{t+1}}$. No-arbitrage between loans and capital implies that

$$E_t \left[\Lambda_{t+1} R_{t+1}\right] = E_t \left[\Lambda_{t+1} \frac{P^K_{t+1} + D_{t+1}}{P^K_t}\right] \tag{2.36}$$

has to hold in equilibrium. The free entry condition for final good firms takes the form

$$\left(1 - \frac{1}{\mu_{x,t}}\right) \frac{P_{x,t} Y_{x,t}}{N_{x,t}} = b_x \psi_t \tag{2.37}$$

where the term on the left side reflects profits of the individual firm.

**R&D and Adoption**

Aggregating over all adopters, the number of blueprints in sector x evolves according to:

$$Z_{x,t+1} = \left[\chi_x \left(\frac{S_{x,t}}{\psi_t}\right)^{\rho} + \phi\right] Z_{x,t} \tag{2.38}$$

Though the individual innovator has access to a constant returns to scale technology, the congestion effect of total R&D expenditures on individual R&D productivity implies that the aggregate elasticity of the number of inventions with respect to R&D is $0 < \rho < 1$. Symmetry implies that all adopters spend the same amount on adoption per variety. The number of implemented varieties thus follows:

$$A_{x,t+1} = \phi \lambda_{x,t} \left[Z_{x,t} - A_{x,t}\right] + \phi A_{x,t} \tag{2.39}$$

Every period, the adopters succeed to implement a fraction $\lambda_{x,t}$ of the not yet implemented varieties, while a fraction $1 - \phi$ of all intermediate inputs becomes obsolete. The free entry condition into R&D aggregates to:

$$\phi E_t \left\{\Lambda_{t+1} J_{x,t+1} \left(Z_{x,t+1} - \phi Z_{x,t}\right)\right\} = S_{x,t} \tag{2.40}$$

It states, that the discounted value of R&D has to equal the total cost. Free entry into adoption implies that

$$1 = \lambda' \left( \frac{A_{x,t}}{\psi_t} H_{x,t} \right) \frac{A_{x,t}}{\psi_t} \phi E_t \left\{ \Lambda_{t+1} \left[ V_{x,t+1} - J_{x,t+1} \right] \right\} \tag{2.41}$$

has to hold, where $H_{x,t}$ is the amount all adopters spend per blueprint. The value of an adopted variety $V_{x,t}$ follows

$$V_{x,t} = \left( 1 - \frac{1}{\nu} \right) \gamma \frac{1}{\mu_{x,t}} \frac{P_{x,t} Y_{x,t}}{A_{x,t}} + E_t \left\{ \Lambda_{t+1} V_{x,t+1} \right\} \tag{2.42}$$

where the first term is the flow profits generated by one intermediate good in period t. The value of unadopted varieties, $J_{x,t}$, obeys:

$$J_{x,t} = -H_{x,t} + \phi E_t \left\{ \Lambda_{t+1} \left[ \lambda_{x,t} V_{x,t+1} + (1 - \lambda_{x,t}) J_{x,t+1} \right] \right\} \tag{2.43}$$

**Productivity and Technology**

The relative price of the intermediate good composite is given by:

$$P^M_{x,t} = \nu \left( A_{x,t} \right)^{1-\nu} \tag{2.44}$$

Since the markup factor $\nu$ is bigger than one, it depends negatively on the number of implemented intermediate goods. By this mechanism, production in this economy becomes more efficient as the state of technology progresses. In order to see how technological change manifests in productivity, consider the Solow residual and the relative price of capital as measures for embodied and disembodied productivity. Denote by $Y^v_{x,t}$ net value added produced in sector x at time t

$$Y^v_{x,t} = \theta_{x,t} K^{\alpha}_{x,t} L^{1-\alpha}_{x,t} \tag{2.45}$$

where $\theta_{x,t}$ is the Solow residual in sector x, given by:

$$\theta_{x,t} = \left(1 - \frac{\gamma}{\nu\mu_{x,t}}\right) N_{x,t}^{\mu_{x,t}-1} P_{x,t}^{1/(1-\gamma)} \left(\frac{\gamma}{\mu_{x,t} P_{x,t}^{M}}\right)^{\frac{\gamma}{1-\gamma}} U_t^{\alpha} - \frac{b_x \psi_t}{K_{x,t}^{\alpha} L_{x,t}^{1-\alpha}} \tag{2.46}$$

Disembodied productivity depends negatively on markups and the relative price of the intermediate good composite and positively on the number of firms, reflecting the returns to variety mentioned above. Thus, movements in the Solow residual do not merely reflect technological change, but are also driven by non-technological variables.

Along the BGP, $N_{x,t}$, $\mu_{x,t}$ and $U_t$ are constant and the steady decline in $P_{x,t}^{M}$, caused by the increase in the stock of intermediate inputs $A_{x,t}$ leads to increasing disembodied productivity.

Through the mechanisms discussed above, stochastic innovations to the wage markup demanded by households lead to countercyclical variations in final good markups and procyclical variations in firm entry and the utilization rate. Further, R&D and adoption expenditures react procyclically, causing countercyclical variations in the relative price of the intermediate good composite. These movements are reflected in procyclical variations in disembodied productivity, that play out over the high-and medium frequencies. The relative price of capital is given by

$$P_t^{K} = \left(\frac{\mu_{k,t}}{\mu_{c,t}}\right) \left(\frac{N_k^{1-\mu_k}}{N_c^{1-\mu_c}}\right) P_t^{I} \tag{2.47}$$

Note that it is increasing in the relative markup demanded in the capital good sector, decreasing in the relative size of the capital good sector and increasing in the efficient price of capital given by

$$P_t^{I} = \left(\frac{P_{k,t}^{M}}{P_{c,t}^{M}}\right)^{\gamma} \tag{2.48}$$

Hence, the relative price of capital reflects the relative efficiencies between the two sectors. These are caused by differences in the state of technology reflected in the efficient price of capital and non-technological differences in markups and the firm number.

Along the BGP, a higher pace of R&D in the capital good sector drives down the relative price of capital, reflecting embodied productivity gains. At the high frequency, the stochastic variations in the wage markup cause variations in markups and firm entry in both sectors. However, investment demand exhibits a stronger cyclicality than consumption demand. Thus, profits are more volatile in the capital good sector and the relative price of capital varies countercyclically at the high frequencies. The stronger cyclicality of profits in the capital good sector also implies that R&D and adoption are more volatile than in the consumption sector, resulting in procyclical variations in the relative state of technology in the capital good sector at the high-and medium frequencies.

# Chapter 3

# Calibration and Solution

## 3.1 Calibration

I choose the same annual calibration as discussed in Comin and Gertler (2006). All standard parameters are set to conventional values. An overview is given in table 3.1.

| | | |
|---|---|---|
| Government | | |
| G/Y | 0.2 | Share of government expenditures in value added |
| Households | | |
| $\beta$ | 0.95 | Discount rate |
| $\xi$ | 1 | Inverse Frisch elasticity of labor supply |
| $\bar{\mu}_w$ | 1.2 | Steady state wage markup factor |
| Final good producers | | |
| $\alpha$ | 0.33 | Capital share |
| $\gamma$ | 0.5 | Intermediate input share |
| $\bar{\mu}_c$,$\bar{\mu}_k$ | 1.1,1.2 | Steady state markup factors |
| $U$ | 0.8 | Steady state utilization rate |
| $\delta(U)$ | 0.08 | Steady state depreciation rate |
| $(\delta^{''}/\delta^{'})U$ | 0.33 | Elasticity of change in $\delta$ w.r.t. $U$ |

Table 3.1: Calibration - Standard Parameters

The non-standard parameters reported in table 3.2 are mostly those that govern the R&D and adoption process. Here, the two scale parameters $\chi_x$ are chosen to match the steady state growth rate of the relative price of capital (-0.026) and real per capita output (0.024) observed in the data. Then, $\phi$, the annual survival rate of intermediate goods is set to 0.97 to match the share of R&D expenditures in output (0.09). Comin and Gertler set the elasticity of new intermediate goods with respect to R&D expenditures, $\rho$, to 0.8 based on evidence in Griliches (1990). The value of 0.1 for $\lambda$ chosen here implies an average diffusion lag of 10 years and is based on evidence Comin and Gertler gathered in an unpublished paper ((Comin and Gertler, 2004)). A measure for adoption expenditures provided by the National Science Foundation (NSF) are the development costs incurred by manufacturing firms that try to make new products usable. Comin and Gertler run a simple regression of these expenditures on the rate of decline of the relative price of capital and obtain an estimate of 0.95, which they set the elasticity of adoption with respect to adoption expenditures, $\rho_a$, equal to.

They set the elasticity of markups with respect to firm entry, $\epsilon$, to unity in order to match the overall medium-term variation in the number of firms. The autocorrelation of the wage markup shock is set to 0.6, such that the model generates an autocorrelation of the gross markup that is consistent with the measures provided in Gali et al. (2002). The scale coefficients in the operating costs, $b_x$, are chosen such as to fix the number of firms active in steady state at unity for both sectors. There is no direct evidence on the markup in the intermediate good market. Comin and Gertler argue that given the specialized nature of the intermediate inputs, an appropriate value for $\nu$ should be at the high end of the range of markup estimates and accordingly choose a value of 1.6.

| | | |
|---|---|---|
| Households | | |
| $\rho_{\mu_w}$ | 0.6 | Autocorrelation of wage markup shock |
| Final good producers | | |
| $b_c$,$b_k$ | 0.11,0.02 | Scale coefficients in operating costs |
| Intermediate good producers | | |
| $\nu$ | 1.6 | Markup factor |
| R&D and Adoption | | |
| $\phi$ | 0.97 | Annual survival rate of intermediate goods |
| $\chi_c$,$\chi_k$ | 2.45,37.24 | Scale coefficients in blueprint creation |
| $\rho$ | 0.8 | Elasticity of new intermediate goods w.r.t. R&D |
| $\lambda_x$ | 0.1 | Steady state adoption rate in both sectors |
| $\rho_a$ | 0.95 | Elasticity of $\lambda_{x,t}$ w.r.t. adoption expenditures |
| $\epsilon$ | 1 | Elasticity of markups w.r.t. entry |
| $\psi$ | 0.56 | Coefficient in TFP |

TABLE 3.2: Calibration - Non-standard Parameters

The precise definition of total factor productivity (TFP) I am going to rely on is

$$\theta_t = \frac{Y_t A_{k,t}^{\psi\alpha(\nu-1)}}{K_t^{\alpha} L_t^{1-\alpha}} \tag{3.1}$$

where the $A_{k,t}$ term captures an externality in the creation of new capital goods. $\psi$ is calibrated to 0.56 in order to match the BGP growth rate of $\theta$ to the long-run growth rate observed in the data. One might refer to $\theta_t$ as measured TFP.

## 3.2 Solution

As the model exhibits sustained growth, I have to appropriately scale the variables to obtain a stationary system. I consider the following transformation

$$\tilde{y}_t = \frac{Y_t}{\Gamma_t^y} \quad \tilde{y}_{c,t} = \frac{Y_{c,t}}{\Gamma_t^y} \quad \tilde{\psi}_t = \frac{\psi_t}{\Gamma_t^y} \quad \tilde{c}_t = \frac{C_t}{\Gamma_t^y} \quad \tilde{w}_t = \frac{W_t}{\Gamma_t^y} \quad \tilde{s}_{x,t} = \frac{S_{x,t}}{\Gamma_t^y} \quad \tilde{g}_t = \frac{G_t}{\Gamma_t^y}$$
$$\tilde{k}_t = \frac{K_t}{\Gamma_t^k} \quad \tilde{k}_{x,t} = \frac{K_{x,t}}{\Gamma_t^k} \quad \tilde{y}_{k,t} = \frac{Y_{k,t}}{\Gamma_t^k}$$
$$\tilde{d}_t = \frac{d_t}{\Gamma_t^p} \quad \tilde{p}_{k,t} = \frac{P_t^K}{\Gamma_t^p} \quad \tilde{p}_t^I = \frac{P_t^I}{\Gamma_t^p}$$
$$\tilde{p}_{x,t}^M = \frac{P_{x,t}^M}{\Gamma_{x,t}^{P^M}} \quad \tilde{m}_{c,t} = \frac{M_{c,t}}{\Gamma_t^{M_c}} \quad \tilde{m}_{k,t} = \frac{M_{k,t}}{\Gamma_t^{M_k}}$$
$$\tilde{a}_{x,t} = \frac{A_{x,t}}{Z_{x,t}} \quad g_{Z_x,t} = \frac{Z_{x,t+1}}{Z_{x,t}}$$
$$\tilde{h}_{c,t} = \frac{H_{c,t}}{\Gamma_t^{H_c}} \quad \tilde{v}_{c,t} = \frac{V_{c,t}}{\Gamma_t^{H_c}} \quad \tilde{j}_{c,t} = \frac{J_{c,t}}{\Gamma_t^{H_c}}$$
$$\tilde{h}_{k,t} = \frac{H_{k,t}}{\Gamma_t^{H_k}} \quad \tilde{v}_{k,t} = \frac{V_{k,t}}{\Gamma_t^{H_k}} \quad \tilde{j}_{k,t} = \frac{J_{k,t}}{\Gamma_t^{H_k}}$$
$$\tilde{\theta}_{x,t} = \frac{\theta_{x,t}}{\Gamma_t^{\theta_x}} \quad \tilde{\theta}_t = \frac{\theta_t}{\Gamma_t^\theta}$$

where

$$\Gamma_t^y = (Z_{c,t})^{\frac{(\nu-1)\gamma[1-\alpha(1-\gamma)]}{(1-\alpha)(1-\gamma)}} (Z_{k,t})^{\frac{(\nu-1)\gamma\alpha}{(1-\alpha)}}$$
$$\Gamma_t^k = (Z_{c,t})^{\frac{(\nu-1)\gamma^2}{(1-\alpha)(1-\gamma)}} (Z_{k,t})^{\frac{(\nu-1)\gamma}{(1-\alpha)}}$$
$$\Gamma_t^p = (Z_{k,t})^{(1-\nu)\gamma} (Z_{c,t})^{(\nu-1)\gamma}$$
$$\Gamma_{x,t}^{P^M} = (Z_{x,t})^{1-\nu}$$
$$\Gamma_t^{M_c} = (Z_{c,t})^{\frac{(\nu-1)[1-\alpha(1-\gamma)(1+\gamma)]}{(1-\alpha)(1-\gamma)}} (Z_{k,t})^{\frac{(\nu-1)\alpha\gamma}{(1-\alpha)}}$$
$$\Gamma_t^{M_k} = (Z_{c,t})^{\frac{(\nu-1)\gamma[1-\alpha(1-\gamma)]}{(1-\alpha)(1-\gamma)}} (Z_{k,t})^{\frac{(\nu-1)[1-\alpha(1-\gamma)]}{(1-\alpha)}}$$
$$\Gamma_t^{H_c} = (Z_{c,t})^{\frac{(\nu-1)\gamma[1-\alpha(1-\gamma)]}{(1-\alpha)(1-\gamma)}-1} (Z_{k,t})^{\frac{(\nu-1)\gamma\alpha}{(1-\alpha)}}$$
$$\Gamma_t^{H_k} = (Z_{c,t})^{\frac{(\nu-1)\gamma[1-\alpha(1-\gamma)]}{(1-\alpha)(1-\gamma)}} (Z_{k,t})^{\frac{(\nu-1)\gamma\alpha}{(1-\alpha)}-1}$$
$$\Gamma_t^{\theta_x} = (Z_{c,t})^{\frac{(\nu-1)\gamma}{(1-\gamma)}}$$
$$\Gamma_t^{\theta} = (Z_{c,t})^{\frac{(\nu-1)\gamma}{(1-\gamma)}} (Z_{k,t})^{(\nu-1)\psi\alpha}$$

All other variables are stationary and do not have to be transformed. In Appendix B1, I derive the scaling factors. Appendix B2 contains the transformed equilibrium system. I solve the model by linearizing it numerically around the non-stochastic steady state in the transformed system. In order to evaluate the results, I subsequently undo the transformation as discussed in Appendix B3.

# Chapter 4

# Numerical Results

## 4.1 Impulse Responses

To gain some intuition in the working of the model, consider a unit shock to wage-markups. Some impulse responses are depicted in Figures 4.1 and 4.2. For each variable, the log difference of its value from the trend it would have followed had no shock occurred is plotted for the 20 years following the shock.
When the equilibrium wage markup factor rises, households effectively demand higher wages. This reduces the profitability of final good producers, who react by decreasing their demand for input factors. Consequently, production, hours and the utilization rate fall below their initial steady state values. Through the endogenous entry mechanism, the decrease in profitability further causes firm exit and increasing markups in both sectors.

All these effects are reflected in an immediate decrease in disembodied productivity and labor productivity. Reacting to their consumption losses, households decrease their savings. R&D and the adoption rate drop below their initial level and the capital stock starts declining. The decline in R&D and the adoption rate slows down the creation and adoption of new intermediate goods and ultimately leads to a persistent decline in the number of implemented intermediate goods relative to trend. This results in a corresponding permanent decrease in total factor productivity and labor productivity. The persistent decline in productivity

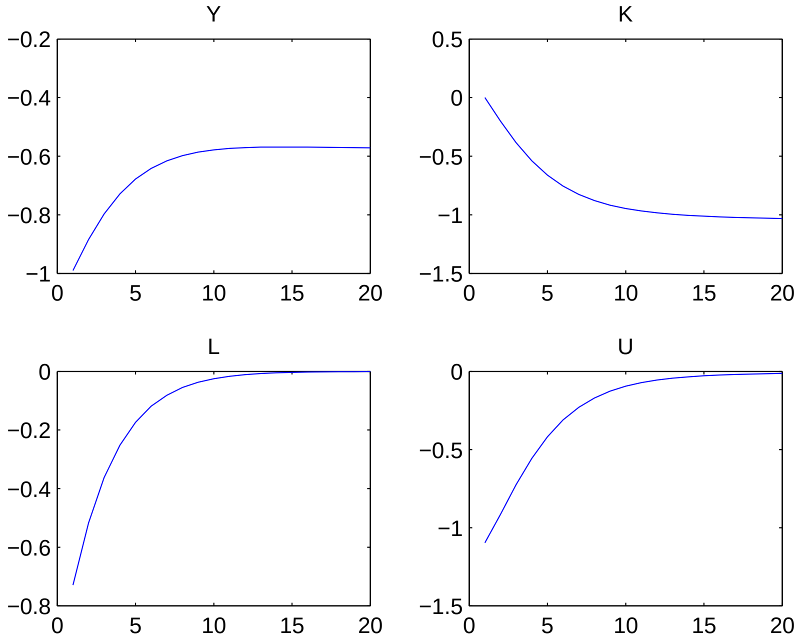


FIGURE 4.1: Impulse responses for factors and output

is mirrored in a persistent decline in hours, output, the capital stock and R&D expenditures.

The relative price of capital increases on impact due to the stronger cyclicality of markups and firm entry in the capital good sector. A stronger response of R&D and adoption in the capital good sector then cause a further increase, that only fades off when markups and the firm number revert back to their steady state values. However, the relative improvement of the state of technology in the consumption good sector persists and leads to a persistent response also in embodied productivity.

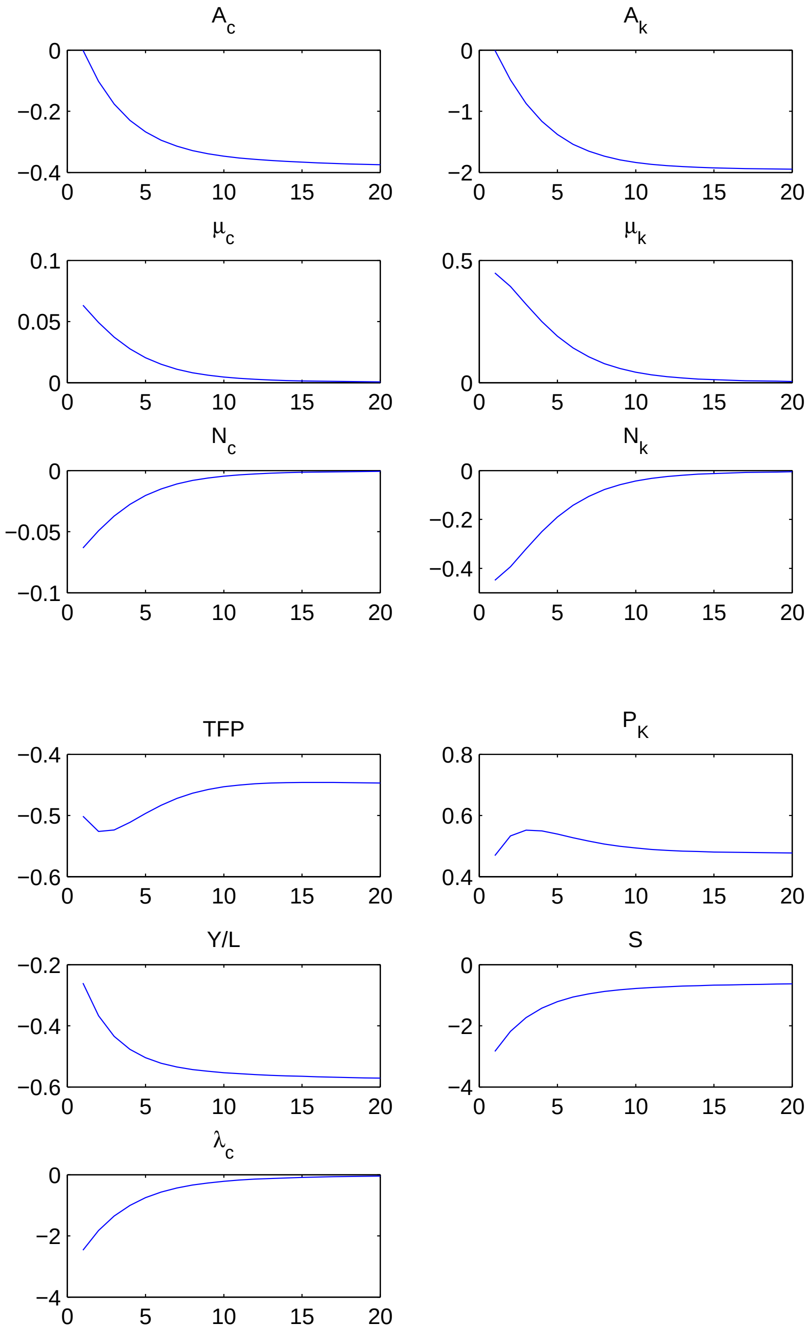


FIGURE 4.2: Impulse responses for productivity

## 4.2 Model Evaluation

### The Data

The data is annual from 1947 to 2001[1]. I did not include any observations past 2001, since the Gordon adjusted price for capital that plays a crucial role in the calibration is only available up to then. I want to evaluate the performance of the model by comparing data moments on standard real business cycle (RBC) variables and technology variables with the moments of the artificial data generated by the model.

The standard RBC variables I consider here are output, consumption, investment, hours, labor productivity, capacity utilization and TFP. As output, I take real total non-farm business output. Consumption is private consumption expenditures on non-durable goods and services and investment is non-residential investment. Hours are total hours worked in the non-farm business sector. All these variables are reported by the Bureau of Economic Analysis (BEA). I scale them by the total civilian non-institutional population as reported by the Bureau of Labor Statistics to convert them into per-capita terms. The data on capacity utilization is provided by the Board of Governors and as TFP, I take the non-capacity utilization adjusted series from Fernald (2012).

Non-standard technology related variables include R&D expenditures, the Gordon adjusted relative price of capital, data on patent applications and two series as adoption indicators. The data on R&D expenditures is constructed from data reported by the NSF. They provide total private R&D expenditures in three categories: Basic R&D, applied research and development. Development is defined as expenses devoted to improving existing products or processes. Comin and Gertler (2006) argue, that only parts of these can be considered as R&D expenditures as considered in the model and only include half of the expenditures in the latter category in their measure for total R&D expenditures. I follow them in that but also did some robustness checks by conducting the analysis below including all

[1]Capacity utilization is only available from 1967 onwards. The two TECH series are available for the period from 1955 to 1997.

development and no development expenditures into R&D. The results were virtually unaffected. The series on the relative price of capital I use is reported by Cummins and Violante (2002). It differs from the series reported by the BEA in being considerably more volatile. The data on patent application is taken from the United States Patent and Trademark Office (USPTO). More specifically, I consider their data on utility patent applications. A utility patent as defined by the USPTO is issued for the invention of a new and useful process, machine, manufacture, or composition of matter, or a new and useful improvement thereof. This definition seems to broadly agree to the concept of blueprints in the model and I use the series to evaluate the model's performance on the creation of new blueprints. The two adoption series are taken from Alexopoulos (2011). In the creation of the two series, she relies on data on new books published in the field of technology. She argues, that new books on technology should be published once the product is first commercialized since books are costly to produce and publishers have an incentive to make them available as soon as possible to maximize returns. Thus, her series can be considered as indicators of diffusion, that are related to the adoption of additional varieties in the model. She reports two series on adoption. The TECH series is based on R.R. Bowker catalogues of new book titles, that report titles published by major book publishers in the US by subject. A potential drawback of the TECH series is that it does not include company manuals, that might be a very important source of information on new technologies. In order to account for this shortcoming, Alexopolous also provides a series based on the catalogue records from the Library of Congress that includes company manuals and titles published by smaller publishers, referred to as TECH2 series. I evaluate the model's performance on the adoption of additional technologies using the two TECH series[2].

One should note that the quality of the data on the technology variables is of course not high. R&D and adoption are hard to measure and the properties of the two adoption series differ strongly as noted below. Patent applications are easily measurable but they might not capture all relevant innovations and

[2] I define newly created blueprints and additionally adopted varieties as the number of blueprints and implemented intermediate goods that were added to the total stock in both sectors in a given period.

be subject to time lags in processing. Still, I would consider it interesting to check whether the model generated behavior of the technology variables agrees with the data, even more so since the technology variables are at the heart of the model mechanism. I detrend the data as follows. First, I convert all series into growth rates. Then, I apply a band-pass filter to filter out all variations at frequencies below 50 years and below 8 years respectively in order to obtain the trend growth rates. This allows me to accumulate a measure of the respective trends in log-levels. Subsequently, I remove the trends from the series in order to obtain measures for the conventional-and the medium-term cycle.

**Persistence**

| | Medium-term cycle | | Conventional cycle | |
|---|---|---|---|---|
| Variable | Data | Model | Data | Model |
| Output | 0.69 | 0.66 | 0.10 | -0.02 |
| | (0.49,0.89) | | (-0.17,0.38) | |
| Consumption | 0.88 | 0.77 | 0.20 | -0.09 |
| | (0.74,1.01) | | (-0.07,0.47) | |
| Investment | 0.78 | 0.69 | 0.15 | 0.07 |
| | (0.61,0.96) | | (-0.13,0.42) | |
| Patents | 0.88 | 0.65 | 0.06 | 0.01 |
| | (0.72,1.04) | | (-0.22,0.34) | |
| R&D | 0.85 | 0.59 | 0.08 | -0.02 |
| | (0.70,0.99) | | (-0.22,0.37) | |
| Hours | 0.62 | 0.58 | 0.11 | -0.04 |
| | (0.40,0.84) | | (-0.17,0.39) | |
| TECH | 0.69 | 0.63 | -0.17 | -0.01 |
| | (0.46,0.91) | | (-0.48,0.14) | |
| TECH2 | 0.76 | 0.63 | 0.01 | -0.01 |
| | (0.56,0.96) | | (-0.30,0.33) | |
| Labor productivity | 0.80 | 0.89 | 0.04 | 0.14 |
| | (0.64,0.95) | | (-0.23,0.31) | |
| Capacity utilization | 0.88 | 0.66 | 0.22 | 0.01 |
| | (0.69,1.07) | | (-0.12,0.57) | |
| TFP | 0.69 | 0.76 | -0.01 | 0.02 |
| | (0.50,0.89) | | (-0.28,0.27) | |
| Relative price of capital | 0.84 | 0.80 | 0.45 | 0.05 |
| | (0.68,1.00) | | (-0.05,0.49) | |

TABLE 4.1: Model Evaluation - Autocorrelations

Table 4.1 contains the autocorrelations of all variables over the medium-term cycle and the conventional cycle. The model autocorrelations are averages over 1000 simulations of a sample size corresponding to the data. For the data, 95% confidence bounds are provided. First, note that for all variables, there is no overlap of the two confidence bounds, confirming that the medium term cycle is indeed significantly more persisitence than the conventional cycle. As the data, the model predicts that the medium-term cycle displays a higher persistence than the conventional cycle. Further, the implied autocorrelations are contained in the 95% confidence bands for all variables but R&D, patents and capacity utilization. I would reason that all in all, the model relying on the propagation mechanism of endogenous technology is succesful in translating the high-frequency variations caused through wage markup shocks into a persistent medium-term cycle.

### Standard Deviations

Table 4.2 contains the standard deviations for all variables considered here over the medium-term cycle and the conventional business cycle. By choosing the standard deviation of the markup shock to match the volatility of a gross markup series, Comin and Gertler (2006) show that the model can generate the kind of volatility documented in the data. I do not replicate this step here. Rather, I only report standard deviations relative to output for all variables. For the data, I further report absolute standard deviations.

| | Medium-term cycle | | | Conventional cycle | | |
|---|---|---|---|---|---|---|
| Variable | Abs Data | Rel Data | Model | Abs Data | Rel Data | Model |
| Output | 3.83 | 1 | 1 | 2.02 | 1 | 1 |
| Consumption | 2.28 | 0.55 | 0.46 | 0.71 | 0.35 | 0.34 |
| Investment | 9.62 | 2.51 | 3.44 | 4.25 | 2.11 | 3.31 |
| Patents | 9.18 | 2.39 | 2.99 | 3.11 | 1.54 | 3.08 |
| R&D | 8.61 | 2.20 | 2.68 | 2.71 | 1.37 | 3.04 |
| Hours | 3.15 | 0.82 | 0.70 | 1.87 | 0.93 | 0.80 |
| TECH | 16.84 | 4.39 | 2.61 | 7.87 | 3.90 | 2.77 |
| TECH2 | 7.92 | 2.06 | 2.61 | 3.56 | 1.76 | 2.77 |
| Labor productivity | 2.42 | 0.63 | 0.5 | 0.88 | 0.43 | 0.24 |
| Capacity utilization | 8.63 | 2.25 | 1.16 | 3.34 | 1.66 | 1.16 |
| TFP | 2.32 | 0.61 | 0.60 | 1.15 | 0.57 | 0.48 |
| Relative price of capital | 4.07 | 0.43 | 0.62 | 1.72 | 0.31 | 0.45 |

Table 4.2: Model Evaluation - Standard Deviations

Overall, the model does well in generating the relative volatilities observed in the data. Amongst the non-technological variables it is furthest off on investment and capacity utilization where it does not come close to the data over both, the medium-term and the conventional cycle. Amongst the technology variables it somewhat overstates the relative volatilities of R&D and Patents. Concerning adoption, the model understates the volatility of adoption when considering the TECH series and overstates it when considering the TECH2 series. Note the huge difference in absolute volatilities of the two TECH series. Interestingly, the model predicts that adoption, R&D and patents are relatively more volatile at the high-frequency while the data suggests the opposite. The model generates a volatility of output over the medium-term cycle relative to the conventional cycle of 1.96, which is very close to the ratio in the data (1.90).

### Cross Correlations

Figure 4.3 portrays how well the model can capture the cyclical comovement of the variables over the medium-term cycle. The top left panel contains the auto-correlation of output over the medium-term at leads and lags of up to ten years. The other panels plot the cross correlation of the respective variable with output at time t. The dashed lines are 95% confidence bounds.

The model does well in capturing the comovement of consumption, investment, hours and utilization with output. Abstracting from small discrepancies, the model can account well for the cyclical dynamics in productivity over the medium-term. However, figure 4.3 also illustrates that it has troubles to capture the comovement of the set of technology related variables with output. The model predicts that R&D closely moves together with output, while such a pattern is not visible in the data. Concerning patent applications, the point estimates suggest that patents follow output over the medium-term, while the model predicts a close comovement. Given the 95% confidence bands of course, the only aspect the data clearly rejects is the contemporaneous correlation of output and newly created blueprints the model predicts. A similar pattern arises when assessing the model's performance on adoption. The TECH series appears to lead output over the medium-term. However, given the width of the confidence bounds this pattern is not significant. The model predicts a clear comovement of adoption with output that is not visible in either, the TECH or the TECH2 series.

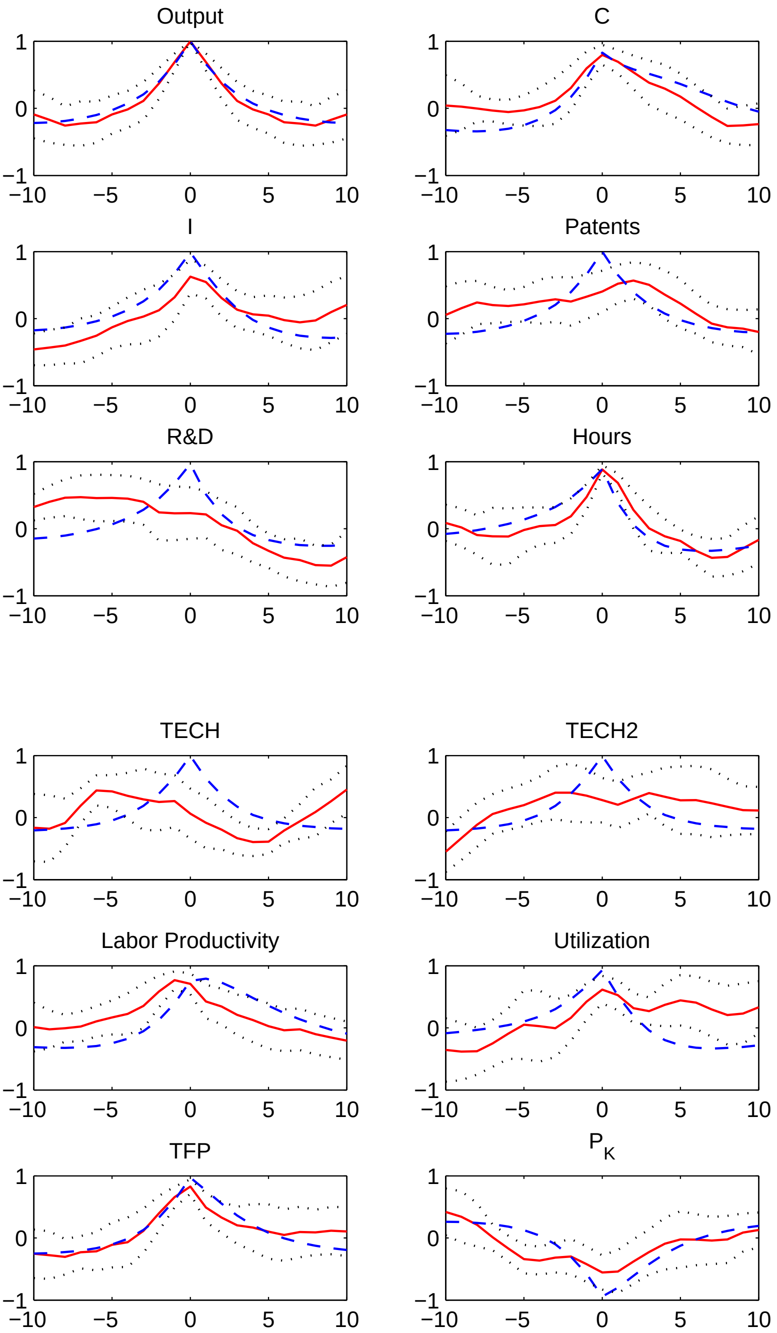


FIGURE 4.3: Cross Correlation with Output: Data(solid), Model(dashed) and 95% Confidence Bands (dotted)

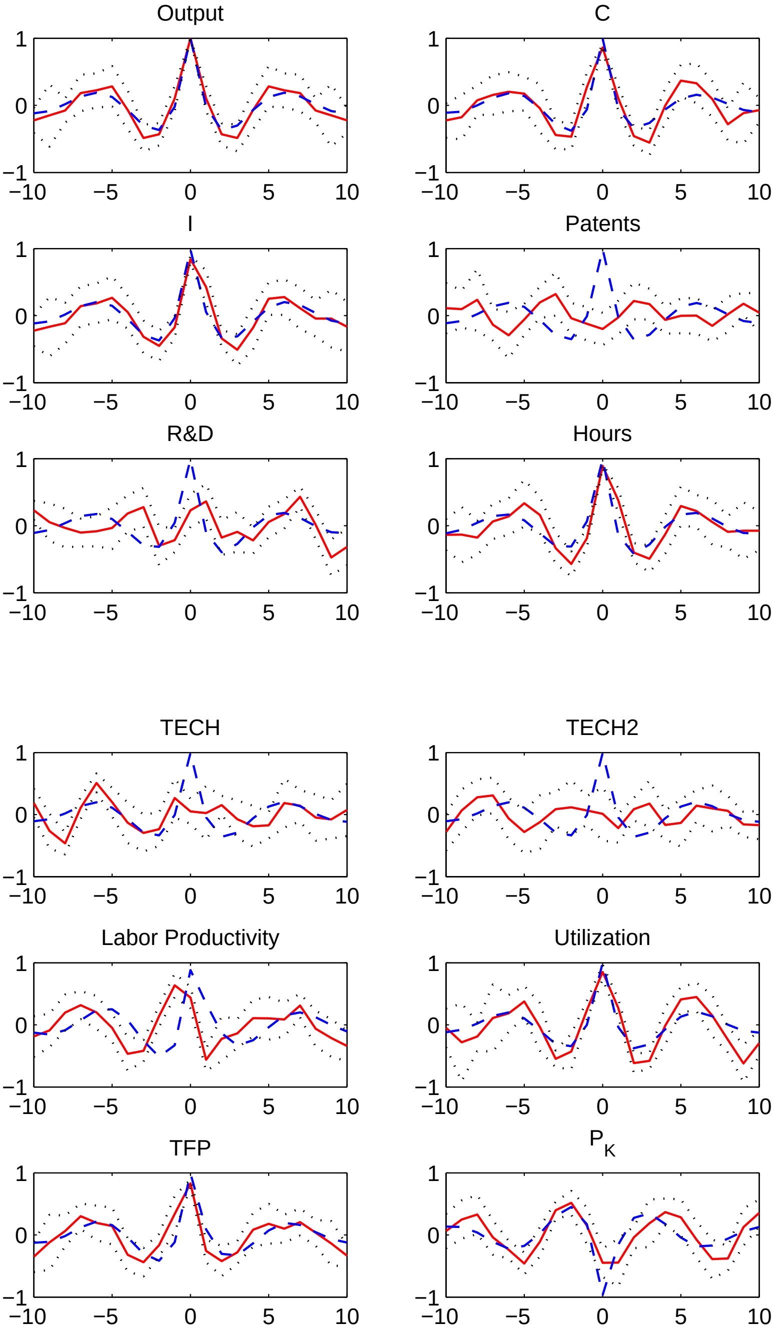


FIGURE 4.4: Cross Correlation with Output: Data(solid), Model(dashed) and 95% Confidence Bands (dotted)

Figure 4.4 plots the comovement of output with the other variables over the traditional business cycle of frequencies of eight years or less. Keeping in mind that the data is annual, one can see that the cyclical dynamics identified using the Band pass filter indeed closely resemble does that are commonly reported.

Again, the model does well in capturing the comovement of most variables. Other than over the medium-term, R&D exhibits a clear cyclical pattern at the high-frequencies. The data suggests a small lead of output over R&D, while the model generates a close comovement of output and R&D and a contemporaneous correlation well outside the confidence bounds. Over the conventional business cycle, output seems to have a lead over patent applications in the data, that appear to be countercyclical. The model predicts that both variables move closely together. At the high frequency, the point estimates suggest that the TECH2 series leads output, while such a pattern is not visible in the TECH series. The model again predicts a close comovement of output and adoption that is not visible in the data.

## 4.3 An Assesement

Comin and Gertler (2006) argue, that their model is able to jointly capture the dynamics of the conventional business cycle and the medium-term cycle. They claim, that through the mechanism of procyclical R&D and adoption, their framework can generate the volatile and persistent variations that characterize the medium-term cycle. As was outlined above, the model indeed generates the persistent medium-term cycle documented in the data and accounts well for the cyclical comovement of output with most other variables. Further, the model is overall successful in replicating the relative volatilities in the data with some exceptions, especially investment and capacity utilization.

I also documented that the model makes prediction on the behavior of the set of technology variables that cannot be documented in the data. However, given the questionable quality of these series, this discrepancy is of course difficult to interpret. For instance, consider the relative volatilities reported in table 4.2. As Comin and Gertler note, the measure of R&D expenditures reported by the NSF is likely to contain a significant overhead cost component, for instance expenditures incurred in the upkeep of research facilities. The model should be evaluated against the variable and thus more volatile component of R&D expenditures, which cannot be retrieved from the available data. Thus, it is difficult to argue whether the model indeed overstates the relative volatility of R&D. A similar case cannot be made for the two TECH series, but their volatilities differ to such a large extent that one should refrain from seriously evaluating the model against the volatility of adoption.

There is some evidence for a phase shift in the technology variables. However, the cross correlations the model predicts are for the most part well within the 95% confidence bounds and the phase-shifts hardly significant. The only aspect the data clearly rejects are the contemporaneous correlations with output the model predicts. Alexopoulos (2011) provides some more evidence that supports a lead of adoption over output. She tests for Granger causality of the two TECH series on output and finds strong evidence that both TECH series Granger-cause output. I applied a similar test to 1000 model generated series in order to check wheter the model generated series display a similar pattern.

| Confidence level | 1% level | 5% level | 10% level |
|---|---|---|---|
| Number of rejections | 23 | 83 | 152 |

Table 4.3: Model Evaluation - Does Adoption Granger-cause Output?

For each model generated series, I estimated a simple VAR of the growth rate of adoption and output with a lag length of one. Table 4.3 displays the number of times I could reject the null of no Granger causality of adoption on output. Apparently, the model cannot account for the fact that adoption Granger-causes output. This originates directly from the model mechanism. A shock to wage markups induces responses in output and adoption that very much resemble each other. On impact, both jump in the same direction to subsequently revert back to a new balanced growth path. This is reflected in the strong comovement of both variables that the model generates. The fact that adoption Granger causes output in the data suggests, that the cyclical variations are not only driven by the wage-markup shock. Rather, it points to the presence of a supply shock that drives fluctuations in output through a direct impact on adoption.

# Chapter 5

# Conclusion

I performed a critical evaluation of the model by Comin and Gertler (2006). They motivate the use of an endogenous technology model that propagates high-frequency fluctuations into a medium-term cycle through the procyclicality of R&D expenditures, which is well documented in the data. However, the mechanism of the model also relies on a procyclical creation of blueprints and procyclical adoption, for which they do not provide evidence. I use data on patent applications and diffusion in order to evaluate the model's performance along this dimension. Further, I evaluate the model against a set of additional standard variables. Overall, I find that it performs well. The comovement of the technology variables with output that is generated by the model does not agree with the data, but given the width of the confidence bands the discrepancies are not robust. The point-estimates on the cross correlations of output with adoption suggest that there might be a lead of adoption over output. Further evidence pointing to this is provided by Alexopoulos (2011), who finds that adoption Granger-causes output, which the model cannot account for. This suggests, that the medium-term cycle is not uniquely driven through markup shocks and points to the presence of a supply shock directly affecting adoption.

# Bibliography


Alexopoulos, Michelle. Read All about It!! What Happens Following a Technology Shock? *American Economic Review*, 101(4):1144–79, September 2011.

Barlevy, Gadi. The Cost of Business Cycles Under Endogenous Growth. *American Economic Review*, 94(4):964–990, September 2004.

Comin, Diego and Gertler, Mark. Procyclical Speed of Technological Diffusion. *NYU mimeo*, 2004.

Comin, Diego and Gertler, Mark. Medium-Term Business Cycles. *American Economic Review*, 96(3):523–551, September 2006.

Cummins, Jason G. and Violante, Giovanni L. Investment-Specific Technical Change in the US (1947-2000): Measurement and Macroeconomic Consequences. *Review of Economic Dynamics*, 5(2):243–284, April 2002.

Fernald, John. A Quarterly, Utilization-Adjusted Series on Total Factor Productivity. Working Paper Series 2012-19, Federal Reserve Bank of San Francisco, 2012.

Gali, Jordi; Gertler, Mark, and Lopez Salido, David. Markups, Gaps and the Welfare Costs of Business Cycles. *Review of Economics and Statistics*, February 2007, Vol. 89, No. 1:44–59, 2002.

Griliches, Zvi. Patent Statistics as Economic Indicators: A Survey. *Journal of Economic Literature*, 28,No. 4. (Dec.):1661–1707, 1990.

Kydland, Finn E and Prescott, Edward C. Time to Build and Aggregate Fluctuations. *Econometrica*, 50(6):1345–70, November 1982.

Lucas, Robert Jr. On the Mechanics of Economic Development. *Journal of Monetary Economics*, 22(1):3–42, July 1988.

Martin, Philippe and Rogers, Carol Ann. Optimal Stabilization Policy in the Presence of Learning by Doing. *Journal of Public Economic Theory*, 2(2): 213–241, 2000. ISSN 1467-9779.

Romer, Paul M. Endogenous Technological Change. *Journal of Political Economy*, 98(5):pp. S71–S102, 1990.

Stadler, George W. Business Cycle Models with Endogenous Technology. *American Economic Review*, 80(4):763–78, September 1990.

# Appendix A

# Derivations

## A.1 Mistakes

Here I discuss mistakes contained in Comin and Gertler (2006).

- Free entry condition

  They specify the free entry condition to the final good sector (eq (34) in their paper) as

  $$\left(1 - \frac{1}{\mu_{x,t}}\right)(N_{x,t})^{-\mu_{x,t}} Y_{x,t} = b_x \psi_t$$

  I think that it should be

  $$\left(1 - \frac{1}{\mu_{x,t}}\right)(N_{x,t})^{-1} P_{x,t} Y_{x,t} = b_x \psi_t$$

  A solution to the firm problem is given in Appendix A2 below.

- Euler Equation

  In their paper, the Euler equation (eq 32) takes the form

  $$1 = E_t \left\{ \Lambda_{t+1} \left[ \frac{D_t + P^K_{t+1}}{P^K_t} \right] \right\}$$

It contains a clear timing mistake. By the convention of the paper, $D_t$ is tied to the marginal product of capital in period t, which is irrelevant to the household when he makes his intertemporal savings decision between period t and t+1.

- Sectoral total factor productivity

  They specify sectoral total factor productivity as

$$\theta_{x,t} = \left(1 - \frac{\gamma}{\nu \mu_{x,t}}\right) N_{x,t}^{\mu_{x,t}-1} \left(\frac{\gamma}{\mu_{x,t} P_{x,t}^M}\right)^{\frac{\gamma}{1-\gamma}} U_t^\alpha - \frac{b_x \psi_t}{K_{x,t}^\alpha L_{x,t}^{1-\alpha}}$$

  I think that the price charged in sector x should also make an appearance, such that $\theta_{x,t}$ becomes

$$\theta_{x,t} = \left(1 - \frac{\gamma}{\nu \mu_{x,t}}\right) N_{x,t}^{\mu_{x,t}-1} P_{x,t}^{1/(1-\gamma)} \left(\frac{\gamma}{\mu_{x,t} P_{x,t}^M}\right)^{\frac{\gamma}{1-\gamma}} U_t^\alpha - \frac{b_x \psi_t}{K_{x,t}^\alpha L_{x,t}^{1-\alpha}}$$

  A derivation is included below.

## A.2 Derivations

Each final good producer solves a two stage problem. Given the quantity produced $Y_{x,t}^j$ and given factor prices, he chooses the cost minimizing input quantities. In a second stage, he solves for the optimal price given demand. Thus, the first stage problem takes the form

$$\min_{L_{x,t}^j, K_{x,t}^j, U_{x,t}^j, M_{x,t}^j} W_t L_{x,t} + \left[D_t + \delta(U_{x,t}^j)\right] K_{x,t}^j + P_{x,t}^m M_{x,t}^j$$

subject to

$$Y_{x,t}^j = \left[\left(U_{x,t}^j K_{x,t}^j\right)^{1-\alpha} (L_{x,t}^j)^\alpha\right]^{1-\gamma} \left[M_{x,t}^j\right]^\gamma$$

The first order conditions are

$$\begin{aligned}
\alpha(1-\gamma)\frac{Y_{x,t}^{j}}{K_{x,t}^{j}}\lambda_{x,t} &= \left[D_t + \delta(U_{x,t}^{j})P_t^K\right] \\
(1-\alpha)(1-\gamma)\frac{Y_{x,t}^{j}}{L_{x,t}^{j}}\lambda_{x,t} &= W_t \\
\alpha(1-\gamma)\frac{Y_{x,t}^{j}}{U_{x,t}^{j}}\lambda_{x,t} &= \delta^{'}(U_{x,t}^{j})P_t^K K_{x,t}^{j} \\
\gamma\frac{Y_{x,t}^{j}}{M_{x,t}^{j}}\lambda_{x,t} &= P_{x,t}^{M}
\end{aligned}$$

where $\lambda$ denotes the Lagrange multiplier. The firm optimally chooses a factor combination such that the marginal costs incurred by expanding production through increasing the intensity of one factor are equal for all production factors. Plugging into the cost function yields $\lambda Y_{x,t}^{j}$. Thus, marginal costs are independent of the quantity produced, an artifact of the CRTS property. The demand faced by each firm takes the standard Dixit-Stiglitz form

$$Y_{x,t}^{j} = \left(\frac{P_{x,t}^{j}}{P_{x,t}}\right)^{\frac{\mu_{x,t}}{1-\mu_{x,t}}} Y_{x,t}$$

In a second stage, the firm chooses the profit maximizing price given demand, i.e. it solves

$$\max_{P_{x,t}^{j}} \left(\frac{P_{x,t}^{j}}{P_{x,t}}\right)^{\frac{\mu_{x,t}}{1-\mu_{x,t}}} Y_{x,t} - \lambda \left(\frac{P_{x,t}^{j}}{P_{x,t}}\right)^{\frac{\mu_{x,t}}{1-\mu_{x,t}}} Y_{x,t}$$

The associated optimality condition is

$$P_{x,t}^{j} = \mu_{x,t}\lambda_{x,t}$$

Firms set prices as a markup over their marginal costs. Plugging in for $\lambda$ into the profit function yields

$$\left(1-\frac{1}{\mu_{x,t}}\right) P_{x,t}^{j} \left(\frac{P_{x,t}^{j}}{P_{x,t}}\right)^{\frac{\mu_{x,t}}{1-\mu_{x,t}}} Y_{x,t}$$

Since marginal costs are independent of the quantity produced, all firms charge the same price $\bar{P}_{x,t}$. One can solve for the optimal price index as

$$\begin{aligned} P_{x,t} &= \left[\int_0^{N_{x,t}} \left(P^j_{x,t}\right)^{\frac{1}{1-\mu_{x,t}}} dj\right]^{1-\mu_{x,t}} \\ &= \bar{P}_{x,t} N_{x,t}^{1-\mu_{x,t}} \end{aligned}$$

Solving for $\bar{P}_{x,t}$ and plugging into the profit function, firm profits can be written as

$$\left(1 - \frac{1}{\mu_{x,t}}\right) \frac{P_{x,t} Y_{x,t}}{N_{x,t}}$$

Thus, the free entry condition becomes

$$\left(1 - \frac{1}{\mu_{x,t}}\right) \frac{P_{x,t} Y_{x,t}}{N_{x,t}} = b_x \psi_t$$

Next, define sectoral net value added $Y^v_{x,t}$ as

$$Y^v_{x,t} = P_{x,t} Y_{x,t} - \frac{P^M_{x,t} M_{x,t}}{\nu} - b_x \psi_t$$

according to the definition of aggregate net value added in the text. Solve the FOC (ref) for $P_{x,t} M_{x,t}$ and plug in to obtain

$$Y^v_{x,t} = \left(1 - \frac{\gamma}{\nu \mu_{x,t}}\right) P_{x,t} Y_{x,t} - b_x \psi_t$$

Using the definition of $Y_{x,t}$ from (2.26), this can be rewritten as

$$Y^v_{x,t} = \left[\left(1 - \frac{\gamma}{\nu \mu_{x,t}}\right) N_{x,t}^{\mu_{x,t}-1} P_{x,t} \frac{M^\gamma_{x,t}}{\left[(U_t K_{x,t})^\alpha L^{1-\alpha}_{x,t}\right]^\gamma} - \frac{b_x \psi_t}{K^\alpha_{x,t} L^{1-\alpha}_{x,t}}\right] K^\alpha_{x,t} L^{1-\alpha}_{x,t}$$

Solving (2.30) for $M_{x,t}$ and expanding the second fraction by $M_{x,t}^{\frac{\gamma}{1-\gamma}}$ one arrives at

$$Y^v_{x,t} = \left[\underbrace{\left(1 - \frac{\gamma}{\nu \mu_{x,t}}\right) N_{x,t}^{\mu_{x,t}-1} P_{x,t}^{1/(1-\gamma)} \left(\frac{\gamma}{\mu_{x,t} P^M_{x,t}}\right)^{\frac{\gamma}{1-\gamma}} U^\alpha_t - \frac{b_x \psi_t}{K^\alpha_{x,t} L^{1-\alpha}_{x,t}}}_{\theta_{x,t}}\right] K^\alpha_{x,t} L^{1-\alpha}_{x,t}$$

where $\theta_{x,t}$ is the Solow residual in sector x.

# Appendix B

# Transformation

## B.1 Stationary Transformation

The trending model variables are net value added and its components $Y, C, G, P_k Y_k$, the aggregate capital stock $K$, capital and intermediate good inputs $K_x, M_x$ and their (rental) prices $P^M, P^K, D$, the social price and value of the capital stock $P^I, \psi$, the number of blueprints and implemented varieties $Z_x, A_x$ and their respective values $J_x, V_x$, aggregate and sectoral total factor productivity $\theta_x, \theta$ and R&D and adoption expenditures $S_x, H_x$. All other variables are constant along the BGP. According to their steady state growth rates, 14 groups of variables can be distinguished.

1. $Y, C, G, Y_c, S_x, \psi, w$
2. $K, K_x, Y_k$
3. $P^K, P^I, D$
4. $H_c, V_c, J_c$
5. $H_k, V_k, J_k$
6. $Z_c, A_c$
7. $Z_k, A_k$
8. $M_c$
9. $M_k$
10. $P_c^M$
11. $P_k^M$
12. $\theta_x$
13. $\theta$
14. $\mu_{x,t}, N_{x,t}, U_t, L_t, L_{x,t}, \lambda_{x,t}$

where group 14 contains the non-trending variables. Denote by $g^x$ the BGP gross-growth rate of variable x and suppose that all variables grow at a constant rate indefinitely. Recall the definition and demand for value added $Y_t$ from (2.22) and (2.24). Along the BGP, all components of $Y_t$ grow at a common rate:

$$\begin{aligned} g^Y = g^C = g^G = g^{P_K} g^{Y_K} = g^{S_x} = g^{H_x} g^{Z_x} = g^{H_x} g^{A_x} \\ = g^{P_K} g^{Y_k} = g^{Y_c} = (g^{A_x})^{1-\nu} g^{M_x} = g^{\psi} \end{aligned} \tag{B.1}$$

Note that the BGP growth rate of the social value of the capital stock $g^{\psi}$ and sectoral output $g^{P_x} g^{Y_x}$ coincide. Together with the free entry condition (2.37) and the inverse relationship between markups and firm entry, this implies that markups $\mu_{x,t}$ and the firm number in sector x, $N_{x,t}$ have to be constant along the BGP. Further, (B.1) implies that that the product of the number of implemented intermediate inputs and adoption expenditures $A_{x,t} H_{x,t}$ grows at the rate of the social value of the capital stock. Comparing to the definition of the adoption rate in (2.18), this implies that the adoption rate is time-invariant along the BGP. The same holds for hours, since the model does not allow for population growth. To sum up:

$$g^{\mu_x} = g^{\lambda_x} = g^{N_x} = g^L = g^{L_x} = 1 \tag{B.2}$$

Next, consider the factor prices and input allocations. The Euler equation (2.36) implies that consumption can only grow at a constant rate if the real interest rate is time-constant, requiring that the capital compensation $D_t$ and the relative price of capital $P_t^K$ grow at a common rate. Recall the formulation of the relative price of capital in (2.47). Together with (B.2), it implies that the efficient price of capital $P_t^I$ grows at the same rate too :

$$g^D = g^{P_t^K} = g^{P_I} \tag{B.3}$$

Capital market clearing in (2.33) requires that the share of capital employed in sector x remains constant along the BGP:

$$g^{K_c} = g^{K_k} = g^K \tag{B.4}$$

Together with (B.1) and the first order condition with respect to utilization (2.29), this implies that the utilization rate $U_t$ is time-invariant along the BGP, i.e. $g^U = 1$. The BGP growth rate of the real wage coincides with the one of value added $Y_t$ as can be seen from (2.28) and (B.1), $g^W = g^Y$. Now, turn to the technology side of the model. Dividing through by $V_{x,t}$ in (2.42), the Bellman equation for un-adopted varieties, one obtains

$$1 = \left[\left(1 - \frac{1}{\nu}\right)\frac{1}{\bar{\mu}_x}\frac{P_{x,t}Y_{x,t}}{A_{x,t}V_{x,t}}\right] + \phi \underbrace{\Lambda_{t+1}}_{const} \underbrace{\frac{V_{x,t+1}}{V_{x,t}}}_{const} \tag{B.5}$$

where $\bar{\mu}_x$ denotes the steady state value of markups. If the value of adopted varieties and consumption grow at a constant rate, then sectoral output $P_{x,t}Y_{x,t}$ and the total value of adopted varieties $A_{x,t}V_{x,t}$ have to grow at the same rate. (B.1) then implies that the value of adopted intermediate goods and adoption expenditures per variety $H_{x,t}$ have to grow at a common rate. Dividing through by $V_{x,t}$ in (2.41), the first order condition for adoption expenditures one obtains

$$\frac{J_{x,t}}{V_{x,t}} = -\underbrace{\frac{H_{x,t}}{V_{x,t}}}_{const} + \phi \underbrace{\Lambda_{t+1}}_{const}\left[\bar{\lambda}_x \underbrace{\frac{V_{x,t+1}}{V_{x,t}}}_{const} + (1-\bar{\lambda}_x)\frac{J_{x,t+1}}{V_{x,t}}\right] \tag{B.6}$$

where $\bar{\lambda}_x$ denotes the steady state value of the adoption rate. Hence, the value of unadopted and adopted varieties grow at the same rate along the BGP. (B.1) implies that $A_{x,t}$ and $Z_{x,t}$ grow at a common rate. It also implies that R&D expenditures $S_{x,t}$ and the social value of the capital stock grow at a common rate. Comparing to the evolution of the stock of blueprints in (2.38), this yields a constant BGP growth rate of the stock of blueprints.

Summing up, the non-trending variables in the model are markups, firm numbers, adoption rates, hours and the utilization rate. Amongst the trending variables, one can distinguish several groups by BGP growth rate. In order to find the appropriate transformations by the number of blueprints, it suffices to solve for the growth rates of value added $Y_t$, the capital stock $K_t$, the relative price of capital $P_t^K$, the prices for intermediate inputs and intermediate input quantities $P_{x,t}^M$, $M_{x,t}$,the adoption expenditures per variety $H_{x,t}$ and the growth rates of $\theta_{x,t}$

and $\theta_t$.

As was already remarked above

$$g^{Z_x} = g^{A_x} \tag{B.7}$$

has to hold in steady state. (B.1) implies that

$$(g^{A_x})^{1-\nu} g^{M_x} = g^P g^K \tag{B.8}$$

The steady state growth rate of $P^I$ is given by

$$g^{P^I} = \left(\frac{g^{A_k}}{g^{A_c}}\right)^{(1-\nu)\gamma} = \left(\frac{g^{Z_k}}{g^{Z_c}}\right)^{(1-\nu)\gamma} \tag{B.9}$$

where the second equality follows with (B.7). Further, since markups, the number of firms and labor input are non-trending, the steady state growth rate of output from sector x is

$$g^{Y_x} = (g^K)^{\alpha(1-\gamma)}(g^{M_x})^\gamma \tag{B.10}$$

Since $g^K = g^{Y_k}$, (B.9) implies

$$g^K = (g^{M_k})^{\frac{\gamma}{1-\alpha(1-\gamma)}} \tag{B.11}$$

Plugging (B.9) and (B.10) into (B.8) then yields

$$g^{M_k} = \left(g^{Z_c}\right)^{\frac{(\nu-1)\gamma[1-\alpha(1-\gamma)]}{(1-\alpha)(1-\gamma)}} \left(g^{Z_k}\right)^{\frac{(\nu-1)[1-\alpha(1-\gamma)]}{(1-\alpha)}} \tag{B.12}$$

Now, use (B.12) to arrive at

$$g^K = \left(g^{Z_c}\right)^{\frac{(\nu-1)\gamma^2}{(1-\alpha)(1-\gamma)}} \left(g^{Z_k}\right)^{\frac{(\nu-1)\gamma}{(1-\alpha)}} \tag{B.13}$$

Exploiting that $g^{Y_c} = g^P g^K$ as implied by (B.1), one obtains

$$g^{Y_c} = \left(g^{Z_c}\right)^{\frac{(\nu-1)\gamma[1-\alpha(1-\gamma)]}{(1-\alpha)(1-\gamma)}} \left(g^{Z_k}\right)^{\frac{(\nu-1)\gamma\alpha}{(1-\alpha)}} \tag{B.14}$$

(B.1) alos implies that $(g^{A_c})^{1-\nu} g^{M_c} = (g^{A_k})^{1-\nu} g^{M_k}$ which we can know use to solve for $g^{M_c}$ as

$$g^{M_c} = \left(g^{Z_c}\right)^{\frac{(\nu-1)[1-\alpha(1-\gamma)(1+\gamma)]}{(1-\alpha)(1-\gamma)}} \left(g^{Z_k}\right)^{\frac{(\nu-1)\alpha\gamma}{(1-\alpha)}} \tag{B.15}$$

The price of the intermediate input composite in sector x grows at

$$g^{P_x^M} = \left(g^{Z_x}\right)^{1-\nu} \tag{B.16}$$

$g^{H_c}$ and $g^{H_k}$ can be obtained from (B.1) as

$$g^{H_c} = \left(g^{Z_c}\right)^{\frac{(\nu-1)\gamma[1-\alpha(1-\gamma)]}{(1-\alpha)(1-\gamma)}-1} \left(g^{Z_k}\right)^{\frac{(\nu-1)\gamma\alpha}{(1-\alpha)}} \tag{B.17}$$

and

$$g^{H_k} = \left(g^{Z_c}\right)^{\frac{(\nu-1)\gamma[1-\alpha(1-\gamma)]}{(1-\alpha)(1-\gamma)}} \left(g^{Z_k}\right)^{\frac{(\nu-1)\gamma\alpha}{(1-\alpha)}-1} \tag{B.18}$$

Using the definition of sectoral total factor productivity from (2.46) and (B.14), (B.11), one can solve for the BGP growth rate as

$$g^{\theta_x} = \left(g^{Z_c}\right)^{\frac{(\nu-1)\gamma}{(1-\gamma)}} \tag{B.19}$$

From (3.1) and (B.14), (B.11), the growth rate of $\theta$ is

$$g^{\theta} = \left(g^{Z_c}\right)^{\frac{(\nu-1)\gamma}{(1-\gamma)}} \left(g^{Z_k}\right)^{(\nu-1)\psi\alpha} \tag{B.20}$$

## B.2 Transformed System

Define as $\epsilon_{xc}$ the power of the transformation of variable x by $Z_c$ and define $\epsilon_{xk}$ accordingly. From the derivations above:

$$\epsilon_{cc} = \frac{(\nu-1)\gamma\left[1-\alpha(1-\gamma)\right]}{(1-\alpha)(1-\gamma)} \tag{B.21}$$

$$\epsilon_{ck} = \frac{(\nu-1)\gamma\alpha}{(1-\alpha)} \tag{B.22}$$

$$\epsilon_{kk} = \frac{(\nu-1)\gamma}{(1-\alpha)} \tag{B.23}$$

$$\epsilon_{kc} = \frac{(\nu-1)\gamma^2}{(1-\alpha)(1-\gamma)} \tag{B.24}$$

The transformed system is given by

- Wage markups

$$\mu_{w,t} = (1-\rho_w)\bar{\mu}_w + \rho_w \mu_{w,t-1} + \epsilon_{w,t} \tag{B.25}$$

- Resource Constraint and Production

$$\tilde{y}_t = \sum_{x=c,k} \left[\tilde{p}_{x,t}\tilde{y}_{x,t} + (\tilde{a}_{x,t})^{1-\nu}\,\tilde{m}_{x,t} - b_x\tilde{\psi}_t\right] \tag{B.26}$$

$$\tilde{y}_t = \tilde{c}_t + \tilde{p}_{k,t}\tilde{y}_{k,t} + \tilde{g}_t + \sum_{x=c,k}\left[\tilde{s}_{x,t} + (1-\tilde{a}_{x,t})\,\tilde{h}_{x,t}\right] \tag{B.27}$$

$$\tilde{y}_{x,t} = (N_{x,t})^{1-\mu_{x,t}}\left[\left(U_t\tilde{k}_{x,t}\right)^{\alpha}(L_{x,t})^{1-\alpha}\right]^{1-\gamma}[\tilde{m}_{x,t}]^{\gamma} \tag{B.28}$$

- Evolution of endogenous states

$$\tilde{k}_{t+1} = \left[(1-\delta(U_t))\,\tilde{k}_t + \tilde{y}_{k,t}\right] g_{zc,t+1}^{-\epsilon_{kc}} g_{zk,t+1}^{-\epsilon_{kk}} \tag{B.29}$$

$$\tilde{a}_{x,t+1} = \left[\lambda_{x,t}\phi\left(1-\tilde{a}_{x,t}\right)+\phi a_{x,t}\right]/g_{zx,t+1}; \tag{B.30}$$

$$g_{zx,t+1} = \chi_x\left(\frac{\tilde{s}_{x,t}}{\tilde{\psi}_{x,t}}\right)^{\rho} + \phi; \tag{B.31}$$

- Factor Market Clearing and Saving

$$\alpha(1-\gamma)\frac{\tilde{p}_{x,t}\tilde{y}_{x,t}}{\tilde{k}_{x,t}} = \mu_{x,t}\left[\tilde{d}_t + \delta(U_t)\tilde{p}_{k,t}\right] \tag{B.32}$$

$$(1-\alpha)(1-\gamma)\frac{\tilde{p}_{x,t}\tilde{y}_{x,t}}{L_{x,t}} = \mu_{x,t}\tilde{w}_t \tag{B.33}$$

$$\alpha(1-\gamma)\frac{\tilde{p}_{x,t}\tilde{y}_{x,t}}{U_t} = \mu_{x,t}\delta'(U_t)\tilde{p}_{k,t}\tilde{k}_{x,t} \tag{B.34}$$

$$\gamma\frac{\tilde{p}_{x,t}\tilde{y}_{x,t}}{\tilde{m}_{x,t}} = \mu_{x,t}\tilde{p}^M_{x,t} \tag{B.35}$$

$$\mu_{w,t}\tilde{c}_t L_t^{\zeta} = \tilde{w}_t \tag{B.36}$$

$$\tilde{p}^M_{x,t} = \nu\left(\tilde{a}_{x,t}\right)^{1-\nu} \tag{B.37}$$

$$\beta E_t\left\{\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\frac{\tilde{d}_{t+1}+\tilde{p}_{k,t+1}}{\tilde{p}_{k,t}}\left(g_{zc,t+1}\right)^{-\epsilon_{cc}+\gamma(\nu-1)}\left(g_{zk,t+1}\right)^{-\epsilon_{ck}+\gamma(1-\nu)}\right\} = 1 \tag{B.38}$$

$$L_t = L_{c,t} + L_{k,t} \tag{B.39}$$

$$\tilde{k}_t = \tilde{k}_{c,t} + \tilde{k}_{k,t} \tag{B.40}$$

- Optimal Adoption and R&D decision

$$\lambda'_{x,t}\frac{\tilde{a}_{x,t}}{\tilde{\psi}_t}\phi\beta E_t\left\{\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\left[\tilde{v}_{x,t+1} - \tilde{j}_{x,t+1}\right]\right\} = g_{zx,t+1} \tag{B.41}$$

$$\phi\beta E_t\left\{\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\tilde{j}_{x,t+1}\left(1 - g_{zx,t+1}^{-1}\right)\right\} = \tilde{s}_{x,t} \tag{B.42}$$

- Bellman Equations for adopted and un-adopted varieties

$$\left(1 - \frac{1}{\nu}\right)\gamma\frac{1}{\mu_{x,t}}\frac{\tilde{p}_{x,t}\tilde{y}_{x,t}}{\tilde{a}_{x,t}} + \frac{1}{g_{zx,t+1}}\phi\beta E_t\left\{\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\tilde{v}_{x,t+1}\right\} = \tilde{v}_{x,t} \tag{B.43}$$

$$-\tilde{h}_{x,t} + \frac{1}{g_{zx,t+1}}\phi\beta E_t\left\{\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\left[\lambda_{x,t}\tilde{v}_{x,t+1} + (1-\lambda_{x,t})\tilde{j}_{x,t+1}\right]\right\} = \tilde{j}_{x,t} \tag{B.44}$$

where

$$\lambda_{x,t} = \lambda\left(\frac{\tilde{a}_{x,t}\tilde{h}_{x,t}}{\tilde{\psi}_t}\right) \tag{B.45}$$

- Social value of capital stock and free entry into final good production

$$\tilde{p}_t^I = \left(\frac{\tilde{a}_{k,t}}{\tilde{a}_{c,t}}\right)^{(1-\nu)\gamma} \tag{B.46}$$

$$\tilde{\psi}_t = \tilde{p}_t^I\tilde{k}_t \tag{B.47}$$

$$\left(1 - \frac{1}{\mu_{x,t}}\right)\frac{\tilde{p}_{x,t}\tilde{y}_{x,t}}{N_{x,t}} = b_x\tilde{\psi}_t \tag{B.48}$$

- Productivity

$$\tilde{\theta}_{x,t} = \frac{\tilde{y}_t^v}{\tilde{k}_{x,t}^{\alpha} L_{x,t}^{1-\alpha}} \tag{B.49}$$

$$\tilde{\theta}_t = \frac{\tilde{y}_t \tilde{a}_{k,t}^{\psi\alpha(\nu-1)}}{\tilde{k}_t^{\alpha} L_t^{1-\alpha}} \tag{B.50}$$

$$\tilde{p}_{k,t} = \left(\frac{\mu_{k,t}}{\mu_{c,t}}\right)\left(\frac{N_{k,t}^{1-\mu_{k,t}}}{N_{c,t}^{1-\mu_{c,t}}}\right)\tilde{p}_t^I \tag{B.51}$$

## B.3 Undoing the Transformation

Let $\tilde{x}_t$ denote the stationary transformation of variable $x_t$ and let $\Gamma_t$ denote the corresponding scaling factor. Given the percentage deviation of $\tilde{x}_t$ from its steady state value, $\hat{\tilde{x}}_t$, I want to solve for the percentage deviation of $x_t$ from its trend value $x_t^{'}$, $\hat{x}_t$.

$$\hat{\tilde{x}}_t = log\left(\frac{x_t/\Gamma_t}{x_t^{'}/\Gamma_t^{'}}\right) \tag{B.52}$$

$$= log\left(\frac{x_t}{x_t^{'}}\right) + log\left(\frac{\Gamma_t^{'}}{\Gamma_t}\right) \tag{B.53}$$

$$= \hat{x}_t + log\left(\frac{\Gamma_t^{'}}{\Gamma_t}\right) \tag{B.54}$$

Let $\epsilon_{xc}$ be defined as above. $\Gamma_t^{'}/\Gamma_t$ can be written as

$$\Gamma_t = \left(\frac{Z_{c,t}^{'}}{Z_{c,t}}\right)^{\epsilon_{xc}}\left(\frac{Z_{k,t}^{'}}{Z_{k,t}}\right)^{\epsilon_{xk}} \tag{B.55}$$

Suppose that I start my simulation at t in steady state. I can back out $\hat{x}_i$ from $\hat{\tilde{x}}_i$ by accumulating the log-deviations of the growth rates of $Z_{k,j}$ and $Z_{c,j}$ from steady state from t to i, multiplied by $\epsilon_{xc}$ and $\epsilon_{xk}$.

$$\hat{x}_t = \hat{\hat{x}}_t + \epsilon_{xc} \sum_{t+1 \leq j \leq i} \tilde{g}_{zc,j} + \epsilon_{xk} \sum_{t+1 \leq j \leq i} \tilde{g}_{zk,j} \tag{B.56}$$